

Recent progress in quests of Spin and Spin-isospin excitations

COMEX6, October 29, Cape Town , South Africa

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1. Introduction
2. Competitions between IS and IV pairing correlations in $N=Z$ nuclei
Superfluidity phase: IS spin-triplet pairing interaction
3. Spin and Gamow-Teller transitions from High Spin Isomers
4. Summary and future perspectives



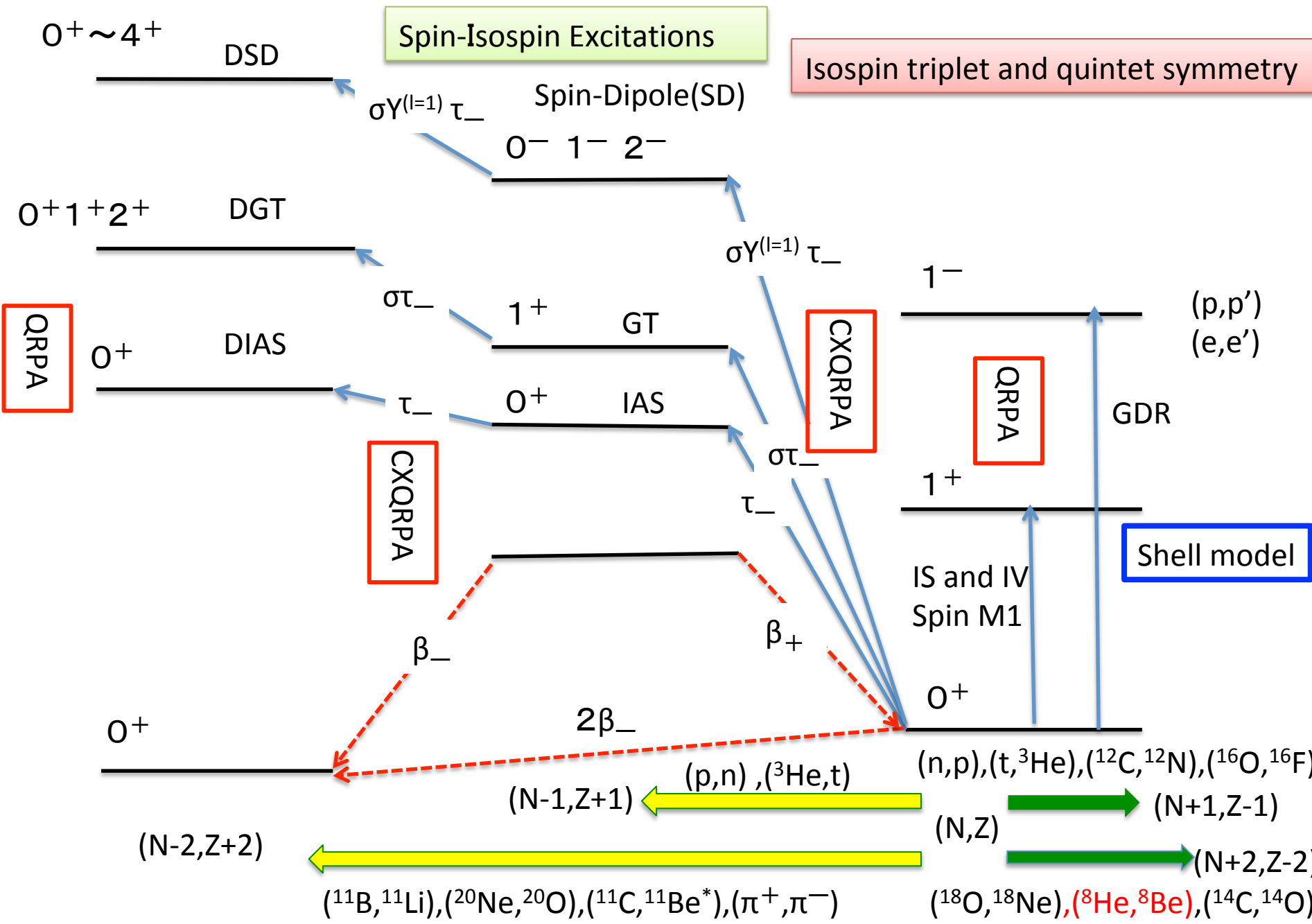
Three dimensions in research of Spin-Isospin modes

- T → high isospin (radioactive beams), isospin Fermisphere
- J → high spin (isomer beams), spin Fermisphere
- dilute-density (halo, skin)
→ n-p pair, alpha condensation
- pair transfer reactions (nn, pp, pn)

- Charge-exchange reactions (Single and double)
→ spin-isospin responses (GT, SD, DGT, DSD...)

Light ions: (p,n), (n,p), (^3He ,t), (t, ^3He)

Heavy ions: (^{11}B , ^{11}Li), (^{20}Ne , ^{20}O), (^{11}C , $^{11}\text{Be}^*$), (^{18}O , ^{18}Ne), (^8He , ^8Be), (^{14}C , ^{14}O)



Spin-Isospin Excitations

Isospin triplet and quintet symmetry

QRPA

CXQRPA

CXQRPA

QRPA

Shell model

$0^+ \sim 4^+$

DSD

Spin-Dipole(SD)

$\sigma\gamma^{(l=1)}\tau_-$

$0^- 1^- 2^-$

$0^+ 1^+ 2^+$

DGT

$\sigma\gamma^{(l=1)}\tau_-$

0^+

DIAS

$\sigma\tau_-$

1^+

GT

1^-

(p,p')
(e,e')

τ_-

0^+

IAS

$\sigma\tau_-$

1^+

GDR

0^+

β_-

β_+

$2\beta_-$

0^+

(n,p), (t, ^3He), ($^{12}\text{C}, ^{12}\text{N}$), ($^{16}\text{O}, ^{16}\text{F}$)

(N-1, Z+1) (p,n), (^3He , t)

(N, Z) (N+1, Z-1)

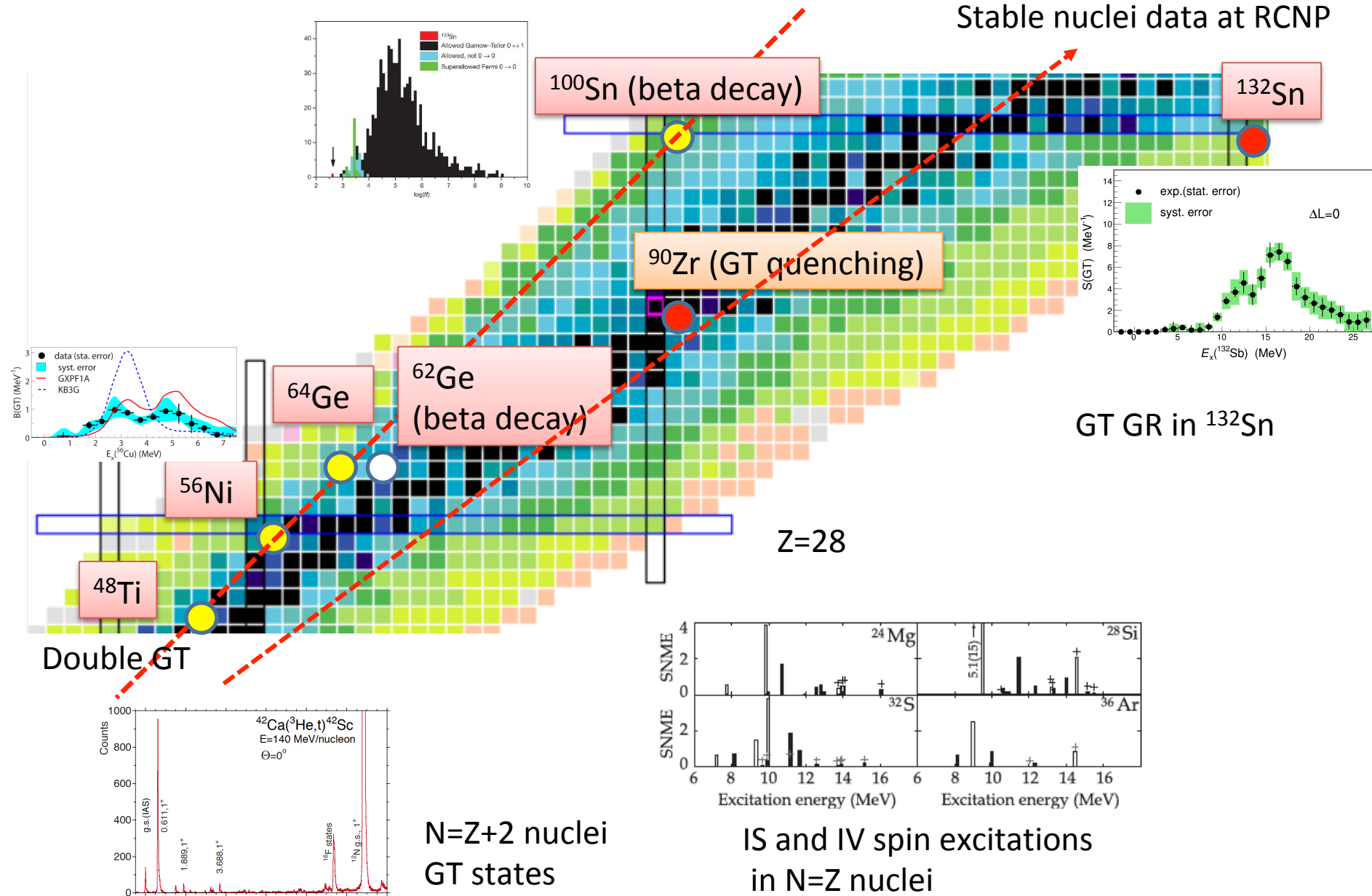
(N-2, Z+2)

