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Faculty of Physics

Department of Nuclear and Particle Physics

A study of the nuclear structure in the even-even Yb isotopes

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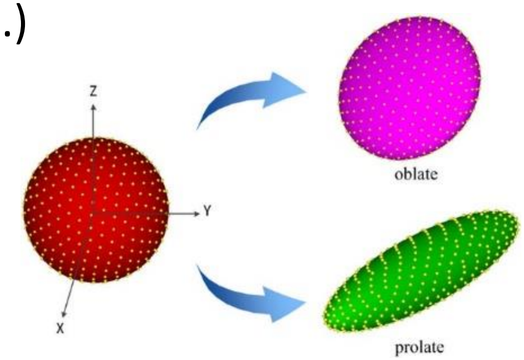
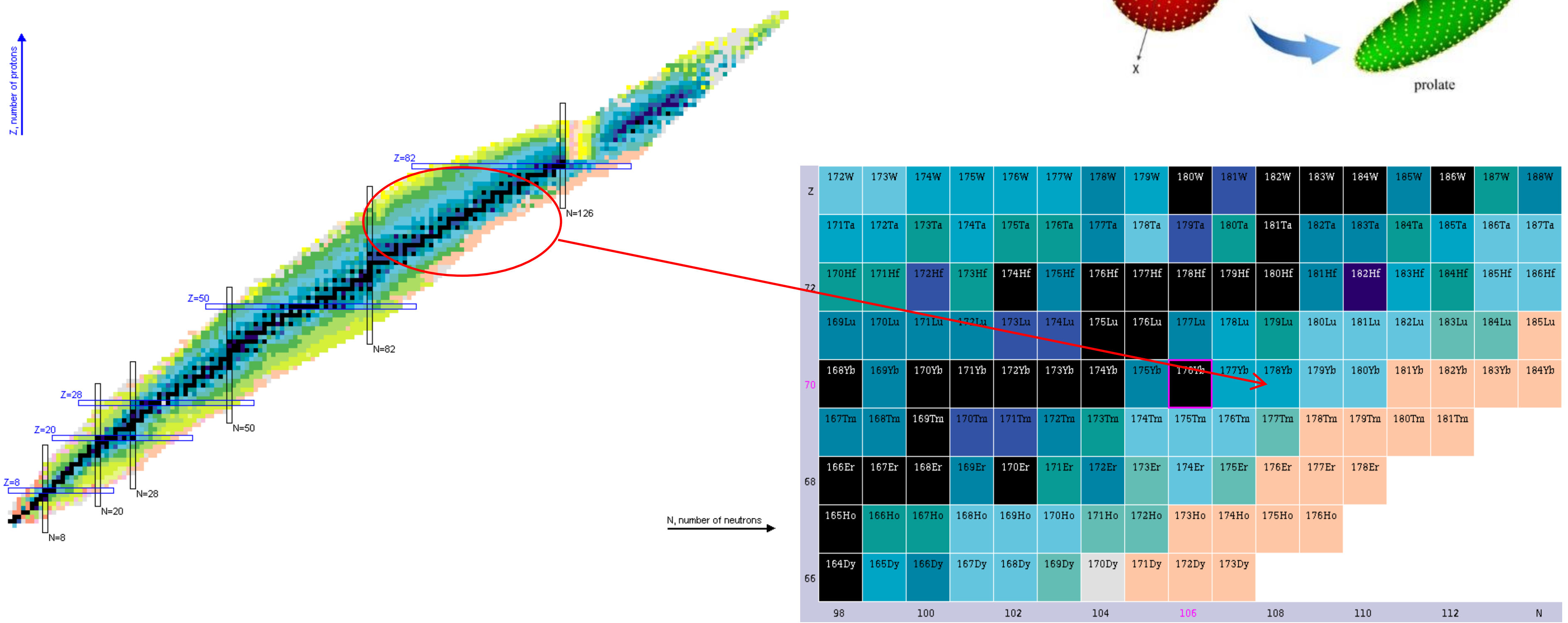
Presentation Outline

1. Area of interest
2. Motivation
3. Related Quantities & Theoretical Models
4. Results
5. Conclusion & Future Steps

1. Area of Interest

1. Nuclear deformation
2. Shape coexistence, proxy-SU(3) (e.g., ^{156}Sm, ^{166}Dy, ^{176}Yb ...)
3. The isotopes of $_{70}$Yb with A~170 appear to have rotational properties





Source: National Nuclear Data Center, http://www.nndc.bnl.gov/

2. Motivation

- Lack of experimental data for lifetimes of ^{178}Yb and energy states in unstable ^{180}Yb
- Focus on the calculation of B(E2; $0^+ \rightarrow 2^+$) values, related quadrupole moments Q and deformation parameter β2, which exhibit quadratic distortion
- The results are compared to available experimental data[1] and can serve as a guide for future experimental studies in the unstable neutron–rich Yb isotopes

