Davydov-Chaban Hamiltonian within the formalism of deformation-dependent mass for Kratzer potential

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1. Abstract

In this work, we modify the Davydov-Chaban Hamiltonian describing the collective motion of a γ-rigid atomic nucleus by allowing the mass to depend on nuclear deformation. Exact analytical expressions are derived for energy spectra as well as normalized wave functions for Kratzer potential. The model called Z(4)-DDM (Deformation Dependent Mass), is achieved by using the Asymptotic Iteration Method (AIM). The numerical calculations for energy spectra and B(E2)transition probabilities are compared to the experimental data of $^{192-196}$Pt isotopes.

2. Formulation of the model

In the model of Davydov-Chaban, the Hamiltonian operator with deformation dependent mass can be written as [1, 2]

$$\left[-\frac{\sqrt{f}}{\beta^3}\frac{\partial}{\partial\beta}\beta^3 f\frac{\partial}{\partial\beta}\sqrt{f}+\frac{f^2}{4\beta^2}\sum_{k=1,2,3}\frac{Q_k^2}{\sin^2(\gamma-\frac{2}{3}\pi k)}\right]\Psi(\beta,\Omega) + v_{eff}\Psi(\beta,\Omega)=\epsilon\Psi(\beta,\Omega), \quad (1)$$

where β is the collective coordinate and γ a parameter, while Q_k are the components of angular momentum in the intrinsic frame. The effective potential is given by

$$v_{eff}=v(\beta)+\frac{1}{4}(1-\delta-\lambda)f\bigtriangledown^2 f+\frac{1}{2}(\frac{1}{2}-\delta)(\frac{1}{2}-\lambda)(\bigtriangledown f)^2, \quad (2)$$

where δ and λ are free parameters originated from the construction procedure of the kinetic term within DDM formalism, while, the reduced energies and potentials are defined as $\epsilon=\frac{B_0}{\hbar^2}E$, $v(\beta)=\frac{B_0}{\hbar^2}V(\beta)$, respectively.

▶ The Kratzer potential [3]:

$$v(\beta)=-\frac{1}{\beta}+\frac{\beta_0}{2\beta^2}, \quad (3)$$

↪β_0 : The position of the minimum of the potential.

▶ According to specific form of the potential, choose the deformation function in the following special form

$$f(\beta)=1+a\beta, \qquad a<<1. \quad (4)$$

↪ a : free parameter

▶ The wave functions can be written as :

$$\Psi(\beta,\Omega)=\chi(\beta)\phi(\Omega). \quad (5)$$

▶ The separation of variables leads to :

$$\left[-\frac{\sqrt{f}}{\beta^3}\frac{\partial}{\partial\beta}\beta^3 f\frac{\partial}{\partial\beta}\sqrt{f}+\frac{f^2}{\beta^2}\Lambda+2v_{eff}\right]\chi(\beta)=2\epsilon\chi(\beta), \quad (6)$$

$$\left[\frac{1}{4}\sum_{k=1,2,3}\frac{Q_k^2}{\sin^2(\gamma-\frac{2}{3}\pi k)}-\Lambda\right]\phi(\Omega)=0, \quad (7)$$

where Λ is the separation constant and $\epsilon=2BE/\hbar^2$.
The above equation has been solved by Meyer-ter-Vehn [4] with the results

$$\Lambda=L(L+1)-\frac{3}{4}\alpha^2, \quad (8)$$

$$\phi^L_{\mu,\alpha}(\Omega)=\sqrt{\frac{2L+1}{16\pi^2(1+\delta_{\alpha,0})}}\left[\mathcal{D}^{(L)}_{\mu,\alpha}+(-1)^L\mathcal{D}^{(L)}_{\mu,-\alpha}\right], \quad (9)$$

where $\mathcal{D}(\Omega)$ denotes Wigner functions of the Euler angles, L is the total angular momentum quantum number, μ and α are the quantum numbers of the projections of angular momentum on the laboratory fixed z-axis and the body-fixed x'-axis, respectively.

3. The energy spectrum

▶ Solving the radial equation Eq. (6) through the AIM, we obtain the radial energy eigenvalues

$$\epsilon_{n_\beta n_w L}=\frac{1}{2}\left[k_0+\frac{a^2}{4}-\left(\frac{k_{-1}+an_\beta^2+a\eta(1+2n_\beta)}{2(\eta+n_\beta)}\right)^2\right]. \quad (10)$$

where n_β is the principal quantum number of β vibrations.

$$\begin{aligned}k_0 &=\frac{a^2}{2}\left[4+2\Lambda+2(\frac{1}{2}-\delta)(\frac{1}{2}-\lambda)+3(1-\delta-\lambda)\right],\\ k_{-1} &=a[3+2\Lambda+\frac{3}{2}(1-\lambda-\delta)]-2,\\ k_{-2} &=\frac{3}{4}+\Lambda+\beta_0. \end{aligned} \quad (11)$$

▶ The bands in the present model are classified by the quantum numbers n, n_γ and $n_w=L-\alpha$:
↪The ground state band (gsb) : $n=n_\gamma=n_w=0$,
↪The β band : $n=1, n_\gamma=0, n_w=0$,
↪The γ band : $n=n_\gamma=0$, if L even $n_w=2$ and if L odd with $n_w=1$.