($^{11}\text{B}, ^{11}\text{Li}$), ($^{20}\text{Ne}, ^{20}\text{O}$), ($^{11}\text{C}, ^{11}\text{Be}^*$), (π^+, π^-)

($^{18}\text{O}, ^{18}\text{Ne}$), ($^8\text{He}, ^8\text{Be}$), ($^{14}\text{C}, ^{14}\text{O}$)

Recent Progresses in spin-isospin excitations

Stable nuclei data at RCNP

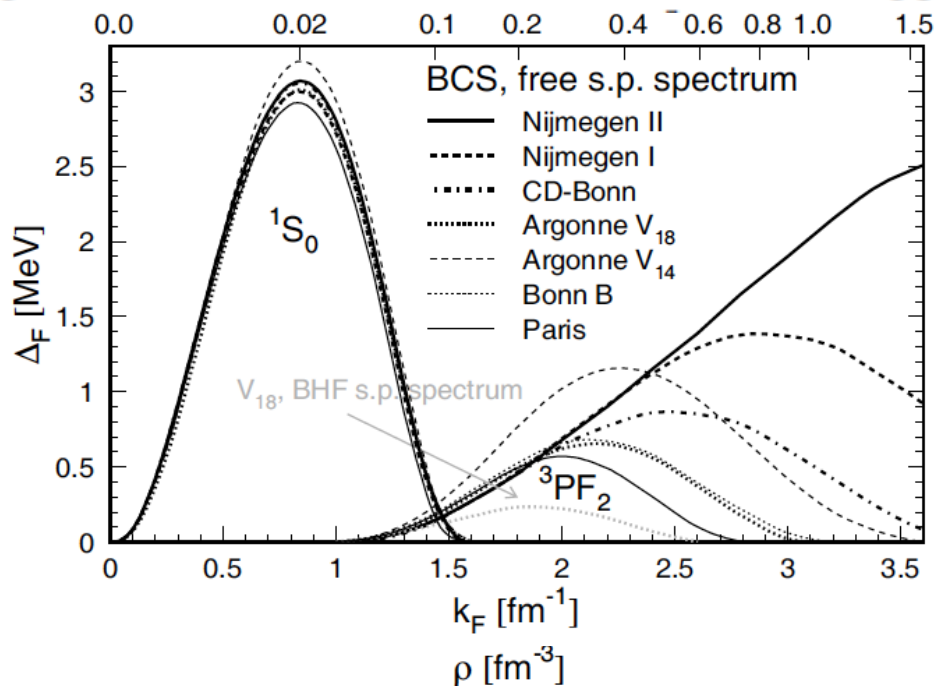


Competition between IS spin-triplet and IV spin-single pairing correlations

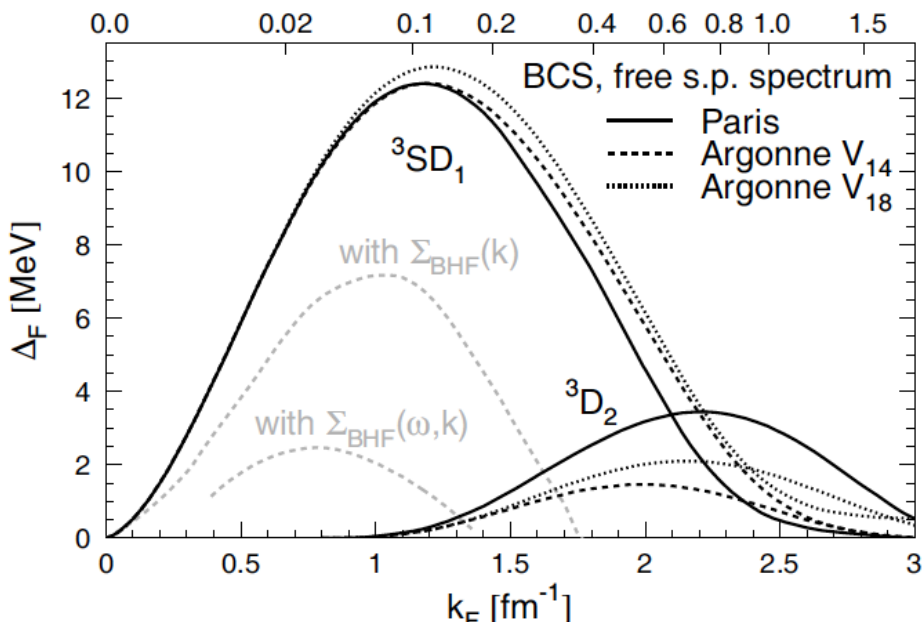
Gamow-Teller transitions in $N=Z+2$ nuclei

n-p pair condensation in nuclei with $N \sim Z$

Isospin $T = 1$ 1S_0 and 3PF_2 gaps in neutron matter evaluated in BCS approximation



Int. Journ. of Mod. Phys.
E14, 513 (2005)
U. Lombardo et al..



Isospin $T = 0$ 3SD_1 and 3D_2 gaps in symmetric nuclear matter

Gamow-Teller Transitions in nuclei with N=Z+2

C.L. Bai, HS, G. Colo, Y. Fujita et al.,

PRC90, 054335 (2014)

HFB+QRPA with T=1 and T=0 pairing

T=1 pairing in HFB

T=0 pairing in QRPA

$$\hat{O}(GT) = \sigma\tau_{\pm}$$

σ , τ and $\sigma\tau$ are generators of SU(4)

Supermultiplet : Wigner SU(4) symmetry

(E. Wigner 1937, F. Hund 1937)

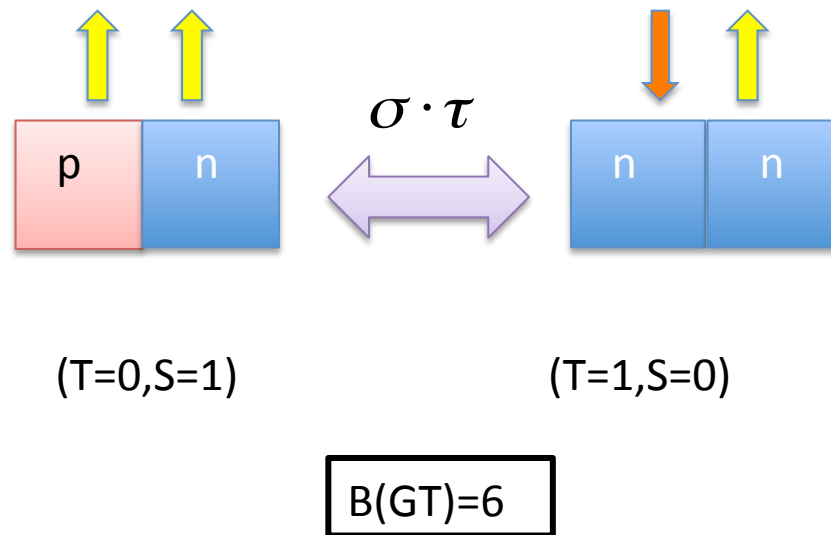
(T=1, S=0) \rightarrow (T=0, S=1) GT transition is allowed and enhanced .

$$V_{T=1}(\mathbf{r}_1, \mathbf{r}_2) = V_0 \frac{1 - P_{\sigma}}{2} \left(1 - \frac{\rho(\mathbf{r})}{\rho_0}\right) \delta(\mathbf{r}_1 - \mathbf{r}_2), \quad (1)$$

$$V_{T=0}(\mathbf{r}_1, \mathbf{r}_2) = fV_0 \frac{1 + P_{\sigma}}{2} \left(1 - \frac{\rho(\mathbf{r})}{\rho_0}\right) \delta(\mathbf{r}_1 - \mathbf{r}_2), \quad (2)$$

Supermultiplet : Wigner SU(4) symmetry
(T=1, S=0) \rightarrow (T=0, S=1) GT transition is allowed and enhanced .