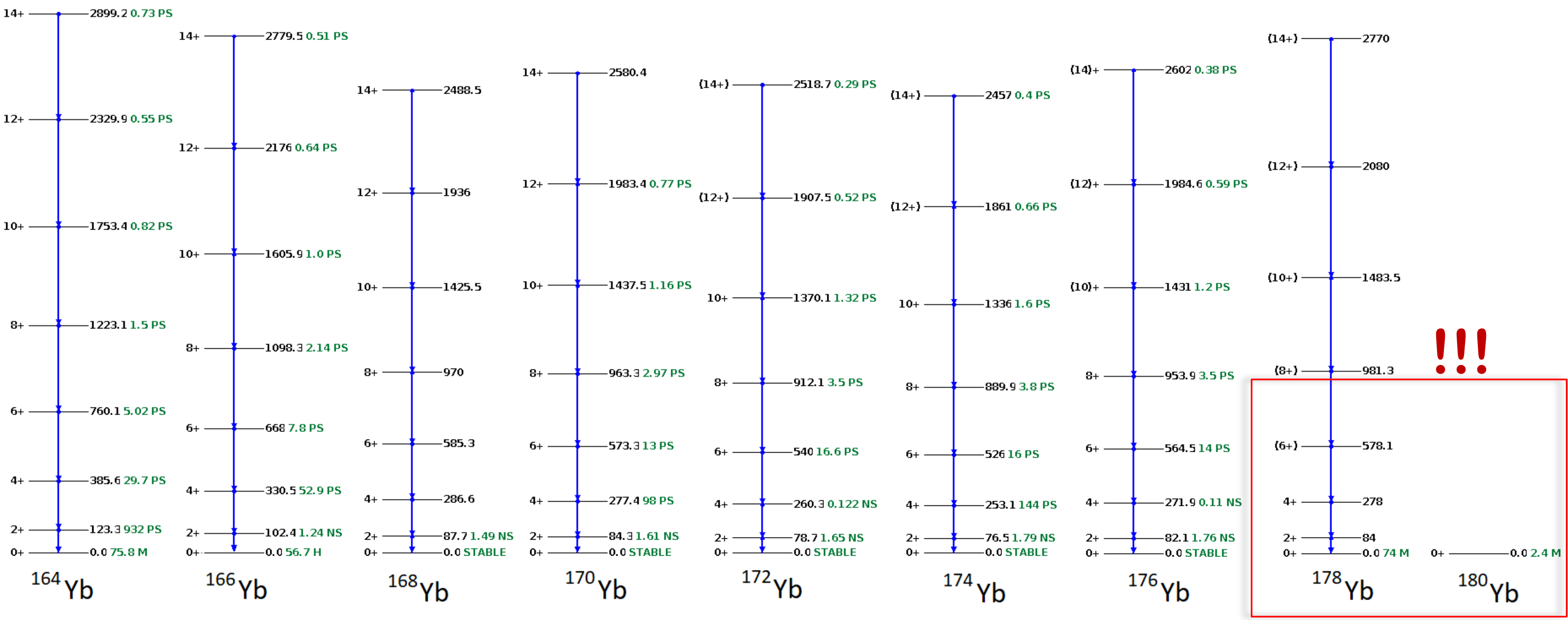


Image Source: National Nuclear Data Center, http://www.nndc.bnl.gov/

[1] B. Pritychenko *et al. , Atom. Data Nucl. Data Tabl., 107:1–139, (2016)*

3. Related Quantities & Theoretical Models

- ✓ The intrinsic electric quadrupole moment (Q_0) is defined in the intrinsic frame of the nucleus. The reduced electric quadrupole transition probability B(E2) contains information about the structure of the low–lying levels of nuclei. Another quantity that is quite useful, despite being model–dependent, is the deformation parameter, β_2

$$Q_0 = \left[\frac{16\pi}{5}\frac{B(E2)\uparrow}{e^2}\right]^{1/2} \text{ (in barns)} \qquad Q(I,K) = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)}Q_0 \qquad \beta_2 = \left(\frac{4\pi}{3ZR_0^2}\right)\left[\frac{B(E2)\uparrow}{e^2}\right]^{1/2}$$

- ✓ Nine (9) theoretical models were used in this work: PhM, FRDM, HFBCS-MSk7, HFBM-Gogny, RHF-NL3*, HFB–UNEDF1, IBM-1 and the recently developed Proxy and Pseudo SU(3) Models

