

# Magnetic giant resonances in the relativistic energy density functional theory

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## Abstract

Magnetic dipole (M1) excitations as a fundamental mode of transitions in nuclei showed, as well, their relevance for nuclear models in astrophysics. We have established a theoretical framework based on relativistic nuclear energy density functional (RNEDF) where the nuclear ground state (g.s.) is calculated by using the relativistic Hartree-Bogoliubov (RHB) model while excitations are modelled with the relativistic quasiparticle random phase approximation (RQRPA). The RQRPA is established using density-dependent point-coupling (DD-PC1) interaction extended with isovector-pseudovector channel which describes unnatural parity transitions. Introduced RHB+RQRPA framework has been validated by M1 sum rule and employed on various nuclear systems. In case of open-shell nuclei, like <sup>50</sup>Ti, we reproduce experimentally observed double-peaked structure in reduced strength spectra. Recent experimental investigations of even-even nuclei in <sup>100–140</sup>Sn isotope chain showed an interplay between single- and double-peaked transition strength, which we have theoretically reproduced. In addition, the spin gyromagnetic factor  $g_{free}^{\sigma}$  is less quenched than in previous theoretical investigations. We have shown that pairing correlations strongly affects the M1 excitation spectra. The experimental data on M1 transitions, together with theoretical description along the isotope chain allow us to discern the pairing properties in finite fermion systems.

## Introduction

Electromagnetic excitations in finite nuclei represent one of the most important probes of relevance in nuclear structure and dynamics, as well as in nuclear astrophysics. In particular, various aspects of magnetic dipole (M1) mode have been considered due to their relevance for diverse nuclear properties associated e.g., to unnatural-parity states and spin-orbit splittings. Specifically, M1 spin-flip excitations are analog of Gamow-Teller (GT) transitions, meaning that, at the operator level, the dominant M1 isovector component is the synonym to the zeroth component of GT transitions, and can serve as probe for calculations of inelastic neutrino-nucleus cross section which is hard to measure but it is essential in supernova physics, as well as in the r-process nucleosynthesis calculations.

## Formalism

We have introduced a novel approach to describe M1, 0<sup>+</sup> ground state to 1<sup>+</sup> excited state transitions, in even-even nuclei, based on the RHB+ RQRPA framework with the relativistic point-coupling interaction [2, 4, 3] described by effective Lagrangian,

$$\mathcal{L}_{RMF} = \bar{\Psi}_N(i\gamma^\mu\partial_\mu - m_{0N})\Psi_N - \frac{1}{2}\alpha_S(\rho)(\bar{\Psi}_N\Psi_N)(\bar{\Psi}_N\Psi_N) - \frac{1}{2}\alpha_V(\rho)(\bar{\Psi}_N\gamma^\mu\Psi_N)(\bar{\Psi}_N\gamma_\mu\Psi_N) - \frac{1}{2}\alpha_{TV}(\rho)(\bar{\Psi}_N\vec{\tau}\gamma^\mu\Psi_N) \cdot (\bar{\Psi}_N\vec{\tau}\gamma_\mu\Psi_N) - \frac{1}{2}\delta_S(\partial_\nu\bar{\Psi}_N\Psi_N)(\partial^\nu\bar{\Psi}_N\Psi_N) - e\bar{\Psi}_N(\gamma^\mu A_\mu(\vec{x}))\Psi_N,$$

and the central part of the Gogny pairing correlations,

$$V^{pp}(1,2) = \sum_{i=1,2} e^{[(\vec{r}_1 - \vec{r}_2)/\mu_i]^2} (W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau), \quad (1)$$

where parameters  $\mu_i$ ,  $W_i$ ,  $B_i$ ,  $H_i$  and  $M_i$  ( $i = 1,2$ ) are adjusted by D1S set as in Ref. [7]. The  $V^{pp}$  particle-particle correlations in the RQRPA excitation channel has the same phenomenological Gogny form and parametrization as in RHB ground state.