Spacial symmetry is the same between the initial and final states



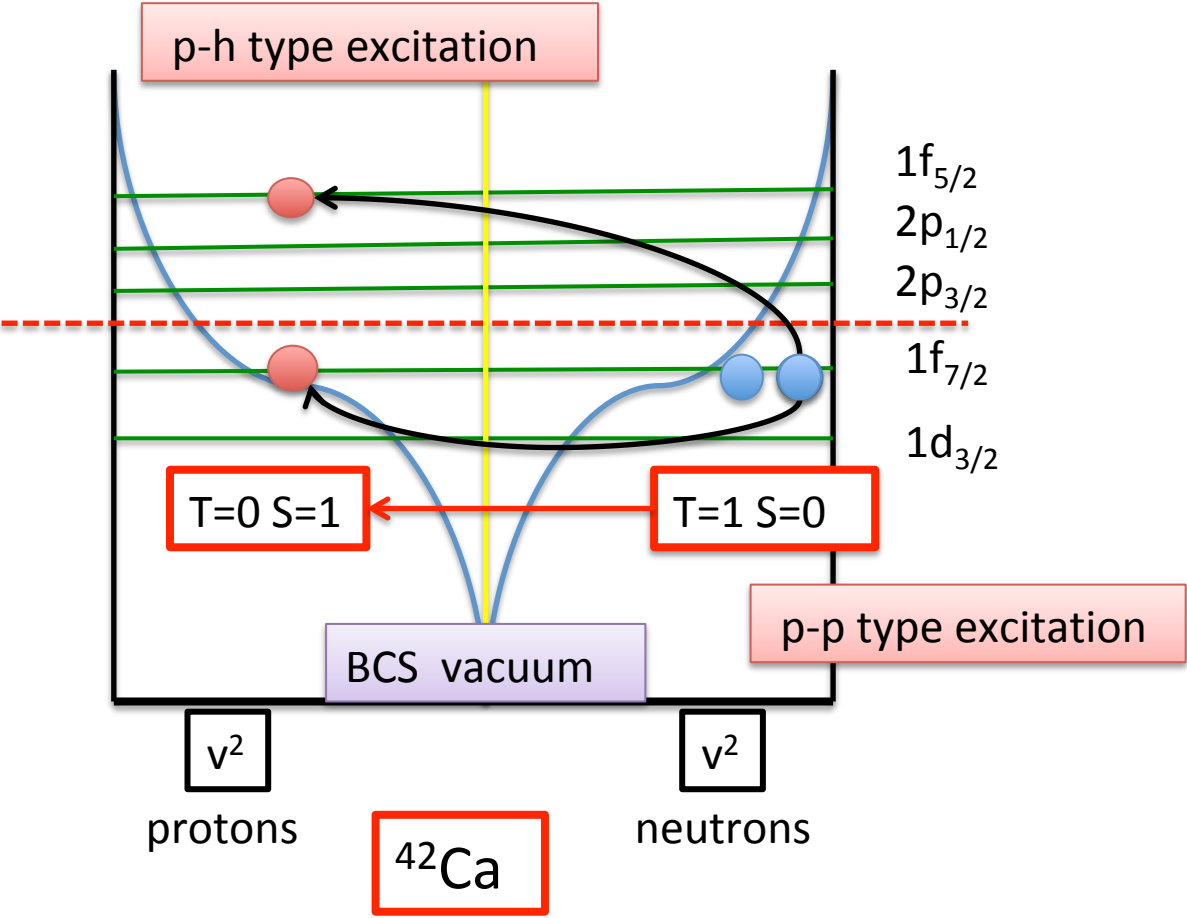
Well-known in light p-shell nuclei (LS coupling dominance)

What happens in **pf shell nuclei** with strong spin-orbit and spin-triplet pairing interactions?

Gamow-Teller transitions in $N=Z+2$ pf nuclei

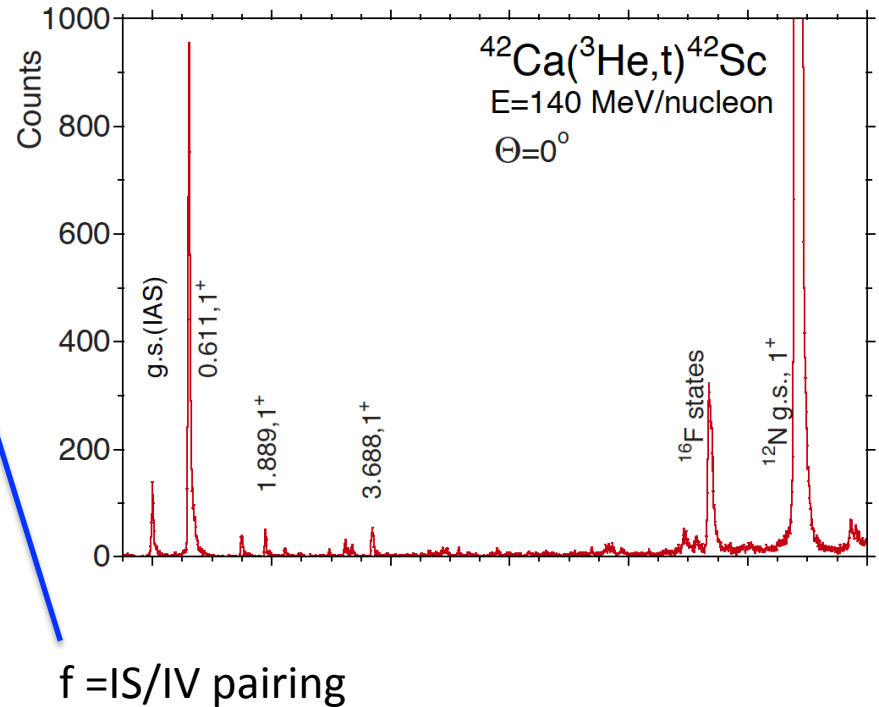
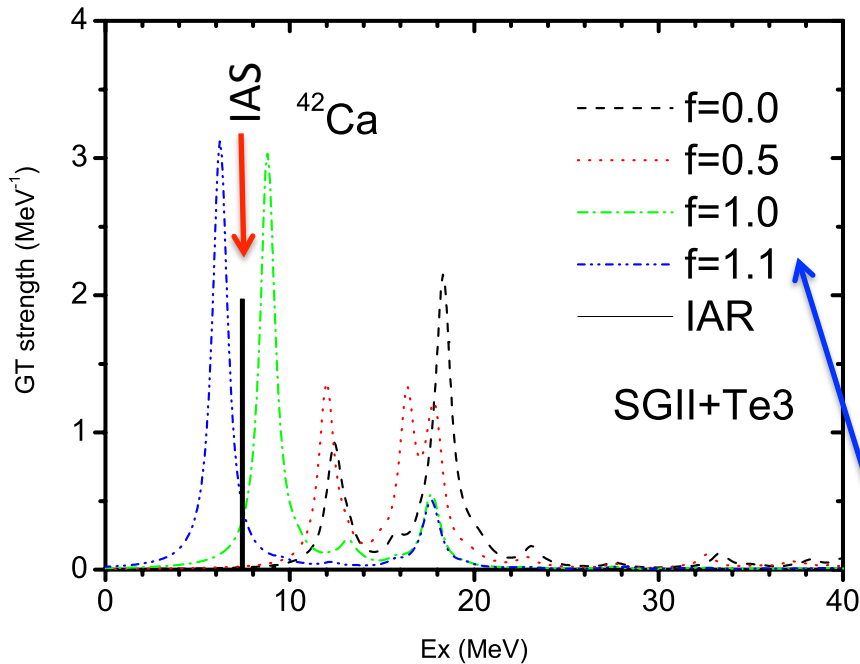
$$\hat{O}(GT) = \sigma\tau_{\pm}$$

Fermi energy



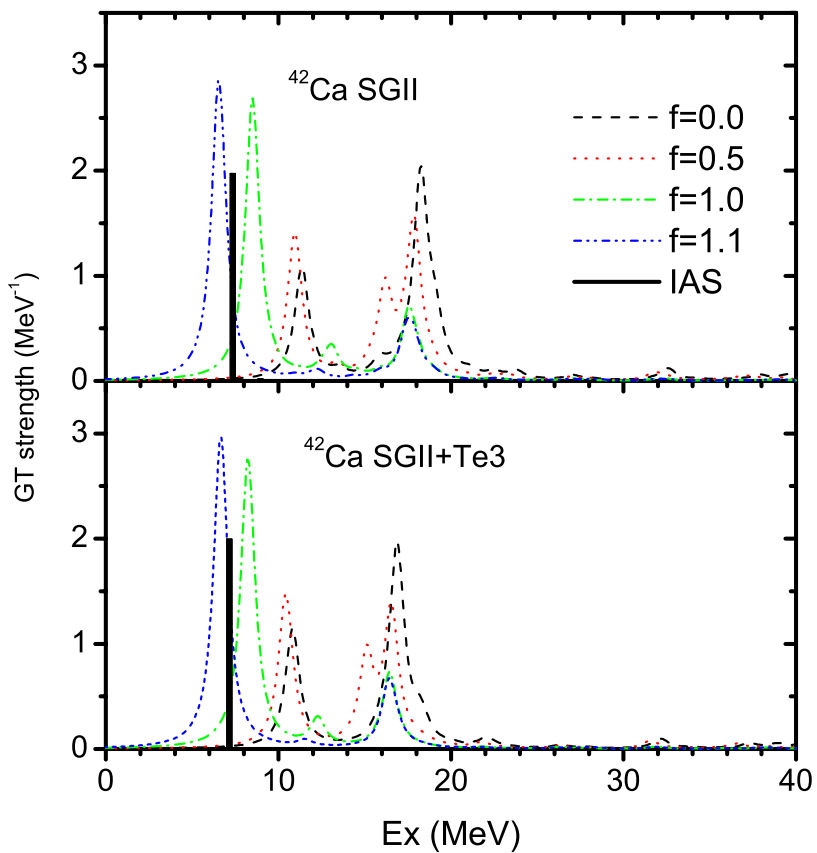
A pair of SU(4) supermultiplet

Spin-spin interaction is strongly repulsive \rightarrow higher energy IAS
 \rightarrow collective Gamow-Teller states
 \rightarrow SU(4) symmetry restoration