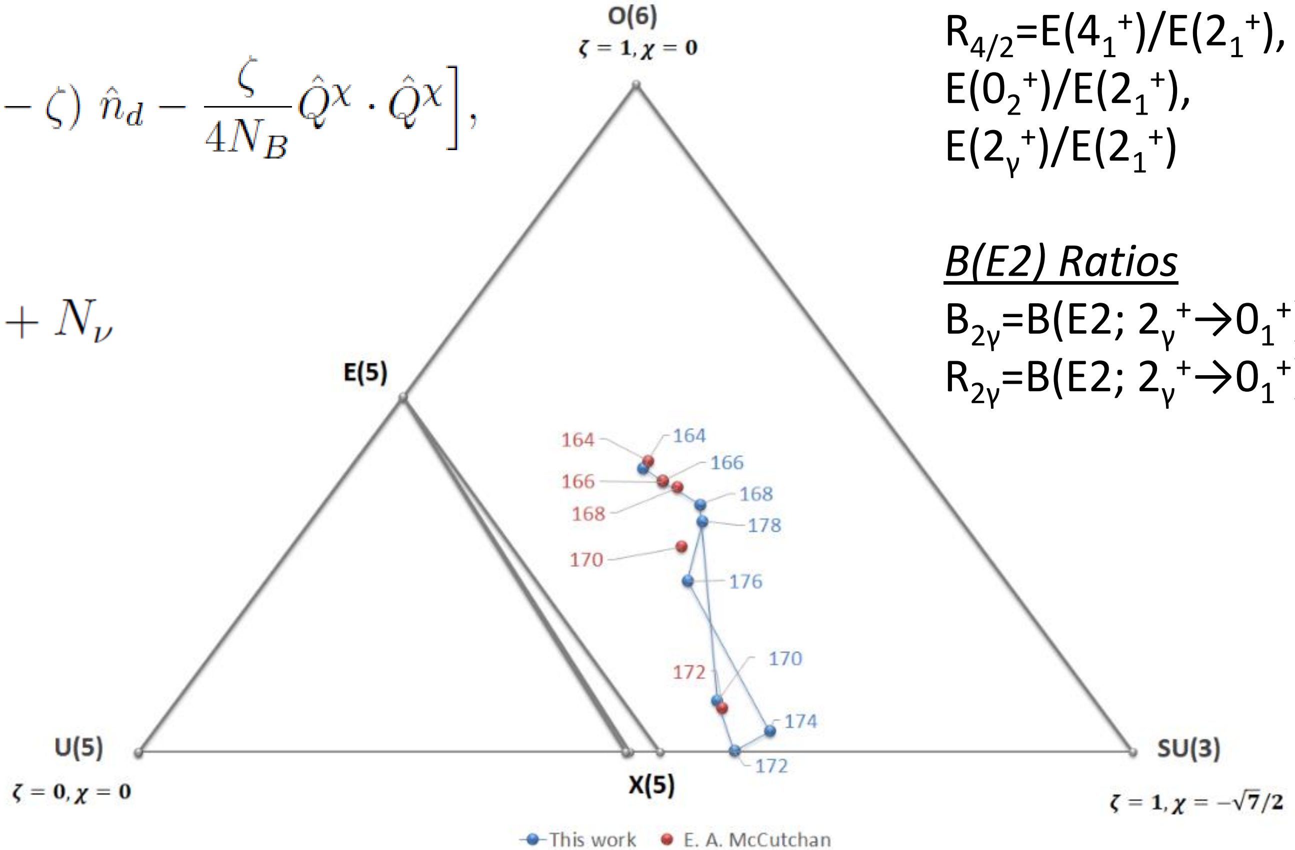
- ✓ Numerical results for energy ratios with the Exactly Separable David-son (ESD), Exactly Separable Morse, Exactly Separable Woods Saxon, Deformation Dependent Mass Davidson (DDMD) and Deformation Dependent Mass Kratzer (DDMK)

Interacting Boson Model (IBM-1) Calculations

For each N_B (N_B = 12 - 17) nearly 20'000 calculations have been carried out with the IBAR program. Results were obtained by fitting five ratios to experimental data for each isotope. The method leaves ζ and χ as free parameters in the range (0.00 to 1.00) for ζ and ($-\sqrt{7}/2$ to 0.00) for χ with step 0.01, where $\chi = -\sqrt{7}/2$ is the limit of SU(3) symmetry.

$$H(\zeta) = c\left[(1-\zeta)\ \hat{n}_d - \frac{\zeta}{4N_B}\hat{Q}^{\chi}\cdot\hat{Q}^{\chi}\right],$$

$$N_B = N_\pi + N_\nu$$



Energy Ratios:

$R_{4/2}=E(4_1^+)/E(2_1^+)$,
$E(0_2^+)/E(2_1^+)$,
$E(2_\gamma^+)/E(2_1^+)$

B(E2) Ratios

$B_{2\gamma}=B(E2;\ 2_\gamma^+\rightarrow 0_1^+)/B(E2;\ 2_1^+\rightarrow 0_1^+)$
$R_{2\gamma}=B(E2;\ 2_\gamma^+\rightarrow 0_1^+)/B(E2;\ 2_\gamma^+\rightarrow 2_1^+)$

A. Zyriliou *et al., (*2021) (under review)
E. A. McCutchan *et al.* Phys. Rev. C, 69:064306 (2004)

Interacting Boson Model (IBM-1) Calculations

Table 11: Experimental[a] [2] and calculated $B(E2; J_i \to J_f)(e^2b^2)$ values for $_{70}$Yb isotopes.

