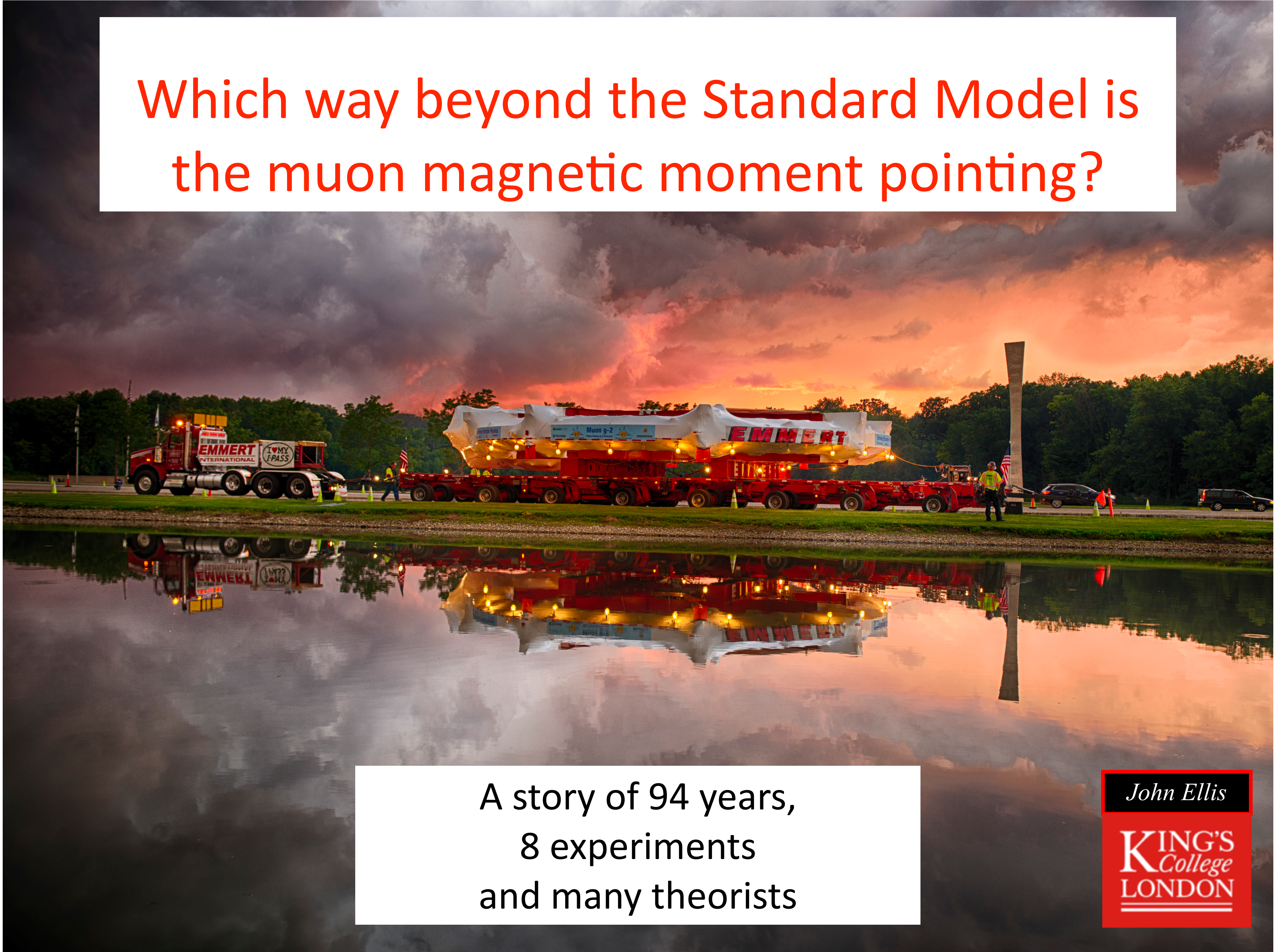
Which way beyond the Standard Model is the muon magnetic moment pointing?



A story of 94 years,
8 experiments
and many theorists

John Ellis
KING'S College LONDON

It began with Dirac …

- Two fundamental papers:
    - "The quantum theory of the electron" (1928)

        The Dirac equation: $(i\gamma_\mu \partial^\mu - m)\psi = 0$ predicts that the electron's magnetic moment $g = 2$

        "That was really an unexpected bonus for me, completely unexpected"

    - "The Quantum Theory of the Emission and Absorption of Radiation" (1927)

        The basis for QED (and all of quantum field theory) enables the calculations of the anomaly: $g \neq 2$

The Quantum Theory of the Electron.

By P. A. M. Dirac, St. John's College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received January 2, 1928.)

The new quantum mechanics, when applied to the problem of the structure of the atom with point-charge electrons, does not give results in agreement with experiment. The discrepancies consist of "duplexity" phenomena, the observed number of stationary states for an electron in an atom being twice the number given by the theory. To meet the difficulty, Goudsmit and Uhlenbeck have introduced the idea of an electron with a spin angular momentum of half a quantum and a magnetic moment of one Bohr magneton. This model for the electron has been fitted into the new mechanics by Pauli,* and Darwin,† working with an equivalent theory, has shown that it gives results in agreement with experiment for hydrogen-like spectra to the first order of accuracy.

The question remains as to why Nature should have chosen this particular model for the electron instead of being satisfied with the point-charge. One would like to find some incompleteness in the previous methods of applying quantum mechanics to the point-charge electron such that, when removed, the whole of the duplexity phenomena follow without arbitrary assumptions. In the present paper it is shown that this is the case, the incompleteness of the previous theories lying in their disagreement with relativity, or, alternatetively, with the general transformation theory of quantum mechanics. It appears that the simplest Hamiltonian for a point-charge electron satisfying the requirements of both relativity and the general transformation theory leads to an explanation of all duplexity phenomena without further assumption. All the same there is a great deal of truth in the spinning electron model, at least as a first approximation. The most im...
to be that the magnitude of the resultant ...
electron moving in an orbit in a central fiel...
model leads one to expect.

* Pauli, 'Z. f. Physik,' vol. 43, p. 601 (1927).
† Darwin, 'Roy. Soc. Proc.,' A, vol. 116, p. 227 (1927).

$$\left[p_0 + \frac{e}{c}A_0 + \rho_1\left(\sigma, p + \frac{e}{c}A\right) + \rho_3 mc\right]\psi = 0$$

(2) Observations have been made of the electric fields and field changes associated with 18 distant and 5 near thunderstorms. The sudden changes of field due to distant lightning discharges (> 8 km.) were predominantly negative in sign, those due to near discharges (< 6 km.) predominantly positive. The relative frequencies of positive and negative changes were 1 : 5 in the former case and 4·3 : 1 in the latter. The steady electric fields below the 5 near storms were all strongly negative.

(3) It is shown that these results indicate that the thunderclouds were bi-polar in nature and that the polarity was generally, if not always, positive, the upper pole being positive and the lower pole negative. It is doubtful if any active storms of opposite polarity were observed at all.

(4) The electric moments of the charges removed by 82 lightning discharges have been measured. The mean value is 94 coulomb-kilometres.

The Quantum Theory of the Emission and Absorption of Radiation.

By P. A. M. Dirac, St. John's College, Cambridge, and Institute for Theoretical Physics, Copenhagen.

(Communicated by N. Bohr, For. Mem. R.S.—Received February 2, 1927.)

§ 1. *Introduction and Summary.*

The new quantum theory, based on the assumption that the dynamical variables do not obey the commutative law of multiplication, has by now been developed sufficiently to form a fairly complete theory of dynamics. One can treat mathematically the problem of any dynamical system composed of a number of particles with instantaneous forces acting between them, provided it is describable by a Hamiltonian function, and one can interpret the mathematics physically by a quite definite general method. On the other hand, hardly anything has been done up to the present on quantum electrodynamics. The
... system in which the forces are propa-
...f instantaneously, of the production of
...ctron, and of the reaction of this field
...hed. In addition, there is a serious
difficulty in making the theory satisfy all the requirements of the restricted

… and then Schwinger

- First calculation of leading-order contribution to $g - 2$ in QED: $\frac{\alpha}{2\pi}$ (1947)

- Inscribed on his tombstone

[1] Walke, Thompson, and Holt, Phys. Rev. **57**, 171 (1940).
[2] Solomon, Gould, and Anfinsen, Phys. Rev. **72**, 1097 (1947).
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[4] Glendenin, Nucleonics, in press for January, 1948.
[5] Marshall and Ward, Can. J. Research **15**, 29 (1939).
[6] This result is in good agreement with a value of 250 kev, given in *Radioisotopes, Catalog and Price List No. 2*, revised September, 1947, distributed by Isotopes Branch, United States Atomic Energy Commission. Unfortunately, the Atomic Energy Commission's result is not supported by any published experimental evidence.

On Quantum-Electrodynamics and the Magnetic Moment of the Electron

JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts
December 30, 1947

ATTEMPTS to evaluate radiative corrections to electron phenomena have heretofore been beset by divergence difficulties, attributable to self-energy and vacuum polarization effects. Electrodynamics unquestionably requires revision at ultra-relativistic energies, but is presumably accurate at moderate relativistic energies. It would be desirable, therefore, to isolate those aspects of the current theory that essentially involve high energies, and are subject to modification by a more satisfactory theory, from aspects that involve only moderate energies and are thus relatively trustworthy. This goal has been achieved by transforming the Hamiltonian of current hole theory electrodynamics to exhibit explicitly the logarithmically divergent self-energy of a free electron, which arises from the virtual emission and absorption of light quanta. The electromagnetic self-energy of a free electron can be ascribed to an electromagnetic mass, which must be added to the mechanical mass of the electron. Indeed, the only meaningful statements of the theory involve this combination of masses, which is the experimental mass of a free electron. It might appear, from this point of view, that the divergence of the electromagnetic mass is unobjectionable, since the individual contributions to the experimental mass are unobservable. However, the transformation of the Hamiltonian is based on the assumption of a weak interaction between matter and radiation, which requires that the electromagnetic mass be a small correction ($\sim(e^2/\hbar c)m_0$) to the mechanical mass m_0.

The new Hamiltonian is superior to the original one in essentially three ways: it involves the experimental electron mass, rather than the unobservable mechanical mass; an electron now interacts with the radiation field only in the presence of an external field, that is, only an accelerated electron can emit or absorb a light quantum;* the interaction energy of an electron with an external field is now subject to a *finite* radiative correction. In connection with the last point, it is important to note that the inclusion of the electromagnetic mass with the mechanical mass does not avoid all divergences; the polarization of the vacuum produces a logarithmically divergent term proportional to the interaction energy of the electron in an external field. However, it has long been recognized that such a term is equivalent to altering the value of the electron charge by a constant factor, only the final value being properly identified with the experimental charge. Thus the interaction between matter and radiation produces a renormalization of the electron charge and mass, all divergences being contained in the renormalization factors.

The simplest example of a radiative correction is that for the energy of an electron in an external magnetic field. The detailed application of the theory shows that the radiative correction to the magnetic interaction energy corresponds to an additional magnetic moment associated with the electron spin, of magnitude $\delta\mu/\mu = (\frac{1}{2}\pi)e^2/\hbar c = 0.001162$. It is indeed gratifying that recently acquired experimental data confirm this prediction. Measurements on the hyperfine splitting of the ground states of atomic hydrogen and deuterium[1] have yielded values that are definitely larger than those to be expected from the directly measured nuclear moments and an electron moment of one Bohr magneton. These discrepancies can be accounted for by a small additional electron spin magnetic moment.[2] Recalling that the nuclear moments have been calibrated in terms of the electron moment, we find the additional moment necessary to account for the measured hydrogen and deuterium hyperfine structures to be $\delta\mu/\mu = 0.00126 \pm 0.00019$ and $\delta\mu/\mu = 0.00131 \pm 0.00025$, respectively. These values are not in disagreement with the theoretical prediction. More precise conformation is provided by measurement of the g values for the $^2S_{\frac{1}{2}}$, $^2P_{\frac{1}{2}}$, and $^2P_{3/2}$ states of sodium and gallium.[3] To account for these results, it is necessary to ascribe the following additional spin magnetic moment to the electron, $\delta\mu/\mu = 0.00118 \pm 0.00003$.

$\mathcal{O}(\frac{\alpha}{\pi})^2$: André Petermann

- Co-inventor (with Stueckelberg) of the renormalisation group
- First to submit a paper proposing quarks (a few days before Gell-Mann and Zweig), not widely known because written in French, and publication delayed > year!

8.B Nuclear Physics 63 (1965) 349—352; © North-Holland Publishing Co., Amsterdam

PROPRIÉTÉS DE L'ÉTRANGETÉ ET UNE FORMULE DE MASSE POUR LES MÉSONS VECTORIELS

A. PETERMANN
CERN, Genéve 23

Reçu le 30 décembre 1963

Abstract: A mass-formula for vector mesons is proposed, and the role of strangeness in mass-formulae discussed.

- Pioneer of $g_\mu - 2$ calculations: $a_\mu \equiv \frac{(g_\mu - 2)}{2} = 1 + \frac{\alpha}{2\pi} + 0.75(\frac{\alpha}{\pi})^2$ (1957)
- First correct calculation of $\mathcal{O}(\alpha^2)$ contributions to $g_e - 2,\ g_\mu - 2$

(also Sommerfeld; Suura & Wichmann, previous work by Karplus & Kroll)

Fourth order magnetic moment of the electron

by **A. Petermann.**

CERN. Theoretical Study Division. Institute for theoretical Physics. Copenhagen.

(17. VIII. 1957.)

In connection with the upper and lower bounds analysis done by the author[1]), which indicated a clear discrepancy with the Karplus and Kroll's result for the 4th order magnetic moment[2]), we have performed an analytic evaluation of the five independent diagrams contributing to this moment in fourth order*). The results are the following:

$$\mu_{\mathrm{I}} = \frac{\alpha^2}{\pi^2}\left(\frac{1}{6} + \frac{13}{36}\pi^2 + \frac{5}{4}\zeta(3) - \frac{5}{6}\pi^2 \operatorname{Log} 2\right) = -0.467\frac{\alpha^2}{\pi^2}. \quad (1)$$

$$\mu_{\mathrm{II_a}} = \frac{\alpha^2}{\pi^2}\left(\frac{11}{48} + \frac{\pi^2}{18}\right) = 0.778\frac{\alpha^2}{\pi^2}. \quad (2)$$

$$\mu_{\mathrm{II_c}} = \frac{\alpha^2}{\pi^2}\left(-\frac{67}{24} + \frac{\pi^2}{18} - \frac{1}{2}\zeta(3) + \frac{1}{3}\pi^2 \operatorname{Log} 2 - \frac{1}{2}\operatorname{Log}\frac{\lambda^2}{m^2}\right) = -0.564\frac{\alpha^2}{\pi^2} - \frac{1}{2}\frac{\alpha^2}{\pi^2}\operatorname{Log}\frac{\lambda^2}{m^2}. \quad (3)$$

$$\mu_{\mathrm{II_d}} = \frac{\alpha^2}{\pi^2}\left(\frac{11}{24} - \frac{\pi^2}{18} + \frac{1}{2}\operatorname{Log}\frac{\lambda^2}{m^2}\right) = -0.090\frac{\alpha^2}{\pi^2} + \frac{1}{2}\frac{\alpha^2}{\pi^2}\operatorname{Log}\frac{\lambda^2}{m^2}. \quad (4)$$

$$\mu_{\mathrm{II_e}} = \frac{\alpha^2}{\pi^2}\left(\frac{119}{36} - \frac{\pi^2}{3}\right) = 0.016\frac{\alpha^2}{\pi^2}. \quad (5)$$

$$\mu^{(4)}_{\mathrm{total}} = \frac{\alpha^2}{\pi^2}\left(\frac{197}{144} + \frac{\pi^2}{12} + \frac{3}{4}\zeta(3) - \frac{1}{2}\pi^2 \operatorname{Log} 2\right) = -0.328\frac{\alpha^2}{\pi^2}. \quad (6)$$

Compared with the values given in their original paper by Karplus and Kroll, one can see that two terms were in error: μ_{I} differs by

$$\frac{\alpha^2}{\pi^2}\frac{1}{32} = 0.031\frac{\alpha^2}{\pi^2};$$

$$\mu_{\mathrm{II_c}} \text{ by } \frac{\alpha^2}{\pi^2}\left(\frac{32}{3} - \frac{61}{8}\pi^2 + \frac{17}{2}\pi^2 \operatorname{Log} 2 - \frac{109}{4}\zeta(3)\right) - 2.614\frac{\alpha^2}{\pi^2}.$$

The three other terms check. The error in μ_{I} remained of course undetected in the upper and lower bound analysis owing to its small-

*) The terminology of ref. 2 is used throughout this paper.

ness. But the large discrepancy in $\mu_{\mathrm{II_c}}$ was that pin-pointed out in the previous paper.

A summary of the most important electromagnetic observables, the theoretical values of which are modified by the new value of the magnetic moment, is now given:

Moment of the electron: $\frac{\mu_e}{\mu_0} = 1.0011596 = 1 + \frac{\alpha}{2\pi} - 0.328\frac{\alpha^2}{\pi^2}$.

Franken and Liebes' value for it: $\mu_e/\mu_0 = 1.001167 \pm 0.000005$*).

g-factor of the μ-meson (electromagnetic):

$$2\,(1.0011654) = 2\left(1 + \frac{\alpha}{2\pi} + 0.75\frac{\alpha^2}{\pi^2}\right).$$

Last Lederman's value: $2\,(1.0021 \pm 0.0008)$ *).

$2^2 S_{1/2} - 2^2 P_{1/2}$ (Hydrogen): (1057.94 ± 0.15) Mc/s; observed: $(1057{\cdot}77 \pm 0.10)$ Mc/s.

$2^2 S_{1/2} - 2^2 P_{1/2}$ (Deuterium): (1059.22 ± 0.15) Mc/s; observed: (1059.00 ± 0.10) Mc/s.

Fine structure constant: $1/\alpha = 137.0384$; (previously: 137.0365).

The new fourth order correction given here is in agreement with:

a) The upper and lower bounds given by the author[1]).

b) A calculation using a different method, performed by C. Sommerfield[3]).

c) A recalculation done by N. M. Kroll and collaborators*).

The author thanks Prof. Niels Bohr for the hospitality at the Institute.

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[3]) C. Sommerfield, Phys. Rev. In the press.

*) Private Communication.

$\mathcal{O}(\frac{\alpha}{\pi})^2$: Charles Sommerfield

seems justifiable to assume that the transition to the B^{12} ground state is allowed in the usual sense.

The next question is that of transitions to excited, bound states of B^{12} in the μ-capture process. The fact that only 13% of all absorptions lead to bound states of B^{12} implies that high excitations are favored. Appreciable formation of excited states would wash out the orientation in the ground state because of the smearing over magnetic quantum numbers that occurs in the process of de-excitation by γ-ray emission. Fortunately, the situation here seems favorable. There are only four known excited states below the threshold for particle emission.[9] While no firm arguments can be made, what is known of the spins and parities of these states makes it seem probable that the large majority of μ-capture events leading to *bound* states of B^{12} actually go directly to the ground state.

Another effect which must be considered is possible depolarization of the B^{12} nucleus due to hyperfine interaction with the atomic electrons. Rough estimates indicate that the atom is probably ionized due to recoil at the instant of absorption of the μ meson. If the atom is always ionized and then re-forms again after it stops, we can calculate the depolarization under the assumption that the fine-structure substates are populated statistically. This gives, for the resultant B^{12} polarization,

$$\langle \mathbf{J} \rangle = \tfrac{2}{3}(0.54)\langle \boldsymbol{\sigma} \rangle = 0.36\langle \boldsymbol{\sigma} \rangle. \quad (2)$$

Thus, if $|\langle \boldsymbol{\sigma} \rangle|$ equals 15%, the final polarization $|\langle \mathbf{J} \rangle|$ of the B^{12} is probably closer to 5% than to the value of 10% given above.

There is an additional depolarization due to the environment in which the B^{12} atom finds itself. But the relaxation time for this effect in graphite is presumably longer than the mean life of B^{12} since metals show relaxation times of the order of tens of milliseconds. In any event, such solid-state effects can be essentially eliminated by a suitable choice of organic material as target.

* This work was supported, in part, by the Office of Naval Research and the U. S. Atomic Energy Commission.
† Visiting Guggenheim Fellow, on leave of absence from McGill University, Montreal, Canada.

[1] T. D. Lee and R. P. Feynman, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics*, April, 1957 (to be published).
[2] R. L. Garwin, L. Lederman, and co-workers at Columbia have observed the longitudinal polarization of the electrons in μ decay (L. Lederman, in reference 1). On the basis of a theory of μ decay the direction of the μ meson's polarization can then be inferred.
[3] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957).
[4] Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957).
[5] T. N. K. Godfrey, Princeton University thesis, 1954 (unpublished).
[6] J and J' are the final and initial nuclear spins respectively, while $\lambda_{J'J}$ is a numerical factor defined in the appendix of Jackson, Treiman, and Wyld, Phys. Rev. **106**, 517 (1957). For a transition with $\Delta J=0$, the polarization of the daughter nucleus is of the form of Eq. (1) with the factor $\lambda_{J'J}$ replaced by $N/(1+b)$, the coefficients N and b being given in the above reference (with $E_e=m$, and the sign appropriate for electrons).
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Magnetic Dipole Moment of the Electron

CHARLES M. SOMMERFIELD*
Harvard University, Cambridge, Massachusetts
(Received May 6, 1957)

THE fourth-order radiative corrections to the magnetic dipole moment of the electron were calculated by Karplus and Kroll in 1949.[1] Their result is contained in the complete expression for the moment,

$$\mu_e/\mu_0 = 1 + (\alpha/2\pi) - 2.973(\alpha^2/\pi^2) = 1.0011454, \quad (1)$$

where μ_0 is the Bohr magneton.

The calculation has been redone in the present instance using the mass-operator formalism of Schwinger.[2] We consider a single electron moving in a constant (in space and time) electromagnetic field. The expectation value of the mass operator in the lowest state represents the self or proper energy of the electron. The magnetic moment is identified from that part of the self-energy which is linear in the external field.

The electron Green's function G, the photon Green's function $\mathcal{G}$, and the interaction operator Γ, which appear in the symbolic expression for the mass operator,

$$M = m_e + ie^2\,\mathrm{Tr}\gamma G \Gamma \mathcal{G},$$

are computed in the presence of (as functions of) the external field. To do this it is sufficient to replace the electron's momentum operator, p, where it occurs, by the combination $\Pi = p - eA$, provided that full account is taken of the commutation properties of Π. Units are such that $\hbar = c = 1$. Renormalized quantities are used throughout the perturbation calculation.

The fourth-order contribution to the moment is found to be

$$\frac{\mu_e^{(4)}}{\mu_0} = \frac{\alpha^2}{\pi^2}\left(\frac{197}{144} + \frac{\pi^2}{12} + \tfrac{3}{4}\zeta(3) - \tfrac{1}{2}\pi^2 \ln 2\right) = -0.328\frac{\alpha^2}{\pi^2}, \quad (2)$$

where $\zeta(3)$ is the Riemann zeta function of 3. Thus

$$\mu_e/\mu_0 = 1.0011596.$$

The discrepancy between (1) and (2) has been traced to the term $\mu^{I} + \mu^{IIe}$ of Karplus and Kroll. In other words, terms μ^{IIe} and $\mu^{IIa} + \mu^{IId}$ appear unchanged in the new result. A further point-by-point comparison of the two answers is not readily accomplished because the grouping of the terms differs markedly in the two cases. The present calculation has been checked several times and all of the auxiliary integrals have been done in at least two different ways.

The theoretical magnetic moment may be compared with the experimental moment; it is also used in determining the fine-structure constant α; and it contributes to the Lamb shift. The magnetic moment is measured by determining μ_e/μ_p and μ_p/μ_0, where μ_p is the proton moment. The measurements of μ_e/μ_p have been quite accurate.[3] On the other hand, there are two conflicting experimental determinations[4,5] of μ_p/μ_0, which result in two different values for the magnetic moment:

References	μ_e/μ_0
3 and 4	1.001146±0.000012
3 and 5	1.001165±0.000011.

The theoretical value[6] for the hyperfine splitting in hydrogen is proportional to the quantity

$$\alpha^2(\mu_p/\mu_0)(\mu_e/\mu_0) = \alpha^2(\mu_p/\mu_e)(\mu_e/\mu_0)^2.$$

Since there is agreement on the experimental value of μ_p/μ_e, we use the second form, together with the present value of α,[7] to determine a new value. This turns out to be

$$1/\alpha = 137.039.$$

The theoretical Lamb shifts in hydrogen, deuterium, and singly ionized helium are affected by the changes in both α and μ_e. Incorporating these changes into the calculations of Salpeter,[8] along with the proton-recoil recoil corrections of Fulton and Martin,[9] and the proton-structure corrections of Aron and Zuchelli,[10] we obtain the following results in Mc/sec:

	Theoretical	Experimental	Reference
S_H	1057.99±0.13	1057.77± 0.10	11
S_D	1059.23±0.13	1059.00± 0.10	11
$S_D - S_H$	1.24±0.04	1.23± 0.15	11
S_{He}	14055.9 ±2.1	14043 ±13.0	12

The experimental values[11,12] have been listed for comparison. There remain several uncomputed theoretical effects which are expected to be of the same order of magnitude as the indicated theoretical uncertainties.

The magnetic moment of the μ meson, as computed by Suura and Wichmann, and Petermann,[13] would be changed to read

$$\mu_\mu = \left(1 + \frac{\alpha}{2\pi} + 0.75\frac{\alpha^2}{\pi^2}\right)\frac{e\hbar}{2m_\mu c}.$$

I would like to thank Professor J. Schwinger, Professor P. C. Martin, Professor E. M. Purcell, and Professor R. J. Glauber, and Dr. K. A. Johnson for their helpful comments and discussion related to this work.

Note added in proof.—Petermann[14] has placed upper and lower bounds on the separate terms of Karplus and Kroll. He finds that their value for μIIc does not lie within the appropriate bounds. Assuming the other terms to be correct, he concludes that μ^4/μ_0 =(− value

* N

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Allowed Capture-Positron Branching Ratios

P. F. ZWEIFEL
Knolls Atomic Power Laboratory, Schenectady, New York*
(Received April 18, 1957)

IN a previous paper,[1] tables of allowed K capture-positron branching ratios were presented. However, it was pointed out by Wapstra[2] and Perlman[3] that numerical errors existed in the table. These errors appear in the first, third, and fifth columns of Table II of reference 1, each entry of which should be multiplied by the factors of 0.5018, 1.2244, and 0.6462, respectively. In Table I of this communication, the corrected table of allowed K to positron branching ratios is given. In this work, the effect of the finite nuclear size on the bound electron wave functions, which was ignored in reference 1, was taken into account.[4] This effect, which is negligible for low Z, reduces the branching ratio by about 10% for $Z=84$ and by about 15% for $Z=92$. Effects of finite size on the positron wave functions was ignored, since it is a considerably smaller effect.[5]

As in reference 1, the bound electron wave functions were taken from Reitz's thesis[6] except for $Z=16$, for

TABLE I. Allowed K to positron branching ratios.

$W_0/mc^2 \diagdown Z$	16	29	49	84	92
1.28	46.6	707	1.208×10^4	4.56×10^5	8.92×10^5
1.44	8.65	112	1.58×10^3	4.50×10^4	8.41×10^4
1.60	2.83	33.6	425	1.03×10^4	1.84×10^4
1.76	1.24	13.9	164	3.57×10^3	5.01×10^3
1.92	0.641	6.91	77.6	1.57×10^3	2.67×10^3
2.08	0.373	3.91	42.3	807	1.36×10^3
2.40	0.190	1.60	16.4	289	479
2.88	0.0613	0.597	5.86	96.4	158
3.84	0.0169	0.160	1.51	23.6	39.0
4.80	7.00×10^{-3}	0.0648	0.603	9.10	15.7
5.76	3.56×10^{-3}	0.0328	0.302	4.82	8.05
6.72	2.06×10^{-3}	0.0188	0.173	2.82	4.75
7.68	1.30×10^{-3}	0.0118	0.109	1.80	3.06
8.64	8.85×10^{-4}	7.93×10^{-3}	0.0729	1.23	2.10
9.60	6.29×10^{-4}	5.60×10^{-3}	0.0513	0.879	1.52
10.56	4.48×10^{-4}	4.09×10^{-3}	0.0377	0.652	1.13
11.52	3.37×10^{-4}	3.07×10^{-3}	0.0281	0.498	0.869
12.48	2.60×10^{-4}	2.37×10^{-3}	0.0219	0.393	0.685

Lederman *et al.*: First Measurement of $g_\mu - 2$

PHYSICAL REVIEW VOLUME 109, NUMBER 3 FEBRUARY 1, 1958

Magnetic Moment of the Free Muon*†

T. Coffin, R. L. Garwin,‡ S. Penman, L. M. Lederman, and A. M. Sachs
Columbia University,§ New York, New York

(Received October 1, 1957)

The magnetic moment of the positive μ meson has been measured in several target materials by a magnetic resonance technique. Muons were brought to rest with their spins parallel to a magnetic field. A radio-frequency pulse was applied to effect a spin reorientation which was detected by counting the decay electrons emerging after the pulse in a fixed direction. Results are expressed in terms of a g factor which for a spin $\frac{1}{2}$ particle is the ratio of the actual moment to $e\hbar/2m_\mu c$. The most accurate result obtained in a $CHBr_3$ target, is that $g=2(1.0026\pm0.0009)$ compared to the theoretical prediction of $g=2(1.0012)$. Less accurate measurements yielded $g=2.005\pm0.005$ in a copper target and $g=2.00\pm0.01$ in a lead target.