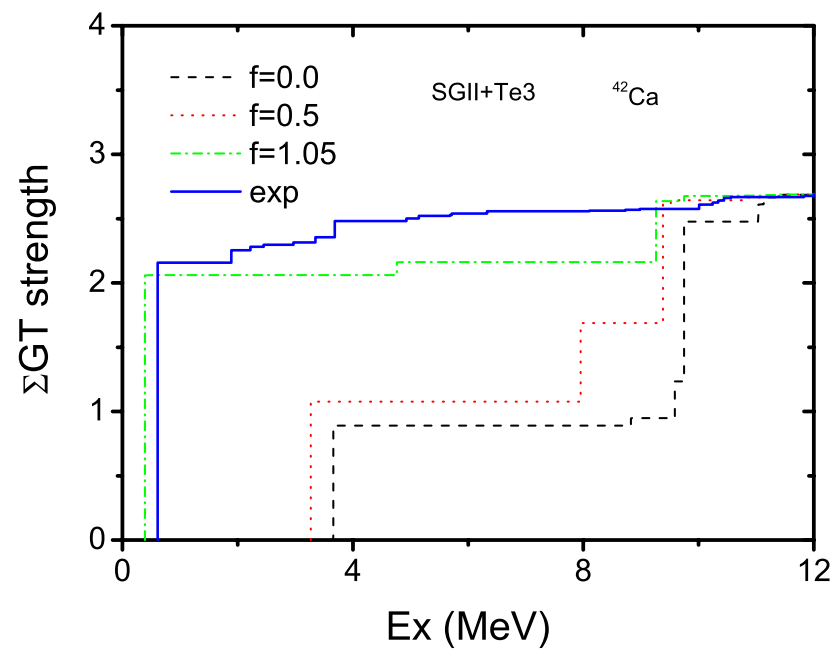
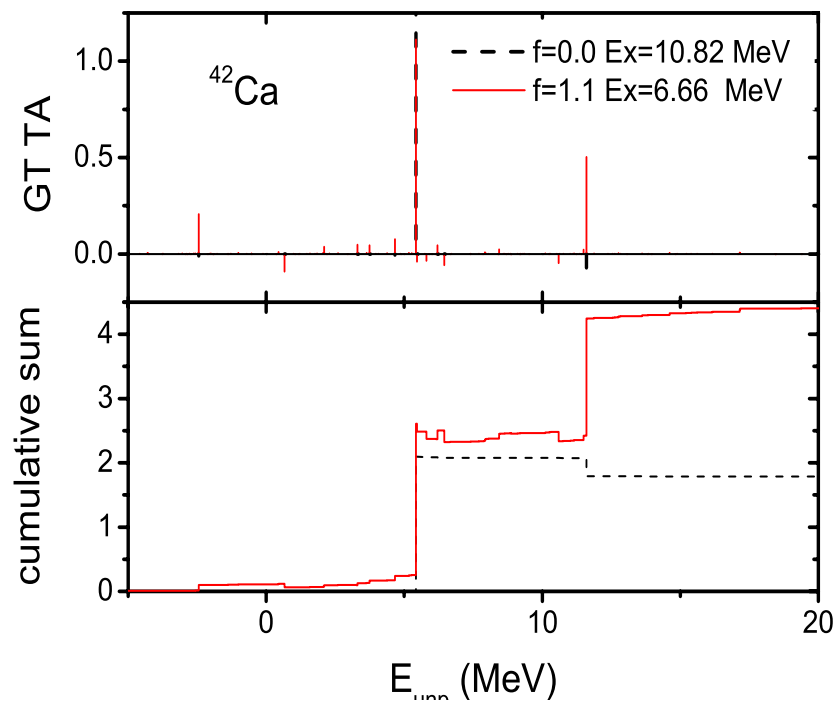


HFB+QRPA with T=1 and T=0 pairing

T=0 pairing strength in QRPA is changed as a parameter f .

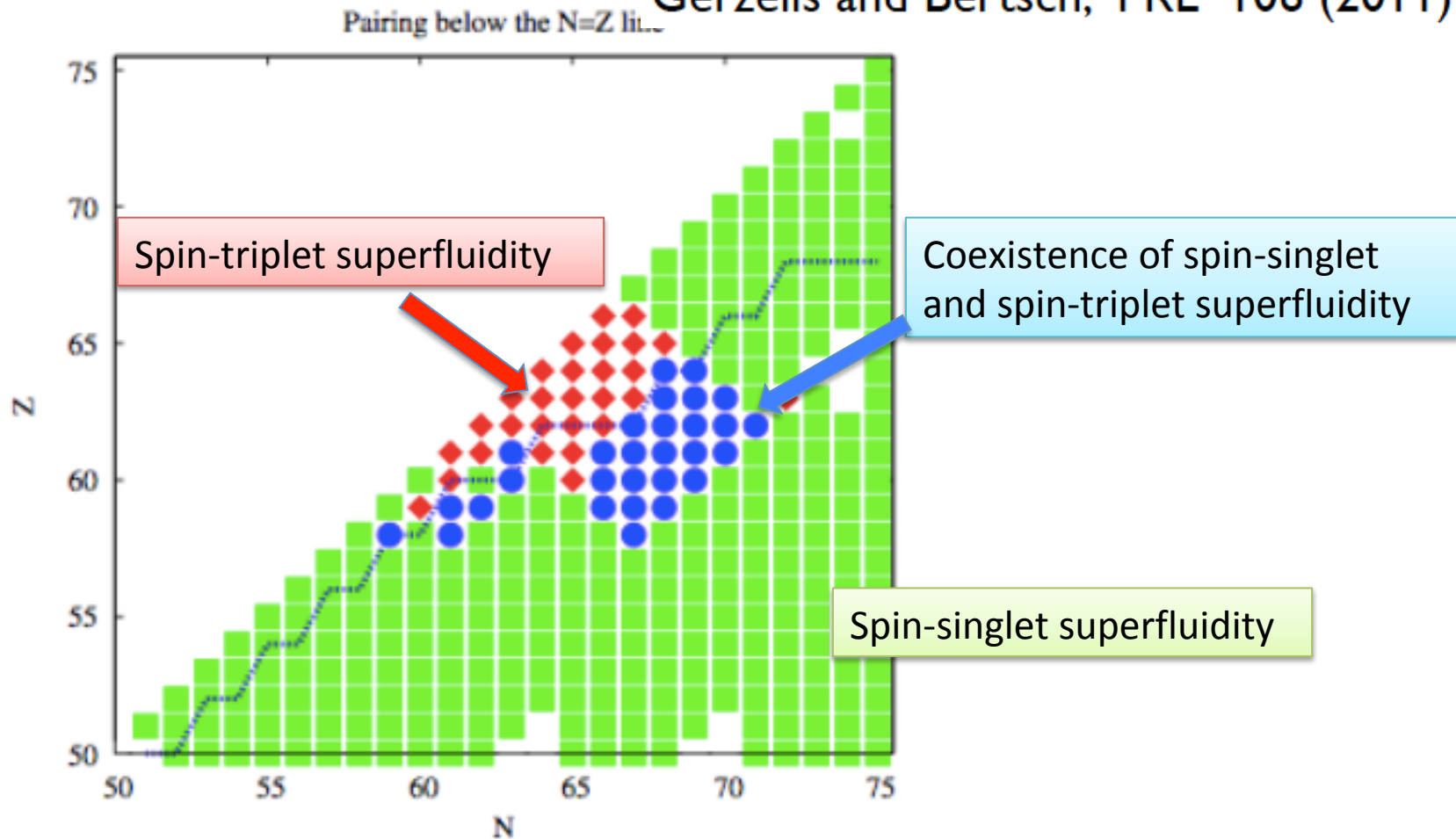


Effect of tensor correlations is small in ^{42}Ca .



Neutron-proton pair condensates

Gerzelis and Bertsch, PRL 106 (2011)



Source	v_s (MeV fm ³)	v_t (MeV fm ³)	Ratio
<i>sd</i> shell [8]	280	465	1.65
<i>fp</i> shell [9]	291	475	1.63

G.F. Bertsch and Y. Luo, PRC81, 064320 (2010)

Deformed HFB calculations with a realistic interaction in N=Z nuclei: a competition between T=0 and T=1 pairing interactions

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RIKEN, Nishina Center for Accelerator-Based Science,

Eunja Ha, Myung-Ki Cheoun, H. Sagawa, Phys. Rev. C, **97** 024320 (2018).

Eunja Ha, Myung-Ki Cheoun, H. Sagawa, Phys. Rev. C, **97** 064322 (2018).

+preprint (2018)

Deformed HFB with a realistic interaction (CD Bonn)

T=1 channel nn,pp,np

T=0 channel np

Nuclear Hamiltonian

$$H = H_0 + H_{\text{int}} ,$$

$$H_0 = \sum_{\rho_\alpha \alpha \alpha'} \epsilon_{\rho_\alpha \alpha \alpha'} c_{\rho_\alpha \alpha \alpha'}^\dagger c_{\rho_\alpha \alpha \alpha'} ,$$

$$H_{\text{int}} = \sum_{\rho_\alpha \rho_\beta \rho_\gamma \rho_\delta, \alpha \beta \gamma \delta, \alpha' \beta' \gamma' \delta'} V_{\rho_\alpha \alpha \alpha' \rho_\beta \beta \beta' \rho_\gamma \gamma \gamma' \rho_\delta \delta \delta'} c_{\rho_\alpha \alpha \alpha'}^\dagger c_{\rho_\beta \beta \beta'}^\dagger c_{\rho_\delta \delta \delta'} c_{\rho_\gamma \gamma \gamma'} ,$$

HFB transformation

$$a_{\rho_\alpha \alpha \alpha''}^\dagger = \sum_{\rho_\beta \beta \beta'} (u_{\alpha \alpha'' \beta \beta'} c_{\rho_\beta \beta \beta'}^\dagger + v_{\alpha \alpha'' \beta \beta'} c_{\rho_\beta \beta \beta'}) ,$$

$$a_{\rho_\alpha \bar{\alpha} \alpha''} = \sum_{\rho_\beta \beta \beta'} (u_{\bar{\alpha} \alpha'' \beta \beta'} c_{\rho_\beta \beta \beta'} - v_{\bar{\alpha} \alpha'' \beta \beta'} c_{\rho_\beta \beta \beta'}^\dagger) . \quad (6)$$