	^{164}Yb		^{166}Yb		^{168}Yb		^{170}Yb	
Transition $J_i \to J_f$	Experiment	IBM	Experiment	IBM	Experiment	IBM	Experiment	IBM
$0_1^+ \to 2_1^+$	4.33 (14)	4.769	5.20 (20)	5.139	5.75 (12)	5.409	5.721 (70)	5.600
$2_1^+ \to 0_1^+$	0.866 (28)	0.954	1.035 (54)	1.028	1.151 (38)	1.082	1.124 (34)	1.120
$4_1^+ \to 2_1^+$	1.381 (48)	1.381[b]	1.474 (49)	1.474[b]	–	1.543	1.607[d]	1.607[b]
$6_1^+ \to 4_1^+$	1.472 (53)	1.530	1.577 (65)	1.630	–	1.688	2.302[d]	1.772
$8_1^+ \to 6_1^+$	1.706 (586)	1.582	1.734 (217)	1.686	–	1.740	2.014 (168)	1.842
$10_1^+ \to 8_1^+$	1.600 (640)	1.573	1.680 (867)	1.684	–	1.739	1.991 (140)	1.860
$12_1^+ \to 10_1^+$	1.600 (586)	1.513	1.463 (759)	1.637	–	1.698	1.499 (117)	1.839
$14_1^+ \to 12_1^+$	1.280 (373)	1.410	1.355 (813)	1.550	–	1.624	–	1.783
$16_1^+ \to 14_1^+$	–	1.266	–	1.427	–	1.518	–	1.696
$2_\gamma^+ \to 2_1^+$	–	0.183	–	0.128	0.051 (6)	0.073	0.027 (6)	0.021
$2_\gamma^+ \to 0_1^+$	–	0.037	–	0.035	0.028 (4)	0.028[b]	0.015 (3)	0.015

	^{172}Yb		^{174}Yb		^{176}Yb		^{178}Yb	
Transition $J_i \to J_f$	Experiment	IBM	Experiment	IBM	Experiment	IBM	Experiment	IBM
$0_1^+ \to 2_1^+$	6.09 (15)	5.965	5.85 (16)	5.676	5.189 (89)	5.516	–	4.705
$2_1^+ \to 0_1^+$	1.205 (11)	1.193	1.160 (40)	1.135	1.072 (41)	1.103	–	0.941
$4_1^+ \to 2_1^+$	1.710 (114)	1.710[b]	1.616 (52)	1.616[b]	1.582 (146)	1.582[b]	–	1.342
$6_1^+ \to 4_1^+$	1.818 (170)	1.885	2.135 (288)	1.766	1.746 (129)	1.741	–	1.470
$8_1^+ \to 6_1^+$	2.273 (227)	1.963	2.239 (121)	1.825	1.758 (293)	1.811	–	1.518
$10_1^+ \to 8_1^+$	2.131 (131)	1.989	1.933 (127)	1.838	1.875 (176)	1.833	–	1.524
$12_1^+ \to 10_1^+$	2.443 (341)	1.977	2.129 (133)	1.821	1.816 (234)	1.819	–	1.499
$14_1^+ \to 12_1^+$	2.239 ($^{+341}_{-256}$)	1.931	1.846 (462)	1.779	1.640 (352)	1.776	–	1.447
$16_1^+ \to 14_1^+$	–	1.856	–	1.714	–	1.705	–	1.371
$2_\gamma^+ \to 2_1^+$	0.012 (1)[c,e]	0.015	0.014 (3)	0.010	–	0.064	–	0.052
$2_\gamma^+ \to 0_1^+$	0.008 (1)	0.010	0.006 (1)	0.006	0.010 (1)	0.010	–	0.021

Lack of experimental data!

Table 8: Possible nuclear shapes as a function of the deformation parameters β_2 and γ.

Spherical:	$\beta_2 = 0$
Prolate:	$\beta_2 > 0, \gamma = 0°$
Oblate:	$\beta_2 > 0, \gamma = 60°$
Triaxial:	$\beta_2 > 0,\ 0° < \gamma < 60°$

$$R = \frac{E(2_\gamma^+)}{E(2_1^+)} \qquad \sin(3\gamma) = \frac{3}{2\sqrt{2}}\sqrt{1 - \left(\frac{R-1}{R+1}\right)^2}$$

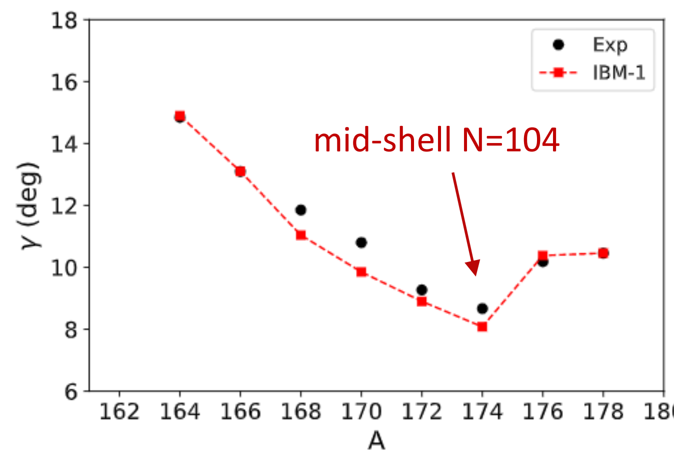


Fig. 5: IBM-1 predictions for γ, obtained from Eq. (31), compared to experimental values extracted from ratios of the γ bandhead to the first 2^+ state. Experimental uncertainties are smaller than the data symbols.