I. INTRODUCTION

THE μ meson has often been described as one of the more baffling of elementary particles. It alone, among the unstable particles, has no strong interaction. Aside from its usefulness as a tool in the study of nuclear structure and the details of parity violation in weak interactions it appears to play no essential role in any organization of fundamental particles. A precise measurement of the magnetic moment of the muon offers some promise for clarification of this situation.

The Dirac equation predicts precisely 2 for the g value of a spin $\frac{1}{2}$ particle. Including corrections due to the interaction of the particle with its radiation field, one obtains[1,2]

$$g_\mu = 2\left(1+\frac{\alpha}{2\pi}+0.75\frac{\alpha^2}{\pi^2}+\cdots\right) \qquad (1)$$
$$=2(1.0012).$$

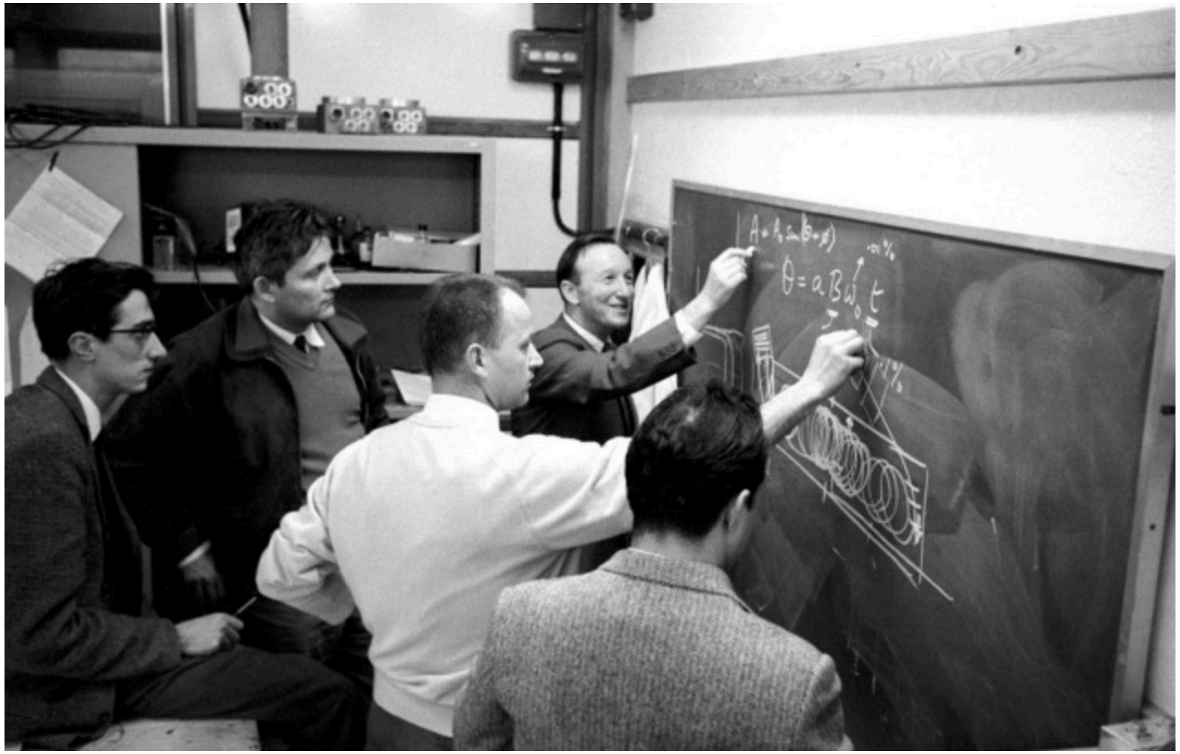
an energy λ would alter the g value as follows

$$g=2\left\{1+\left[1-\frac{2}{3}\left(\frac{m_\mu}{\lambda}\right)^2\right]\frac{\alpha}{2\pi}+\cdots\right\}. \qquad (2)$$

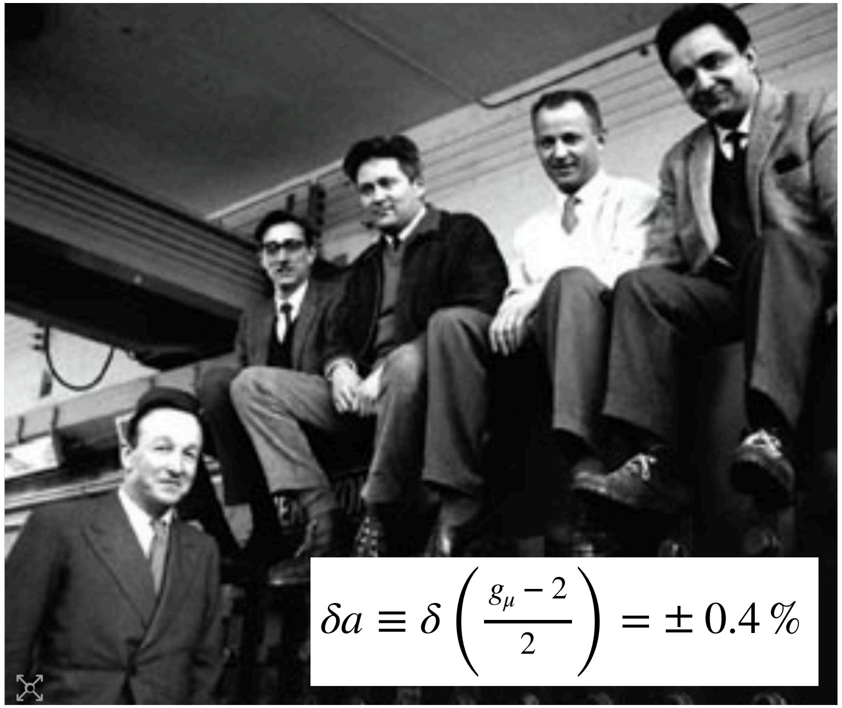
It might be remarked[4] that the model used in reference 3 implies a modification in the scattering of one Dirac particle by another. Such a modification can be described by a mean square radius, the appropriate relation being $\langle r^2\rangle_e=6(\hbar/\lambda c)^2$. Qualitatively, at least, the measured proton radius should constitute an upper limit for such an "electrodynamic radius." Hence the fractional alteration of the muon moment from such a presumed breakdown of quantum electrodynamics should not exceed $\sim0.02(\alpha/2\pi)$.

- Columbia Nevis and Carnegie Institute of Technology cyclotrons
- Agreement between theory and experiment

First $g_\mu - 2$ Experiment at CERN



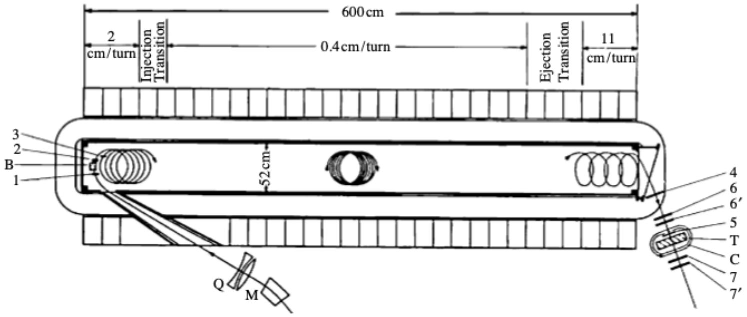
(1958 - 1962)



$$\delta a \equiv \delta\left(\frac{g_\mu - 2}{2}\right) = \pm\, 0.4\ \%$$

Georges Charpak   Francis Farley

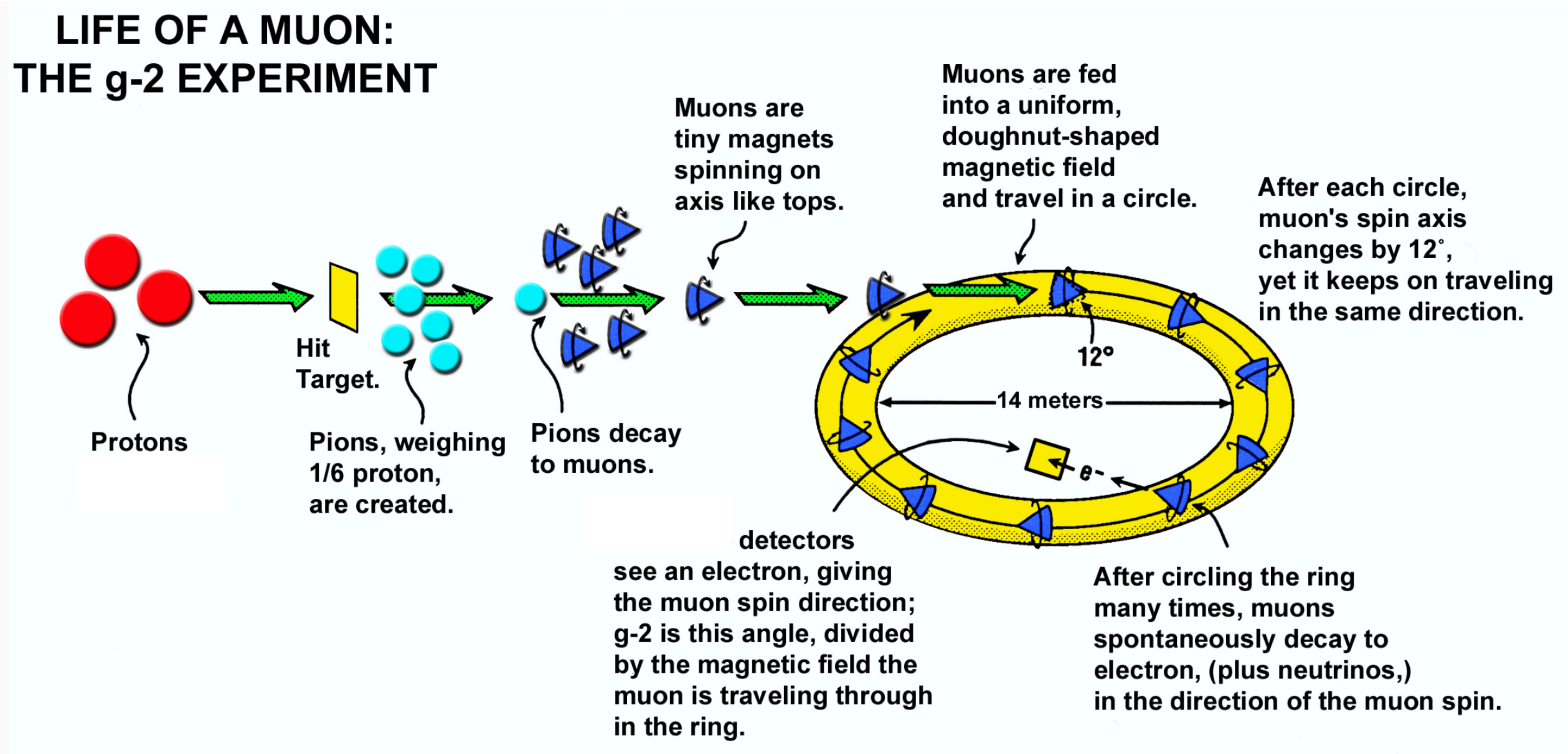
Hans Sens   Theo Muller   Nino Zichichi



(Suggested by Leon Lederman)

(Also experiment at Berkeley Cyclotron)

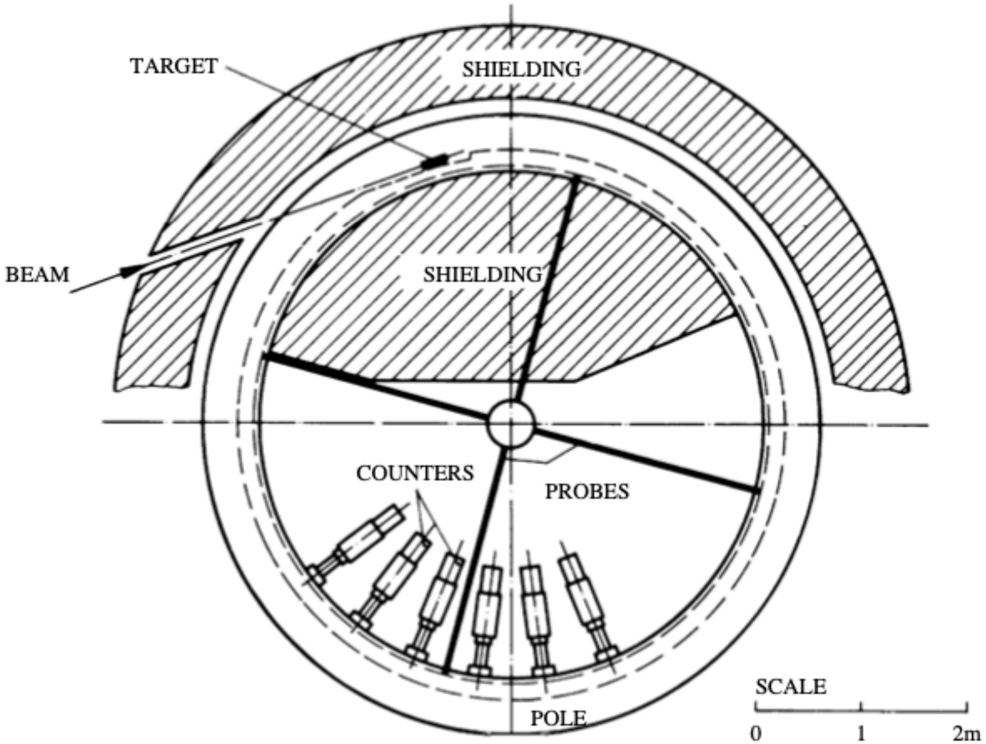
Experimental Principle of Storage Ring Experiments



First Storage Ring Experiment at CERN

(1962 - 1968)

Design



Under construction

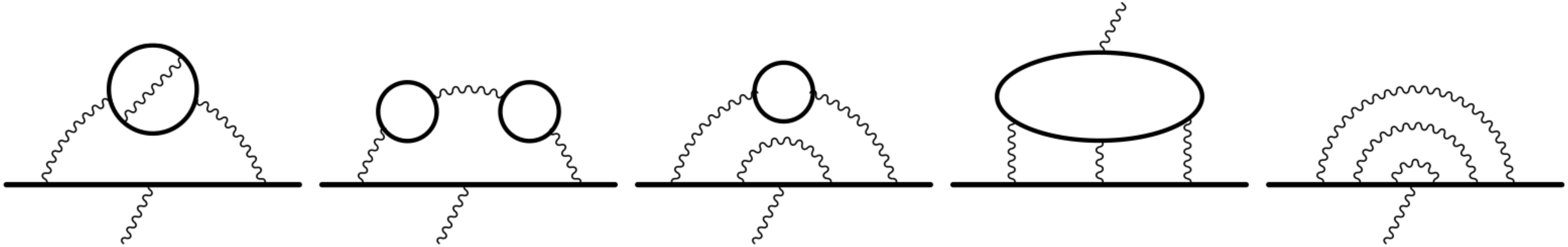
$$\delta a = \pm\, 270\,\text{ppm}$$

Agreement with theory after inclusion of light-by-light scattering
(Aldins, Kinoshita, Brodsky, Dufner)

$\mathcal{O}(\frac{\alpha}{\pi})^3$ Calculations

(Kinoshita, 1967)

(Lautrup, 1968)
(+ De Rafael)

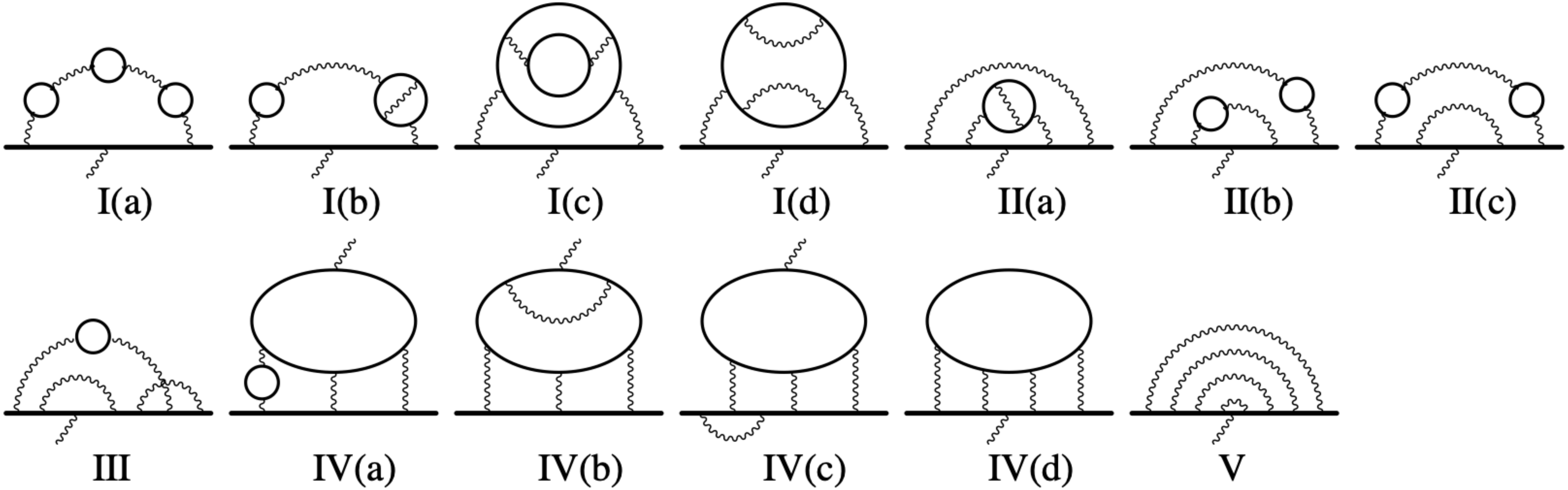


$$
\begin{aligned}
A_2^{(6)}(m_\mu/m_e) &= \frac{2}{9}\log^2 x - \left(\zeta(3) - \frac{2}{3}\pi^2\log 2 + \frac{7\pi^2}{9} + \frac{31}{27}\right)\log x + \frac{97\pi^4}{360} \\
&- \frac{2}{9}\pi^2\log^2 2 - \frac{8}{3}a_4 - \frac{\log^4 2}{9} - 6\zeta(3) + \frac{5}{3}\pi^2\log 2 - \frac{85\pi^2}{18} + \frac{1219}{216} \\
&+ x\left(-\frac{4}{3}\pi^2\log x - \frac{604}{9}\pi^2\log 2 + \frac{54079\pi^2}{1080} - \frac{13\pi^3}{18}\right) \\
&+ x^2\left[\frac{2}{3}\log^3 x + \left(\frac{\pi^2}{9} - \frac{10}{3}\right)\log^2 x + \left(\frac{16\pi^4}{135} + 4\zeta(3) - \frac{32\pi^2}{9} + \frac{194}{9}\right)\log x \right. \\
&\left. + \frac{4}{3}\zeta(3)\pi^2 - \frac{61\pi^4}{270} + \zeta(3) + \frac{197\pi^2}{36} - \frac{2809}{108} - \frac{14}{3}\pi^2\log 2\right] + O(x^3) \\
&= 22.868\,379\,98(20)\,,
\end{aligned}
$$

$\mathcal{O}(\frac{\alpha}{\pi})^4$ Calculations

(Kinoshita)

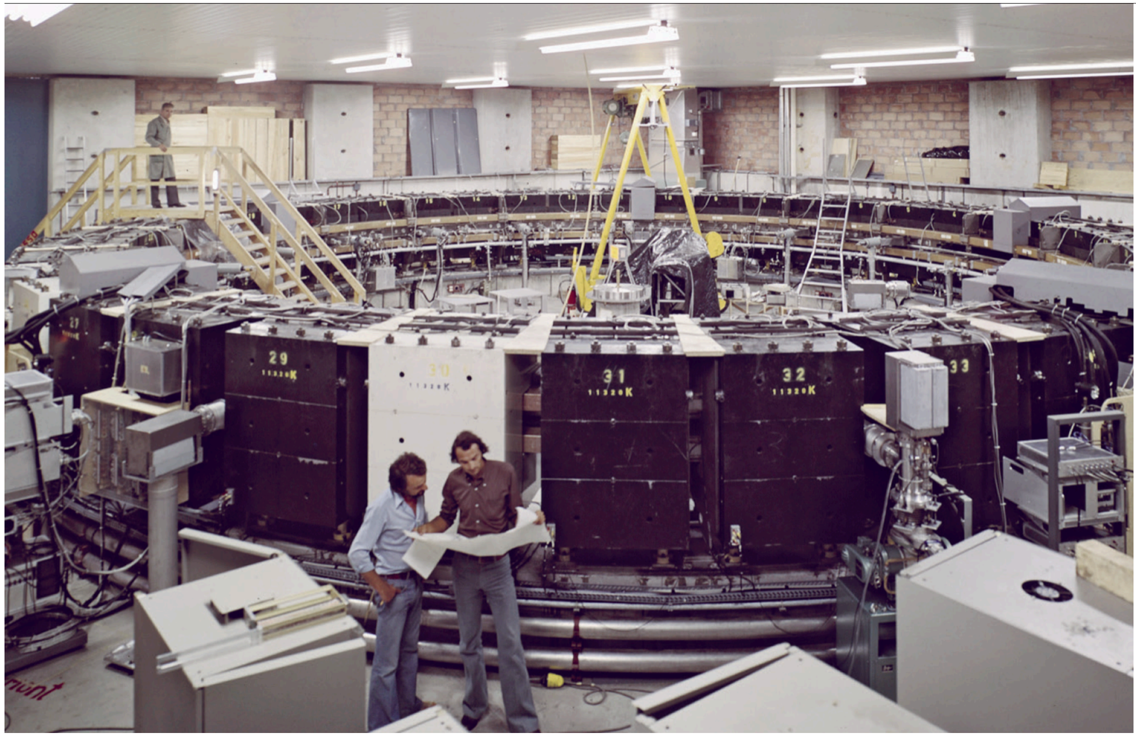
(Lautrup, 1972)

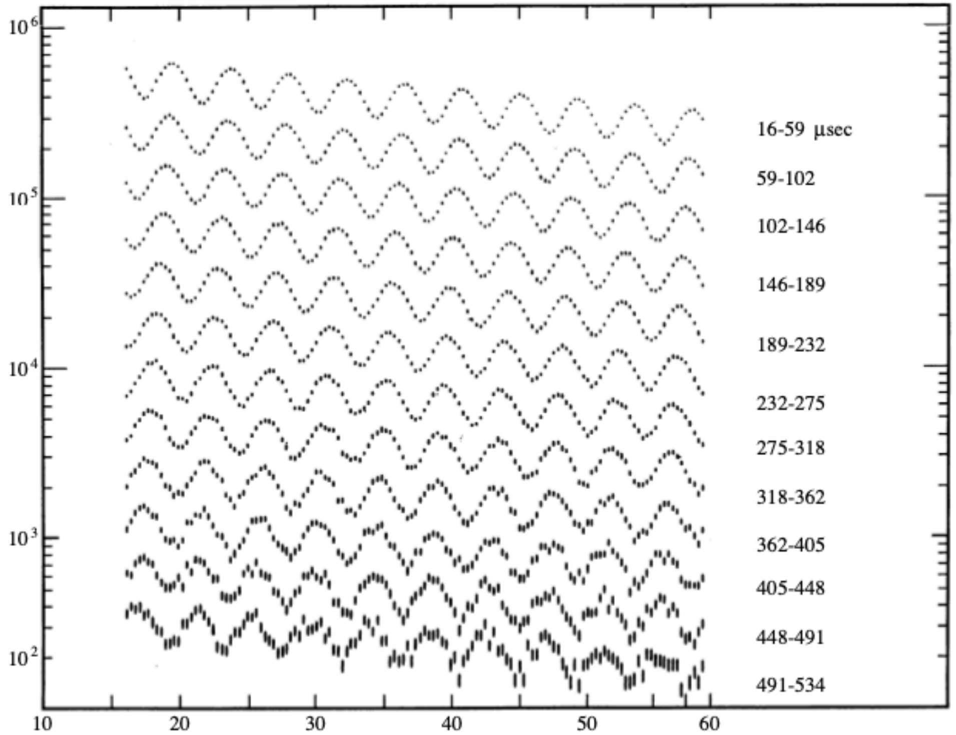


$$A_2^{(8)}(m_\mu/m_e) = 123.785\,51(44) + 8.8997(59) = 132.6852(60)$$

Second Storage Ring Experiment at CERN

(1969 - 1976)





Precession frequency

$$\vec{\omega}_a \equiv \vec{\omega}_s - \vec{\omega}_c = -\frac{q}{m_\mu}\left[a_\mu \vec{B} - a_\mu \left(\frac{\gamma}{\gamma+1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right].$$

$\beta\,.\,\mathbf{B} = 0$

Magic energy
$E = 3.094$ GeV
$\gamma = 29.3$

Precession frequency $\propto a_\mu$

$\delta a_\mu = 8$ ppm

(Emilio Picasso → LEP)

$g_\mu - 2$ in Supersymmetry

- One-loop contribution from smuon/neutralino loop

$$\Delta(g-2)_\mu = -ab(\cos\alpha \sin\alpha/4\pi^2)(m_\mu/m_{\tilde{G}}) \times \{1/(1-\eta_1) + 2\eta_1/(1-\eta_1)^2 + [2\eta_1/(1-\eta_1)^3]\log\eta_1 - (\eta_1 \leftrightarrow \eta_2)\},$$

- where $\eta_i \equiv (m^2_{s\mu_i}/m^2_{\tilde{G}})$

- and $\mathcal{L} = a\sqrt{2}\, s_\mu \bar{\mu}_L \tilde{G} + b\sqrt{2}\, t_\mu \bar{\mu}_R \tilde{G}$

Volume 116B, number 4 PHYSICS LETTERS (1982)

SPIN-ZERO LEPTONS AND THE ANOMALOUS MAGNETIC MOMENT OF THE MUON

John ELLIS, John HAGELIN and D.V. NANOPOULOS
CERN, Geneva, Switzerland

Received 14 June 1982

The anomalous magnetic moment of the muon $(g-2)_\mu$ imposes constraints on the masses and mixing of spin-zero leptons (sleptons). We develop the predictions of models of spontaneous supersymmetry breaking for the slepton mass matrix, and show that they are comfortably consistent with the $(g-2)_\mu$ constraints.

During the present resurgence of interest in supersymmetry broken at low energies [1] new significance is attached to the classical phenomenological playgrounds of gauge theories such as the anomalous magnetic moments of the electron and muon [2], flavour-changing neutral interactions [3–5] parity [6] and *CP* violation [7,8] in the strong interactions. The three latter phenomena make life rather difficult [3,7] for the most general form of soft supersymmetry breaking, whereas simple models [9–11] of spontaneously broken supersymmetry naturally [3,4 7] respect the $\Delta F \neq 0$, *P* and *CP* violation constraints. As for the anomalous magnetic moments of the leptons, it has long been known that they vanish in an exactly supersymmetric theory [12], and Fayet [2] showed that in his model of supersymmetry breaking $(g-2)_\mu$ would be compatible with experiment if the spin-zero muon (smuon) masses were heavier than 15 GeV. Direct experimental searches [13] now exclude the existence of lighter smuons. Fayet's analysis [2] was in the context of a model with a very light photino $\tilde{\gamma}$ (see fig. 1a), and Grifols and Méndez [14] have recently made the interesting observation that his analysis is significantly altered for massive gauginos (see figs. 1b, 1c). They show that there are potentially nontrivial constraints on the smuon masses in models of broken supersymmetry.

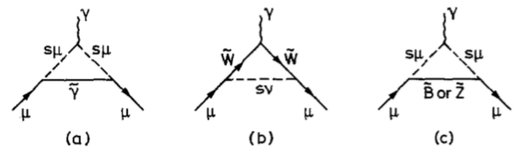


Fig. 1. One-loop diagrams contributing to $(g-2)_\mu$: (a) essentially massless photino ($\tilde{\gamma}$) exchange, (b) $\tilde{W}$ and sneutrino ($s\nu$) exchange, and (c) $\tilde{B}$ or $\tilde{Z}$ exchange.