$\alpha, \beta, \gamma, \delta$: real (bare) s.p. states with Ω

α', β' : isospin quantum number (bare) particle (p and n)

α'', β'' : isospin of quasi-particle (1 and 2)

ρ_α : sign of Ω , $\pm \Omega$ (angular momentum projection on the symmetry axis)

Deformed BCS transformation

$$\begin{aligned}
 a_{\rho_\alpha \alpha \alpha''}^\dagger &= \sum_{\rho_\beta \beta \beta'} (u_{\alpha \alpha'' \beta \beta'} c_{\rho_\beta \beta \beta'}^\dagger + v_{\alpha \alpha'' \beta \beta'} c_{\rho_\beta \bar{\beta} \beta'}), \\
 a_{\rho_\alpha \bar{\alpha} \alpha''} &= \sum_{\rho_\beta \beta \beta'} (u_{\bar{\alpha} \alpha'' \beta \beta'} c_{\rho_\beta \bar{\beta} \beta'} - v_{\bar{\alpha} \alpha'' \beta \beta'} c_{\rho_\beta \beta \beta'}^\dagger). \quad (6)
 \end{aligned}$$

$$\begin{pmatrix} a_1^\dagger \\ a_2^\dagger \\ a_{\bar{1}} \\ a_{\bar{2}} \end{pmatrix}_\alpha = \begin{pmatrix} u_{1p} & u_{1n} & v_{1p} & v_{1n} \\ u_{2p} & u_{2n} & v_{2p} & v_{2n} \\ -v_{1p} & -v_{1n} & u_{1p} & u_{1n} \\ -v_{2p} & -v_{2n} & u_{2p} & u_{2n} \end{pmatrix}_\alpha \begin{pmatrix} c_p^\dagger \\ c_n^\dagger \\ c_{\bar{p}} \\ c_{\bar{n}} \end{pmatrix}_\alpha,$$

where the u and v coefficients are calculated by the following DBCS equation

$$\begin{pmatrix} \epsilon_p - \lambda_p & 0 & \Delta_{p\bar{p}} & \Delta_{p\bar{n}} \\ 0 & \epsilon_n - \lambda_n & \Delta_{n\bar{p}} & \Delta_{n\bar{n}} \\ \Delta_{p\bar{p}} & \Delta_{p\bar{n}} & -\epsilon_p + \lambda_p & 0 \\ \Delta_{n\bar{p}} & \Delta_{n\bar{n}} & 0 & -\epsilon_n + \lambda_n \end{pmatrix}_\alpha \begin{pmatrix} u_{\alpha''p} \\ u_{\alpha''n} \\ v_{\alpha''p} \\ v_{\alpha''n} \end{pmatrix}_\alpha = E_{\alpha\alpha''} \begin{pmatrix} u_{\alpha''p} \\ u_{\alpha''n} \\ v_{\alpha''p} \\ v_{\alpha''n} \end{pmatrix}_\alpha.$$

Pairing Gaps

Δ_{nn}, Δ_{pp} : real

Δ_{np} : complex

$$\Delta_{p\bar{p}\alpha} = \Delta_{\alpha p\bar{\alpha}p} = - \sum_{J,c,d} g_{pp} F_{\alpha a \bar{\alpha} a}^{J0} F_{\gamma c \bar{\delta} c}^{J0} G(aacd, J, T=1) (u_{1pc}^* v_{1pd} + u_{2pc}^* v_{2pd}) ,$$

$$\Delta_{p\bar{n}\alpha} = \Delta_{\alpha p\bar{\alpha}n} = - \sum_{J,c,d} g_{np} F_{\alpha a \bar{\alpha} a}^{J0} F_{\gamma c \bar{\delta} c}^{J0} [G(aacd, J, T=1) \text{Re}(u_{1nc}^* v_{1pd} + u_{2nc}^* v_{2pd}) + iG(aacd, J, T=0) \text{Im}(u_{1nc}^* v_{1pd} + u_{2nc}^* v_{2pd})] ,$$