The residual R(Q)RPA interaction has been extended by the isovector-pseudovector (IV-PV) contact type of interaction that contributes to unnatural parity transitions,

$$\mathcal{L}_{IV-PV} = -\frac{1}{2}\alpha_{IV-PV}[\bar{\Psi}_N\gamma^5\gamma^\mu\vec{\tau}\Psi_N] \cdot [\bar{\Psi}_N\gamma^5\gamma_\mu\vec{\tau}\Psi_N]. \quad (2)$$

The free, density independent,  $\alpha_{IV-PV} = 0.53$  MeV fm<sup>3</sup> parameter is adjusted to reproduce the experimental M1 response of the well studied <sup>48</sup>Ca and <sup>208</sup>Pb referent systems.

The strength function  $B(J, \omega_\nu)$  [2] in the RQRPA for magnetic operator  $\hat{\mu}_{JM}$  of rank J, in case of M1 transitions J=1, is calculated by following expression,

$$B(MJ, \omega_\nu) = \left| \sum_{\kappa\kappa'} \left( X_{\kappa\kappa'}^{\nu,J0} \langle \kappa || \hat{\mu}_J || \kappa' \rangle + (-1)^{j_\kappa - j_{\kappa'} + J} Y_{\kappa\kappa'}^{\nu,J0} \langle \kappa' || \hat{\mu}_J || \kappa \rangle \right) \times \left( u_\kappa v_{\kappa'} + (-1)^J v_\kappa u_{\kappa'} \right) \right|^2,$$

where  $\kappa$  and  $\kappa'$  are quantum numbers denoting single-particle states with  $X_{\kappa\kappa'}^{\nu,J0}$  forward and  $Y_{\kappa\kappa'}^{\nu,J0}$  backward scattering amplitudes and  $u_\kappa$ ,  $v_\kappa$  occupation coefficients.

The MJ transition operator is written in a block diagonal form,

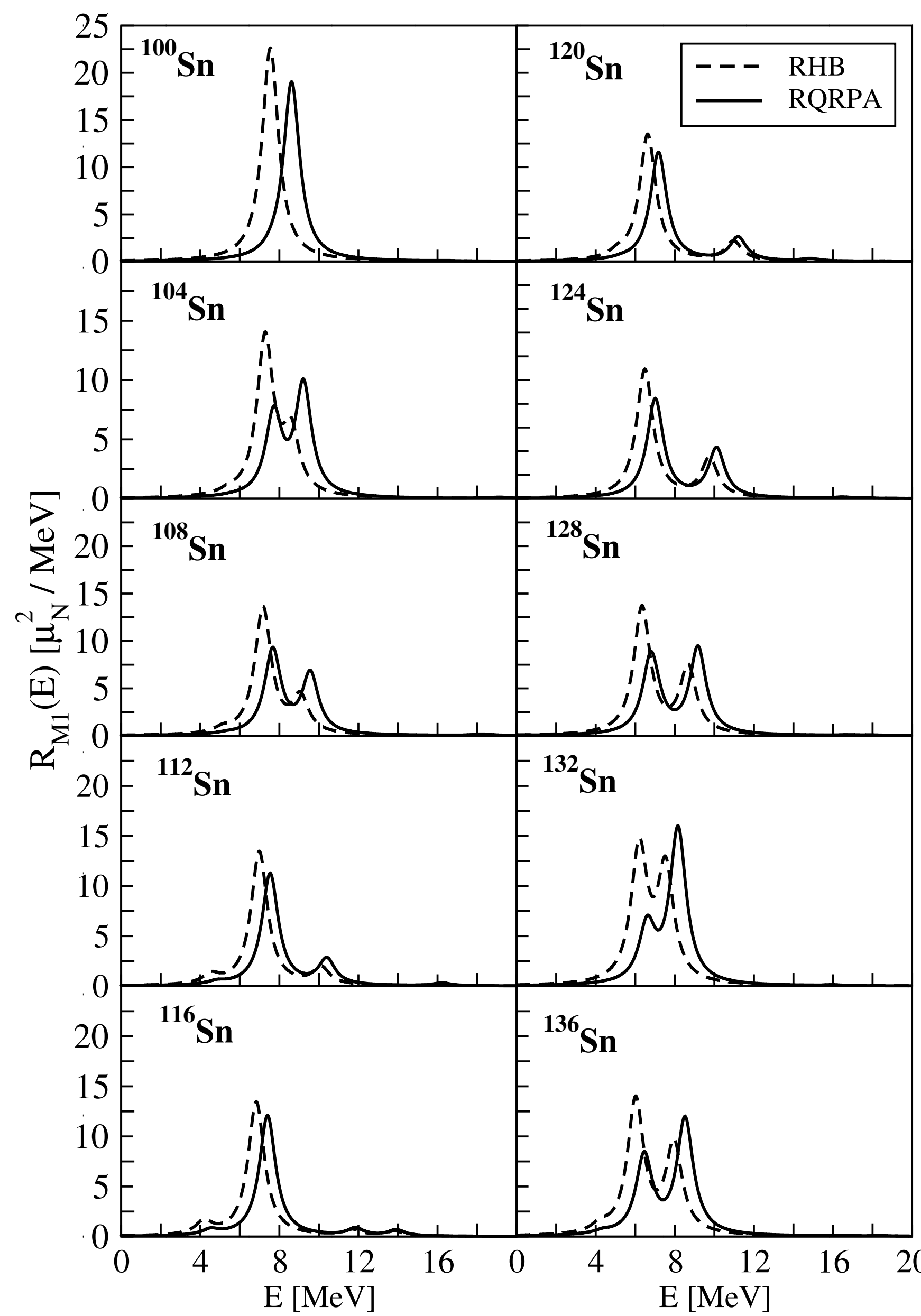
$$\hat{\mu}_{JM}^{(IS/IV)}(ii)_k = \frac{\mu_N}{\hbar} \left( \frac{2}{J+1} g_\ell^{IS/IV} \hat{\ell}_k + g_s^{IS/IV} \hat{s}_k \right) \cdot \nabla (r^J Y_{JM}(\Omega_k)), \quad (3)$$