4. The radial wave functions

▶ The radial functions $R_{n_\beta L}(\beta)$ of Eq. (6) are given by

$$R_{n_\beta L}(\beta)=C_{n_\beta L}\,a^{-\eta}\,2^{n_\beta+\kappa_n}\,(1+t)^\eta\,(1-t)^{-\kappa_n-\eta-n_\beta} \times P_{n_\beta}^{(-1-2(n_\beta+\kappa_n+\eta),2\eta-1)}(t), \quad (12)$$

with:
$\hookrightarrow t=\frac{-1+a\beta}{1+a\beta}$, $\quad\hookrightarrow \eta=\frac{1}{2}(1+\sqrt{1+4k_{-2}})$,
$\hookrightarrow \kappa_n=-\frac{1}{2}\left(1+n_\beta+\eta+\frac{k_{-2}-\frac{k_{-1}}{a}}{n_\beta+\eta}\right)$.

▶ To determine $C_{n_\beta L}$, we use the usual orthogonality relation of Jacobi polynomials, obtaining

$$C_{n_\beta L}=\left[\frac{a^{1+2\eta}}{2}\,\frac{(1+2n_\beta+2\eta+2\kappa_n)(1+2\kappa_n)}{(\eta+n_\beta)}\right]^{\frac{1}{2}} \times\left[\frac{\Gamma(1+n_\beta+2\kappa_n)\Gamma(n_\beta+1)}{\Gamma(n_\beta+2\eta+2\kappa_n)\Gamma(n_\beta+2\eta)}\right]^{\frac{1}{2}}. \quad (13)$$

5. Numerical results

The Z(4) DDM-K model presented in the previous sections has been applied for calculating the energies of the collective states, the reduced $E2$ transition probabilities and the staggering of the γ-band for the $^{192,194,196}Pt$ isotopes.

▶ **Energies of the collective states** :
In this work the theoretical predictions for the levels Eq. (10) are treated equally, depending on two parameters, namely: a the deformation mass parameter and β_0 the minimum of the potential. These parameters are adjusted to reproduce the experimental data by applying a least-squares fitting procedure for each considered isotope.
We evaluate the root mean square (rms) deviation between the theoretical values and the experimental data by

$$\sigma=\sqrt{\frac{\sum_{i=1}^m(E_i(exp)-E_i(th))^2}{(m-1)E(2_1^+)^2}}, \quad (14)$$

where m denotes the number of states, while $E_i(exp)$ and $E_i(th)$ represent the theoretical and experimental energies of the i^{th} level, respectively.

↪ In Table1, we see that the obtained results for the levels belonging to the ground state, β, and γ-bands are in a quite satisfactory agreement with the corresponding experimental data.

	^{192}Pt		^{194}Pt		^{196}Pt	
L	exp	th	exp	th	exp	th
	gsb	gsb	gsb	gsb	gsb	gsb
4	2.479	2.451	2.470	2.506	2.465	2.455
6	4.314	4.129	4.298	4.334	4.290	4.144
8	6.377	5.844	6.392	6.306	6.333	5.877
10	8.624	7.472	8.672	8.280	8.558	7.528
	β_1	β_1	β_1	β_1	β_1	β_1
0	3.776	3.768	3.858	3.806	3.192	3.124
2	4.547	4.472	4.603	4.555	3.828	3.844
	γ_1	γ_1	γ_1	γ_1	γ_1	γ_1
2	1.935	1.900	1.894	1.926	1.936	1.902
3	2.910	2.714	2.809	2.786	2.854	2.719
4	3.795	4.548	3.743	4.806	3.636	4.566
5	4.682	4.748	4.563	5.034	4.526	4.769
6	5.905	6.785		7.434	5.644	6.830
7	6.677	6.637		7.255		6.681
σ		0.500		0.390		0.576
β_0		50.0		70.6		51.0
a		0.002		0.004		0.006

Table 1: Comparison of the present model for energy levels to experimental data [5].