We do not include

Δ_{np} and $\Delta_{\bar{n}\bar{p}}$ explicitly, but include implicitly

multiplying a factor 2 on the T=0 pairing matrix

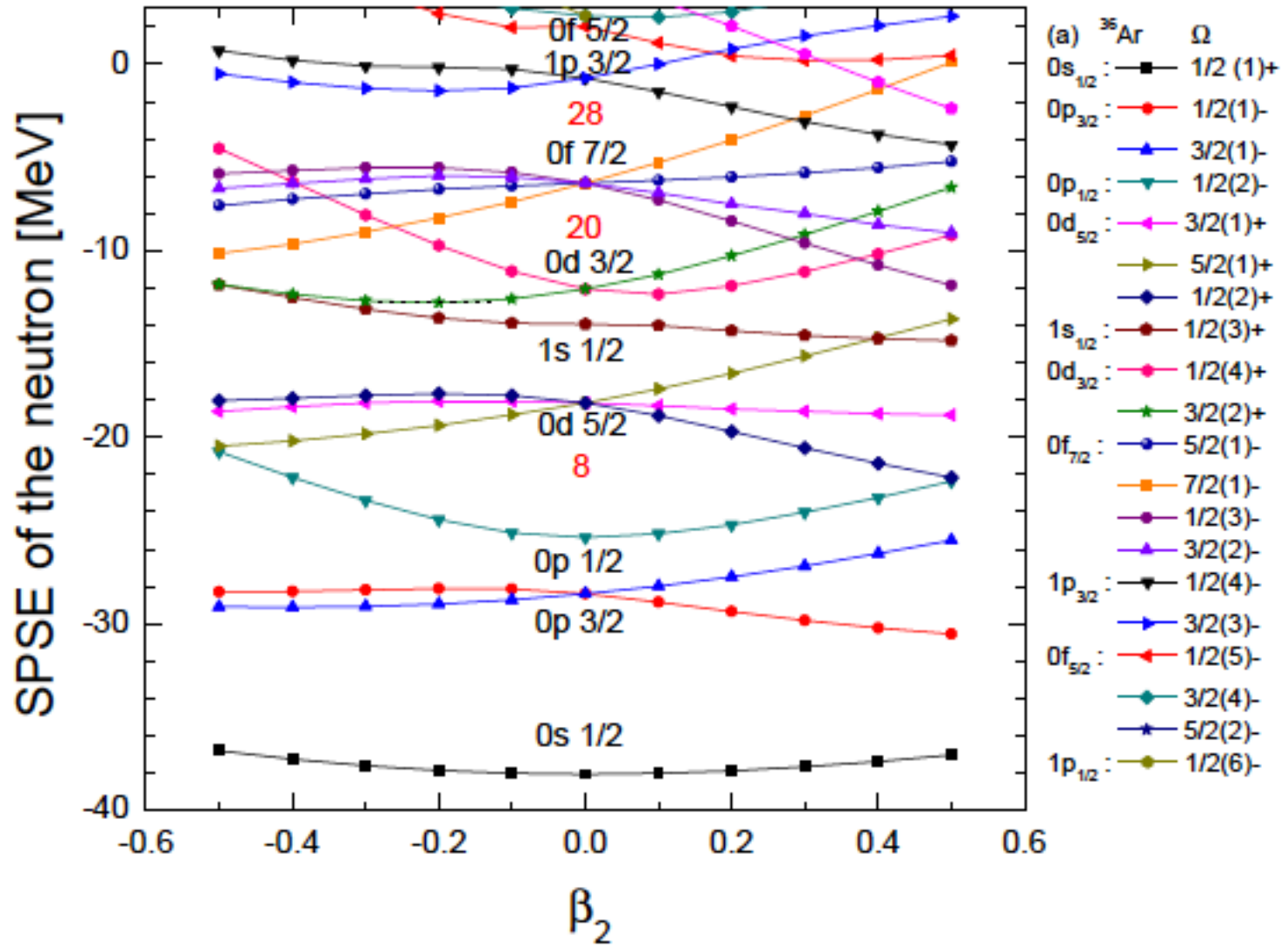
4 point formulas for empirical gaps

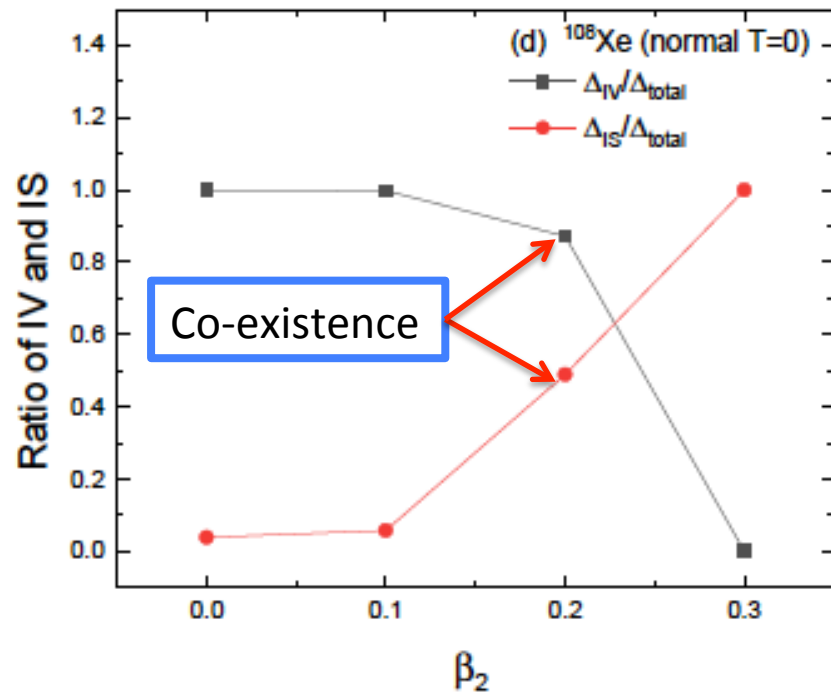
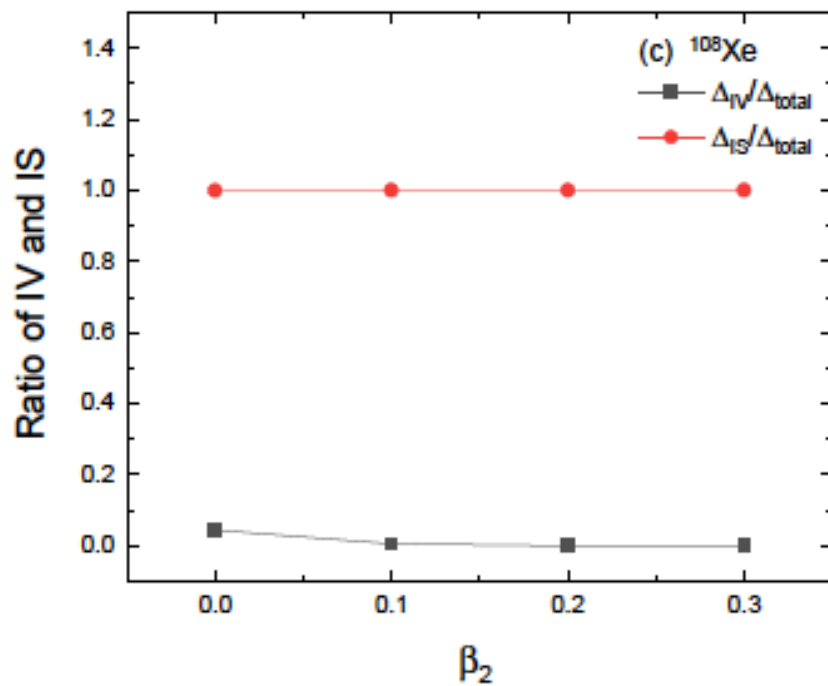
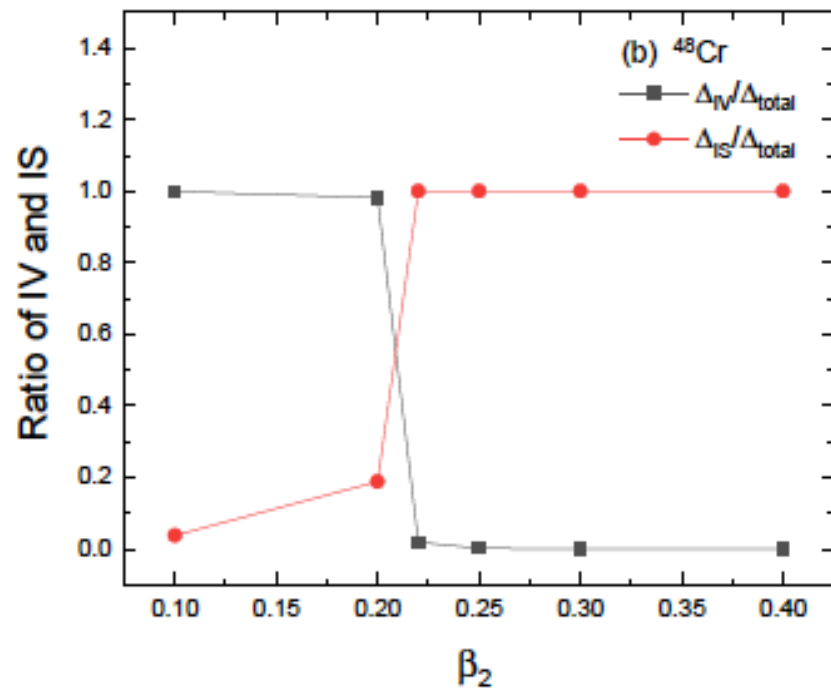
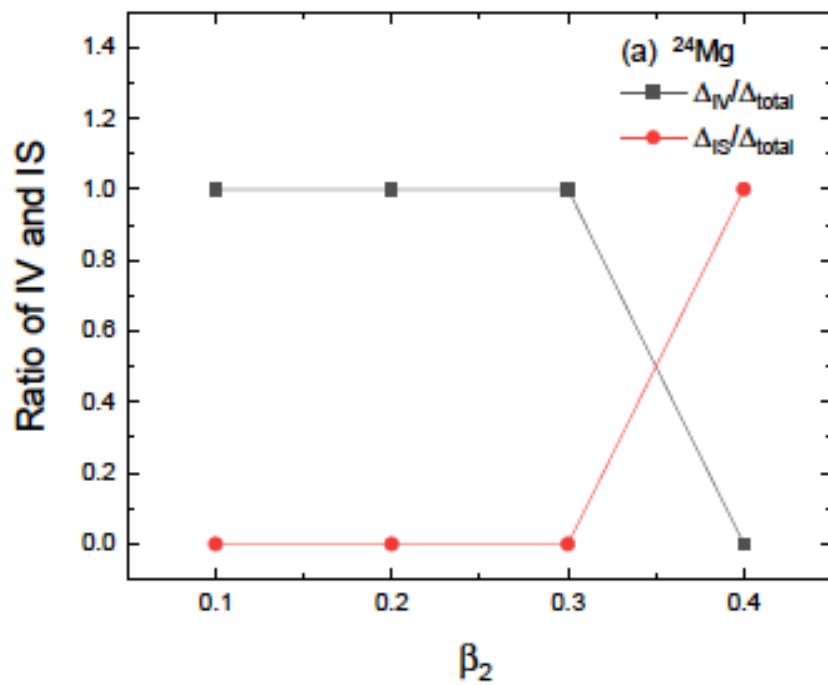
$$\Delta_p^{\text{emp}} = \frac{1}{8} [M(Z+2, N) - 4M(Z+1, N) + 6M(Z, N) - 4M(Z-1, N) + M(Z-2, N)], \quad (14)$$

$$\Delta_n^{\text{emp}} = \frac{1}{8} [M(Z, N+2) - 4M(Z, N+1) + 6M(Z, N) - 4M(Z, N-1) + M(Z, N-2)]. \quad (15)$$

Nucleus	$ \beta_2^{E2} $ [29]	β_2^{RMF} [30]	β_2^{FRDM} [31]	β_2^{Ours}	Δ_p^{emp}	Δ_n^{emp}	δ_{np}^{emp}
^{24}Mg	0.605	0.416	0.	0.300	3.123	3.193	1.844
^{36}Ar	0.256	-0.207	-0.255	-0.200	2.265	2.311	1.373
^{48}Cr	0.337	0.225	0.226	0.200	2.128	2.138	1.442
^{64}Ge	–	0.217	0.207	0.100	1.807	2.141	1.435
^{108}Xe	–	–	0.162	0.100	1.467	1.496	0.605
^{128}Gd	–	0.350	0.341	0.100	1.415	1.393	0.592

Deformed Woods-Saxon potential for s.p. energies in ^{36}Ar





Gamow-Teller transitions from high-spin isomers in $N = Z$ nuclei

H. Z. Liang(梁豪兆),^{1,2} H. Sagawa,^{1,3} M. Sasano,¹ T. Suzuki(鈴木俊夫),^{4,5} and M. Honma³

Phys. Rev. C 98, 014311 (2018) - Published 9 July 2018

Ikeda Gamow-Teller sum rule

=> proton and neutron Fermi sphere

$$S_- - S_+ \equiv \sum_{f,\nu} |\langle f | \hat{O}_\nu^- | i \rangle|^2 - \sum_{f,\nu} |\langle f | \hat{O}_\nu^+ | i \rangle|^2 = 3(N - Z)$$

New sum rule for GT transitions from High Spin Isomers

=> spin-up and spin-down Fermi sphere

the intrinsic frame of deformed nuclei,

$$\hat{O}_\nu^\pm = \sum_{\alpha} \sigma_\nu(\alpha) t_\pm(\alpha),$$

HSI: $I=12^+$ in ^{52}Fe is most likely to be oblate deformation.

Ex=6.96MeV $t_{1/2}=45.9\text{s}$

$I=21^+$ in ^{94}Au Ex=6.67MeV $t_{1/2}=0.4\text{s}$

Combinations of spin-up and spin-down operators

$$\begin{aligned}\Delta S(t_-) &\equiv S(\sigma_{-1}t_-) - S(\sigma_{+1}t_-) \\ &= 2 \sum_{\alpha} \langle i | \sigma_0(\alpha) t_+(\alpha) t_-(\alpha) | i \rangle, \\ \Delta S(t_+) &\equiv S(\sigma_{-1}t_+) - S(\sigma_{+1}t_+) \\ &= 2 \sum_{\alpha} \langle i | \sigma_0(\alpha) t_-(\alpha) t_+(\alpha) | i \rangle.\end{aligned}$$

$$\begin{aligned}\Delta S(t_-) - \Delta S(t_+) &= 4 \sum_{\alpha} \langle i | \sigma_0(\alpha) t_z(\alpha) | i \rangle \\ &= 2(\langle S_n \rangle - \langle S_p \rangle), \\ \Delta S(t_-) + \Delta S(t_+) &= 2 \sum_{\alpha} \langle i | \sigma_0(\alpha) | i \rangle \\ &= 2(\langle S_n \rangle + \langle S_p \rangle),\end{aligned}$$

Sum rule values depend on the spin expectation of protons and neutrons. Notice $|i\rangle$ is High spin isomers.