where  $\hat{\mu}_{JM}^{(IS/IV)}(ii)_k$  are block-diagonal ( $i=1, 2$ ) isoscalar and isovector matrix elements of magnetic operator for  $k^{th}$  nucleon. Finally, a complete expression of MJ operator in the relativistic formalism which acts on Hilbert space with mixed spin-isospin basis is,

$$\hat{\mu}_{JM} = \sum_{k=1}^A \left( \hat{\mu}_{JM}^{(IS)} k \otimes 1_\tau - \hat{\mu}_{JM}^{(IV)} k \otimes \hat{\tau}_3 \right), \quad (4)$$

where  $\hat{\mu}_{JM}^{(IS/IV)} k$  are block-diagonal matrices in spin and  $\hat{\tau}_3$  is Pauli matrix in isospin space.

## Evolution of M1 strength along <sup>100–140</sup>Sn isotope chain



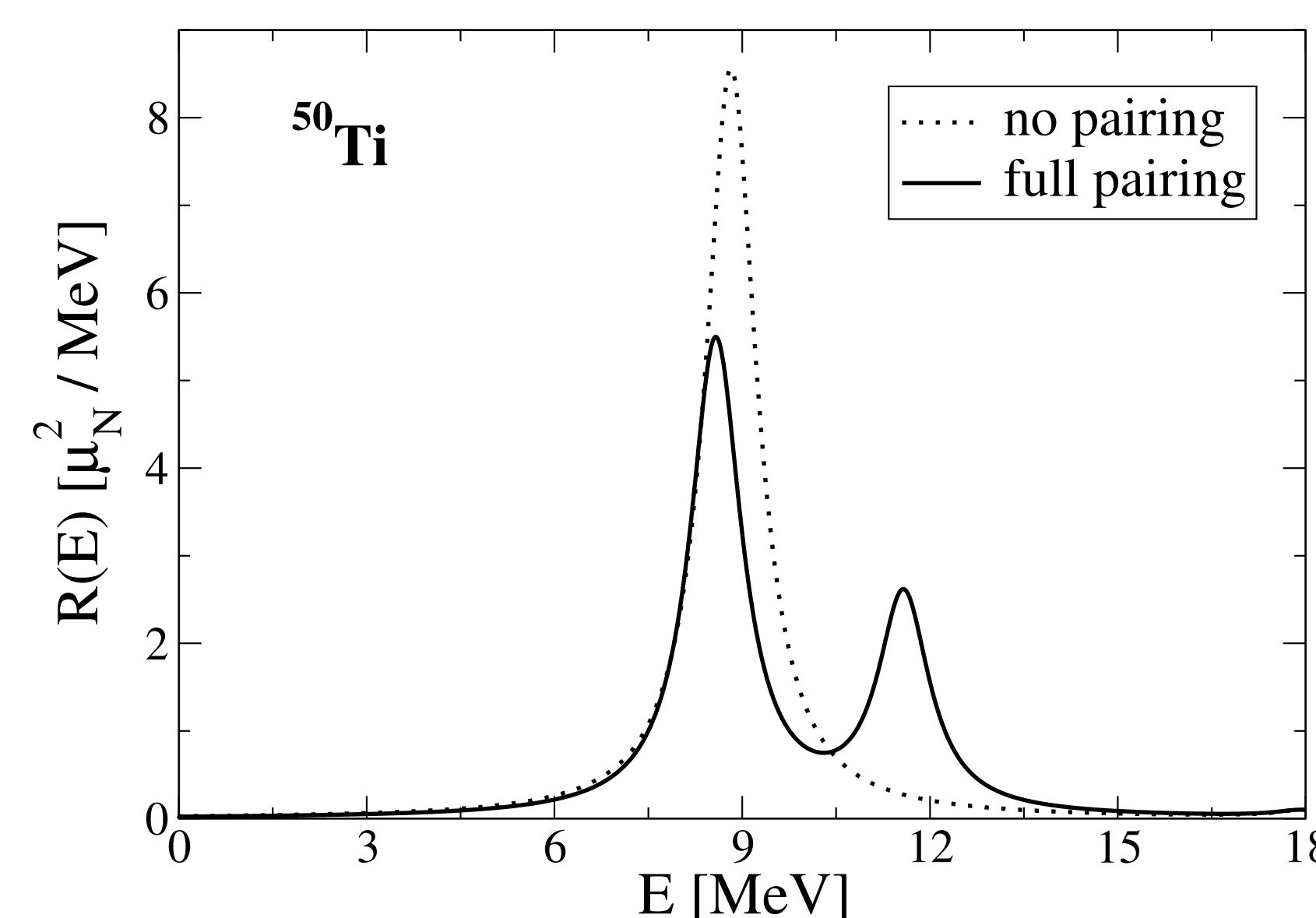
The full M1 response function  $R_{M1}(E)$  spectra reproduced one and double peaked experimental structure observed in even-even nuclei of <sup>100–140</sup>Sn isotope chain.

The spin-quenching factor,

$$\zeta_s = g_{eff}^\sigma / g_{free}^\sigma \approx \sqrt{\frac{B_{M1}^{exp.}(E)}{B_{M1}^{th.}(E)}}, \quad (5)$$

obtained by comparison of the RQRPA calculations Ref. [4] with the recent experimental data from Ref. [6] showed that  $\zeta_s = 0.80 - 0.93$  values are less quenched, closer to unit, compared to previous theoretical investigations.