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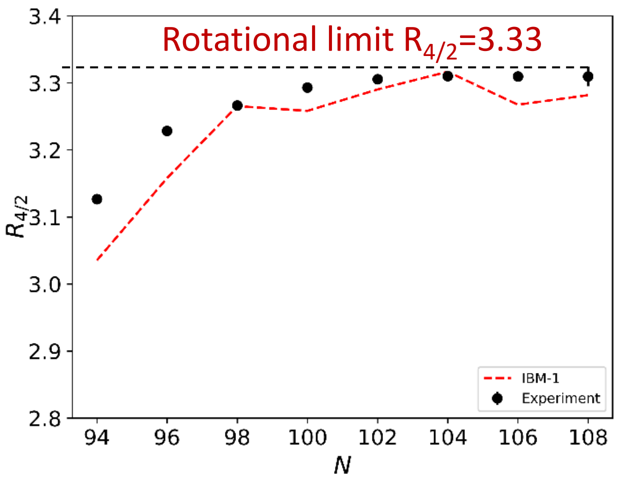
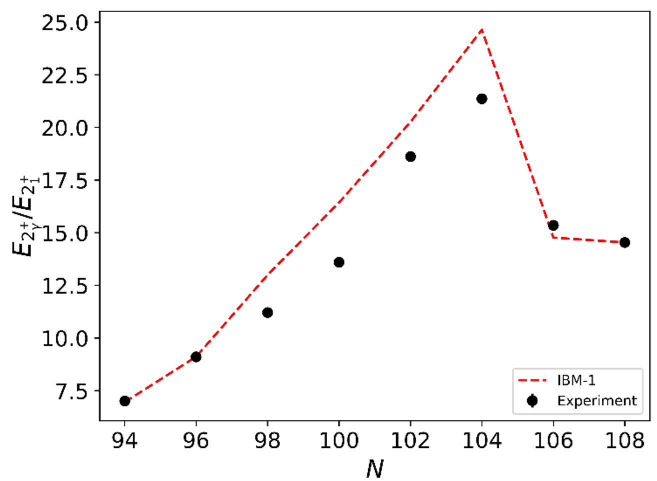
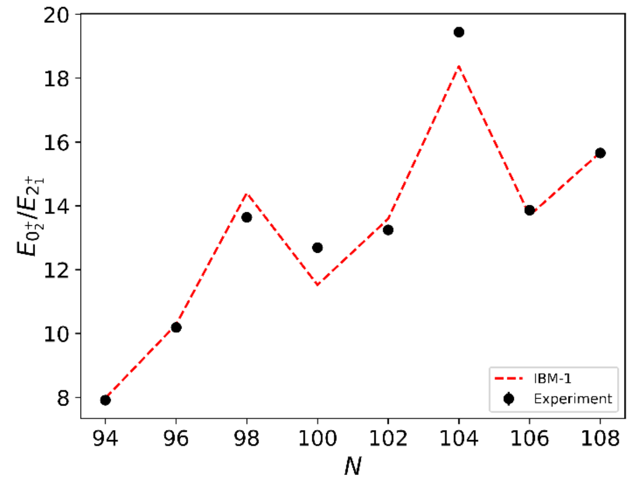
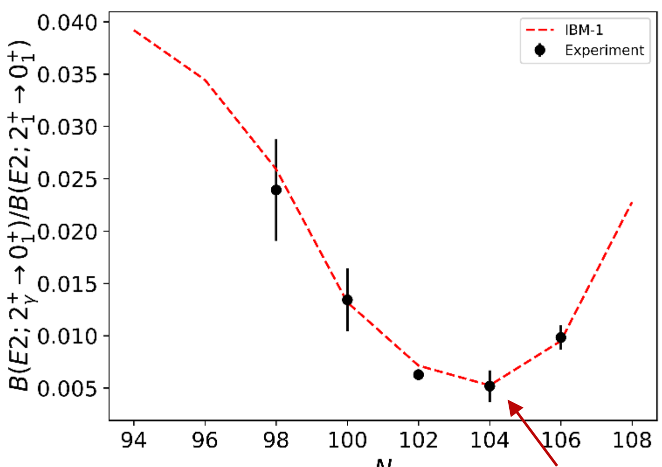
Interacting Boson Model (IBM-1) Calculations

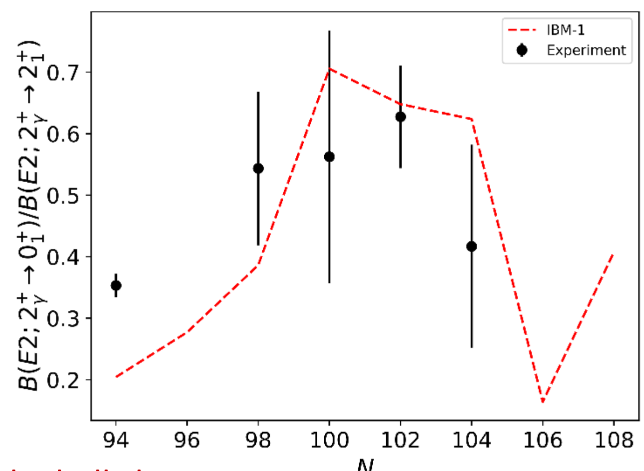
These specific ratios can explain the low–spin structure of the collective even–even nuclei.