Angular momentum projection in Laboratory frame

$$\Phi_{KIM} = \left(\frac{2I+1}{16\pi^2} \right)^{1/2} \times (\Psi_K(q) D_{MK}^I(\omega) + (-)^{I+K} \Psi_{\bar{K}}(q) D_{M-K}^I(\omega))$$

$$\hat{O}_{GT}(1\mu) = \sum_{\nu} \hat{O}_{\nu} D_{\mu\nu}^1(\omega).$$

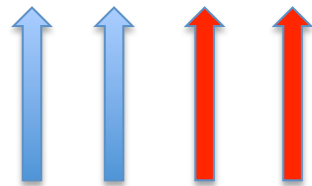
$$\langle K'I' || \hat{O}_{GT} || KI \rangle = (2I+1)^{1/2} \langle IK1\Delta K | I'K' \rangle \langle K' || \hat{O} || K \rangle$$

TABLE II. B_{GT} strengths for the transitions $I \rightarrow I'$ with the 4-qp configuration of protons and neutrons $[Nn_3\Lambda\Omega] = [N0N(\Lambda+1/2)]_{\nu(\pi)}$ and $[N1(N-1)(\Lambda+1/2)]_{\nu(\pi)}$. Sum values in the last line are evaluated with $I = 12$ and $j = 7/2$.

	$I' = I - 1$	$I' = I$	$I' = I + 1$
$\Delta K = -1$	$\frac{2(4j-1)(2I-1)}{j(2I+1)}$	$\frac{2(4j-1)}{j(I+1)}$	$\frac{2(4j-1)}{j(2I+1)(I+1)}$
$\Delta K = 0$	—	$\frac{4I}{I+1}$	$\frac{4}{I+1}$
$\Delta K = +1$	—	—	$\frac{2}{j}$
sum	6.83	4.26	0.90

^{52}Fe Ex=6.958MeV
 life time 45.9s

Seniority four states



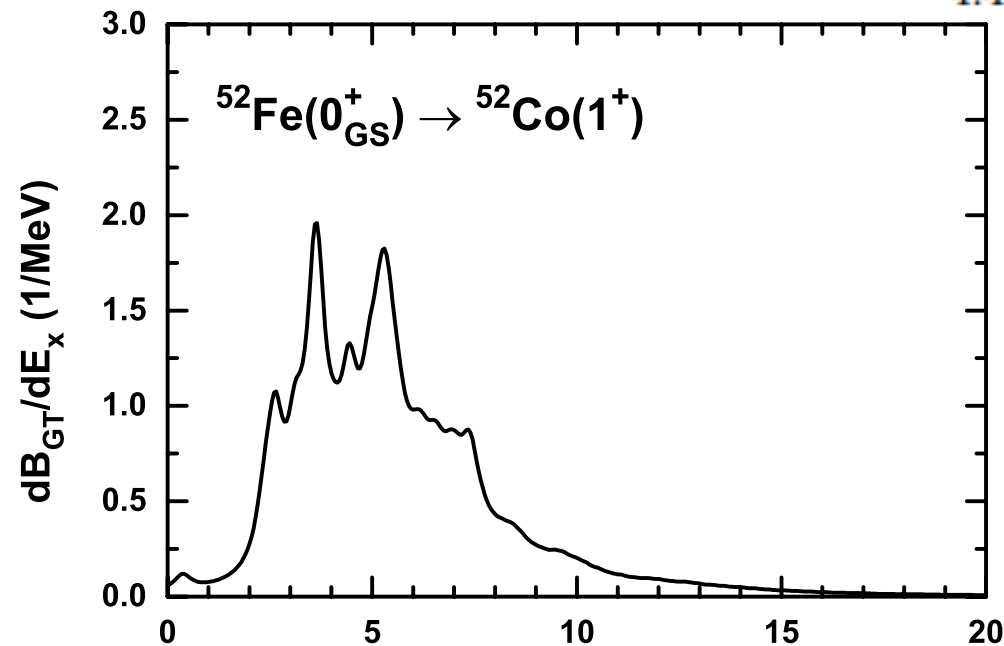
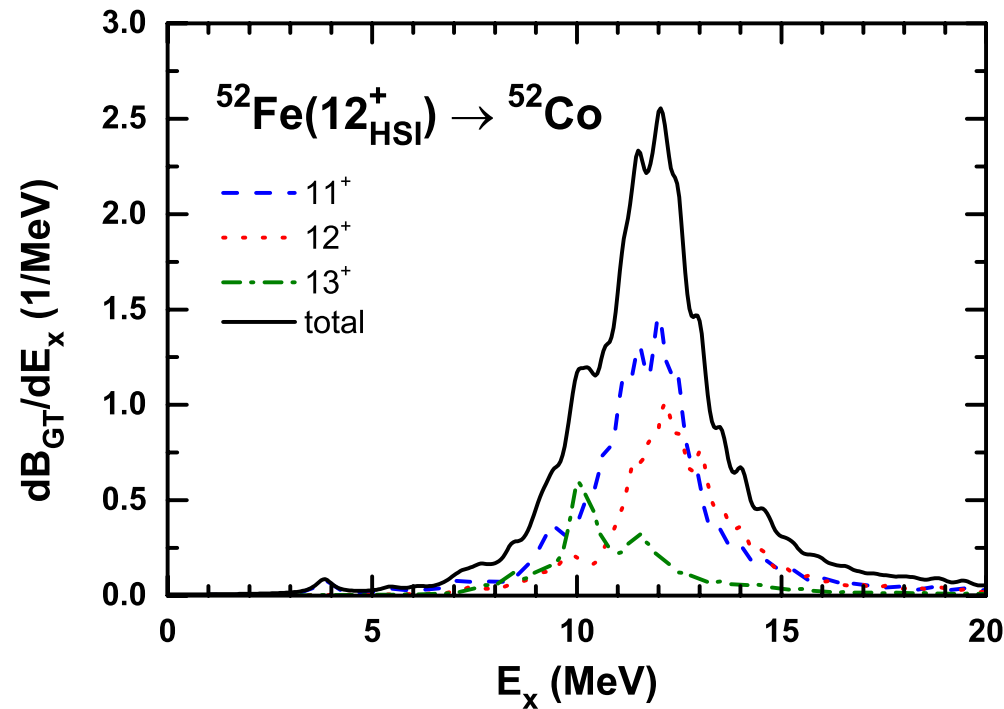
$I_i = 12^+$

$I_n^\pi = 6^+ \quad I_p^\pi = 6^+$

$B(GT)_{sum} :$

4.47, 3.11, and 1.44 for the $I' = 11, 12,$ and 13

Brink-Axel hypothesis
 on GR top of excited states



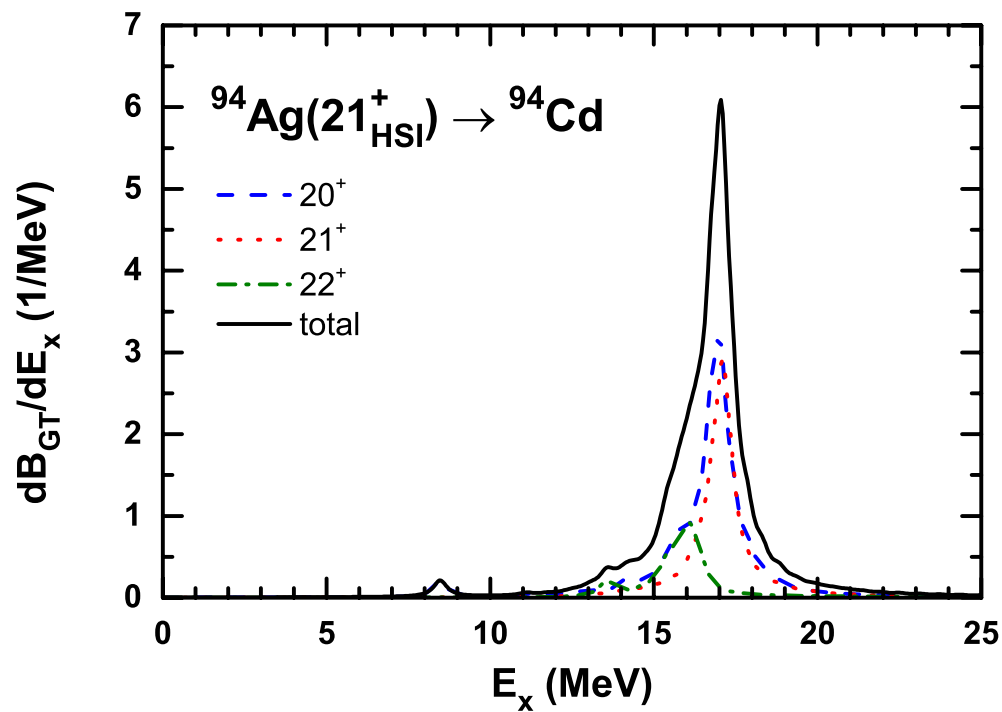
^{94}Ag

$$[Nn_3\Lambda\Omega] = [404\frac{9}{2}]_{\pi(\nu)}, [413\frac{7}{2}]_{\pi(\nu)}, \text{ and } [422\frac{5}{2}]_{\pi(\nu)} \quad 1$$

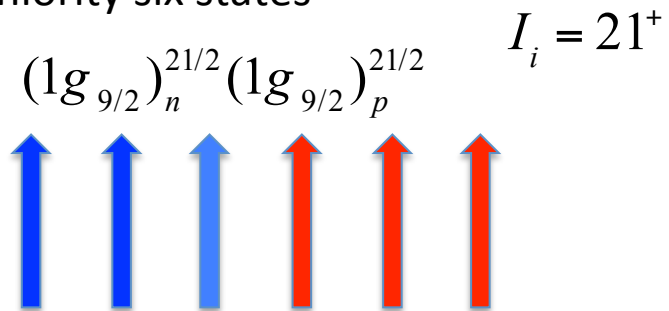
$$I_n^\pi = \frac{21^+}{2} \quad I_p^\pi = \frac{21^+}{2} \quad I_i = 21^+$$

TABLE III. B_{GT} strengths for the transition $I \rightarrow I'$ with the 6-qp configuration of protons and neutrons $[Nn_3\Lambda\Omega] = [N0N(\Lambda + 1/2)]_{\nu(\pi)}$, $[N1(N - 1)(\Lambda + 1/2)]_{\nu(\pi)}$, and $[N2(N - 2)(\Lambda + 1/2)]_{\nu(\pi)}$. The sum values in the last line are evaluated with $I = 21$ and $j = 9/2$.