## Pairing effects on M1 excitations



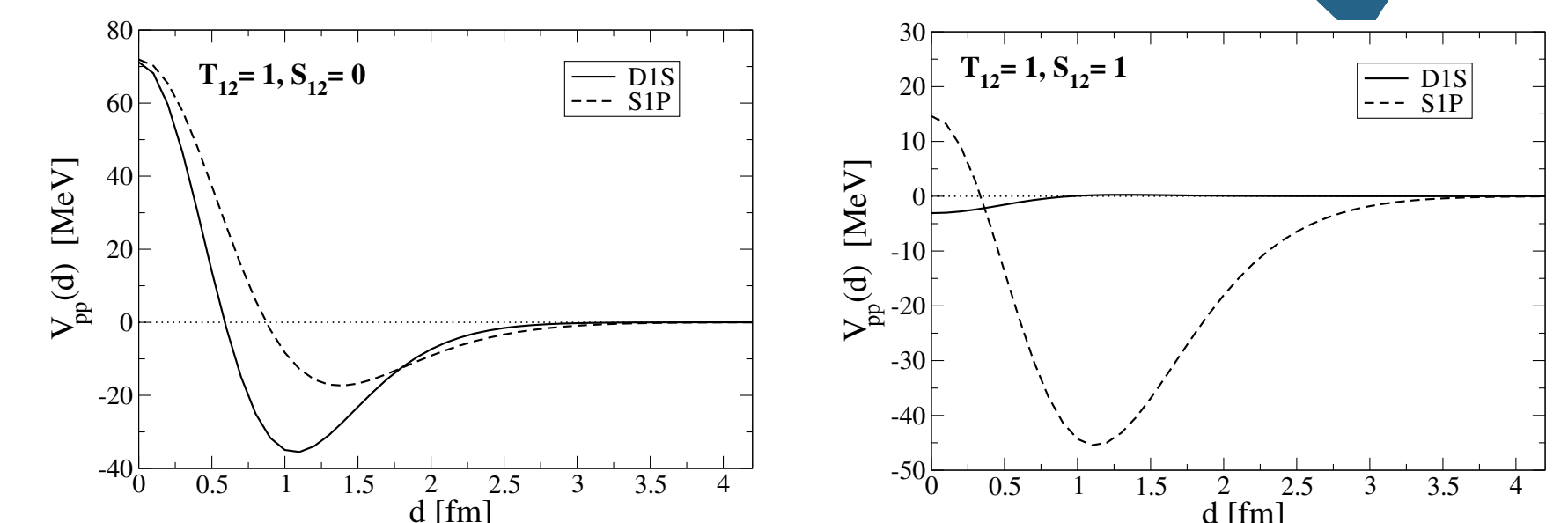
A double-peaked, experimentally observed, response function  $R_{M1}(E)$  of open-shell <sup>50</sup>Ti nuclei is reproduced by DD-PC1+D1S parametrization.

It is shown that D1S parametrization of Gogny interaction is crucial for reproduction of <sup>50</sup>Ti double-peaked response function  $R_{M1}(E)$  spectra as in Ref. [2].