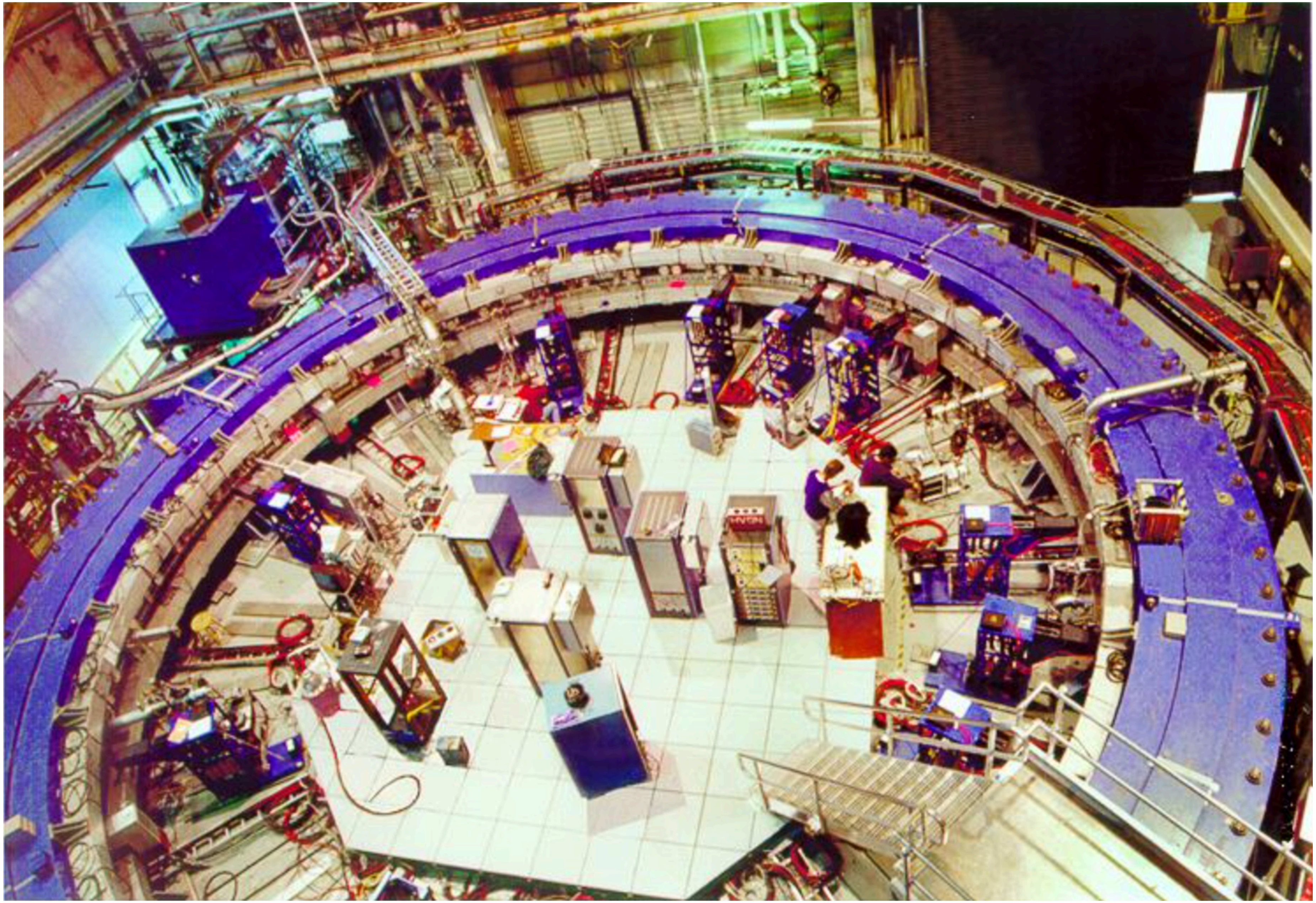
right transition operator there is a GIM [15]-like cancellation between the smuon mass eigenstates in fig. 1c which provides a potential suppression mechanism. We analyze recent models [10,11] of spontaneous supersymmetry breaking originating in the *D* and *F* sectors, respectively. We show that in the former case $(g-2)_\mu$ is suppressed by near degeneracy between the smuon mass eigenstates, while in the latter case $(g-2)_\mu$ is suppressed by small mixing angles between the left- and right-handed smuons. We close with some remarks about $(g-2)_e$ and about parity violation in the strong interactions.

When they examined figs. 1a, 1b and 1c, Grifols and Méndez [14] realized that there was a fundamental difference between the (almost ?) massless $\tilde{\gamma}$ diagram of fig. 1a and the $\tilde{W}$ diagram of fig. 1b as compared to the massive $\tilde{B}$ or $\tilde{Z}$ diagram of fig. 1c. The

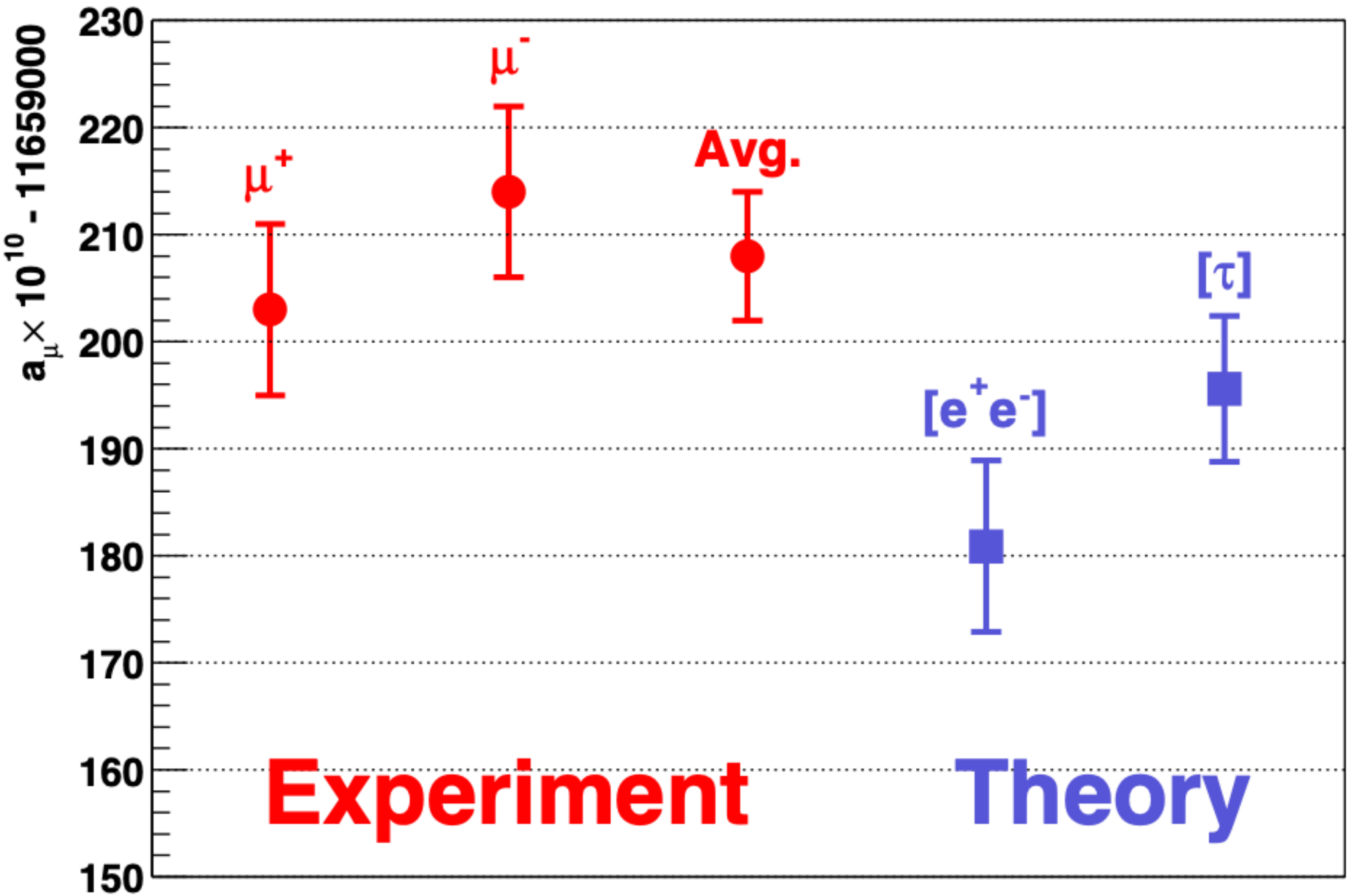
BNL Experimental Team

BNL Experiment

(1984 - 2003)



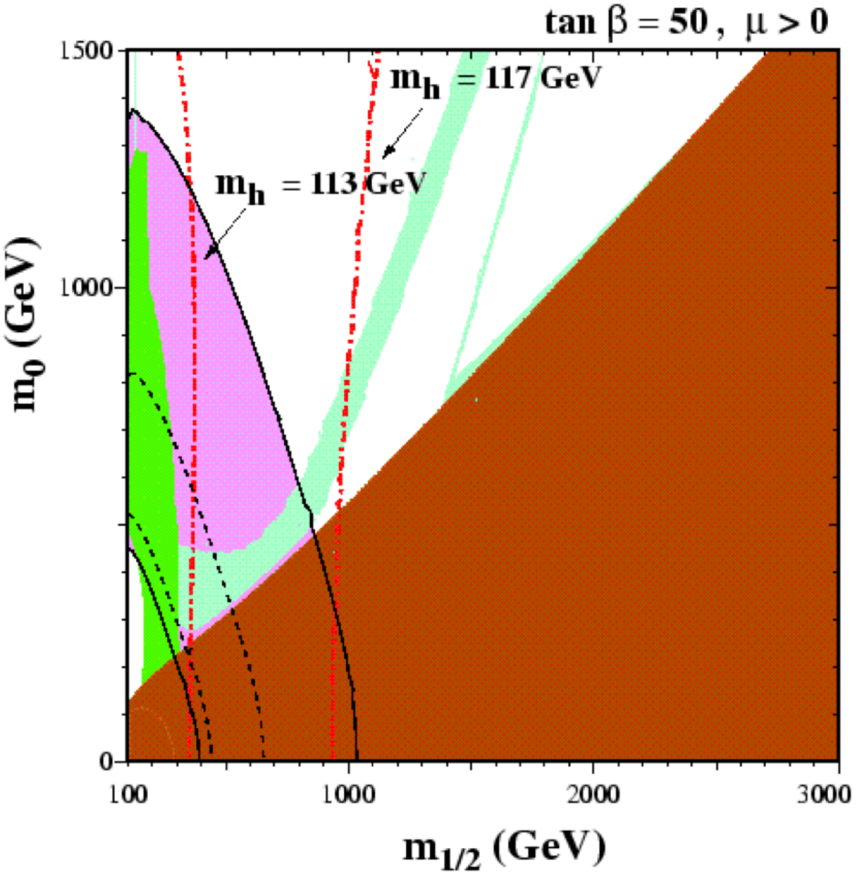
Possible Discrepancy with Theory?



$$\delta a = \pm\, 0.47\ \text{ppm}$$

BNL E821 experiment, 2001 - 2006

$g_\mu - 2$ in Supersymmetry v2: the CMSSM



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24 May 2001
Physics Letters B 508 (2001) 65–73
PHYSICS LETTERS B
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Combining the muon anomalous magnetic moment with other constraints on the CMSSM

John Ellis [a], D.V. Nanopoulos [b,c,d], Keith A. Olive [a,e]

[a] *TH Division, CERN, Geneva, Switzerland*
[b] *Department of Physics, Texas A&M University, College Station, TX 77843, USA*
[c] *Astroparticle Physics Group, Houston Advanced Research Center (HARC), Mitchell Campus, Woodlands, TX 77381, USA*
[d] *Chair of Theoretical Physics, Academy of Athens, Division of Natural Sciences, 28 Panepistimiou Avenue, Athens 10679, Greece*
[e] *Theoretical Physics Institute, School of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455, USA*

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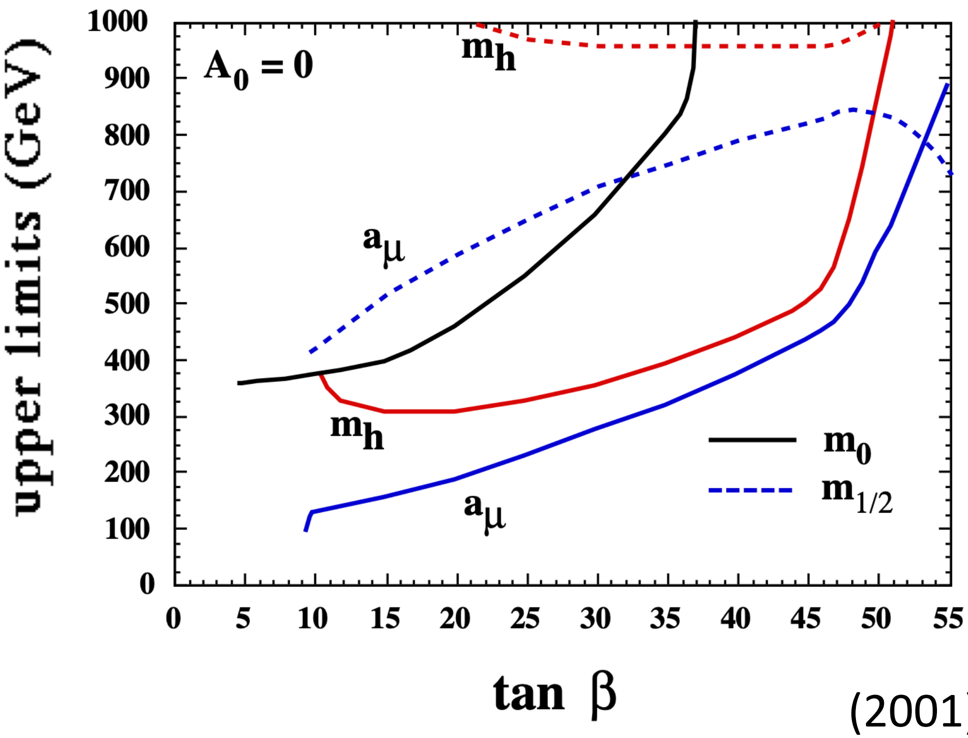
Abstract

We combine the constraint suggested by the recent BNL E821 measurement of the anomalous magnetic moment of the muon on the parameter space of the constrained MSSM (CMSSM) with those provided previously by LEP, the measured rate of $b \to s\gamma$ decay and the cosmological relic density $\Omega_\chi h^2$. Our treatment of $\Omega_\chi h^2$ includes carefully the direct-channel Higgs poles in annihilation of pairs of neutralinos χ and a complete analysis of $\chi - \tilde{\ell}$ coannihilation. We find excellent consistency between all the constraints for $\tan\beta \gtrsim 10$ and $\mu > 0$, for restricted ranges of the CMSSM parameters m_0 and $m_{1/2}$. All the preferred CMSSM parameter space is within reach of the LHC, but may not be accessible to the Tevatron collider, or to a first-generation e^+e^- linear collider with centre-of-mass energy below 1.2 TeV. © 2001 Published by Elsevier Science B.V.



(2001)

$\mathcal{O}(\frac{\alpha}{\pi})^5$ Calculations

Complete Tenth-Order QED Contribution to the Muon $g-2$

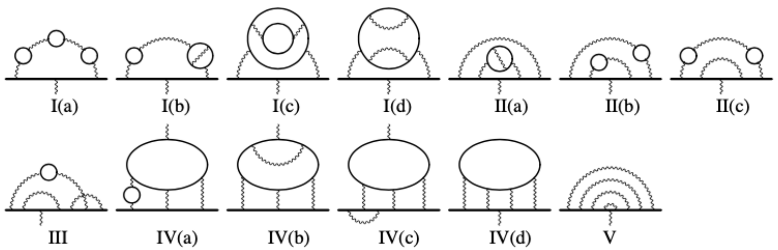
Tatsumi Aoyama,[1,2] Masashi Hayakawa,[3,2] Toichiro Kinoshita,[4,2] and Makiko Nio[2]

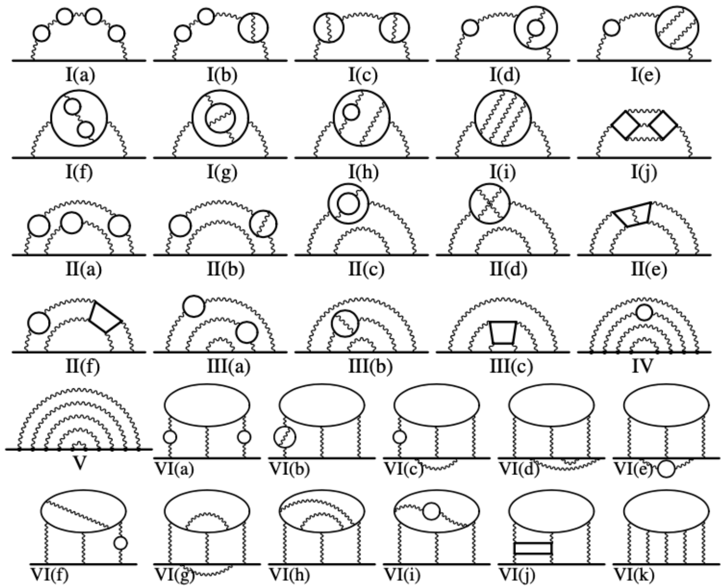
[1] *Kobayashi-Maskawa Institute for the Origin of Particles and the Universe (KMI), Nagoya University, Nagoya, 464-8602, Japan*
[2] *Nishina Center, RIKEN, Wako, Japan 351-0198*
[3] *Department of Physics, Nagoya University, Nagoya, Japan 464-8602*
[4] *Laboratory for Elementary Particle Physics, Cornell University, Ithaca, New York, 14853, U.S.A*

(Dated: August 21, 2012)

We report the result of our calculation of the complete tenth-order QED terms of the muon $g-2$. Our result is $a_\mu^{(10)} = 753.29\ (1.04)$ in units of $(\alpha/\pi)^5$, which is about 4.5 s.d. larger than the leading-logarithmic estimate 663 (20). We also improved the precision of the eighth-order QED term of a_μ, obtaining $a_\mu^{(8)} = 130.8794\ (63)$ in units of $(\alpha/\pi)^4$. The new QED contribution is $a_\mu(\text{QED}) = 116\,584\,718\,951\ (80) \times 10^{-14}$, which does not resolve the existing discrepancy between the standard-model prediction and measurement of a_μ.

PACS numbers: 13.40.Em,14.60.Ef,12.20.Ds

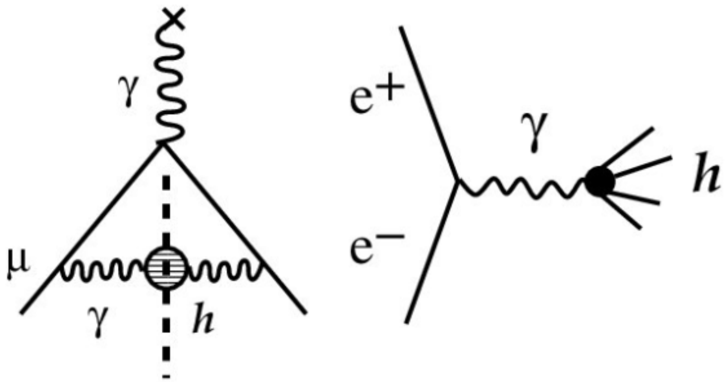




(2012)

Theory Initiative

- Comprehensive review of calculations of the Standard Model contributions to $g_\mu - 2$
- Including discussion of the uncertainties
- Particularly in calculation of leading-order vacuum polarisation



Aoyama et al, arXiv:2006.04822

Physics Reports 887 (2020) 1–166

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The anomalous magnetic moment of the muon in the Standard Model

T. Aoyama [1,2,3], N. Asmussen [4], M. Benayoun [5], J. Bijnens [6], T. Blum [7,8], M. Bruno [9], I. Caprini [10], C.M. Carloni Calame [11], M. Cè [9,12,13], G. Colangelo [14,*], F. Curciarello [15,16], H. Czyż [17], I. Danilkin [12], M. Davier [18,*], C.T.H. Davies [19], M. Della Morte [20], S.I. Eidelman [21,22,*], A.X. El-Khadra [23,24,*], A. Gérardin [25], D. Giusti [26,27], M. Golterman [28], Steven Gottlieb [29], V. Gülpers [30], F. Hagelstein [14], M. Hayakawa [31,2], G. Herdoíza [32], D.W. Hertzog [33], A. Hoecker [34], M. Hoferichter [14,35,*], B.-L. Hoid [36], R.J. Hudspith [12,13], F. Ignatov [21], T. Izubuchi [37,8], F. Jegerlehner [38], L. Jin [7,8], A. Keshavarzi [39], T. Kinoshita [40,41], B. Kubis [36], A. Kupich [21], A. Kupść [42,43], L. Laub [14], C. Lehner [26,37,*], L. Lellouch [25], I. Logashenko [21], B. Malaescu [5], K. Maltman [44,45], M.K. Marinković [46,47], P. Masjuan [48,49], A.S. Meyer [37], H.B. Meyer [12,13], T. Mibe [1,*], K. Miura [12,13,3], S.E. Müller [50], M. Nio [2,51], D. Nomura [52,53], A. Nyffeler [12,*], V. Pascalutsa [12], M. Passera [54], E. Perez del Rio [55], S. Peris [48,49], A. Portelli [30], M. Procura [56], C.F. Redmer [12], B.L. Roberts [57,*], P. Sánchez-Puertas [49], S. Serednyakov [21], B. Shwartz [21], S. Simula [27], D. Stöckinger [58], H. Stöckinger-Kim [58], P. Stoffer [59], T. Teubner [60,*], R. Van de Water [24], M. Vanderhaeghen [12,13], G. Venanzoni [61], G. von Hippel [12], H. Wittig [12,13], Z. Zhang [18], M.N. Achasov [21], A. Bashir [62], N. Cardoso [47], B. Chakraborty [63], E.-H. Chao [12], J. Charles [25], A. Crivellin [64,65], O. Deineka [12], A. Denig [12,13], C. DeTar [66], C.A. Dominguez [67], A.E. Dorokhov [68], V.P. Druzhinin [21], G. Eichmann [69,47], M. Fael [70], C.S. Fischer [71], E. Gámiz [72], Z. Gelzer [23], J.R. Green [9], S. Guellati-Khelifa [73], D. Hatton [19], N. Hermansson-Truedsson [14], S. Holz [36], B. Hörz [74], M. Knecht [25], J. Koponen [1], A.S. Kronfeld [24], J. Laiho [75], S. Leupold [42], P.B. Mackenzie [24], W.J. Marciano [37], C. McNeile [76], D. Mohler [12,13], J. Monnard [14], E.T. Neil [77], A.V. Nesterenko [68], K. Ottnad [12], V. Pauk [12], A.E. Radzhabov [78], E. de Rafael [25], K. Raya [79], A. Risch [12], A. Rodríguez-Sánchez [6], P. Roig [80], T. San José [12,13], E.P. Solodov [21], R. Sugar [81], K. Yu. Todyshev [21], A. Vainshtein [82], A. Vaquero Avilés-Casco [66], E. Weil [71], J. Wilhelm [12], R. Williams [71], A.S. Zhevlakov [78]

[1] Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK), Tsukuba 305-0801, Japan
[2] Nishina Center, RIKEN, Wako 351-0198, Japan
[3] Kobayashi–Maskawa Institute for the Origin of Particles and the Universe (KMI), Nagoya University, Nagoya 464-8602, Japan
[4] School of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, United Kingdom
[5] LPNHE, Sorbonne Université, Université de Paris, CNRS/IN2P3, Paris, France

* Corresponding authors.
E-mail address: MUON-GM2-THEORY-SC@fnal.gov (G. Colangelo, M. Davier, S.I. Eidelman, A.X. El-Khadra, M. Hoferichter, C. Lehner, T. Mibe, A. Nyffeler, B.L. Roberts, T. Teubner).

https://doi.org/10.1016/j.physrep.2020.07.006
0370-1573/

Hadronic Vacuum Polarization

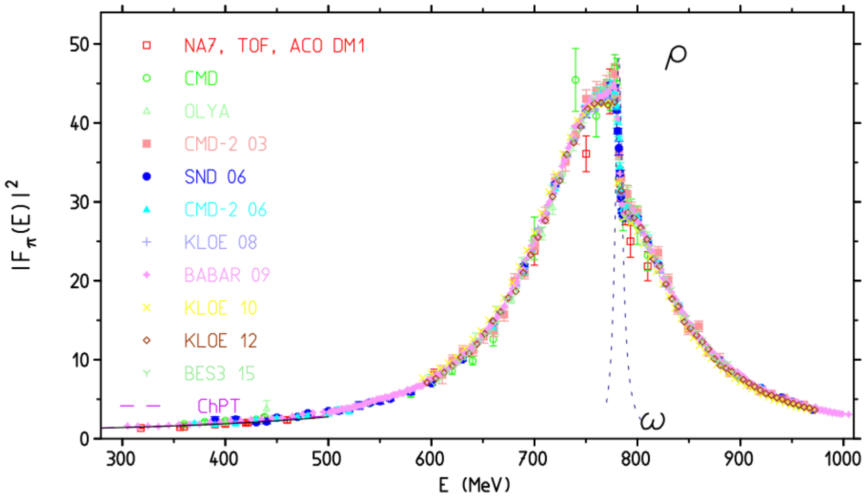
- Most important contribution is from low energies $\lesssim 1$ GeV, dominated by ρ and ω peaks, taking account of interference effects

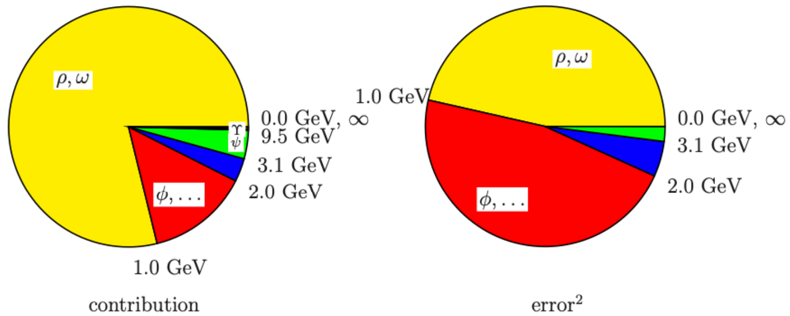
- Uncertainties dominated by ρ and ω region, and by region between 1 and 2 GeV (ϕ, etc.)

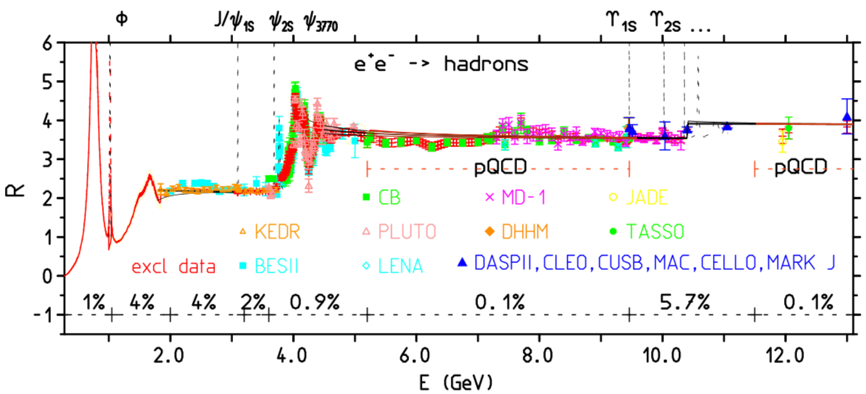
- High energies under good control from perturbative QCD

$$a_\mu^{\mathrm{HVP,\,LO}} = 693.1(2.8)_{\mathrm{exp}}(2.8)_{\mathrm{sys}}(0.7)_{\mathrm{DV+QCD}} \times 10^{-10}$$
$$= 693.1(4.0) \times 10^{-10}\,.$$

Aoyama et al, arXiv:2006.04822







τ Decays?