max boson number, sub-shell closure

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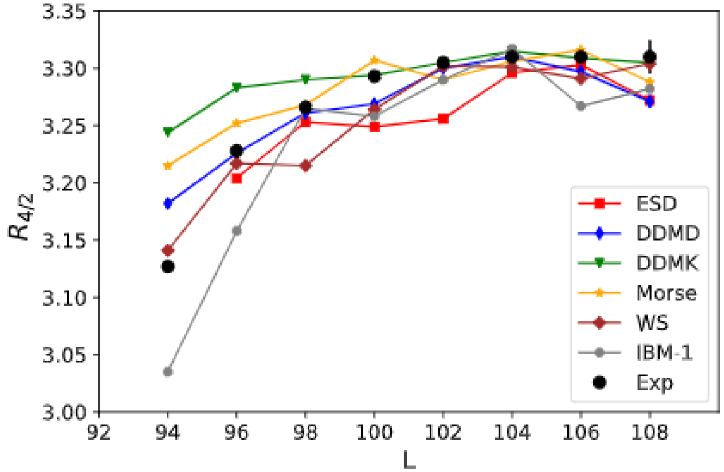
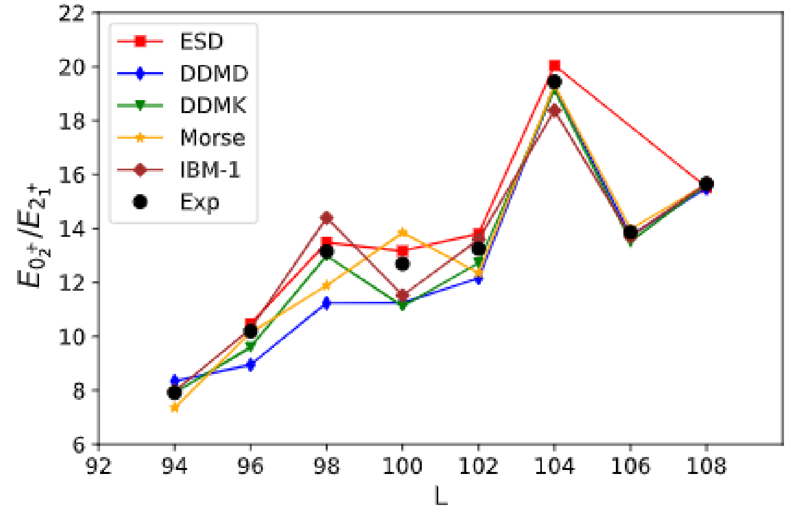
4. Results for $^{164\text{-}180}$Yb isotopes





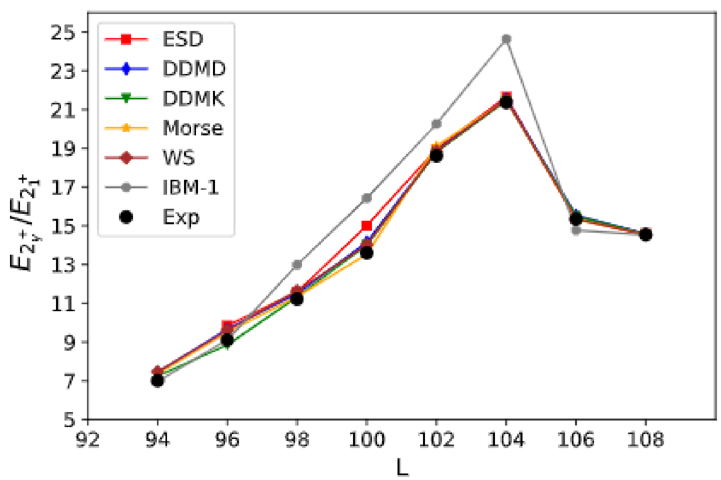


Fig. 13: Numerical results for important energy ratios obtained by ESD, ES Morse, ES Woods–Saxon, DDMD and DDMK analytical solutions of the Bohr Hamiltonian and IBM–1 are compared to experimental values. Experimental uncertainties are smaller than the data symbols.