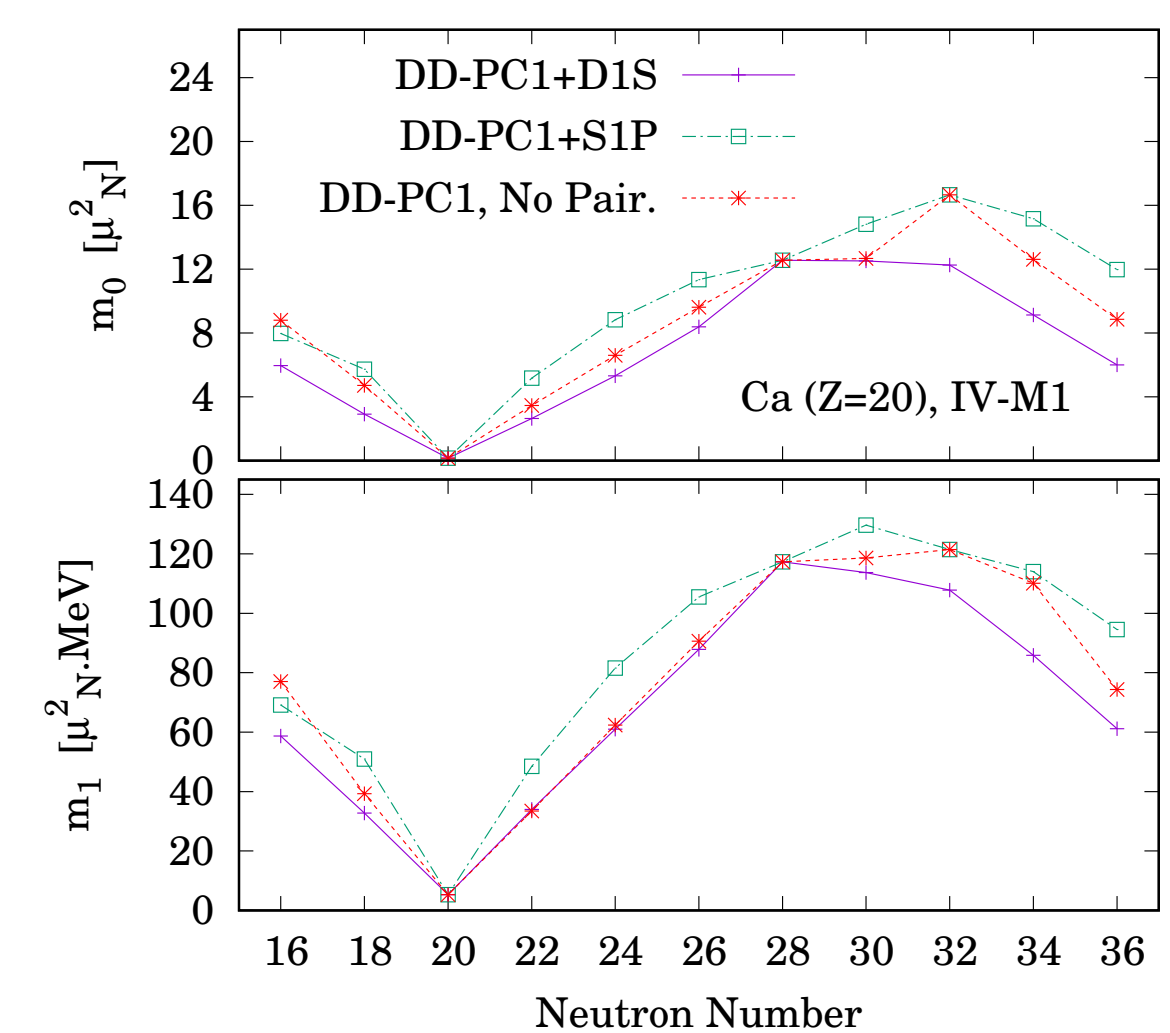
For the analysis of M1 strength spectra energy-weighted moments,

$$m_k = \sum_\nu B(MJ, \omega_\nu) (E_\nu - E_0)^k,$$

are used,  $m_0$  non-energy-weighted (NEWSR) and  $m_1$  energy-weighted (EWSR).



Gogny  $V_{pp}(d)$  pairing potential in particle-particle channel for two different D1S and S1P parametrizations as a function of distance between two nucleons and for spin-isospin ( $S_{12} - T_{12}$ ) singlet(triplet) configurations.



(Top) NEWSR summation  $m_0$  of the M1 strength for the DD-PC1+ D1S or S1P pairing and no pairing cases. (Bottom) Same for the EWSR summation  $m_1$ .

## Conclusions

We have presented a novel approach for the description of M1 transitions in even-even nuclei based on the RHB+R(Q)RPA theoretical framework with relativistic point-coupling interaction. The standard terms of point-coupling interaction with DD-PC1 parametrisation is extended with the isovector-pseudovector type of R(Q)RPA residual interaction which dominantly induces unnatural parity transitions. The single and double-peaked response function  $R_{M1}(E)$  spectra has been successfully reproduced along <sup>100–140</sup>Sn isotope chain. Considering the pairing effect in open-shell nuclei is essential for the reproduction of M1 response function as it is shown in case of <sup>50</sup>Ti. The present analysis shows that smaller quenching in the spin g-factors is needed than obtained in previous studies.

## References

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