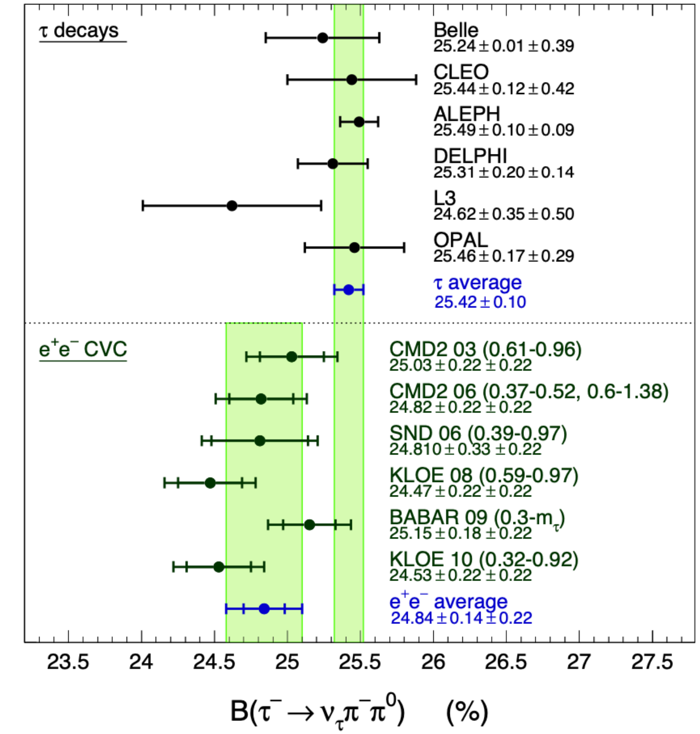
- Relation between τ decays and $I = 1$ portion of hadronic vacuum polarization:

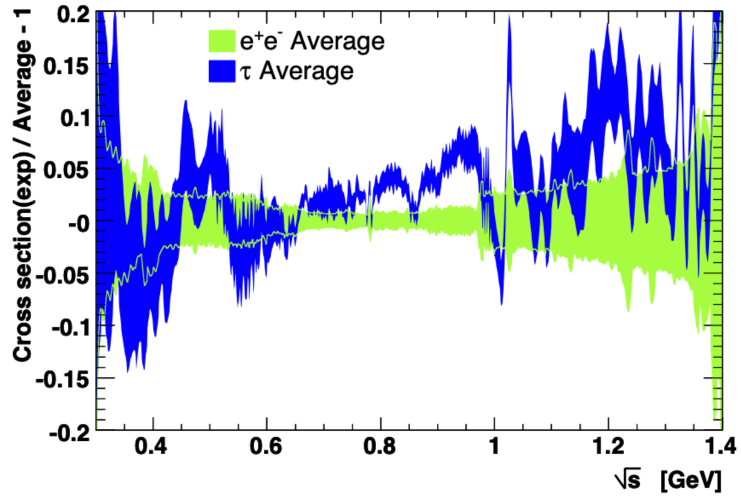
$$\sigma_{X^0}^{I=1}(s) = \frac{4\pi\alpha^2}{s} v_{1,X^-}(s)$$

$$v_{1,X^-}(s) = \frac{m_\tau^2}{6|V_{ud}|^2} \frac{\mathcal{B}_{X^-}}{\mathcal{B}_e} \frac{1}{N_X} \frac{dN_X}{ds} \times \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right)\right]^{-1} \frac{R_{\text{IB}}(s)}{S_{\text{EW}}}$$

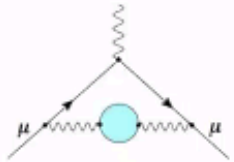
- BUT what about $I = 0$ portion?
- AND what about isospin breaking?
- AND uncertainties in τ decay data?
- NOT INCLUDED by Theory Initiative

Aoyama et al, arXiv:2006.04822

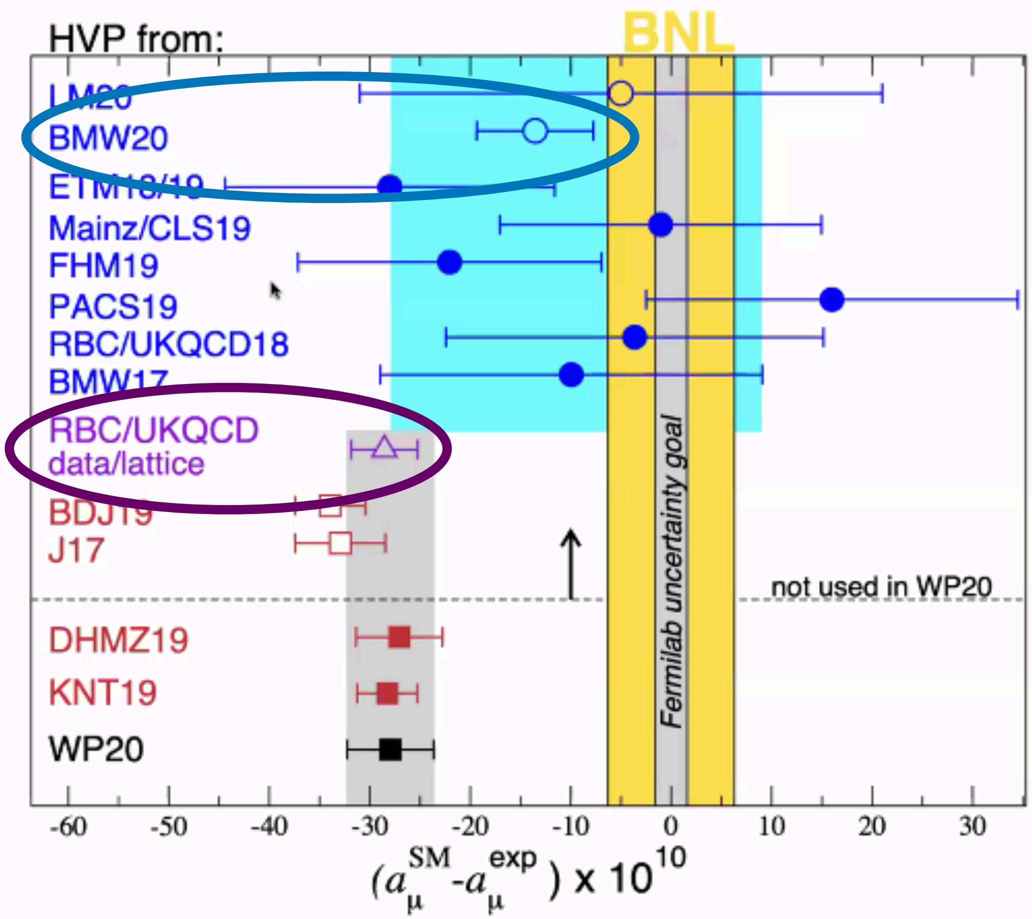




Comparison of Calculations of Hadronic Vacuum Polarization



$$a_\mu^{\mathrm{HVP}} + [a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{Weak}} + a_\mu^{\mathrm{HLbL}}] \;\rangle\; a_\mu^{\mathrm{SM}}$$

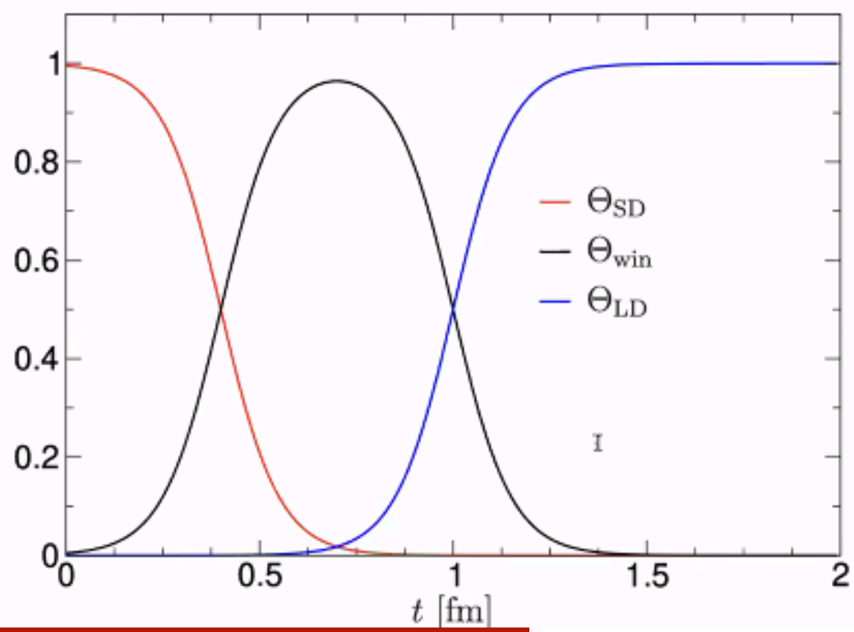


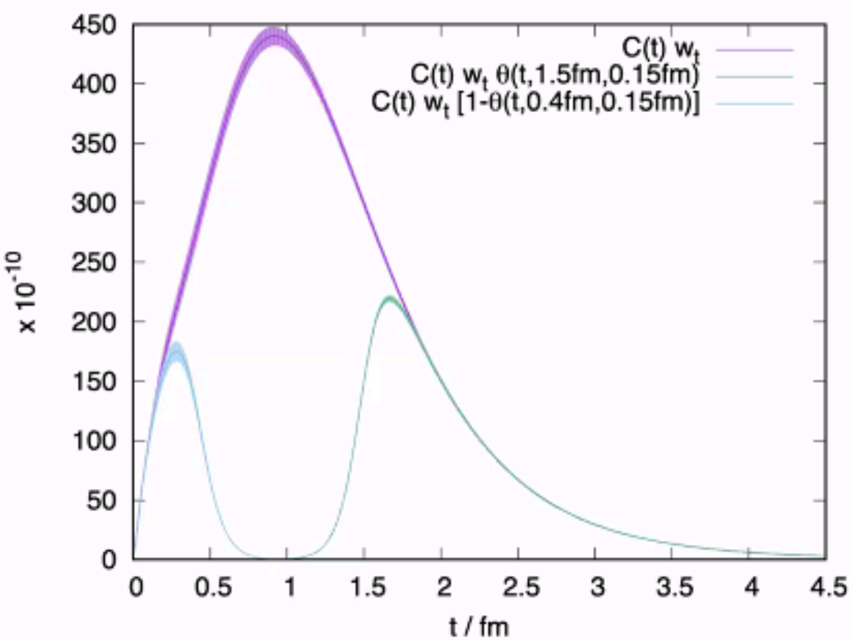
Aoyama et al, arXiv:2006.04822

RBC/UKQCD Hybrid Method

Replace lattice data at very short and long distances
by experimental e+e- scattering data

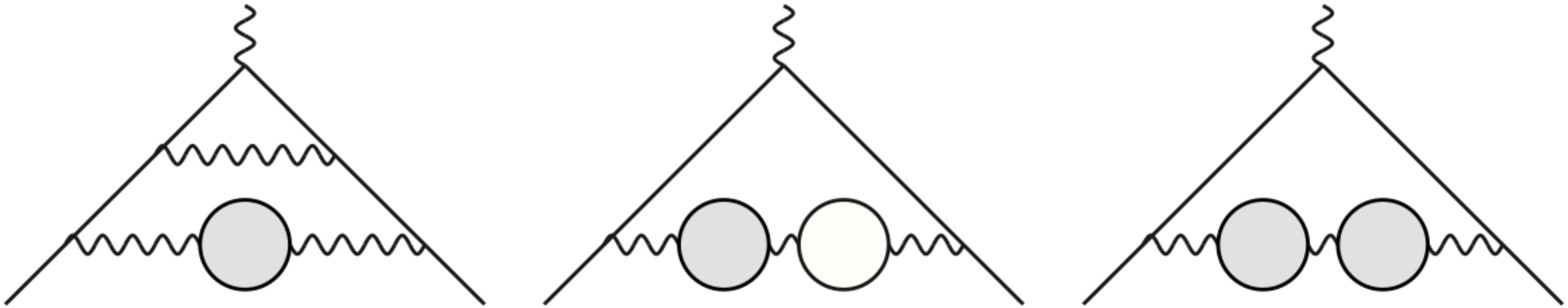
- Convert R-ratio data to Euclidean correlation function (via the dispersive integral) and compare with lattice results for windows in Euclidean time
- intermediate window:
  expect reduced FV effects and discretization errors





Blum et al, arXiv:1801.07224

Higher-Order Hadronic Vacuum Polarization

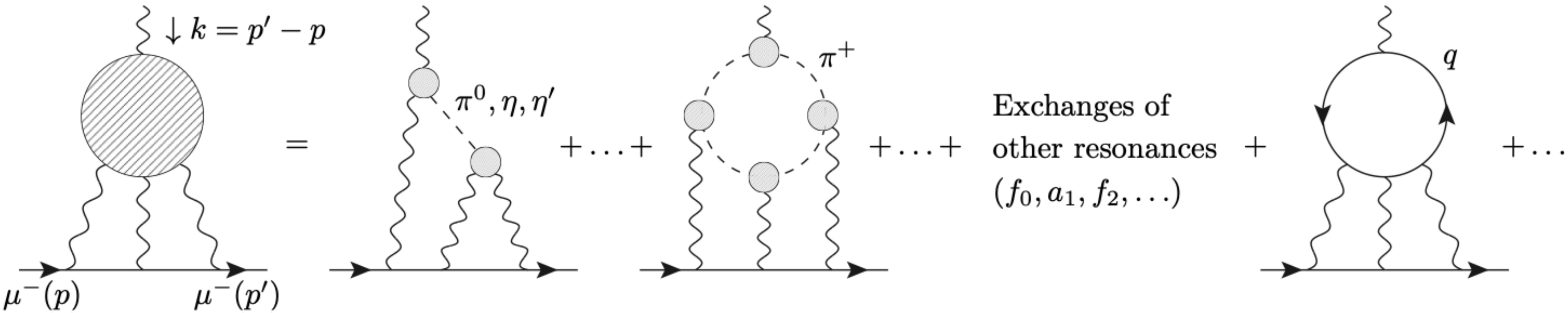


$$a_\mu^{\text{HVP, NLO}} = -9.83(7) \times 10^{-10}$$

$$a_\mu^{\text{HVP, NNLO}} = 1.24(1) \times 10^{-10}$$

Aoyama et al, arXiv:2006.04822

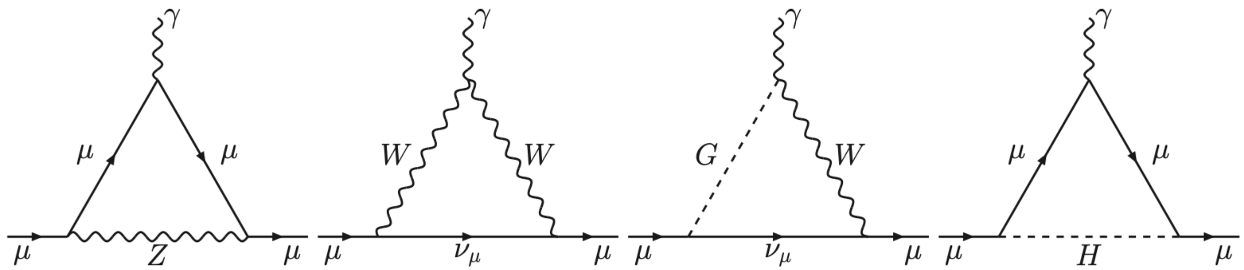
Light-by-Light Scattering



Contribution	PdRV(09) [475]	N/JN(09) [476, 596]	J(17) [27]	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	} − 1(3)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

Aoyama et al, arXiv:2006.04822

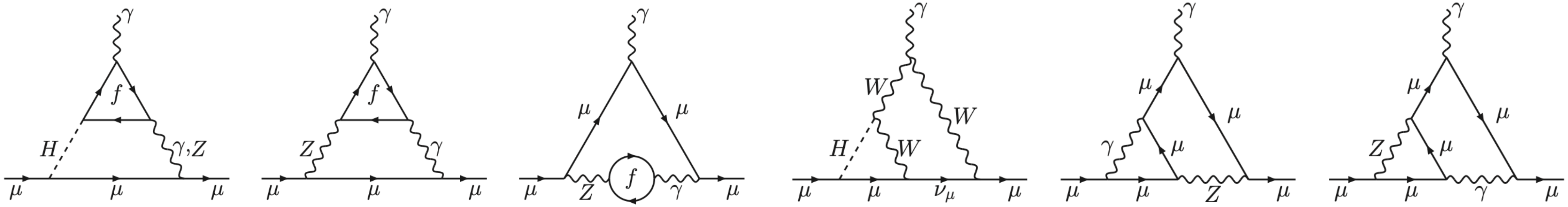
Electroweak Contributions



- Leading one-loop order

$$a_\mu^{\mathrm{EW}(1)} = \frac{G_\mathrm{F}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\left[\frac{5}{3} + \frac{1}{3}(1-4s_\mathrm{W}^2)^2\right] = 194.79(1)\times 10^{-11}$$

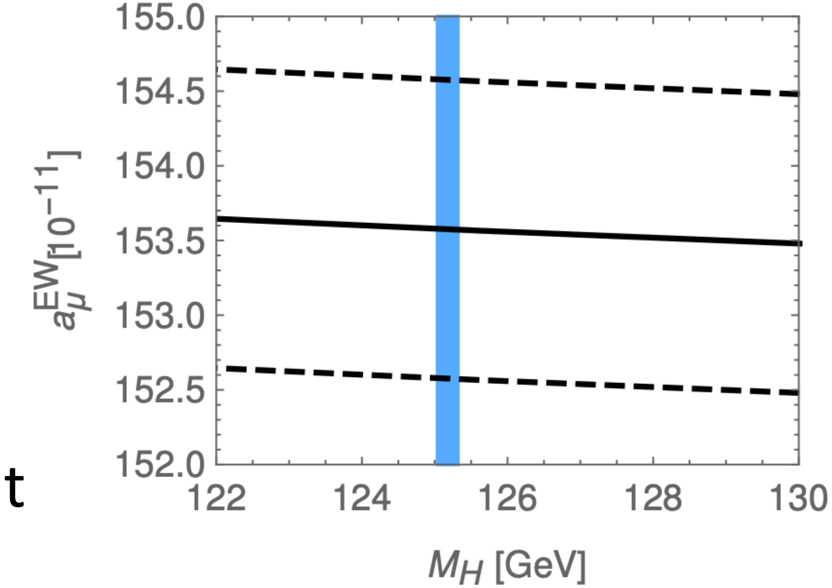
- Two-loop contributions



$$a_\mu^{\mathrm{EW}(2),\,\mathrm{logs}} = -4\frac{\alpha}{\pi}\log\frac{M_Z}{m_\mu}a_\mu^{\mathrm{EW}(1)} + \frac{G_\mathrm{F}m_\mu^2}{8\pi^2\sqrt{2}}\frac{\alpha}{\pi}\log\frac{M_Z}{m_\mu}\left[-\frac{47}{9} - \frac{11}{9}(1-4s_\mathrm{W}^2)^2\right] + \frac{G_\mathrm{F}m_\mu^2}{8\pi^2\sqrt{2}}\frac{\alpha}{\pi}\sum_f \log\frac{M_Z}{\max(m_f, m_\mu)}\left[-6g_A^\mu g_A^f N_f Q_f^2 + \frac{4}{9}g_V^\mu g_V^f N_f Q_f\right]$$

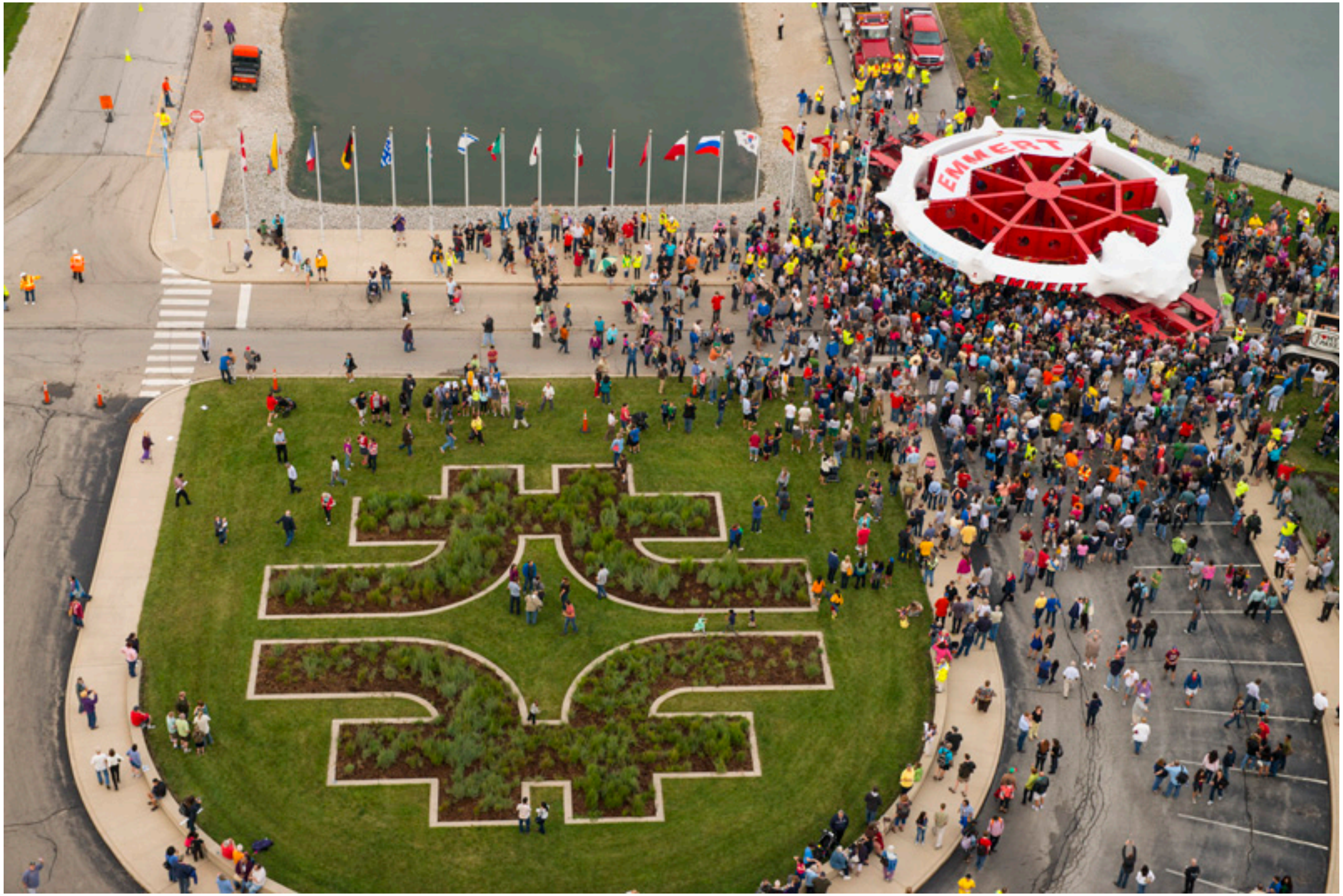
$$= -41.2(1.0)\times 10^{-11}$$

Combined result

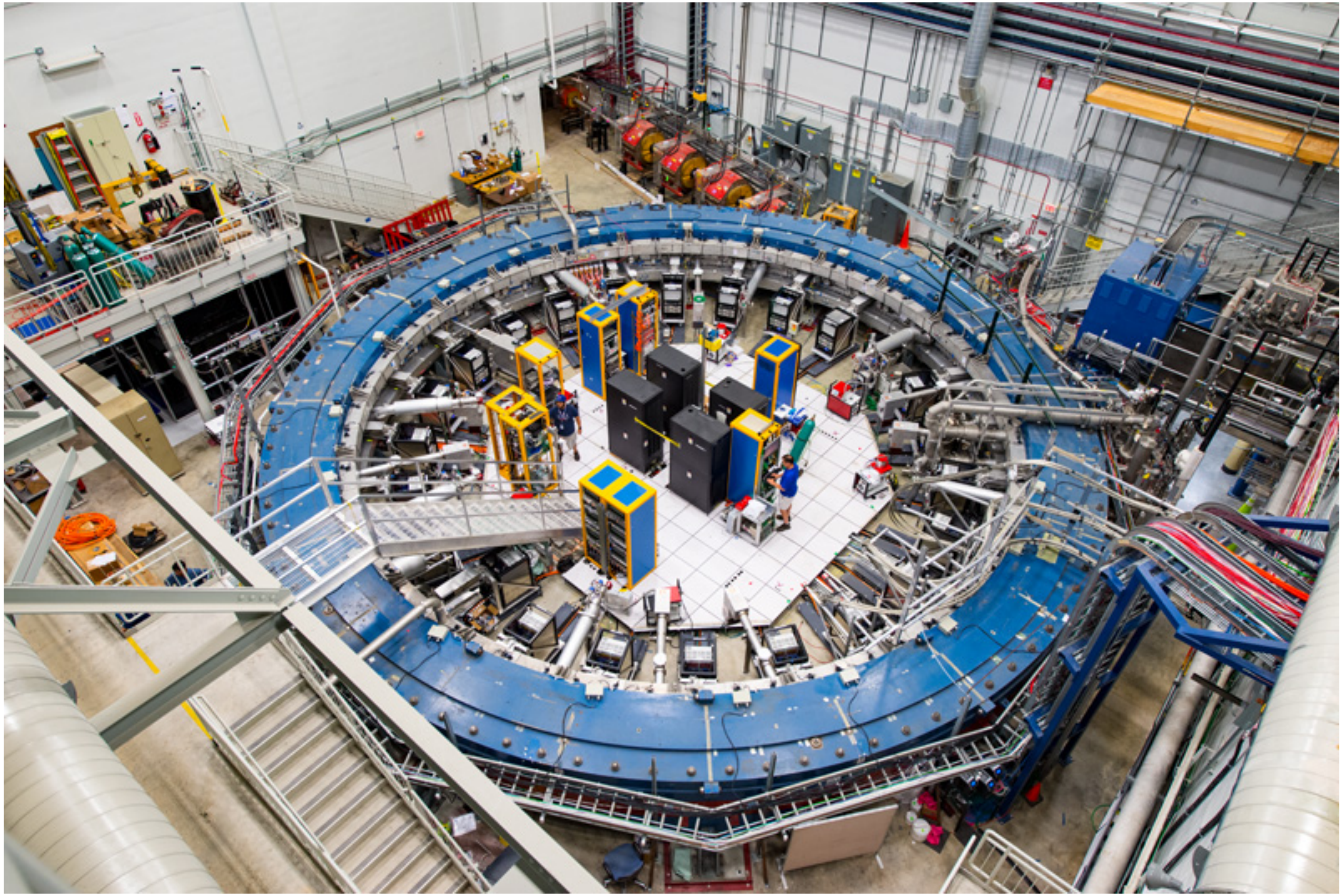


Aoyama et al, arXiv:2006.04822

Fermilab Experiment

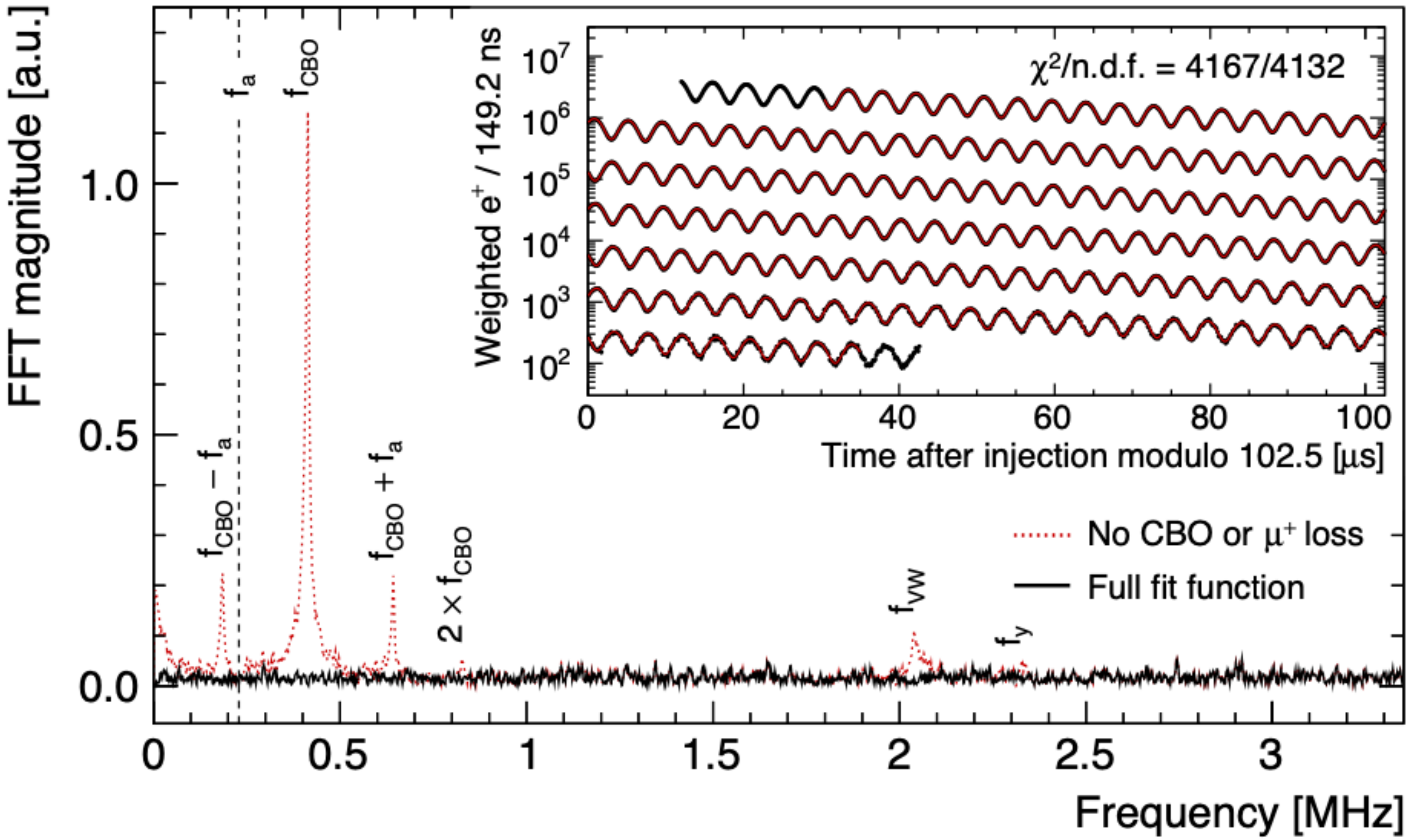


Fermilab Experiment



Does the magnet look familiar?