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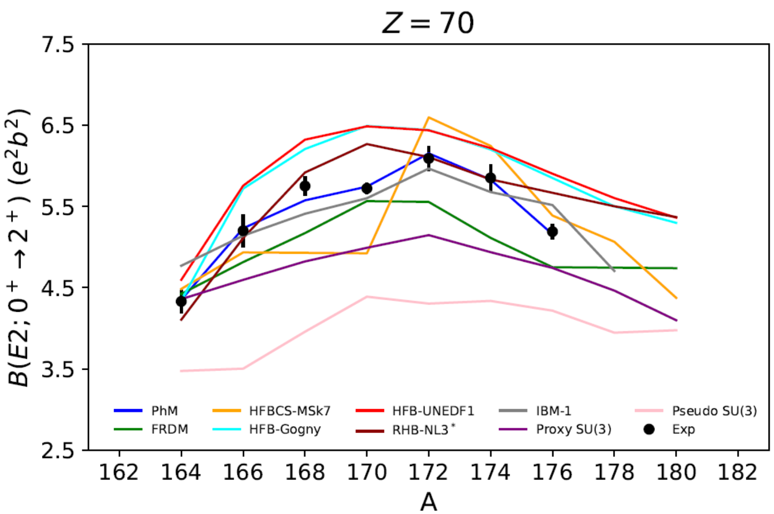
4. Results for $^{164\text{-}180}$Yb isotopes



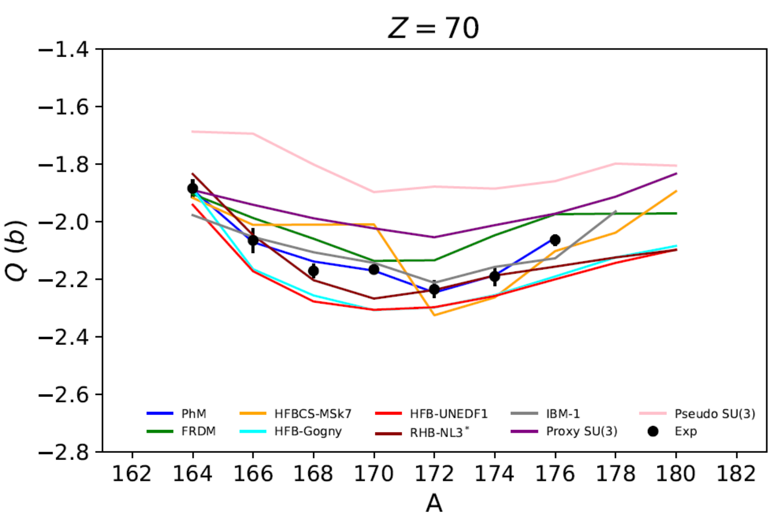


Table 15: Values of Q in Units of b (see Fig. 16(b)). See text, for details of their calculation. ("Exp" values in first column extracted from experimental $B(E2; 0_1^+ \to 2_1^+)$ [27] using rotational model equations 1,3.)

Isotope	"Exp"	PhM	FRDM	HFBCS MSk7	HFB Gogny	HFB UNEDF1	RHB NL3*	IBM-1	Proxy SU(3)	Pseudo SU(3)
^{164}Yb	-1.885 (31)	-1.890	-1.905	-1.918	-1.888	-1.942	-1.835	-1.978	-1.891	-1.688
^{166}Yb	-2.066 (42)	-2.072	-1.988	-2.012	-2.166	-2.173	-2.048	-2.054	-1.942	-1.695
^{168}Yb	-2.172 (25)	-2.139	-2.060	-2.011	-2.257	-2.278	-2.204	-2.107	-1.989	-1.802
^{170}Yb	-2.167 (14)	-2.171	-2.137	-2.010	-2.308	-2.307	-2.268	-2.144	-2.024	-1.898
^{172}Yb	-2.236 (30)	-2.247	-2.135	-2.326	-2.299	-2.298	-2.238	-2.213	-2.055	-1.879
^{174}Yb	-2.191 (33)	-2.187	-2.048	-2.264	-2.256	-2.259	-2.188	-2.158	-2.013	-1.886
^{176}Yb	-2.064 (19)	-2.057	-1.975	-2.103	-2.191	-2.201	-2.157	-2.128	-1.973	-1.860
^{178}Yb	–	–	-1.973	-2.039	-2.125	-2.144	-2.125	-1.965	-1.914	-1.799
^{180}Yb	–	–	-1.972	1.895	-2.085	-2.098	-2.099	–	-1.834	-1.806

Table 16: Predicted Values of $B(E2; 0_1^+ \to 2_1^+)$ in Units of e^2b^2 are compared with adopted ones [27] (see Fig. 17(a)).

Isotope	Exp [27]	PhM	FRDM	HFBCS MSk7	HFB Gogny	HFB UNEDF1	RHB NL3*	IBM-1	Proxy SU(3)	Pseudo SU(3)
^{164}Yb	4.33 (14)	4.351	4.423	4.484	4.342	4.596	4.105	4.769	4.358	3.474
^{166}Yb	5.20 (20)	5.230	4.814	4.933	5.718	5.754	5.110	5.139	4.595	3.502
^{168}Yb	5.75 (12)	5.574	5.171	4.928	6.206	6.321	5.917	5.409	4.822	3.957
^{170}Yb	5.721 (70)	5.743	5.565	4.922	6.491	6.485	6.267	5.600	4.990	4.388
^{172}Yb	6.09 (15)	6.152	5.554	6.595	6.441	6.436	6.106	5.965	5.146	4.303
^{174}Yb	5.85 (16)	5.828	5.112	6.244	6.200	6.217	5.832	5.676	4.936	4.336
^{176}Yb	5.189 (89)	5.157	4.751	5.387	5.849	5.902	5.667	5.516	4.742	4.216
^{178}Yb	–	–	4.745	5.064	5.501	5.602	5.501	4.705	4.463	3.945
^{180}Yb	–	–	4.739	4.374	5.297	5.361	5.368	–	4.098	3.975

