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## Introduction

Recently, it has been shown that nuclei having particle-hole symmetry reveal similar excitation patterns up to $10^{+}$in yrast bands (Sayğı, 2018a; Sayğı,2018b; Sayğ 2019). In this nuclei, almost overlay of the $\gamma$-ray transitions from equal spin and parity states $\left(\mathrm{E}\left(2^{+}\right), \mathrm{E} 4^{+} / \mathrm{E} 2^{+}\right)$ and their related reduced transition probabilities (B(E2)) with equal $\mathrm{N}_{\pi} \mathrm{N}_{\mathrm{v}}$ number are remarkable.

The $\mathrm{N}_{\mathrm{p}} \mathrm{N}_{\mathrm{n}}\left(=4 \mathrm{~N}_{\pi} \mathrm{N}_{v}\right)$ quantity is an indicator of the symmetry and depends on valence nucleon (or holes) number from the nearest closed shell by considering the subshell closures, which simplifies the systematics of excitation energies and related $\mathrm{B}\left(\mathrm{E} 2 ; \mathrm{L}^{+} \rightarrow(\mathrm{L}-)^{+}\right)$values of excited states in nuclei placed in the different mass regions of nuclear chart (Casten, 1985; Casten \& Zamfir, 1999). The Pseudo-Mirror Nuclei (PMN) concept is based on the $\mathrm{N}_{\pi} \mathrm{N}_{v}$ quantity (Moscrop et al., 1988). In the scheme of pseudo-mirror nuclei, if two nuclei possess the same $\mathrm{N}_{\pi} \mathrm{N}_{v}$ quantity, then they are expected to exhibit the same $\mathrm{E}_{4}^{+} / \mathrm{E}_{2}^{+}$therefore similar nuclear structures.

Motivations of the present work are to figure out what the range of symmetrical behaviour PMN couple does extend and how reduced transtision probabilities of high-energy transitions do behave in PMN couple? In order to answer these two questions, we are aiming to investigate the dipole excitation $(2-20 \mathrm{MeV})$ modes in ${ }^{162} \mathrm{Er}$ and ${ }^{166} \mathrm{Yb}$. These features have not been investigated within this perspective so far.

The electric dipole responses of the ${ }^{162} \mathrm{Er}$ and ${ }^{166} \mathrm{Yb}$ PMN couple were studied using quasiparticle, random-phase approximation (QRPA) in $2-20 \mathrm{MeV}$ energy interval. The excitation energies, transition probabilities, splitting behaviors of E1`s into $\mathrm{K}=0$ and $\mathrm{K}=1$, and photoabsorbtion cross-section were investigated. The nuclear model we adopted for our calculation was phenomenological QRPA, which takes into account the pairing effect and where the effective separable interactions are chosen as restoring the broken translational and Galilean symmetries. Microscopic models based on the QRPA are well known for their accuracy at describing collective modes, such as giant resonances (Giai, 1983), so the main limit of our model is it not being fully self-consistent. The QRPA model employed here has already succeeded at describing low-lying collective states in well-deformed (Repko, 2017; Tabar et al., 2020; Demirci Saygı et al., 2020), transitional, and $\gamma$-soft nuclei Guliyev et al., 2002; Linneman et al., 2003).

## 1. Same $\mathbf{N}_{\mathrm{p}} \mathbf{N}_{\mathrm{n}}$ Number <br> 1. $\mathrm{S}_{\mathrm{p}}$

## ${ }^{162}$ Er- ${ }^{166} \mathbf{Y b}$ Nuclei

${ }^{162} \operatorname{Er} \quad 6894$ over 66 mid shell. So, Closer to 82 the situation with respect to 82: 14 less than 82. In this case, 14 protons hole
${ }^{166} \mathrm{Yb} \quad 7096$ over 66 . Closer to 82. the situation with respect to 82: 12 less than 82 . In this case,
12 protons hole
under the 104 middle shell. so it's closer to the 82 closed shell. the situation with respect to 82 : 12 neutrons more. In this case: 12 neutron particles
under the 104 middle shell. so it's closer to the 82 closed shell. the situation with respect to 82: 14 neutrons more. In this case: 14 neutron particles