Fermilab Data



Fit to data + Fourier transform of residuals

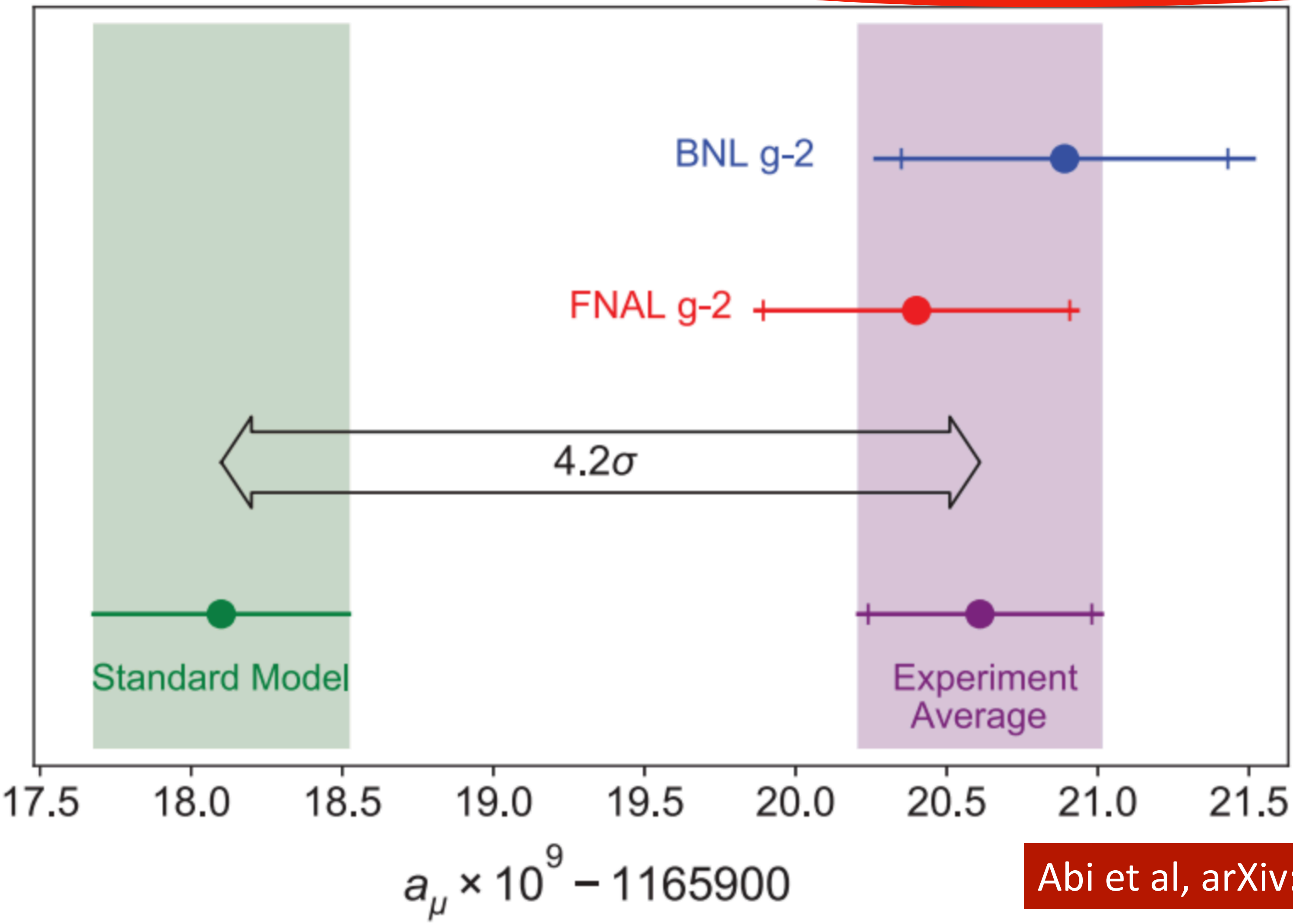
Abi et al, arXiv:2104.03281

Fermilab Measurement

FNAL result: $a_\mu(\text{FNAL}) = 116\,592\,040(54) \times 10^{-11}$ $(0.46\,\text{ppm})$

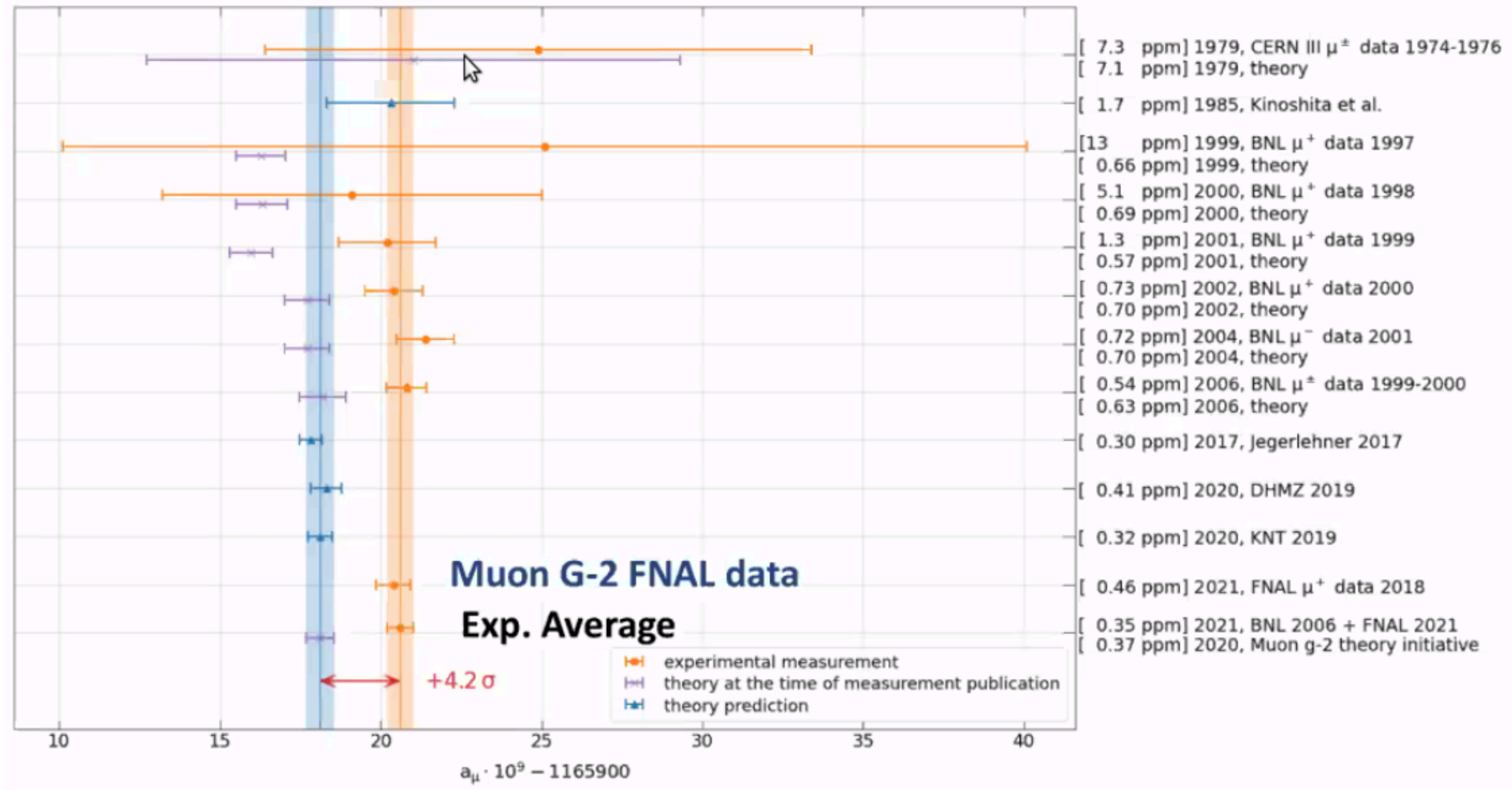
Combined result: $a_\mu(\text{Exp}) = 116\,592\,061(41) \times 10^{-11}$ $(0.35\,\text{ppm})$

Difference from Standard Model: $a_\mu(\text{Exp}) - a_\mu(\text{SM}) = (251 \pm 59) \times 10^{-11}$



Abi et al, arXiv:2104.03281

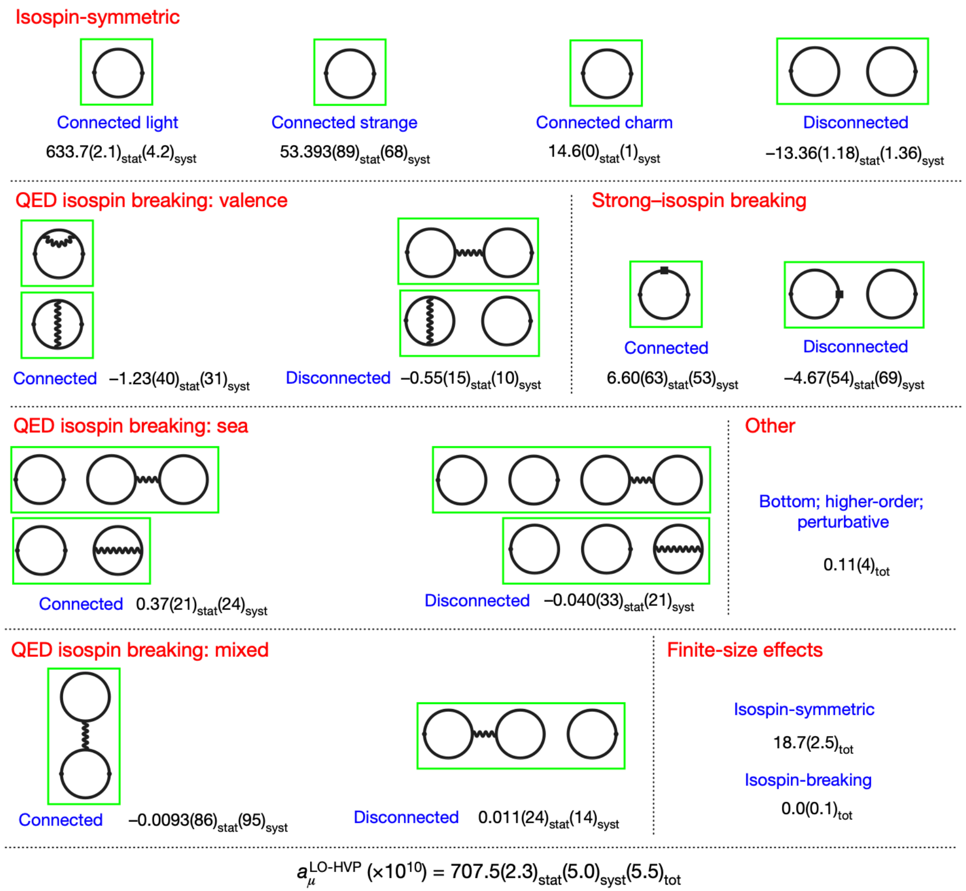
History of Measurements & Predictions

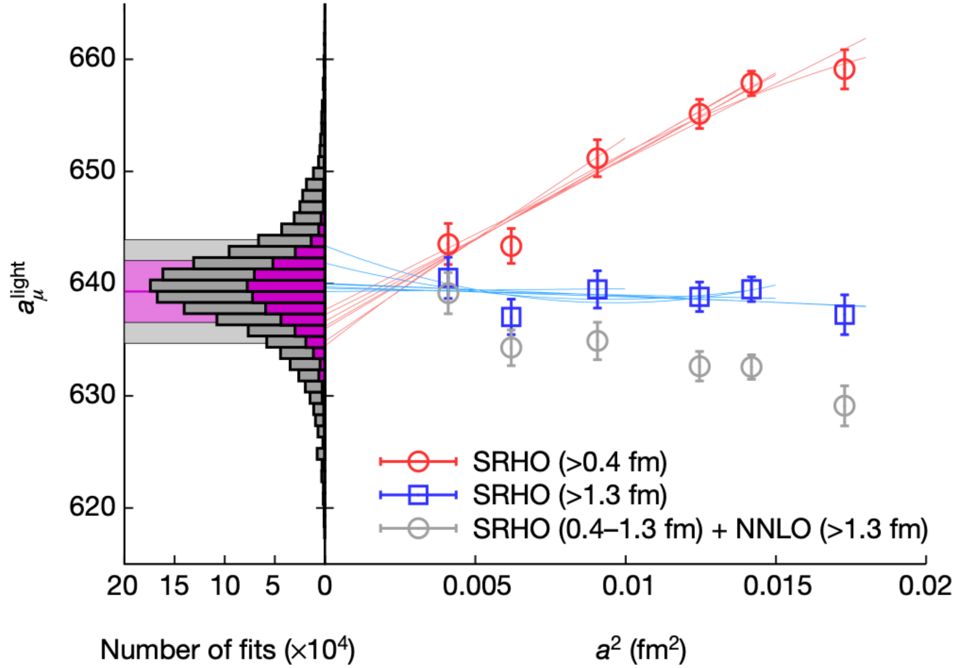


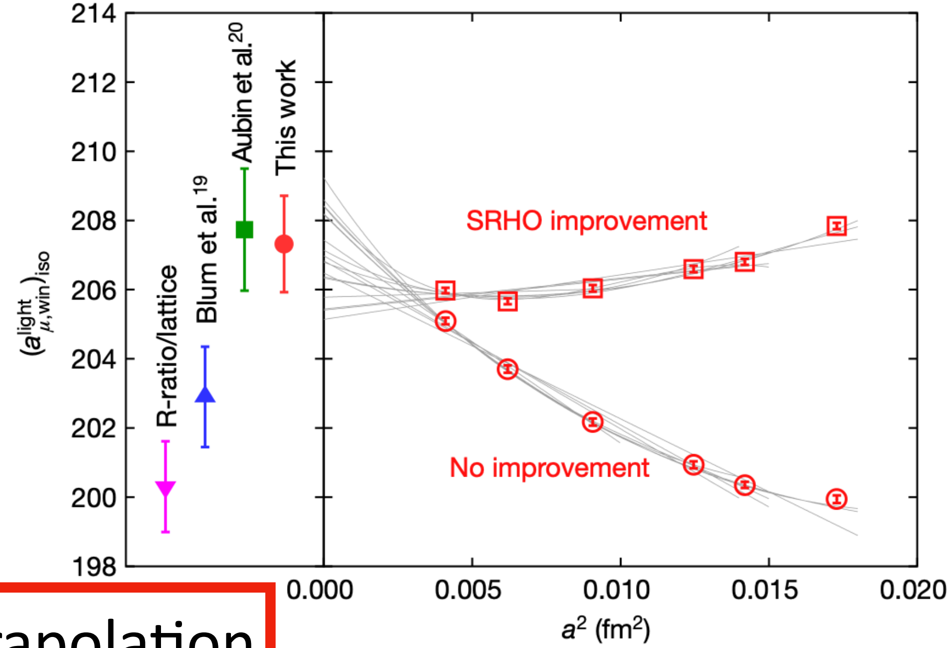
BUT

BMW Lattice Calculation

BMW Collaboration, Borsanyi et al, arXiv:2002.12347



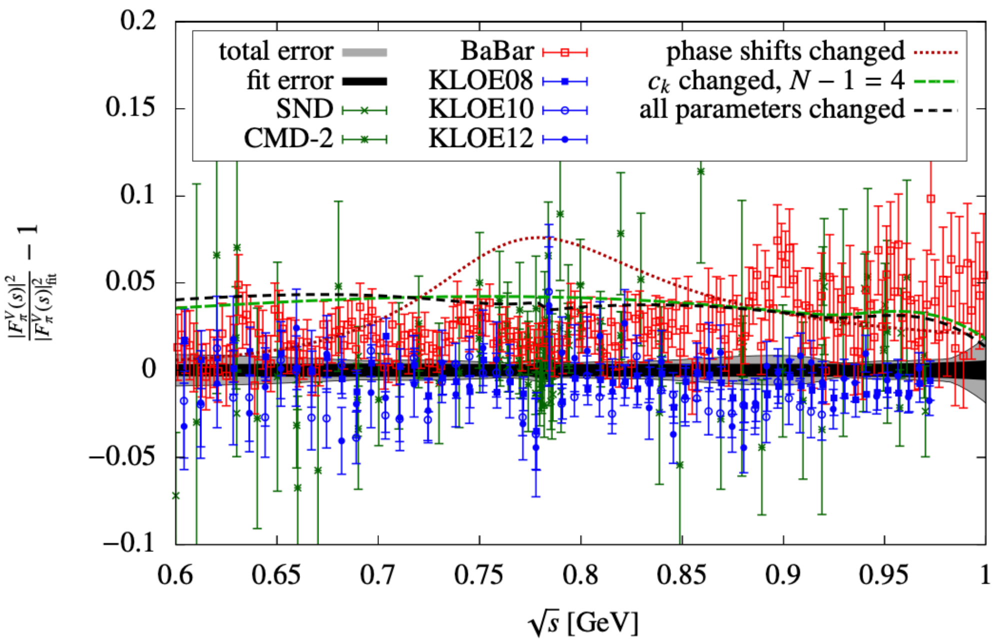


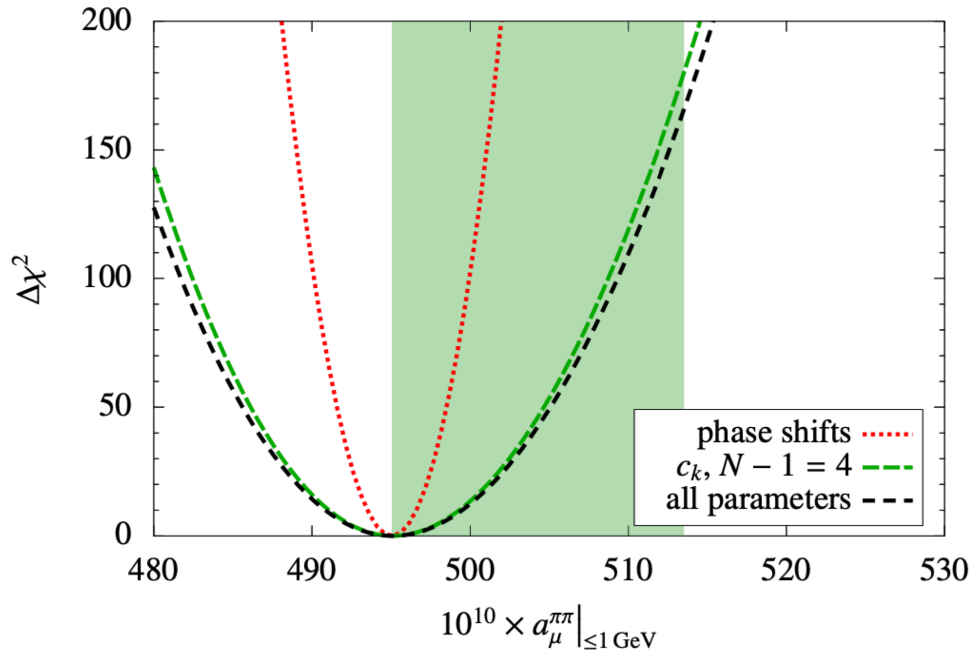


High statistics, accurate continuum extrapolation

$$a_\mu^{\mathrm{EXP}} - a_\mu^{\mathrm{BMW}} = 107(70) \times 10^{-11}$$

How to Accommodate BMW?





- Analyticity and unitarity constrain increase in $\pi^+\pi^-$ cross section < 1 GeV
- Maximum increase conflicts with data, does not change greatly prediction for a_μ
- Increase in cross section at higher energies affects electroweak observables

Colangelo, Hoferichter & Stoffer, arXiv:2010.07943

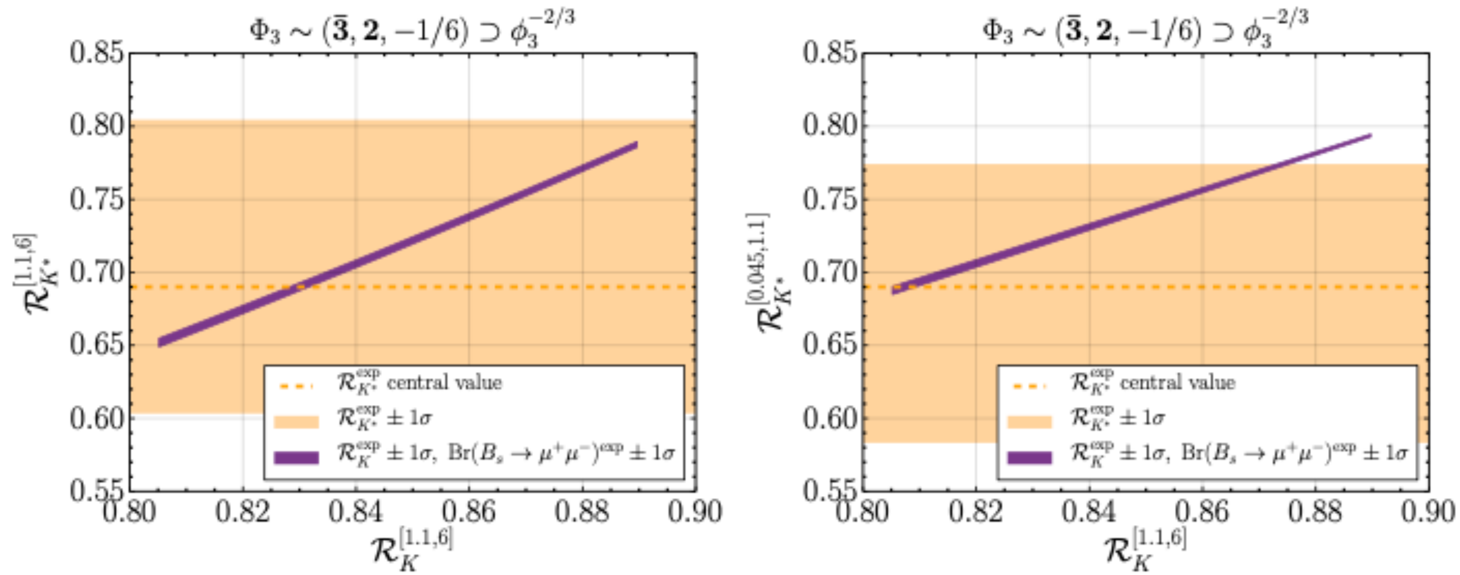
Interpretation Papers

2104.05685	Vector LQ	B	Du	
5656	L_\mu - L_\tau	DM	Borah	
5006	B_q - L_\mu	B	Cen	Leptoquarks
4494	LFV	LFV	Li	
4503	Pseudoscalar	DM, H decays	Lu	Extra U(1)
4456	2HDM	DM	Arcadi	
3542	B-LSSM	H decays	Yang	Extra Higgs
3701	Leptophilic spin 0	H factory	Chun	
3839	SUSY	HL-LHC	Aboubrahim	Supersymmetry
3691	Survey	DM, LHC	Athron	
3705	Seesaw	g_e	Escribano	Axion
3699	Gauged 2HDM	B	Chen	
3239	SUSY	Gravitino DM	Gu	
3284	NMSSM	DM	Cao	
3262	GUT-constrained SUSY	DM, LHC	Wang	
3292	MSSM	CPV	Han	
3296	lepton mass matrix	Flavour	Calibbi	
3280	Z_d	Cs weak charge	Cadeddu	
3334	E_6 3-3-1	H stability	Li	
3242	\mu-\tau-philic H	\tau decays, LHC	Wang	
3259	Anomaly mediation	DM	Yin	
3245	pMSSM	DM, fine-tuning	Van Beekveld	
3274	NMSSM	DM, AMS-02 pbar	Abdughani	
3290	MSSM	DM	Cox	
3367	2HDM	V-like leptons	Ferreira	
3267	Axion	Low-scale	Buen-Abad	
3340	L_\mu - L_tau	AMS-02 positrons	Zu	
3282	ALP	V-like fermions	Brdar	
3301	Lepton portal	DM	Bai	
3276	Dark axion portal	Dark photon	Ge	
3491	GmSUGRA	LHC	Ahmed	
3227	2HDM	LHC	Han	
3302	SUSY	small \mu	Baum	
3238	Scalar	DM, p radius	Zhu	
3489	\mu \nu SSM	B, H decays	Zhang	
3287	pMSSM	ILC	Chakraborti	
3228	DM	B, H decays	Arcadi	

890	Radiative seesaw		Chiang
2103.13991	Scalar LQ	B, H decays	Greljo
2012.11766	DM		D'Agnolo
2012.07894	Axions		Darmé
1812.06851	Charmphilic LQ		Kowalska
2104.04458	GUT-constrained SUSY	DM	Chakraborti
5730	LQ + charged singlet	B, Cabibbo	Marzocca
6320	L-R symmetry		Boyarkin
6858	L_\mu - L_\tau	\nu masses	Zhou
6854	D-brane	U(1), Regge	Anchordoqui
6656	vector LQ	B	Ban
7597	SUSY	LHC, landscape	Baer
7047	3HDM	Fermion masses	Carcamo
7680	Leptophilic Z'	Global analysis	Buras
8289	Custodial symmetry	Light scalar + pseudoscalar	Balkin
9205	U(1)D	Neutrino mass	Dasgupta
8819	Lepton non-universality	Naturalness	Cacciapaglia
8640	2x2x1	Higgses, heavy nus	Boyarkina
8293	Multi-TeV sleptons in FSSM	Extended H, tau decays	Altmannshofer
10114	SO(10)	Yukawa unification	Aboubrahim
7681	U(1)B-L	DUNE	Dev
10324	Gauged lepton number	Dark matter	Ma
10175	2HDM	Lighter Higgs?	Jueid
11229	LQ	Matter unification	Fileviez
15136	U(1)	HE neutrinos, H tension	Alonso
2105.00903	Anomalous 3-boson vertex	W mass	Arbuzov
7655	U(1)T3R	RK(*)	Dutta
8670	Leptoquark	nu mass, LFV	Zhang

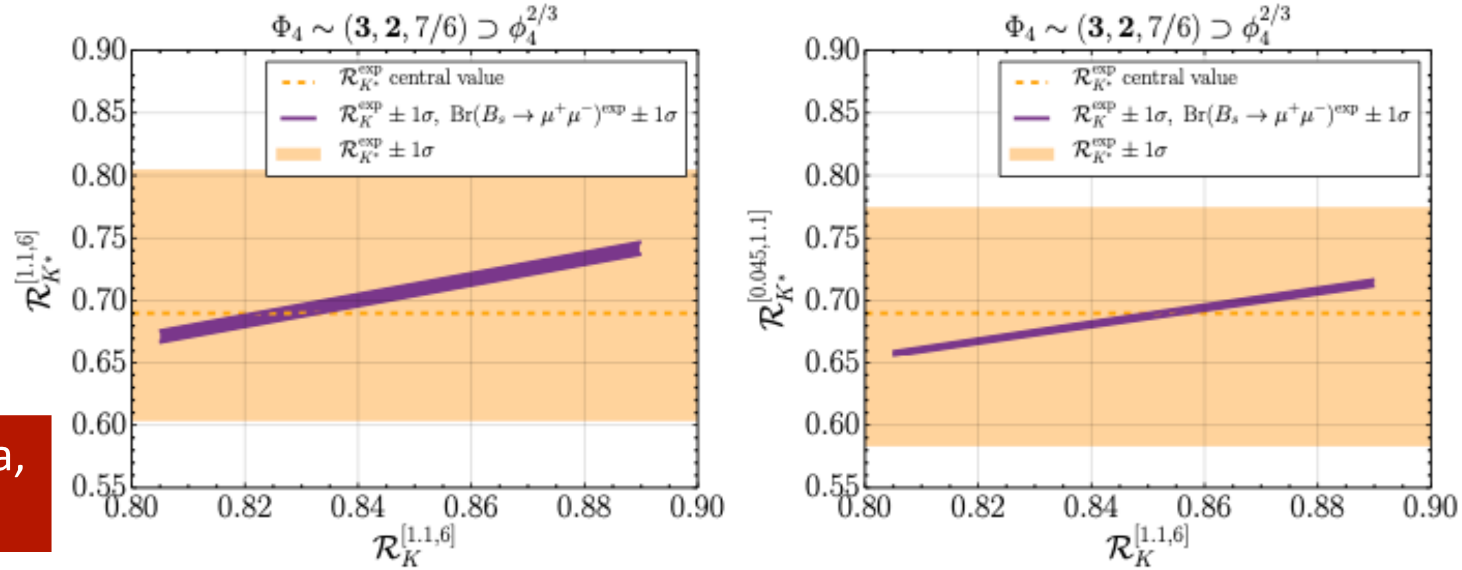
Scalar Leptoquarks

- Consider two types of leptoquarks with couplings



$$-\mathcal{L}_{QL}^{Y} \supset Y_2\, Q_L \Phi_3 (\nu^c)_L + Y_2\, \ell_L \Phi_4 (u^c)_L + Y_4\, \ell_L \Phi_3^\dagger (d^c)_L + Y_4\, Q_L \Phi_4^\dagger (e^c)_L + \text{h.c.}$$

- Consider constraints from $B_s \to \mu^+\mu^-, \mathcal{R}_K, \mathcal{R}_{K^*}$



Fileviez, Margui & Plascencia, arXiv:2103.13397

Lepton Flavour Violation in $B \to K\ell^+\ell^-$ Decays?