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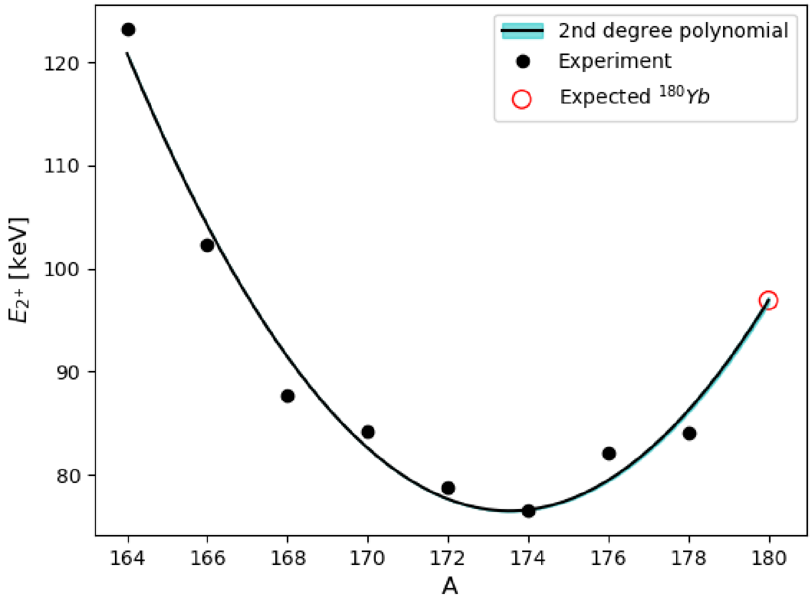
4. Results for $^{164\text{-}180}$Yb isotopes



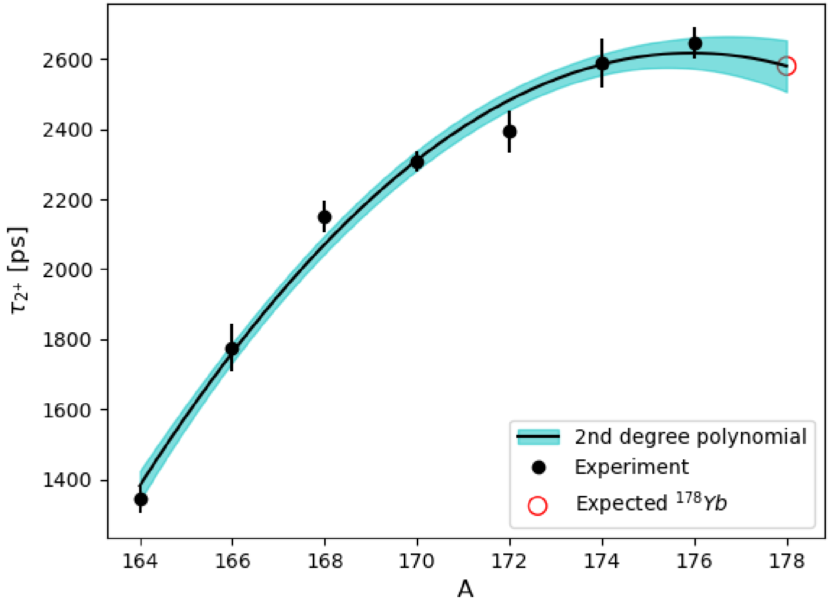


Table 12: Extrapolated values and Raman's global fit values for ^{178}Yb and ^{180}Yb.

A	Energy E_{2^+} (keV)	Local Fit τ_{2^+} (ps)	Global Fit [25] τ_{2^+} (ps)
178	84.0 (3)	2580 (73)	2825 (499)
180	96.90 (33)	–	–

A. Zyriliou *et al., (*2021) (under review)
S. Raman *et al.,* At. Data and Nucl. Data Tables, 78(1):1 – 128, 2001.

5. Conclusion & Future Steps

- ✓ The nuclear properties of the Ytterbium isotopes and their evolution as the neutron number increases has been the major subject in this work
- ✓ The collective behavior of the even–even $^{164-180}$Yb isotopes was investigated using several well–established theoretical models in synergy with available experimental data.
- ✓ The nuclear observables examined in this work (energies, reduced transition probabilities etc.) for a number of permanently deformed Yb isotopes are calculated in agreement with available experimental results
- ✓ Predictions have been made for some observables of ^{178,180}Yb isotopes for which no experimental data currently exist
- ✓ We hope this work can serve as a reference point for future experimental and theoretical work in this mass region, which will provide useful information towards understanding the nuclear structure as one moves closer to the neutron dripline
- ✓ It can also serve as useful input for incorporating nuclear structure in models for nuclear astrophysics processes that involve Yb and other neighboring nuclei (e.g. Hf) in their pathways to produce heavier-mass isotopes in the Universe

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Thank You

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PhD Candidate

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