# Erosion of $N=28$ Shell Closure in Light Nuclei <br> Pankaj Kumar ${ }^{\dagger *}$, Shashi K. Dhiman ${ }^{*}$ <br> *Department of Physics, Himachal Pradesh University Summerhill, Shimla171005, India <br> †pankajdhiman659@gmail.com 

## Introduction

The nuclei in the vicinity of closed shells are generally stable and spherical. A spherical (magic) nucleus exhibit large shell gaps between occupied and valance orbits around Fermi level that prevent any excitations. Away from shell closure, the valance nucleons can scatter into various orbitals near the Fermi surface. However, the change in energy spacing between single nucleon levels lead to reduction of spherical shell gaps and a deformed ground state may appear. $N=28$ is first magic number deriven by the spin orbit interaction which lowers the $1 f_{7 / 2}$ orbital w.r.t. $2 p_{3 / 2}$ one. The disappearence of $N$ $=28$ shell gap have been explored in many experimental and theoretical studies[1, 2, 3, 4].

## Theoretical Framework

The Lagrangian for density-dependent point coupling models includes the isoscalar-scalar, isoscalarvector and isovector-vector four-fermion contact interactions and can be written as [5, 6]

$$
\begin{align*}
\mathcal{L}= & \bar{\psi}(i \gamma \cdot \partial-m) \psi-\frac{1}{2} \alpha_{S}(\rho)(\bar{\psi} \psi)(\bar{\psi} \psi)-\frac{1}{2} \alpha_{V}(\rho)\left(\bar{\psi} \gamma^{\mu} \psi\right)\left(\bar{\psi} \gamma_{\mu} \psi\right) \\
& -\frac{1}{2} \alpha_{T V}(\rho)\left(\bar{\psi} \vec{\tau} \gamma^{\mu} \psi\right)\left(\bar{\psi} \vec{\tau} \gamma_{\mu} \psi\right)-\frac{1}{2} \delta_{S}\left(\partial_{\nu} \bar{\psi} \psi\right)\left(\partial^{\nu} \bar{\psi} \psi\right)-e \bar{\psi} \gamma \cdot \mathbf{A} \frac{1-\tau_{3}}{2} \psi, \tag{1}
\end{align*}
$$

where $m$ is the mass of nucleon, $\alpha_{S}, \alpha_{V}$ and $\alpha_{T V}$ represent the coupling constants for four-fermion contact terms. The microscopic density-dependent scalar and vector self-energies are computed by using following functional form of the couplings.

$$
\begin{equation*}
\alpha_{i}(\rho)=a_{i}+\left(b_{i}+c_{i} x\right) e^{-d_{i} x}, \quad(i=S, V, T V) \tag{2}
\end{equation*}
$$

where $x=\rho / \rho_{\text {sat }}$ denotes the nucleon density in symmetric nuclear matter at saturation point $\rho_{\text {sat }}$. The point coupling CDFT model involve 10 parameters, $\left(a_{S}, b_{S}, c_{S}, d_{S}, a_{V}, b_{V}, d_{V}, b_{T V}, d_{T V}, \delta_{S}\right)$, are given in Ref.[6].
It is necessary to consider pairing correlations for a quantitative description of open-shell nuclei[7, 8]. In the CDFT framework with pairing correlations, the density matrix can be generalized into two densities, the normal density $\hat{\rho}$, and pairing density $\hat{\kappa}$. The relativistic Hartree-Bogoliubov energy density functional can be written as[7]

$$
\begin{equation*}
\mathcal{E}_{R H B}[\hat{\rho}, \hat{\kappa}]=\mathcal{E}_{R M F}[\hat{\rho}]+\mathcal{E}_{\text {pair }}[\hat{\kappa}] . \tag{3}
\end{equation*}
$$

The energies $\mathcal{E}_{R M F}[\hat{\rho}]$ and $\mathcal{E}_{\text {pair }}[\kappa]$ are given by
$\mathcal{E}_{R M F}\left[\psi, \bar{\psi}, \sigma, \omega^{\mu}, \vec{\rho}^{\mu}, A^{\mu}\right]=\int d^{3} r \mathcal{H}(r) \& \mathcal{E}_{\text {pair }}[\hat{\kappa}]=\frac{1}{4} \sum_{n_{1} n_{1}^{\prime}} \sum_{n_{2} n_{2}^{\prime}} \kappa_{n_{1} n_{1}^{\prime}}^{*}\left\langle n_{1} n_{1}^{\prime}\right| V^{P P}\left|n_{2} n_{2}^{\prime}\right\rangle \kappa_{n_{2} n_{2}^{\prime}}$,
The total energy $E_{t o t}(\mathrm{MeV})$ for the nuclear system with $A$ nucleons can be calculated as

$$
\begin{equation*}
E_{t o t}=\mathcal{E}_{R M F}+\mathcal{E}_{\text {pair }}+E_{\text {c.m. }} \tag{4}
\end{equation*}
$$

Here, $E_{c . m \text {. }}$ accounts for the center-of-mass correction is given as $E_{c . m .}=-\frac{\left\langle\mathbf{P}^{2}\right\rangle}{2 A m}$.