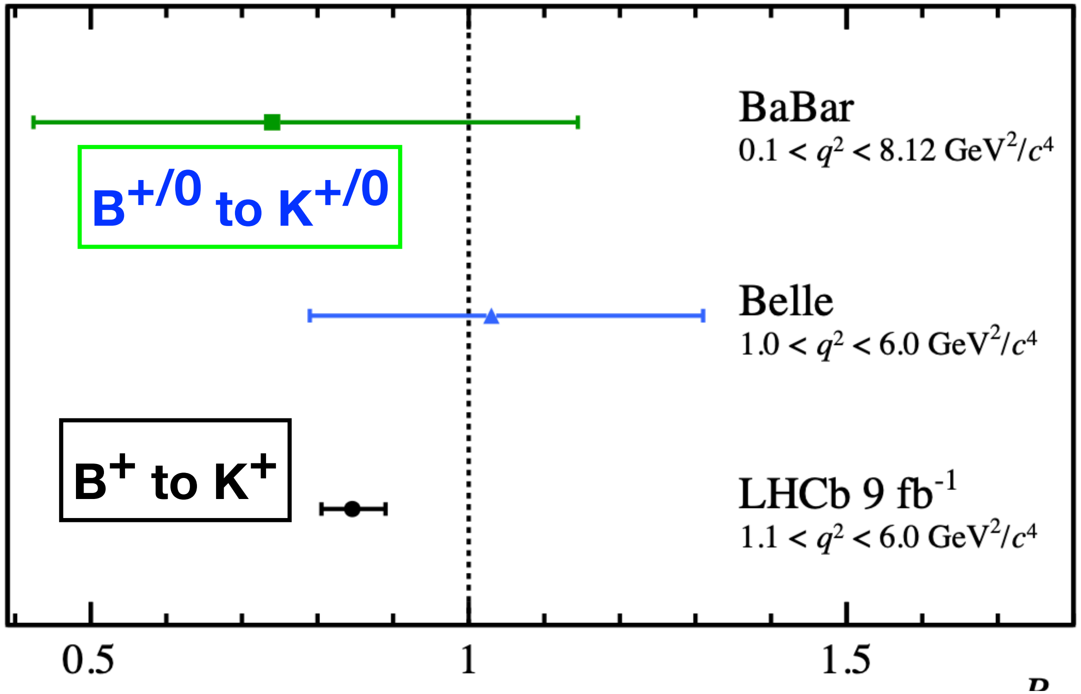
B decays to $e^+e^- > \mu^+\mu^-$

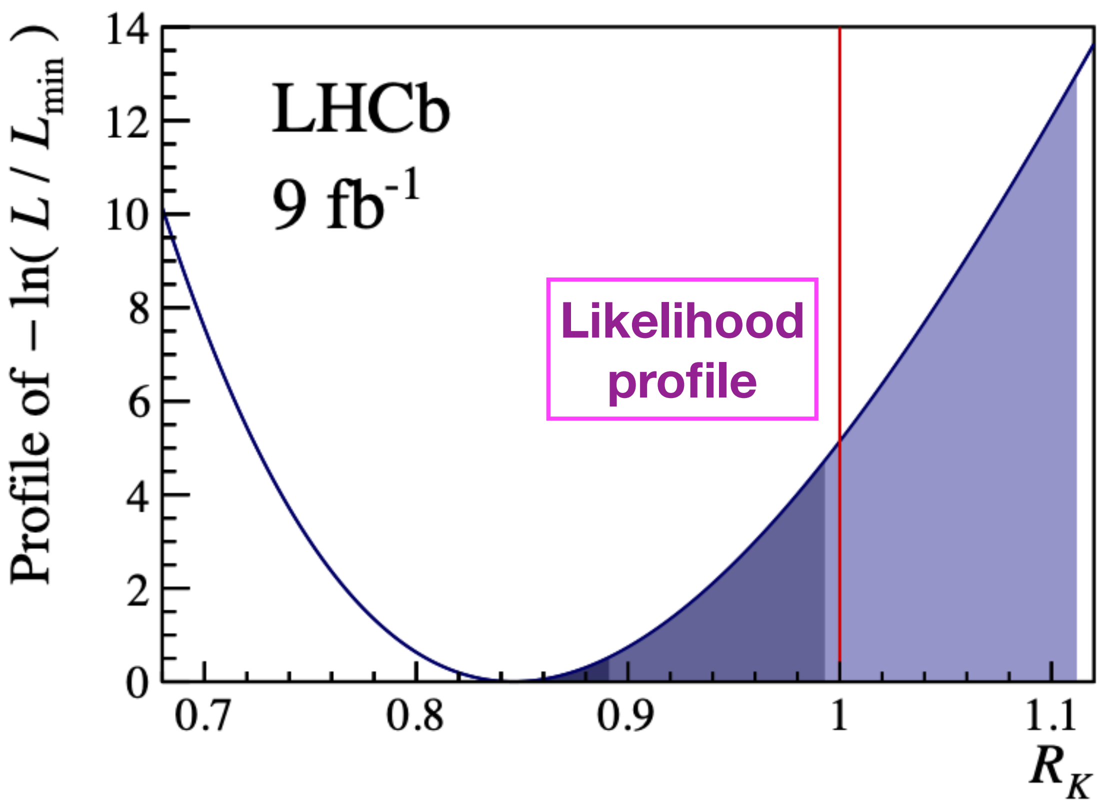
Prima facie violation of lepton universality

SM interactions flavour-universal

Except for Higgs couplings ∝ masses

LHCb Collaboration, arXiv:2103.11769



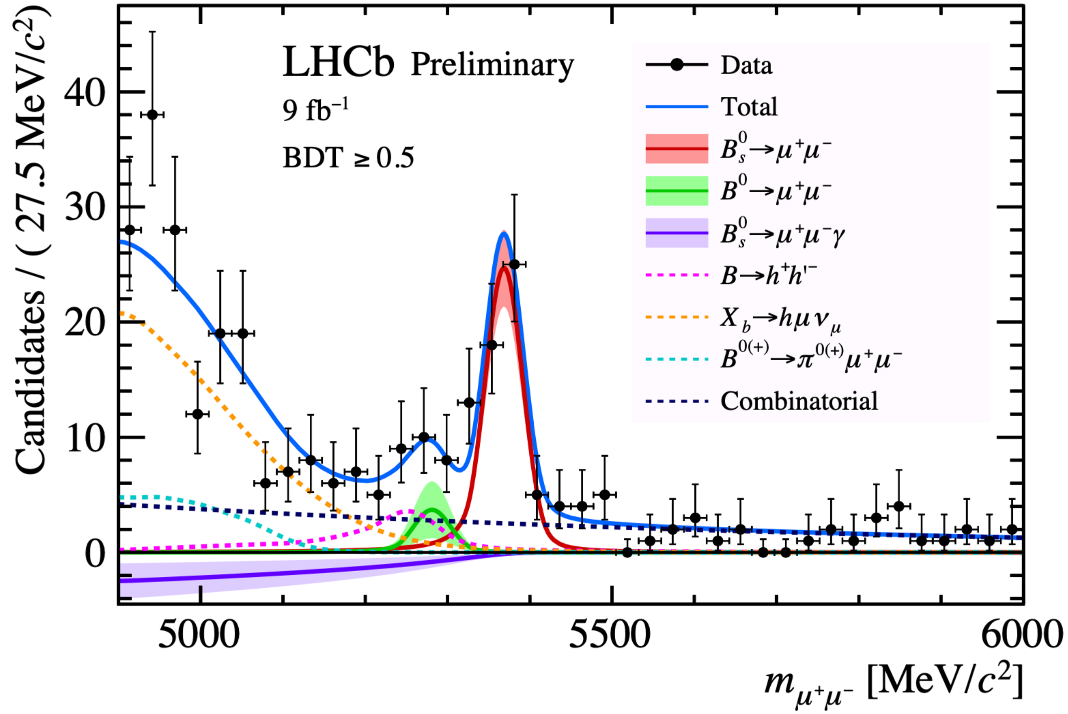


New LHCb $BR(B_s \to \mu^+\mu^-)$ Measurement

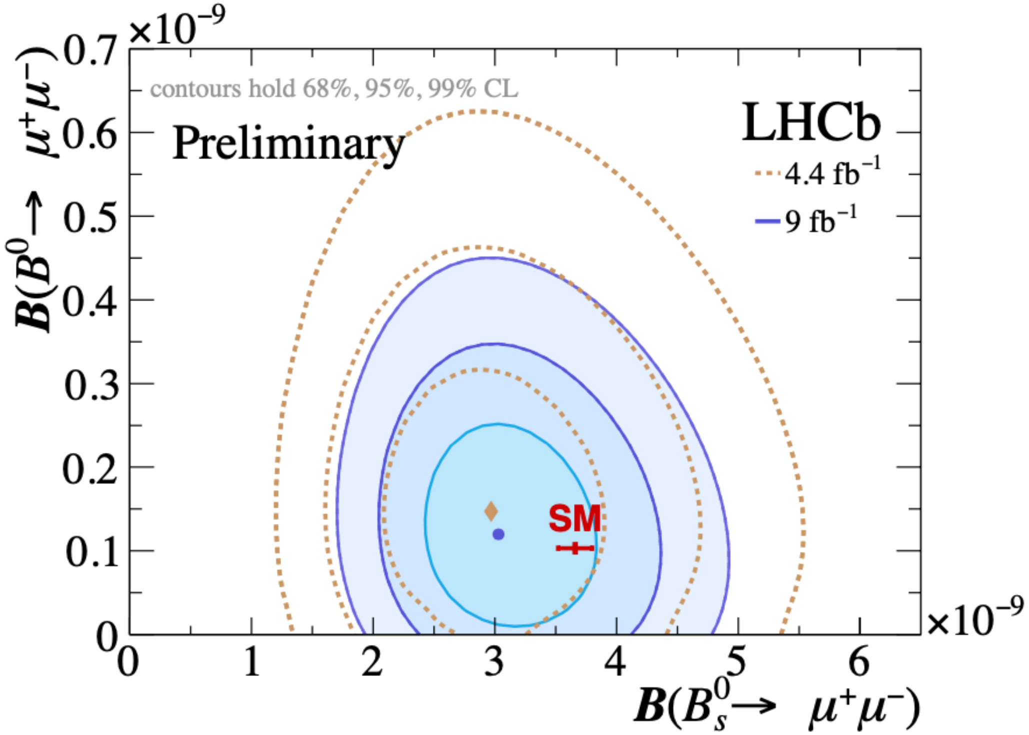
Rare decay induced by loop diagrams in SM

Measured value < SM prediction

Further evidence for new physics associated with the muon?



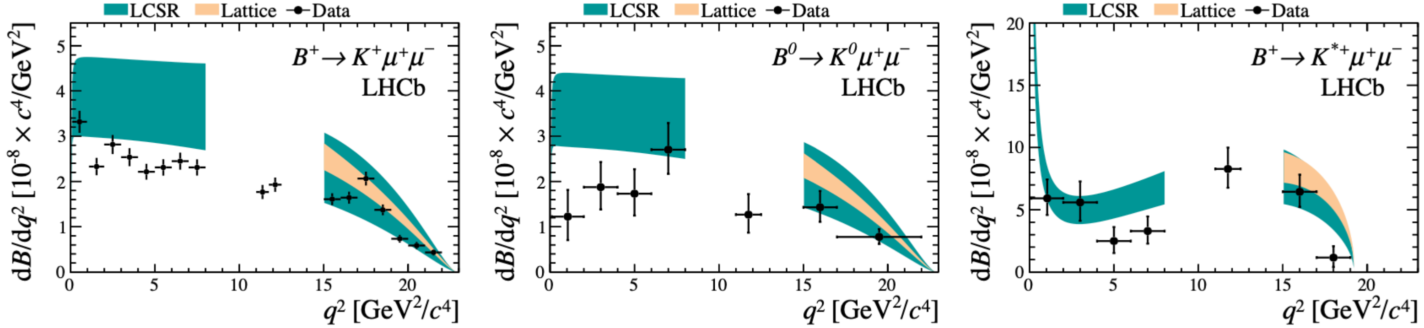
● $\mathcal{B}(B_s^0 \to \mu^+\mu^-) = (3.09^{+0.46+0.15}_{-0.43-0.11}) \times 10^{-9}$ (10.8σ)



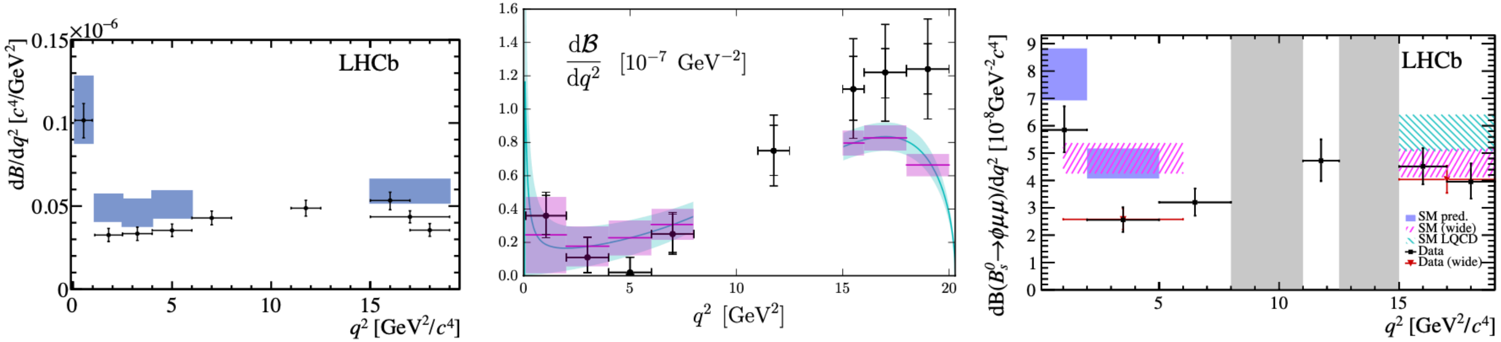
Other Previous Measurements

Rates

[JHEP06(2014)133]



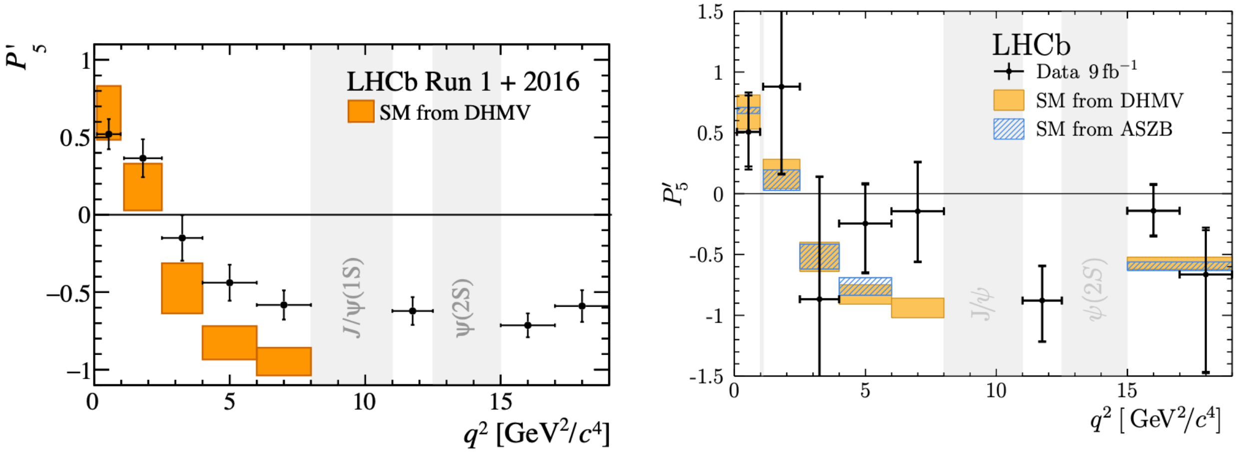
$B^0 \to K^{*0}\mu^+\mu^-$ [JHEP11(2016)047], $\Lambda_b \to \Lambda\mu^+\mu^-$ [JHEP06(2015)115] $B_s \to \phi\mu^+\mu^-$ [JHEP09(2015)179]



- SM predictions suffer from large hadronic uncertainties

Angular distributions

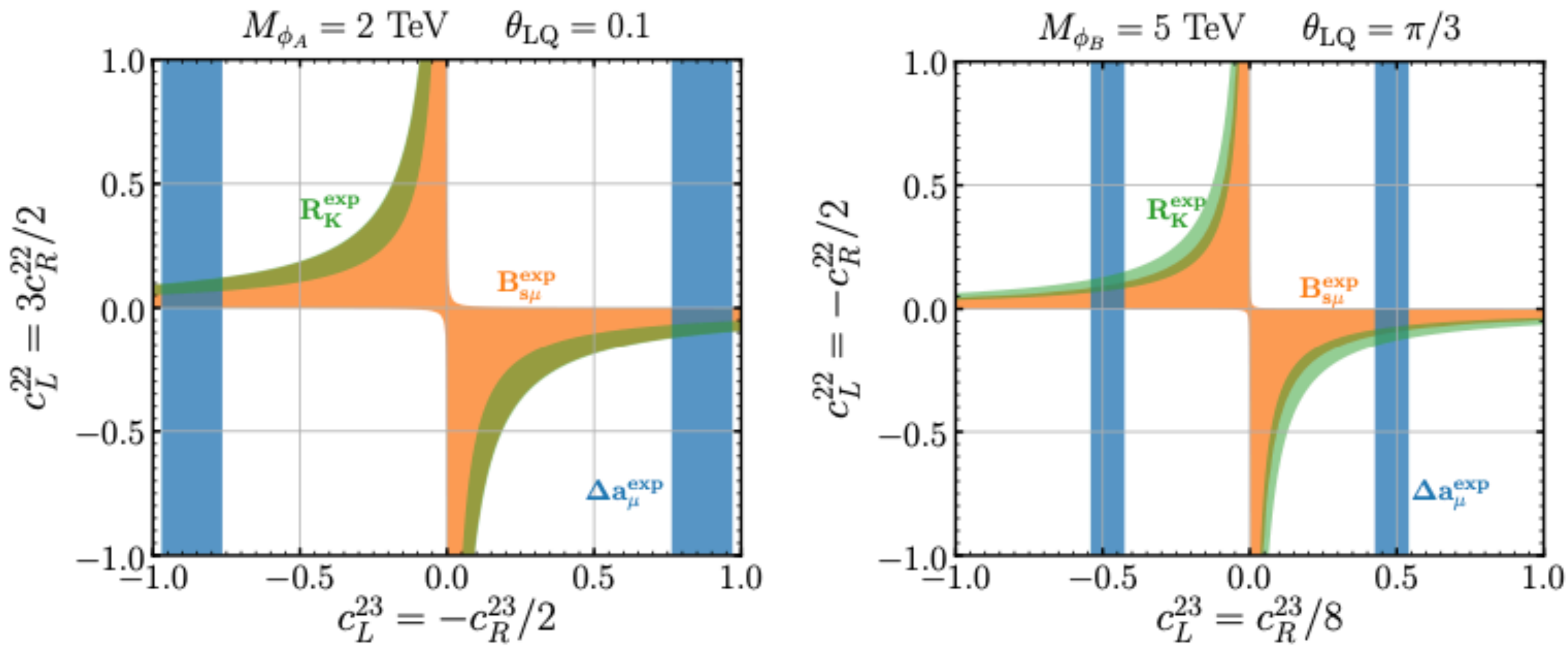
Left: $B^0 \to K^{*0}\mu^+\mu^-$ [PRL125011802(2020)], Right: $B^+ \to K^{*+}\mu^+\mu^-$ [arXiv:2012.13241]



Scalar Leptoquarks

- Consider 2 scenarios for mixing between leptoquarks:

$$-\mathcal{L} \supset \bar{e}^i \left(-\sin\theta_{\rm LQ}\, c_L^{ij}\, P_L + \cos\theta_{\rm LQ}\, c_R^{ij}\, P_R \right) d^j\, \phi_A^{-2/3}$$
$$+\ \bar{e}^i \left(\cos\theta_{\rm LQ}\, c_L^{ij}\, P_L + \sin\theta_{\rm LQ}\, c_R^{ij}\, P_R \right) d^j\, \phi_B^{-2/3} + \text{h.c.}$$



- Constraints from $g_\mu - 2$, $\mathcal{R}_K$, $B_s \to \mu^+\mu^-$

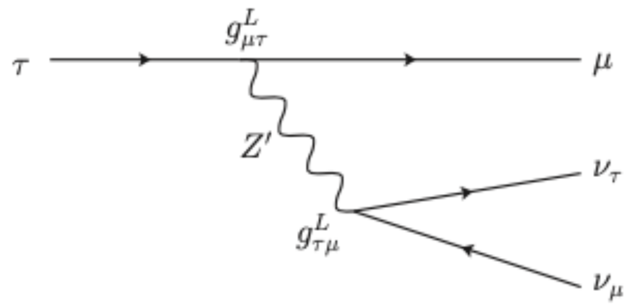
Fileviez, Margui & Plascencia, arXiv:2103.13397

Leptophilic Z’ Gauge Boson

- LHC sets strong bounds only if Z’ boson couples to quarks

- Weaker constraints on Z’ bosons coupled to leptons only

- $\ell \to \ell' \nu\bar{\nu}, \ell \to \ell'\gamma, \ell \to 3\ell'$, mixing with Z and anomalous magnetic moments



- Can explain $g_\mu - 2$ with Z' coupled to $L_\mu - L_\tau$

- Search for lepton flavour annihilation in $\tau \to \mu\nu\bar{\nu}/\tau \to e\nu\bar{\nu}$

$$\left.\frac{\mathcal{A}[\tau \to \mu\nu\bar{\nu}]}{\mathcal{A}[\mu \to e\nu\bar{\nu}]}\right|_{\mathrm{EXP}} = 1.0029 \pm 0.0014$$

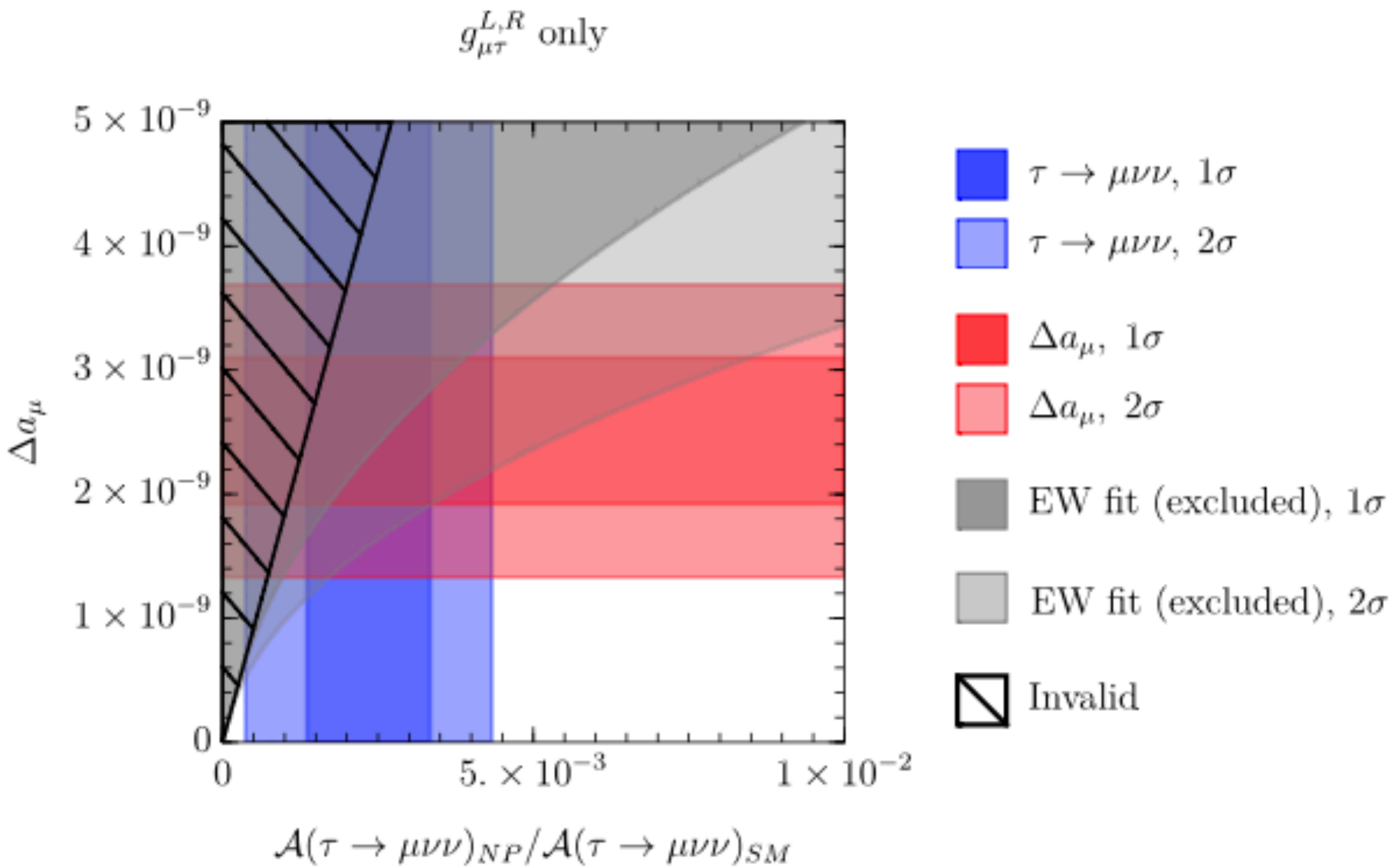
$$\left.\frac{\mathcal{A}[\tau \to \mu\nu\bar{\nu}]}{\mathcal{A}[\tau \to e\nu\bar{\nu}]}\right|_{\mathrm{EXP}} = 1.0018 \pm 0.0014$$

$$\left.\frac{\mathcal{A}[\tau \to e\nu\bar{\nu}]}{\mathcal{A}[\mu \to e\nu\bar{\nu}]}\right|_{\mathrm{EXP}} = 1.0010 \pm 0.0014$$

Buras, Crivellin, Kirk, Mansari & Montull, arXiv:2104.07680

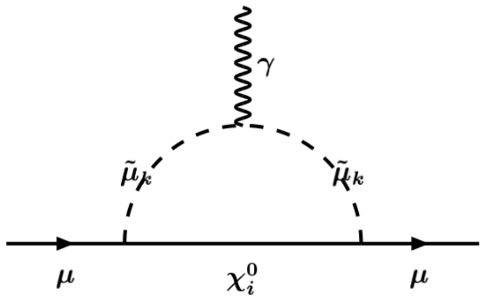
Leptophilic Z' Gauge Boson

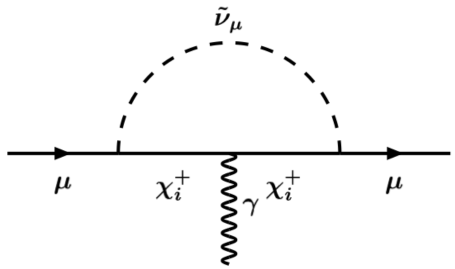
- Scenario with no Z - Z' mixing, left- and right-handed couplings to μ, τ only



Buras, Crivellin, Kirk, Mansari & Montull, arXiv:2104.07680

$g_\mu - 2$ in Supersymmetry





- Muon ψ_f, 4 neutralinos ψ_i, 2 smuons ϕ_k ($\tilde{\mu}_{L,R}$)

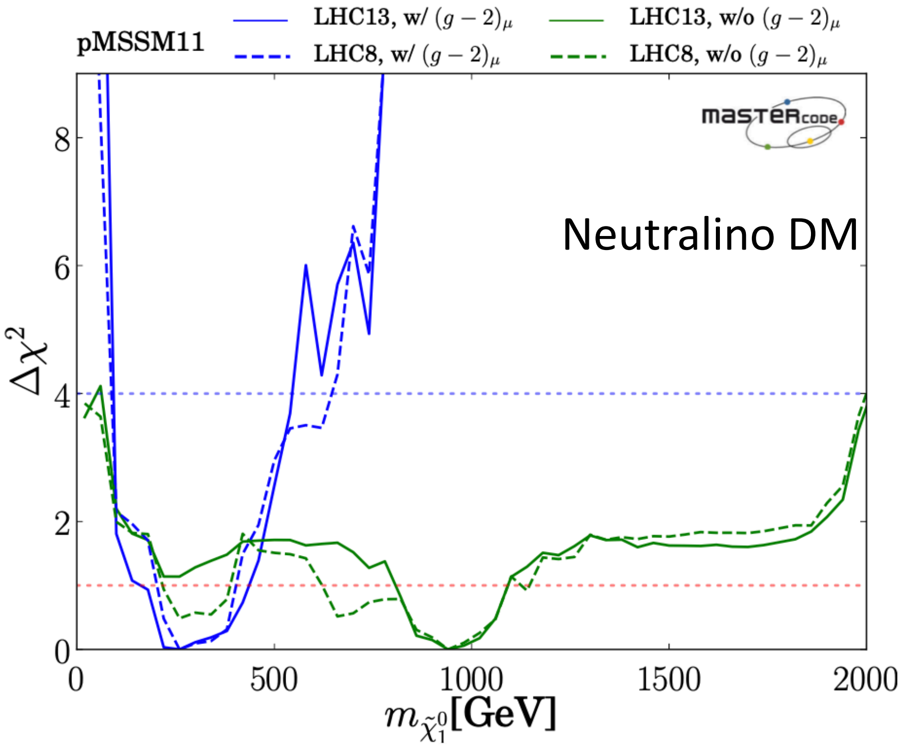
$$-\mathcal{L}_{int} = \sum_{ik} \bar{\psi}_f \left(K_{ik}\frac{1-\gamma_5}{2} + L_{ik}\frac{1+\gamma_5}{2}\right)\psi_i \phi_k + H.c.$$