	$I' = I - 1$	$I' = I$	$I' = I + 1$
$\Delta K = -1$	$\frac{2(2I-1)}{2I+1} \frac{6j-3}{j}$	$\frac{2}{I+1} \frac{6j-3}{j}$	$\frac{2}{(2I+1)(I+1)} \frac{6j-3}{j}$
$\Delta K = 0$	—	$\frac{6I}{I+1}$	$\frac{6}{I+1}$
$\Delta K = +1$	—	—	$\frac{6}{j}$
sum	10.17	6.21	1.61

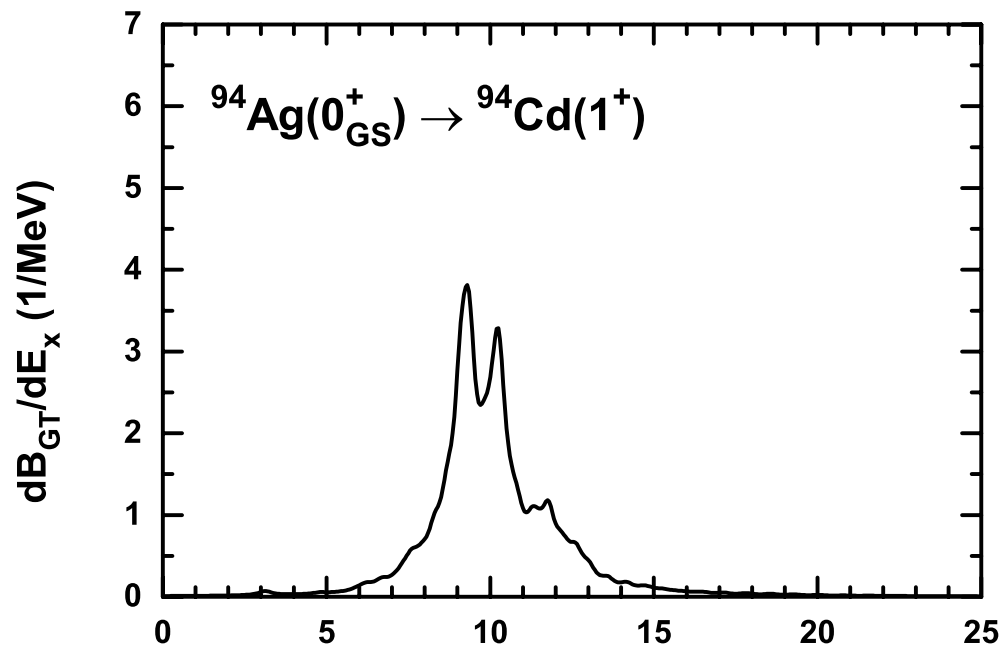


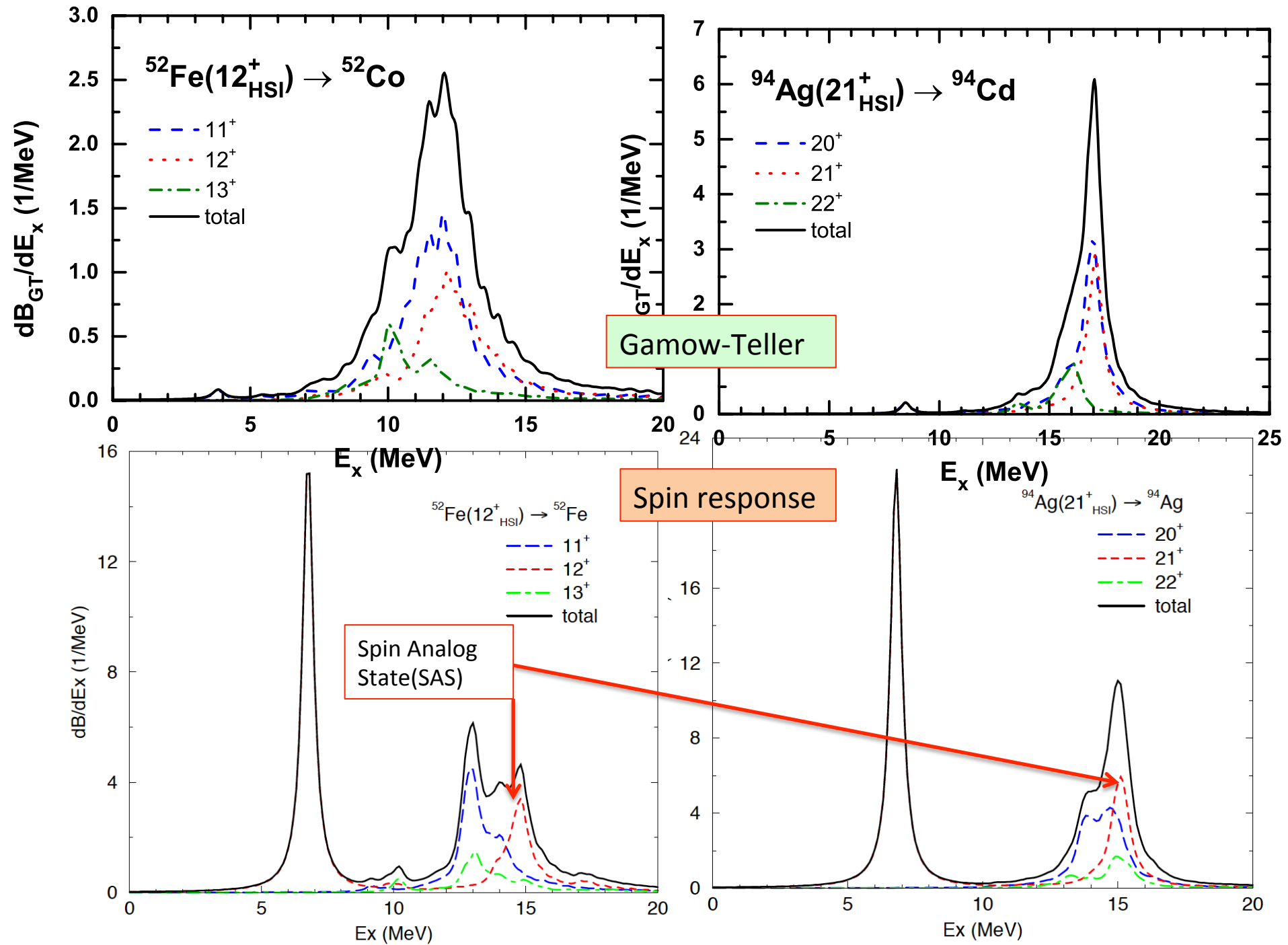
Seniority six states



^{94}Ag Ex=6.67MeV

Life Time 0.4s





Summary: spin-isospin states in $N=Z$ nucleus

1. Cooperative role of $T=0$ and $T=1$ pairings induce (SU(4) symmetry restoration in spin-isospin space)
=>large Gamow-Teller transitions of $N=Z+2$ nuclei at lower energy
2. HFB results: $T=0$ superfluidity may coexist with $T=1$ superfluidity.
The deformation plays an important role to realize spin-triplet superfluid phase in the ground state: surface =>spin-singlet
center => spin-triplet
more theoretical study : Isospin projection and angular momentum projection
3. HIS-GT: new sum rule in the spin-up and spin-down Fermi sphere.
(Ikeda GT sum rule: isospin-up and -down Fermi sphere.
provide effective spin-spin residual and spin-isospin residual interactions in extreme spin polarized space.
4. Fine fittings of energy density functions of spin and spin-isospin channels
(which was done already for Shell model interactions: GPFX1J
BY Toshio Suzuki, Michio Honma)

Recent progress(M1 transitions)

1. For $N=Z$ odd-odd nuclei, a strong competition between $S=0$ and $S=1$ pairing correlations is observed near the ground states.
2. How Spin-triplet superfluidity can be seen in nuclear many-body system: abrupt or smooth (crossover) transitions?
3. Large quenching in the IV spin response was observed which is consistent with magnetic moments and Gamow-Teller beta-decay matrix.
4. IS spin sum rule strength shows much smaller quenching than IV spin ones.
5. Strong spin-triplet pairing gives positive contribution to the spin-spin neutron-proton correlations in $N=Z$ nuclei.