## 2. Overlaying Level schemes



## Theory

## TGI-QRPA

The TGI-QRPA model Hamiltonian (Eq(1)) includes $\mathrm{H}_{\text {sqp }}$ single particle Hamiltonian, $\mathrm{W}_{\text {dip }}$ isovector part of the dipole-dipole ( $\lambda=1$ ) interaction, $\mathrm{h}_{0}$ and $\mathrm{h}_{\Delta}$ separable effective interactions which restoring the broken Translational and Galilean symmetry of single particle Hamiltonian (Pyatov, Salamov, 1977) and these statements are given in following equations

$$
\begin{gathered}
H=H_{s q p}+h_{0}+h_{\Delta}+W_{d i p} \\
H_{s q p}=\sum_{q q^{\prime}} \varepsilon_{s}(\tau) B_{q q}(\tau) \quad h_{\Delta}=-\frac{1}{2 \beta} \sum_{\mu}\left[U_{\Delta}, R_{\mu}\right]^{+}\left[U_{\Delta}, R_{\mu}\right] \\
W_{d i p}=\frac{3}{2 \pi} \chi_{1}\left(\frac{N Z}{A}\right)^{2}\left(\vec{R}_{n}-\vec{R}_{p}\right)^{2} \quad h_{0}=-\frac{1}{2 \gamma} \sum_{\mu}\left[H_{s q p}, P_{\mu}\right]^{+}\left[H_{s q p}, P_{\mu}\right] \\
\text { ere, } \varepsilon_{s}(\tau)=\sqrt{\Delta_{\tau}^{2}+\left(E_{s}(\tau)-\lambda_{\tau}\right)^{2}} \text { is the quasiparticle energy of a nucleon where is th }
\end{gathered}
$$

is the quasiparticle energy of a nucleon where is the

## Here

 BCS paring energy gap, plays the role of the chemical potential which physically is the Fermi level energy. The wave function with a single phonon defined for electrical dipole excitation is determined.$$
\left|\Psi_{i}\right\rangle=Q_{i}^{+}\left|\Psi_{0}\right\rangle=\sum_{s s^{\prime}, \tau}\left[\psi_{s s^{\prime}}^{i}(\tau) A_{s s^{\prime}}^{+}(\tau)-\varphi_{s s^{\prime}}^{i}(\tau) A_{s s^{\prime}}(\tau)\right]\left|\Psi_{0}\right\rangle
$$

The motion equation solution technique used to solve the TGI-QRPA Hamiltonian is used in the second quantization method. $\omega_{i}$ excitation energies and photoabsorbtion cross section are obtained and the details of this approach can be found in (Kuliev et al., 2010;Demirci Saygı et. al, 2020).

Two characteristic excitation based on spin and parity of dipole transition:
$1^{-}=$electric dipole (E1)
$1^{+}=$magnetic dipole (M1)
Electric dipole: Center of mass for neutrons and protons oscillates against each other (macroscopically). The electric dipole (E1) modes of the nuclei are determined by the giant dipole resonance, a highstimulated collective mode above the particle emission threshold (Berman and Fultz, 1975). Electric dipole excitations (according to energy ranges): Low-lying (2-4 MeV ), Pygme Dipole Resonce PDR ( $5-8 \mathrm{MeV}$ ), Giant Dipole Resoance GDR 8-20 MeV),

## Results

 QRPA framework using a deformed Woods-Saxon potential. The neutron (proton) pairing parameters $\Delta_{\mathrm{n}}\left(\Delta_{\mathrm{p}}\right)$ and chemical potentials $\lambda_{\mathrm{n}}\left(\lambda_{\mathrm{p}}\right)$ were determined according to Soloviev's method (Soloviev, 1976), while the mean-field deformation parameters $\delta_{2}$ were calculated according to (Bohr ve Motelson, 1975) using the experimental quadrupole moments (Raman, 2001).| Nuclei | $\Delta_{\mathrm{n}}$ | $\Delta_{\mathrm{p}}$ | $\lambda_{\mathrm{n}}$ | $\lambda_{\mathrm{p}}$ | $\beta_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{162} \mathrm{Er}$ | 1.07 | 1.09 | -8.327 | -5.559 | 0.3222 |
| ${ }^{166} \mathrm{Yb}$. | 1.01 | 0.98 | -8.504 | -4.923 | 0.315 |



Conclusions
Based on our calculations, the following results were obtained.
1- Our calculation shows that the splitting of $K=0$ and $K= \pm 1$ modes due to the deformed structure of ${ }^{162} \mathrm{Er}^{-166} \mathrm{Yb}$ nuclei. (This splitting is clearly seen in the energy versus $\mathrm{B}(\mathrm{E} 1)$ and photo-absorption cross-section graphs given above).
2- E1 excitations in GDR regions (above the 8 MeV ) display a stronger collective structure than low-lying and PDR energy regions.
3. The calculation revealed that the symmetrical behavior tends to continue through highlying excitations in the pseudo mirror nuclei.
4. There are no experimental studies on the photoabsorption cross-sections of ${ }^{162} \mathrm{Er}$ and ${ }^{166} \mathrm{Yb}$. The photoabsorption cross-section graphs of the two nuclei were compared with each other as seen above. The calculated photoabsorption cross-section confirms that these two nuclei having similar structures.
5. The calculations also show that particle-hole symmetry plays a key role not only in the low-lying levels but also in high lying levels.
6. We believe that our calculations will shed light on future experimental works.
7. We recommend taking into account particle-hole symmetry and consider the PMN concept for future calculation.

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