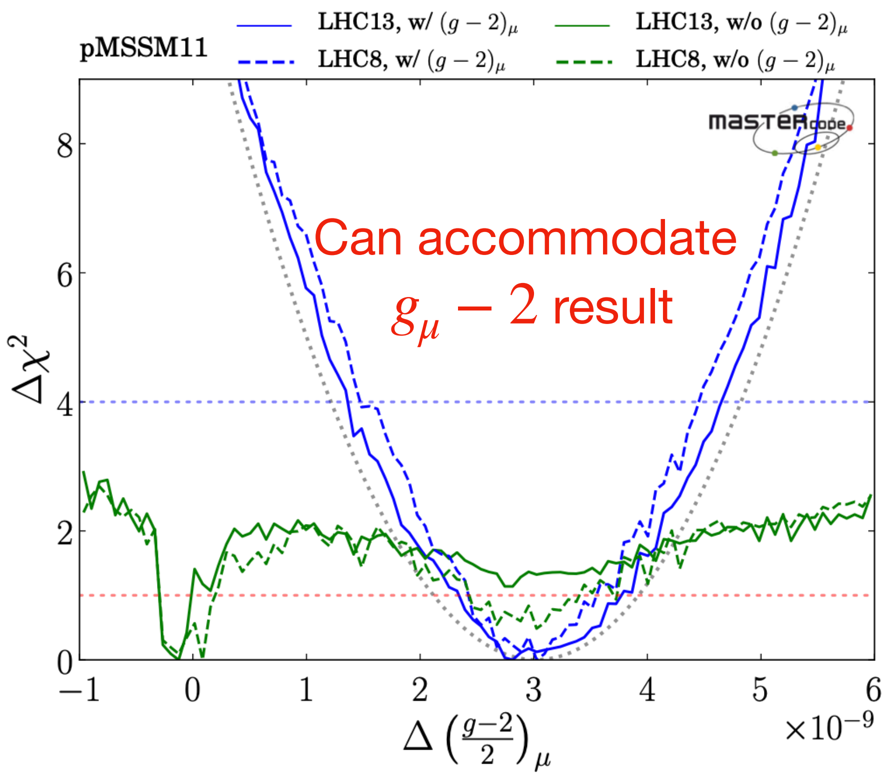
- One-loop contributions from smuon/neutralino loops:
  - Left-right mixing: $a_f^{11} = \sum_{ik} \frac{m_f}{8\pi^2 m_i} Re(K_{ik}L_{ik}^*) I_1\left(\frac{m_f^2}{m_i^2}, \frac{m_k^2}{m_i^2}\right)$ ← Most important
  - Unmixed: $a_f^{12} = \sum_{ik} \frac{m_f^2}{16\pi^2 m_i^2}(|K_{ik}|^2 + |L_{ik}|^2) I_2\left(\frac{m_f^2}{m_i^2}, \frac{m_k^2}{m_i^2}\right)$

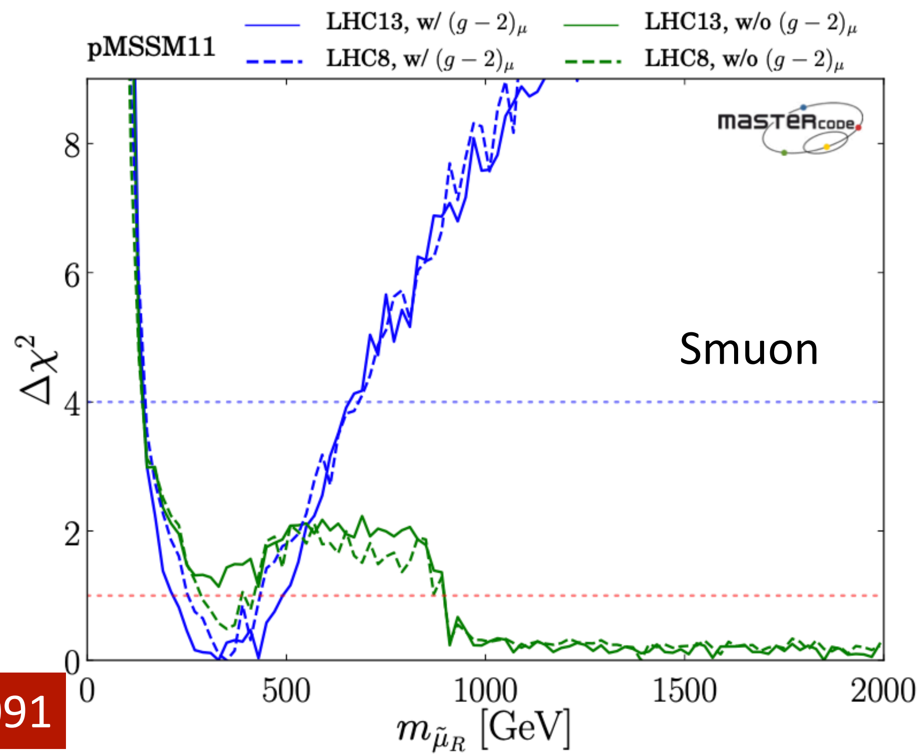
Ibrahim & Nath, hep-ph/9910553

$g_\mu - 2$ in Phenomenological Supersymmetry
(pMSSM11)

No relation between squark/gluino masses and slepton/neutralino masses







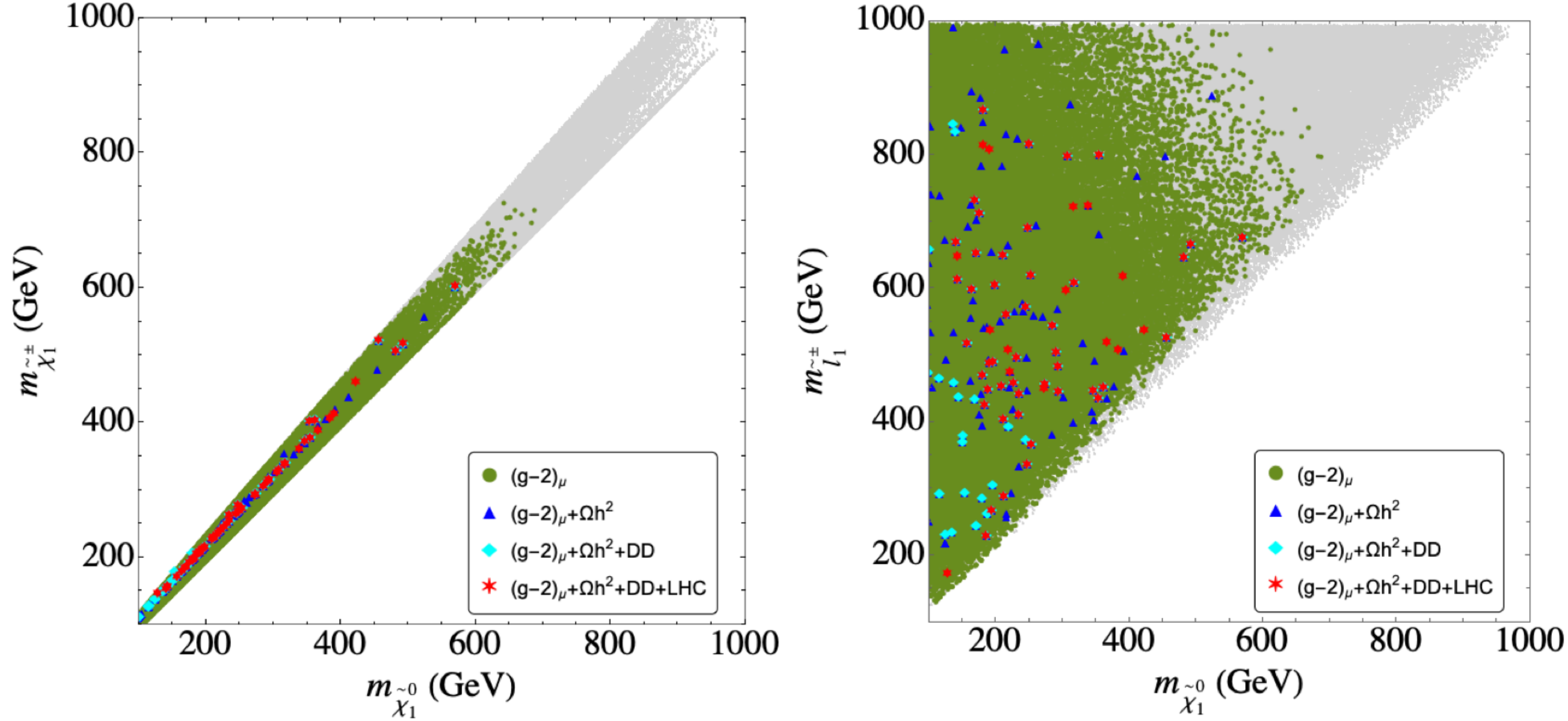
No problem accommodating BNL/FNAL result

Neutralino DM, smuon masses $\sim 300/400$ GeV

MasterCode, E. Bagnaschi, …, JE et al, arXiv:1710.11091

Supersymmetry

- $g_\mu - 2$-friendly scenario with light neutralino, chargino & slepton



- Red star points include all relevant LHC and direct scattering constraints

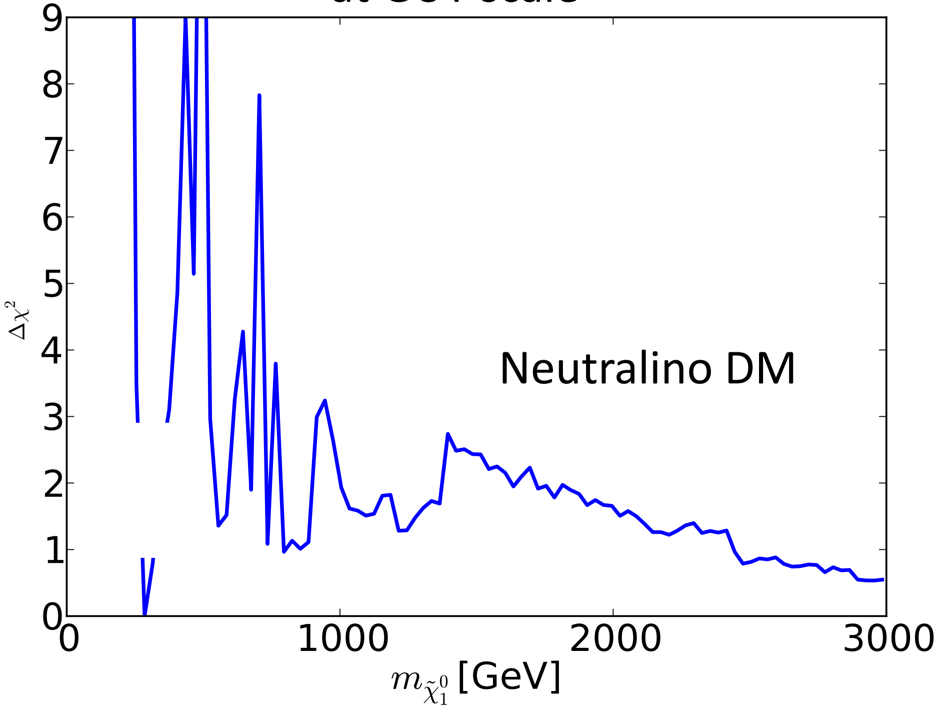
- Prospects for the ILC

Chakraborti, Heinemeyer & Saha, arXiv:2104.03287

$g_\mu - 2$ in Supersymmetric SU(5) GUT

(CMSSM)

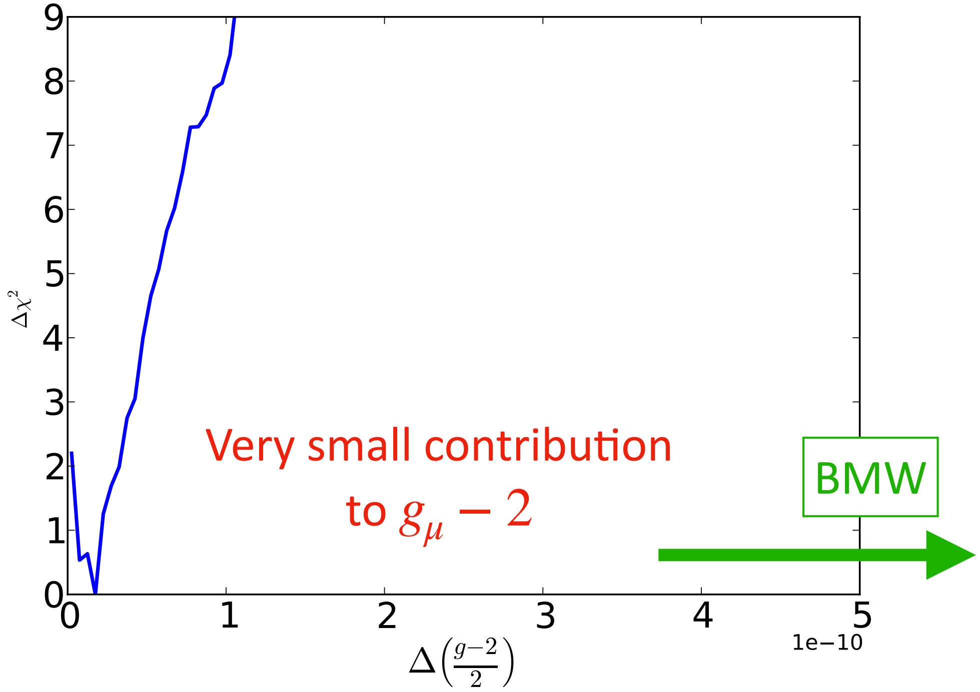
Assume universality between squark & slepton, and between gluon and electroweakino masses at GUT scale

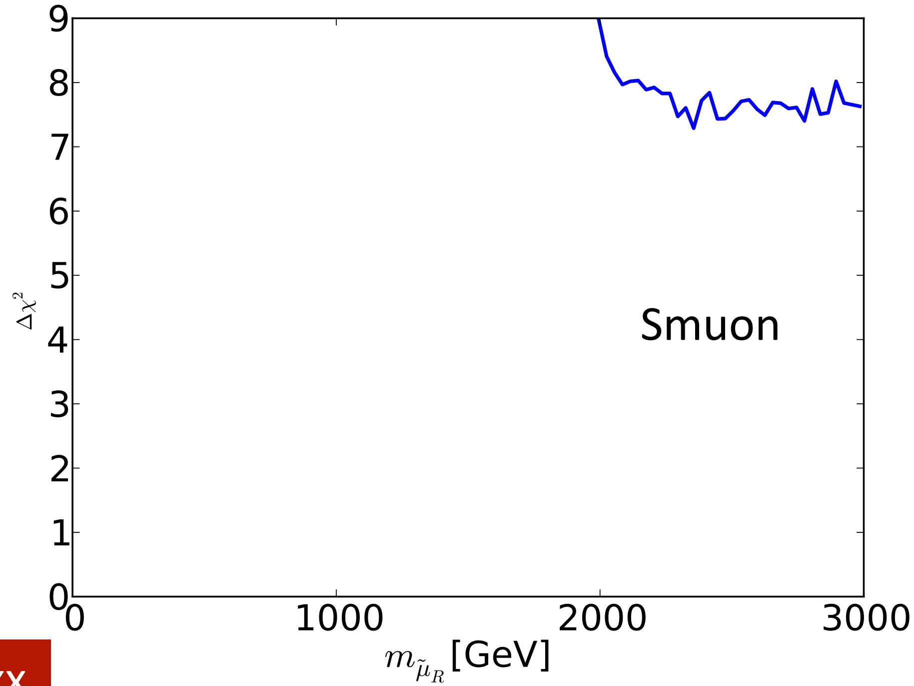


Scenario relates squark/gluino masses to slepton/neutralino masses

Cannot accommodate BNL/FNAL result

Smuon masses $\gtrsim 4$ TeV





MasterCode, E. Bagnaschi, …, JE et al, arXiv:2108.xxxxx

Flipped SU(5) GUT

- Extend GUT SU(5) with additional U(1) [motivated by string theory]

Antoniadis, JE, Hagelin & Nanopoulos, 1987

- "Flipped" fermion assignments to representations:

$$\bar{f}_i(\bar{\mathbf{5}}, -3) = \{U_i^c, L_i\} \ , \quad F_i(\mathbf{10}, 1) = \{Q_i, D_i^c, N_i^c\} \ , \quad l_i(\mathbf{1}, 5) = E_i^c \ , \quad i = 1, 2, 3$$

- Break GUT symmetry with 10-dimensional Higgses, electroweak symmetry with 5-dimensional Higgses:

$$H(\mathbf{10}, 1) = \{Q_H, D_H^c, N_H^c\} \ , \qquad \bar{H}(\bar{\mathbf{10}}, -1) = \{\bar{Q}_H, \bar{D}_H^c, \bar{N}_H^c\}$$

$$h(\mathbf{5}, -2) = \{T_{H_c}, H_d\} \ , \qquad \bar{h}(\bar{\mathbf{5}}, 2) = \{\bar{T}_{\bar{H}_c}, H_u\}$$

- Superpotential:

$$W = \lambda_1^{ij} F_i F_j h + \lambda_2^{ij} F_i \bar{f}_j \bar{h} + \lambda_3^{ij} \bar{f}_i \ell_j^c h + \lambda_4 H H h + \lambda_5 \bar{H} \bar{H} \bar{h} + \lambda_6^{ia} F_i \bar{H} \phi_a + \lambda_7^a h \bar{h} \phi_a + \lambda_8^{abc} \phi_a \phi_b \phi_c + \mu_\phi^{ab} \phi_a \phi_b \ ,$$

- Scan free parameters of model:

$$M_5, \ M_{X1}, \ m_{10}, \ m_5, \ m_1, \ \mu, \ M_A, \ A_0, \ \tan\beta$$

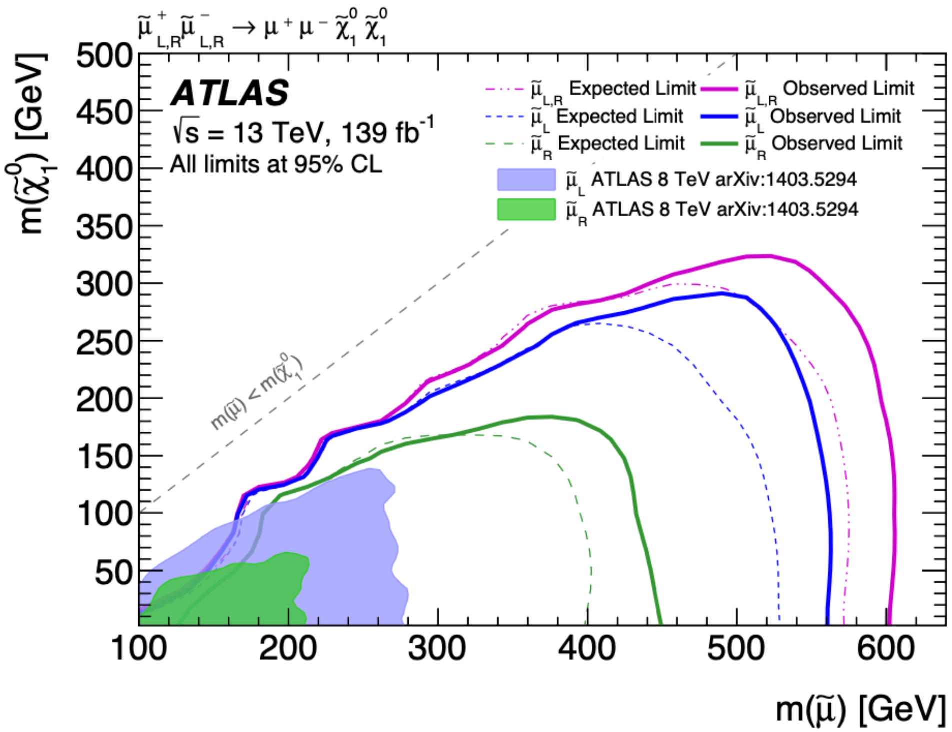
$g_\mu - 2$ in Flipped SU(5)

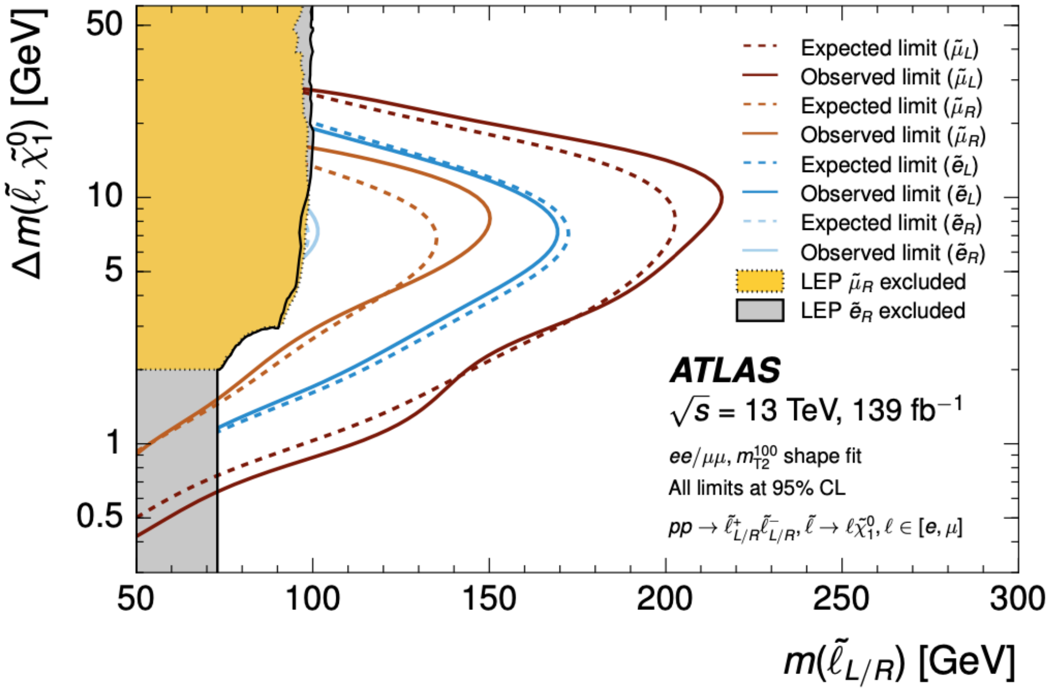
- Extend GUT SU(5) with additional U(1) [motivated by string theory]
- Supersymmetric partner of right-handed muon in singlet representation, mass independent of other sparticle masses
- Lightest supersymmetric particle (LSP) is mixture of neutral gauginos and Higgsinos
- Mass of additional U(1) gaugino is independent of other gauginos
- Smuon and LSP can be much lighter than in conventional SU(5)
- Not subject to strong LHC constraints
- Large contribution to $g_\mu - 2$ is possible

JE, Evans, Nagata, Nanopoulos & Olive, arXiv:2107.03025

LHC vs Supersymmetry

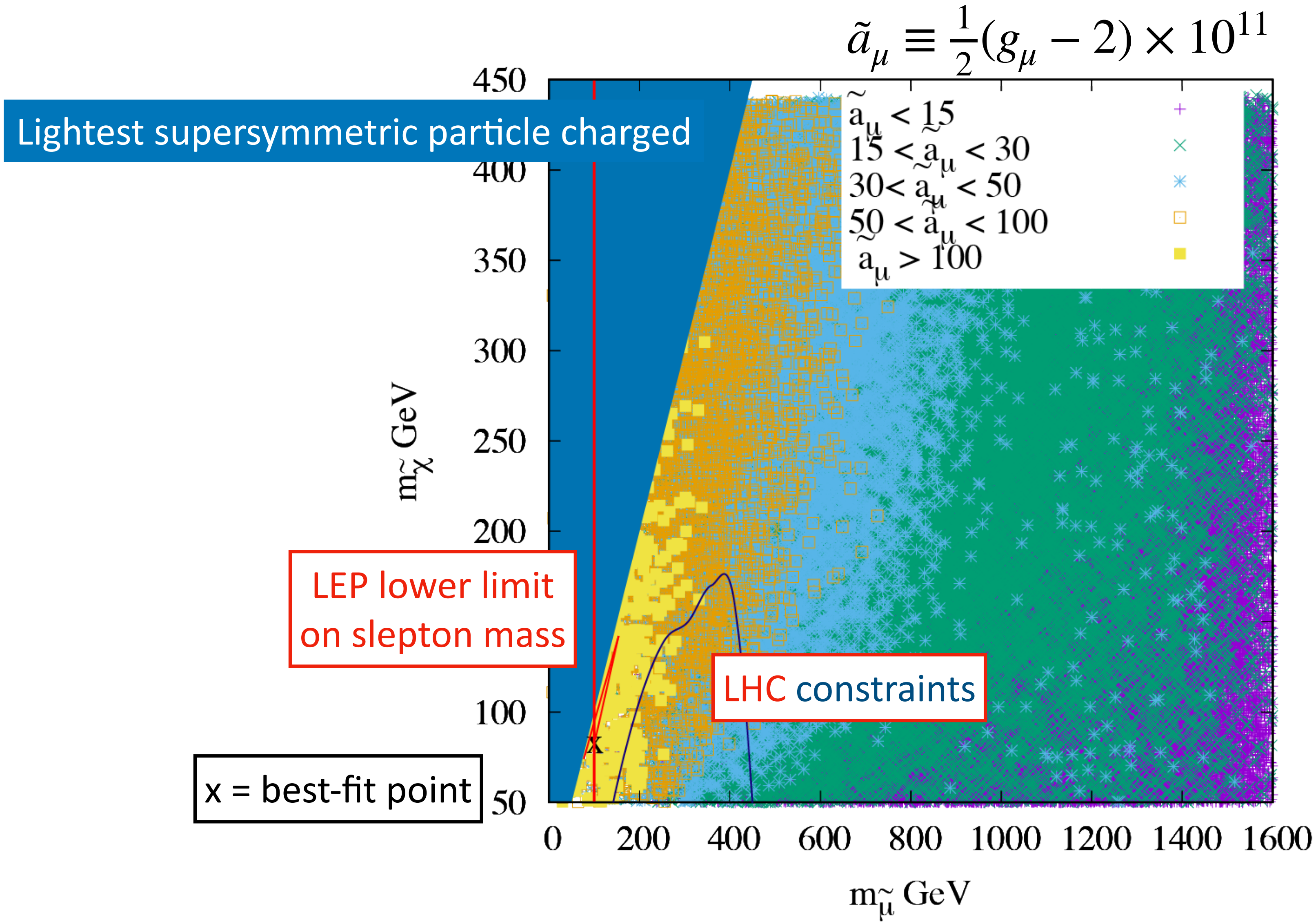
- LHC does not exclude (relatively) light electroweakly-interacting particles, e.g., sleptons





- LHC favours squarks & gluinos > 2 TeV (but loopholes)

$g_\mu - 2$ in Flipped SU(5)

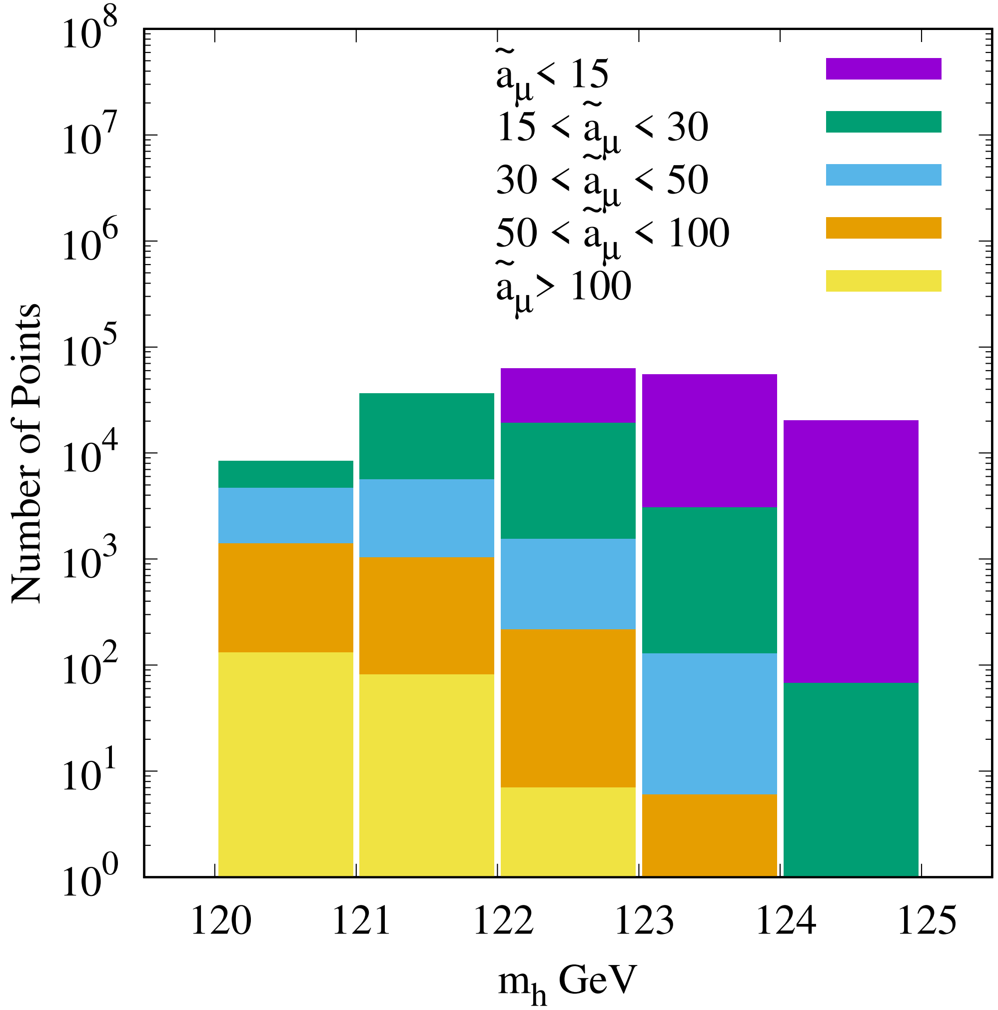


JE, Evans, Nagata, Nanopoulos & Olive, arXiv:2107.03025

$g_\mu - 2$ in Flipped SU(5)