## Results

Fig. 1 presents the 2D contour plots of potential energy surfaces (PESs) in $\beta_{2}-\gamma$ plane, calculated using DD-PCX interaction. The evolution of shapes and fragility of $N=28$ shell gap can be observed in binding energy maps. A well deformed minima have been observed for $N=28$ isotones.

| Nucleus | $\beta_{2}$ | $\gamma$ |
| :--- | :--- | :--- |
| ${ }^{40} \mathrm{Mg}$ | 0.45 | $0^{\circ}$ |
| ${ }^{42} \mathrm{Si}$ | 0.35 | $60^{\circ}$ |
| ${ }^{44} \mathrm{~S}$ | 0.34 | $0^{\circ}$ |
| ${ }^{46} \mathrm{Ar}$ | 0.19 | $60^{\circ}$ |
| ${ }^{48} \mathrm{Ca}$ | 0 | $0^{\circ}$ |

Table 1: The location of absolute minima $\left(\beta_{2}, \gamma\right)$ of the potential energy surfaces for $N=28$ isotones. In case of ${ }^{40} \mathrm{Mg}$, the gorund state minima is observed along prolate side with quadrupole deformation parameter $\beta_{2}=0.45$. While an oblate shape can be depicted for ${ }^{42} \mathrm{Si}$. An interesting case is seen for ${ }^{44} \mathrm{~S}$ which exhibits an oblate-prolate coexistence with prolate minima as deepest solution. An oblate ground-state minima is observed for ${ }^{46} \mathrm{Ar}$ with a $\gamma$-soft behavior. The spherical shape is restored in case of doubly magic ${ }^{48} \mathrm{Ca}$ nucleus. The values of absolute minima for $N=28$ isotones are given in Table 1
The origin of nuclear shapes is governed by evolution of the shell structure of single-particle orbitals. The ground-state minima in the potential energy curve are associated with the effect of low-level density around the Fermi surface. Fig. 2 displays the neutron and proton single-particle energy levels for ${ }^{40} \mathrm{Mg},{ }^{42} \mathrm{Si},{ }^{44} \mathrm{~S}$, and ${ }^{46} \mathrm{Ar}$, respectively. In this figure, the solid (violet) lines correspond to levels with positive parity and dashed (red) lines denote the negative parity levels. The Fermi level is shown by thick black line. The neutron and proton single-particle levels are plotted as a function of deformation parameters along closed paths in $\beta_{2}-\gamma$ plane.


Figure 1: The potential energy surfaces (PESs) of $N=28$ isotones in the $\beta_{2}-\gamma$ plane calculated using CDFT with DD-PCX interaction. The energies are normalized with respect to the binding energy of the absolute minima

Table 2 presents the theoretically calculated neutron $N=28$ spherical shell gaps, and the corresponding quadrupole deformation parameter for the minima of potential energy curves of ${ }^{40} \mathrm{Mg},{ }^{42} \mathrm{Si},{ }^{44} \mathrm{~S}$, ${ }^{46} \mathrm{Ar}$, and ${ }^{48} \mathrm{Ca}$. A clear reduction of the spherical $N=28$ shell gap is seen as one moves towards the proton-deficient side.

| Nucleus | $\Delta_{N=28}^{s p h}$ | $\left(\beta_{2}, \gamma\right)$ |
| :--- | :--- | :--- |
| ${ }^{40} \mathrm{Mg}$ | 1.725 | $\left(0.45,0^{\circ}\right)$ |
| ${ }^{42} \mathrm{Si}$ | 2.871 | $\left(0.35,60^{\circ}\right)$ |
| ${ }^{44} \mathrm{~S}$ | 3.521 | $\left(0.34,0^{\circ}\right)$ |
| ${ }^{46} \mathrm{Ar}$ | 4.319 | $\left(0.19,60^{\circ}\right)$ |
| ${ }^{48} \mathrm{Ca}$ | 4.704 | $\left(0,0^{\circ}\right)$ |

Table 2: The calculated neutron $N=28$ spherical energy gaps, and the corresponding values of ground-state minima of the quadrupole binding energy maps of ${ }^{40} \mathrm{Mg},{ }^{42} \mathrm{Si},{ }^{44} \mathrm{~S},{ }^{46} \mathrm{Ar}$, and ${ }^{48} \mathrm{Ca}$.


Figure 2: Neutron and proton single-particle energy levels of ${ }^{40} \mathrm{Mg},{ }^{42} \mathrm{Si}$, ${ }^{44} \mathrm{~S}$, and ${ }^{46} \mathrm{Ar}$ as functions of the deformation parameters along closed paths in the $\beta_{2}-\gamma$ plane

## Conclusions

- It has been observed from that the isotones of $N=28$ magic number show a well-deformed mini mum rather than spherical minima.
- The evolution of shell structures of single-particle orbitals are seen to play an important role for the occurrence of deformation in a perticular nuclear chain.
- The theory and experiment, both points toward a strong disruption of the $N=28$ spherical gaps as the isotones become more neutron-rich.
- The density functional DD-PCX is found efficient and successful for the description of light nuclei.


## References

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