▶ **Electric quadrupole transitions** :

Similarly, we have calculated the intraband and interband $B(E2)$ transition rates, normalized to the $B(E2;2_g^+\longrightarrow 0_g^+)$ rate. For triaxial nuclei around $\gamma\approx\pi/6$, the quadrupole operator becomes

$$T^{(E2)}_M=t\beta\frac{1}{\sqrt{2}}\left(\mathcal{D}^{(2)}_{M,2}(\theta_i)+\mathcal{D}^{(-2)}_{M,-2}(\theta_i)\right) \quad (15)$$

The $B(E2)$ rates from an initial to a final state are given by

$$B(E2;L_i\alpha_i\rightarrow L_f\alpha_f)=\frac{5}{16\pi}\frac{|\left\langle L_f\alpha_f\,||\,T^{(E2)}\,||\,L_i\alpha_i\right\rangle|^2}{(2L_i+1)} \quad (16)$$

$\frac{L^{(i)}\rightarrow L^{(f)}}{2_g\rightarrow 0_g}$	^{192}Pt		^{194}Pt		^{196}Pt	
	exp	th	exp	th	exp	th
$4_g\rightarrow 2_g$	1.56	1.58	1.73	1.56	1.48	1.63
$6_g\rightarrow 4_g$	1.23	2.38	1.36	2.31	1.80	2.58
$8_g\rightarrow 6_g$		3.32	1.02	3.19	1.92	3.86
$10_g\rightarrow 8_g$		4.61	0.69	4.41		5.91
$2_\gamma\rightarrow 2_g$	1.91	1.60	1.81	1.58		1.65
$2_\gamma\rightarrow 0_g\times 10^3$	9.5	0.0	5.9	0.0	0.4	0.0
$0_\beta\rightarrow 2_g$		0.70	0.01	0.67	0.07	0.9
$2_\beta\rightarrow 0_g\times 10^3$		17.36		25.08	0.06	23.16

Table 2: Comparison of the present model for $B(E2)$ transition rates to experimental data [5].

↪ In Table 2, we compare our theoretical calculations with the available experimental data. The overall agreement is good for transitions within the ground state band with exception of the higher L levels. In what concerns the interband transitions rates between the γ to the g.s. band, our theoretical calculations give good results, while interband transitions from β band to g.s. band tend to be overpredicted.

▶ **The staggering of the γ-band** :
An other sensitive signature for triaxiality structure, which has to be studied, is obviously the odd-even staggering of the level energies within the γ-band, described by the following quantity : $S(J)=[E(J_\gamma^+)+E((J-2)_\gamma^+)-2E((J-1)_\gamma^+)]/E(2_1^+)$.

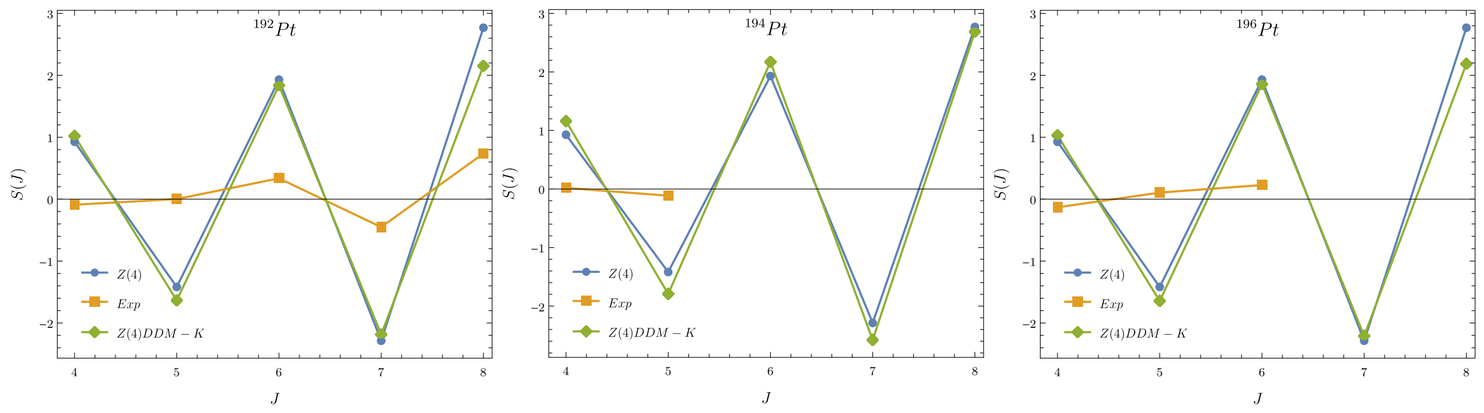

Figure 1: The staggering behavior $S(J)$ of $^{192,194,196}Pt$ (Exp) [5] compared with Z(4) DDM-K and Z(4) [6] models.

↪ In Figure 1, we plotted the function $S(J)$ for the nuclei considered here. As it is shown, the staggering of ^{192}Pt and ^{194}Pt isotopes is well reproduced by Z(4) DDM-K, while for ^{196}Pt the agreement is not good.

6. Conclusion

▶The solution called Z(4) DDM-K, is achieved by means of the Asymptotic Iteration Method. Analytical expressions for the spectra and wave functions have been obtained.
▶ Energy spectra and $B(E2)$ transitions have been calculated for triaxial nuclei and then these have been compared with experimental data.
▶The predicted energy spectra and $B(E2)$ values are in good agreement with the experimental data for $^{192-196}$Pt.
▶ We have investigated the role of Kratzer potential in avoiding the level spacings within the β bands.

7. References

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