Histograms of Higgs mass values

Coloured according to $\tilde{a}_\mu \equiv \frac{1}{2}(g_\mu - 2) \times 10^{11}$ values



Larger m_h requires heavier sparticles

→

smaller $\tilde{\mu}$ mixing

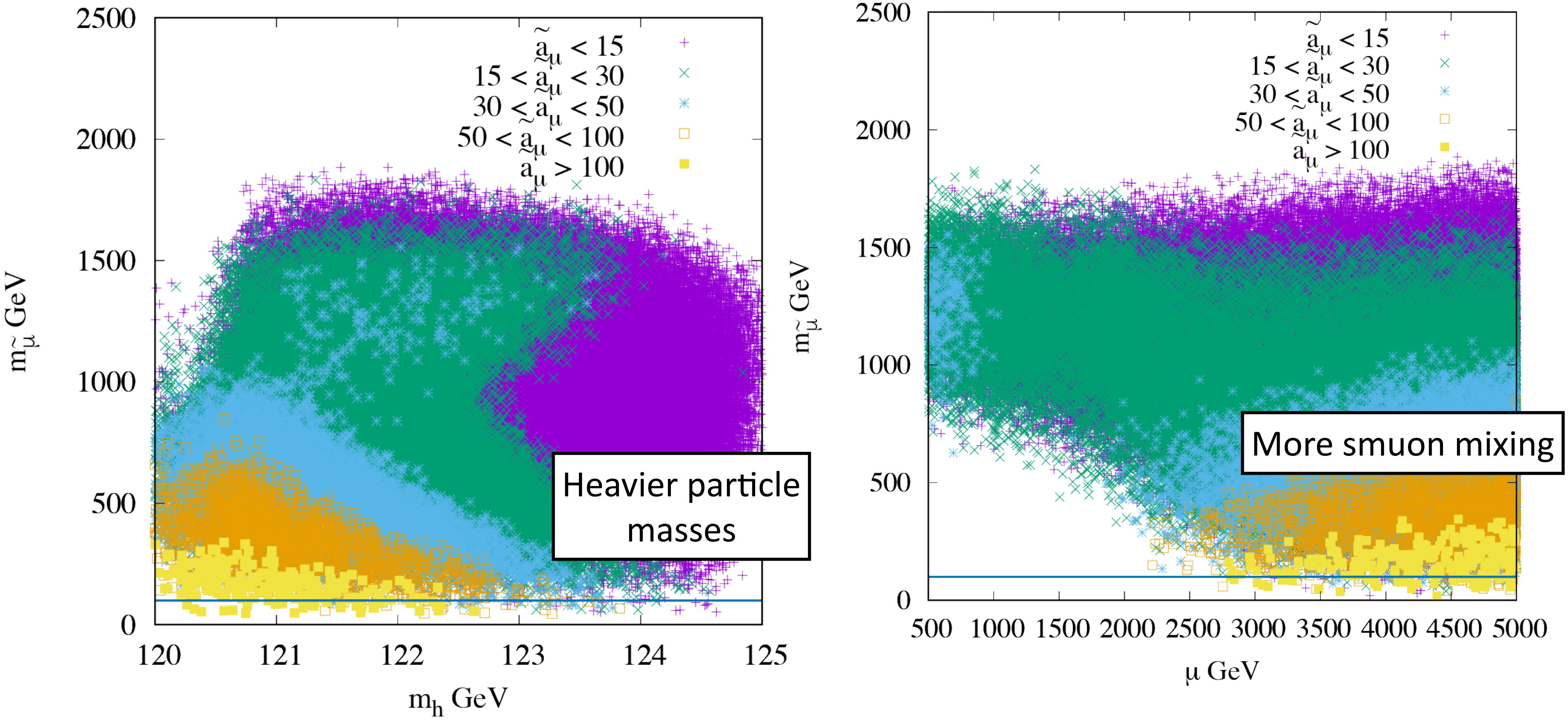
→

smaller $g_\mu - 2$

JE, Evans, Nagata, Nanopoulos & Olive, arXiv:2107.03025

$g_\mu - 2$ in Flipped SU(5)

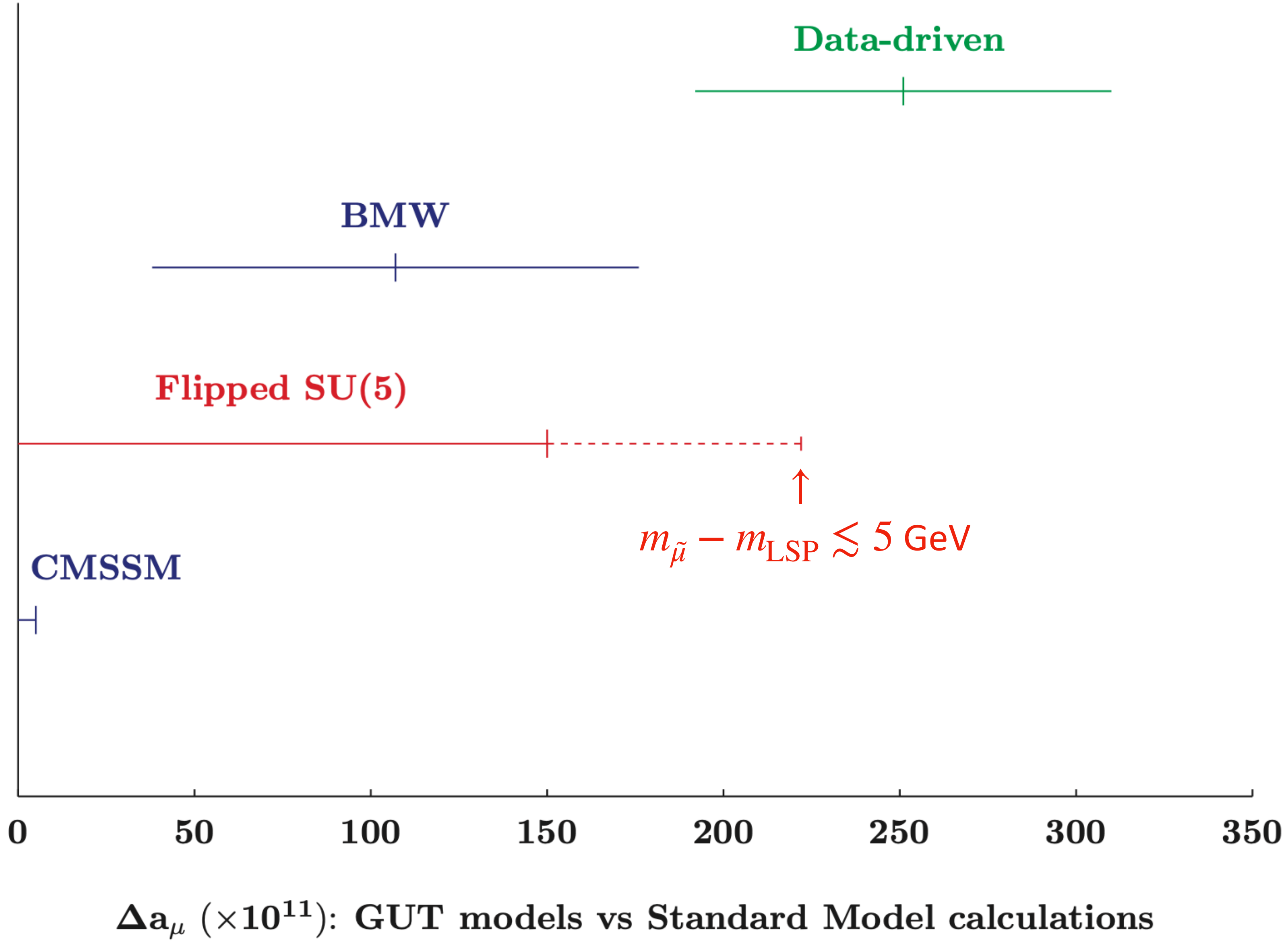
Scan of $\tilde{a}_\mu \equiv \frac{1}{2}(g_\mu - 2) \times 10^{11}$ as function of Higgs mass, Higgs mixing



Larger m_h requires heavier sparticles → smaller $\tilde{\mu}$ mixing → smaller $g_\mu - 2$

JE, Evans, Nagata, Nanopoulos & Olive, arXiv:2107.03025

$g_\mu - 2$ in CMSSM & Flipped SU(5) vs Lattice, Data-Driven Calculation



JE, Evans, Nagata, Nanopoulos & Olive, arXiv:2107.03025

$g_\mu - 2$ in Flipped SU(5)

Parameters & predictions at best-fit point

Input GUT parameters (masses in units of 10^{16} GeV)		
$M_{GUT} = 1.00$	$M_X = 0.79$	$V = 1.13$
$\lambda_4 = 0.1$	$\lambda_5 = 0.3$	$\lambda_6 = 0.001$
$g_5 = 0.70$	$g_X = 0.70$	$m_{\nu_3} = 0.05$ eV
Input supersymmetry parameters (masses in GeV units)		
$M_5 = 2460$	$M_1 = 240$	$\mu = 4770$
$m_{10} = 930$	$m_{\bar{5}} = 450$	$m_1 = 0$
$M_A = 2100$	$A_0/M_5 = 0.67$	$\tan\beta = 35$
MSSM particle masses (in GeV units)		
$m_\chi = 84$	$m_{\tilde{t}_1} = 4030$	$m_{\tilde{g}} = 5090$
$m_{\chi_2} = 2160$	$m_{\chi_3} = 5080$	$m_{\chi_4} = 5080$
$m_{\tilde{\mu}_R} = 101$	$m_{\tilde{\mu}_L} = 1600$	$m_{\tilde{\tau}_1} = 1010$
$m_{\tilde{q}_L} = 4470$	$m_{\tilde{d}_R} = 4250$	$m_{\tilde{u}_R} = 4170$
$m_{\tilde{t}_2} = 4410$	$m_{\tilde{b}_1} = 4170$	$m_{\tilde{b}_2} = 4400$
$m_{\chi^\pm} = 2160$	$m_{H,A} = 2100$	$m_{H^\pm} = 2100$
Other observables		
$\Delta a_\mu = 150 \times 10^{-11}$	$\Omega_\chi h^2 = 0.13$	$m_h = 122$ GeV
Normal-ordered ν masses:	$\tau_{p\to e^+\pi^0}\|_{\rm NO} = 1.1 \times 10^{36}$ yrs	$\tau_{p\to\mu^+\pi^0}\|_{\rm NO} = 1.1 \times 10^{37}$ yrs
Inverse-ordered ν masses:	$\tau_{p\to e^+\pi^0}\|_{\rm IO} = 3.2 \times 10^{37}$ yrs	$\tau_{p\to\mu^+\pi^0}\|_{\rm IO} = 2.3 \times 10^{36}$ yrs

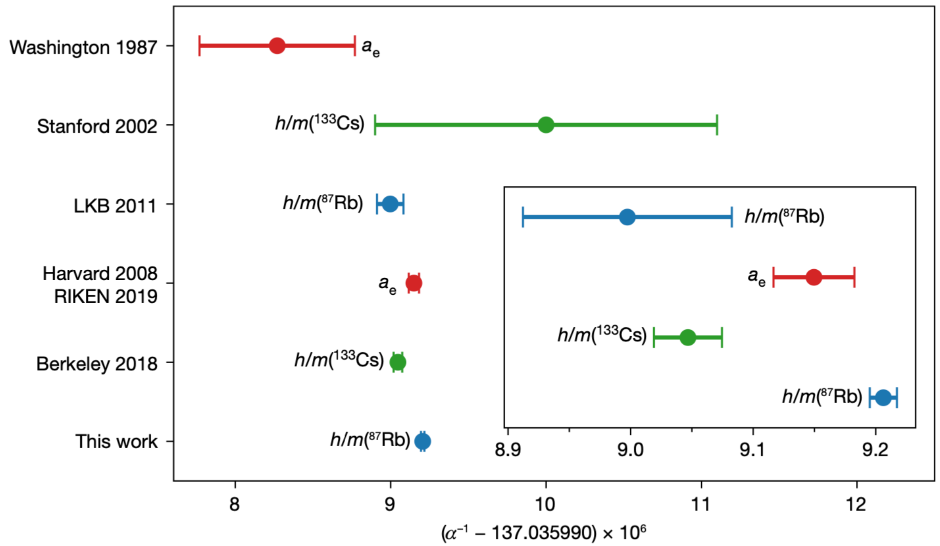
JE, Evans, Nagata, Nanopoulos & Olive, arXiv:2107.03025

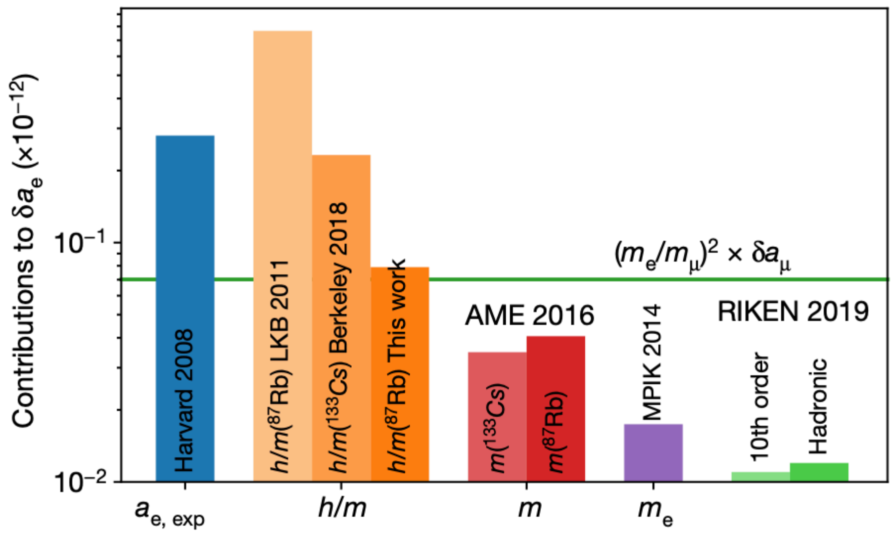
Magnetic Dipole Moment of the Electron

- Discrepancies between determinations of α from atomic measurements and $a_e \equiv (g_e - 2)/2$ + QED

- Could these be due to same new physics in $a_e \equiv (g_e - 2)/2$ as discrepancy in $a_\mu \equiv (g_\mu - 2)/2$?

- $-3.4 \times 10^{-13} < \delta a_e < 9.8 \times 10^{-13}$, comparable to $\delta a_\mu \times (m_e/m_\mu)^2$





Morel , Yao , Cladé & Guellati-Khélifa,

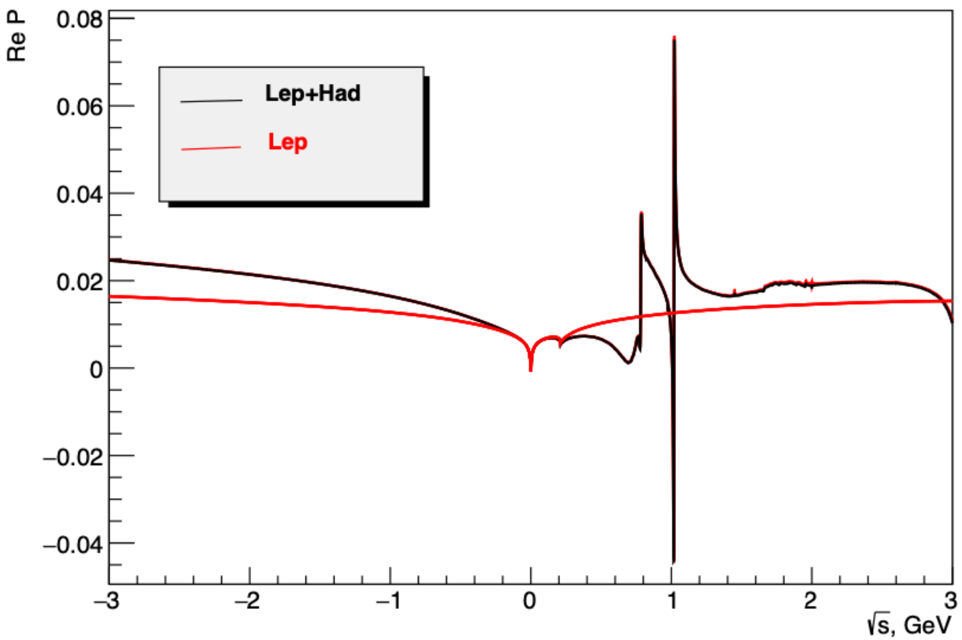
Nature, https://doi.org/10.1038/s41586-020-2964-7

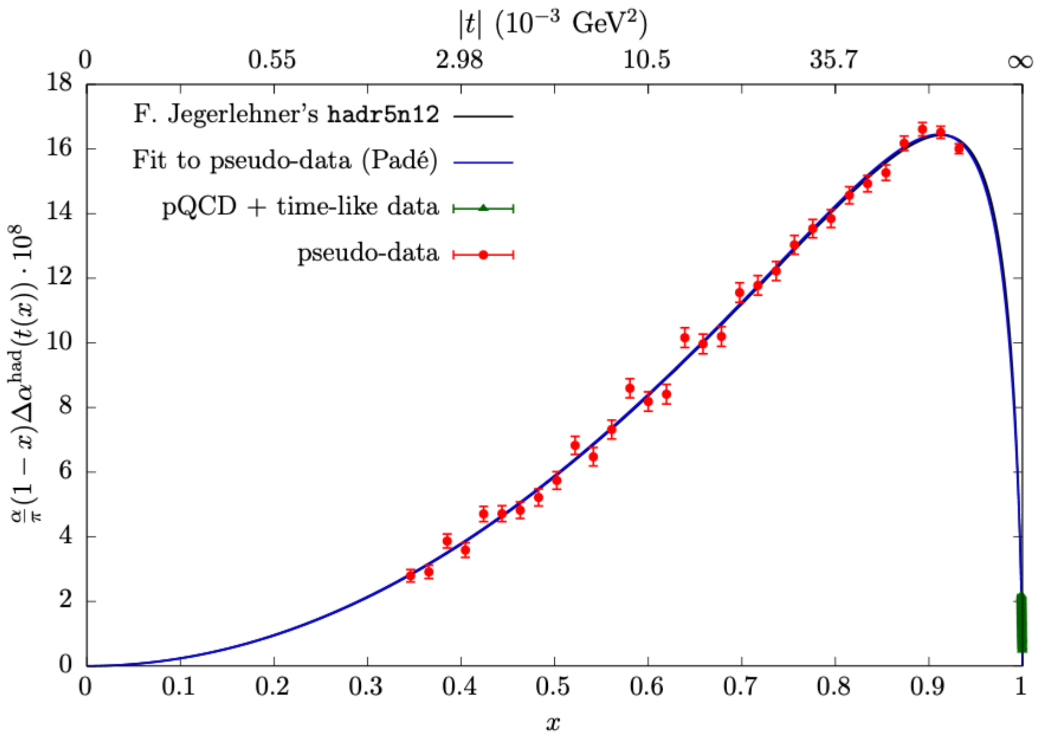
Future

MuonE: Proposed CERN Experiment to Measure HVP in Space-Like Region

Scattering of 150 GeV muons on electrons at CERN SPS

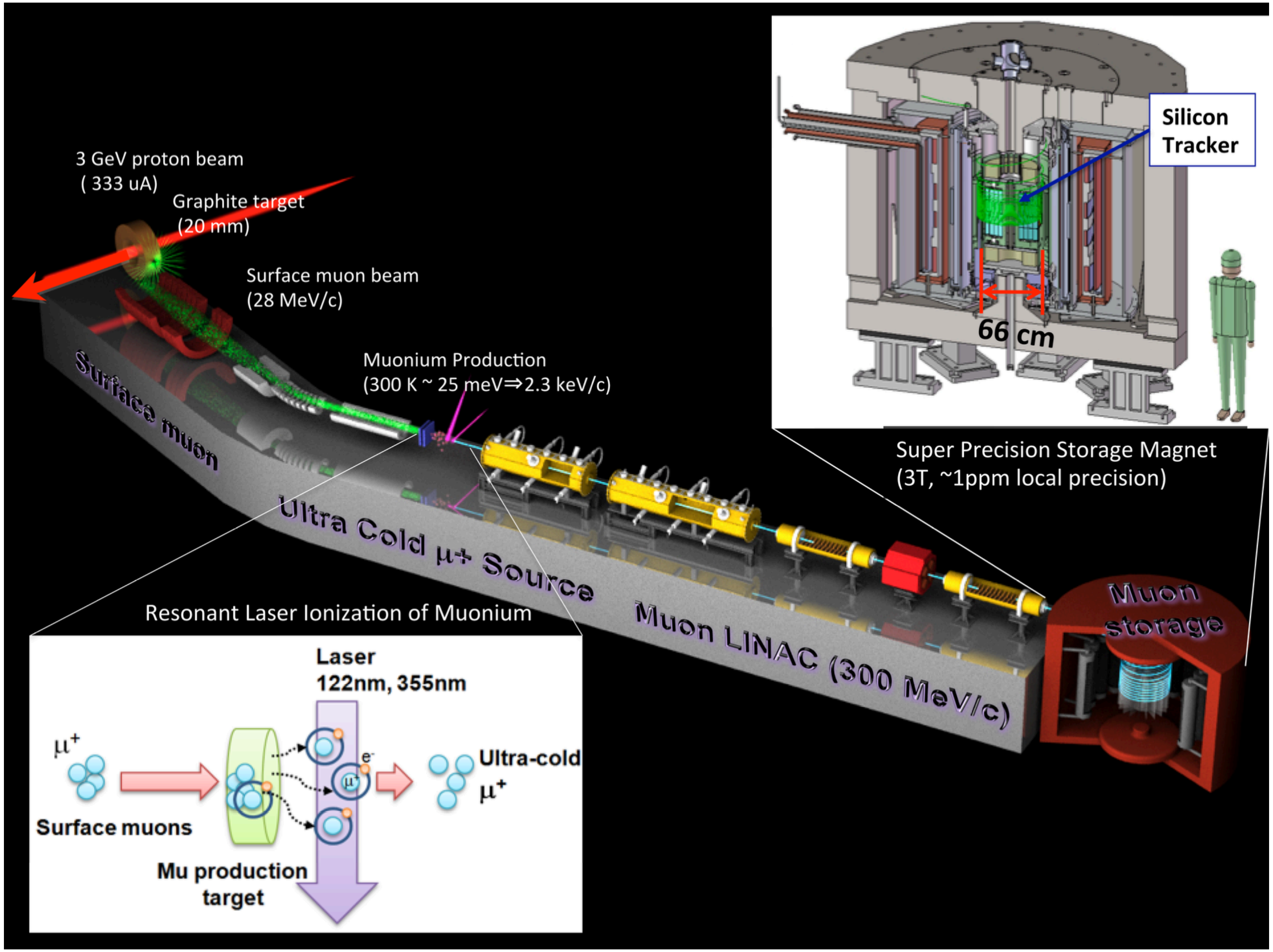
$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\text{had}}[t(x)] \qquad \alpha(t) = \frac{\alpha(0)}{1-\Delta\alpha(t)} \qquad t(x) = -\frac{x^2 m_\mu^2}{1-x} < 0$$





MuonE Theory Initiative, Banerjee et al, arXiv:2004.13663

J-PARC Experiment



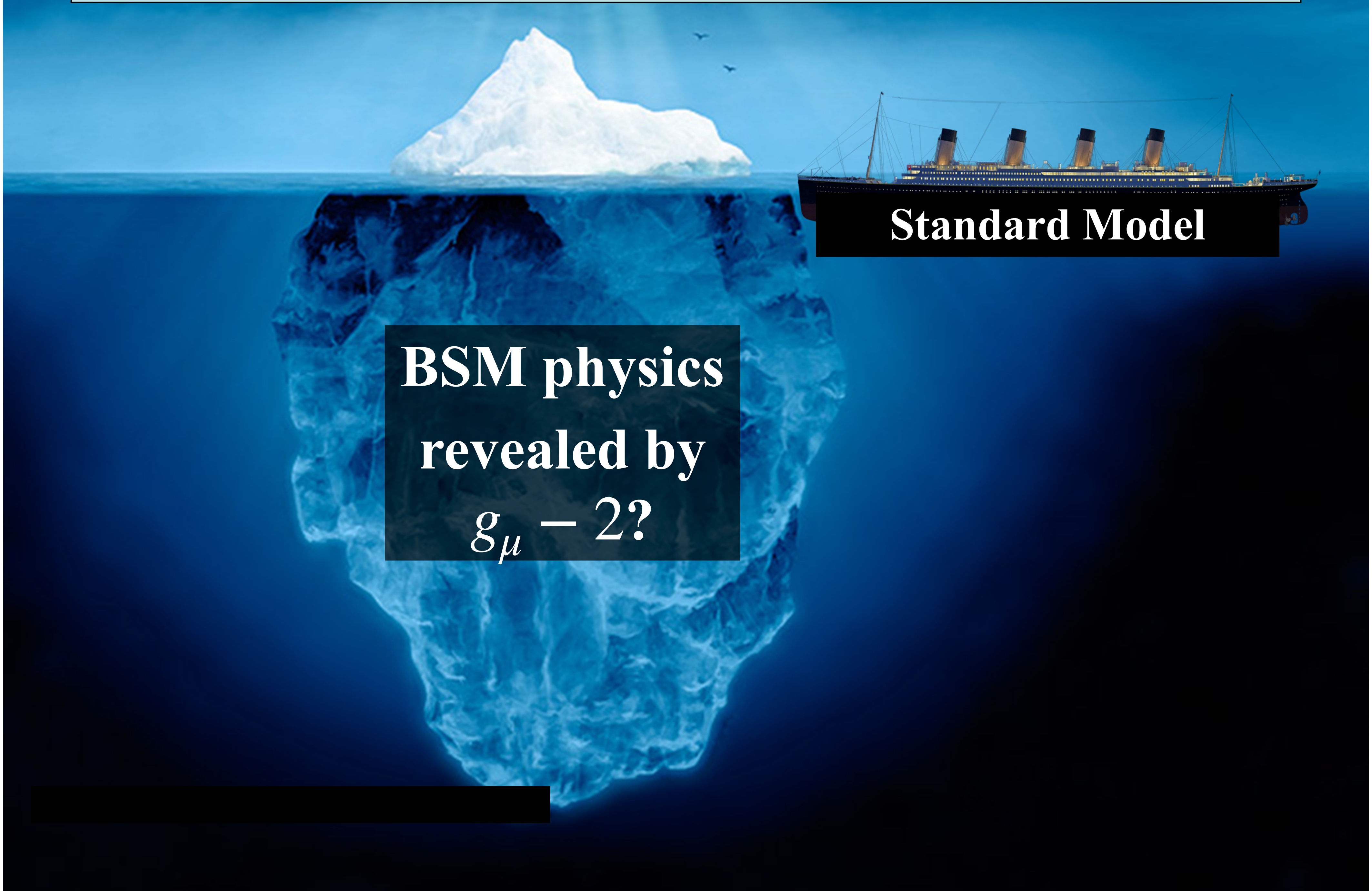
Different technique: ultra-cold muon beam from muonium, accelerate to 300 MeV, inject into storage ring with radius 66cm

M. Abe et al, Progr. Theor. Exp. Phys., 2019, 053C02

Quo vadis $g_\mu - 2$?

- **Never forget**: the (near-) consistency between theory and experiment for $g_\mu - 2$ (and $g_e - 2$) is among the greatest successes of particle physics, particularly quantum field theory
- **Need no reminder**: the discrepancy between theory and experiment for $g_\mu - 2$ may be a window on physics beyond the Standard Model
- Still some **debate** about Standard Model calculation (lattice?)
- **Plenty** of theoretical interpretations proposed: many possible connections to other physics areas (B decays, dark matter, …)
- **More experimental results** on the way: FNAL, J-PARC, MuonE, …
- **A good time to be alive!**

Summary



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