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Université Mohammed V - RABAT -
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Faculté des sciences
كلية العلوم

ESMaR
Equipe Sciences de la Matière et du Rayonnement

Thermodynamic of black holes in a cavity from shadow formalism

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March 21, 2022

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Introduction

- **Einstein field equations**

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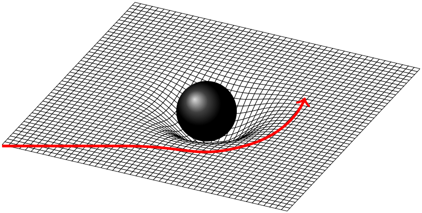

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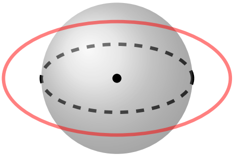

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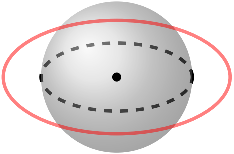

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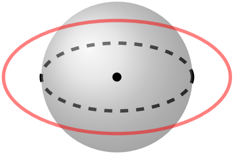

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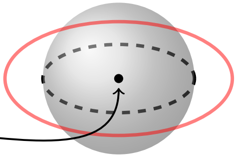

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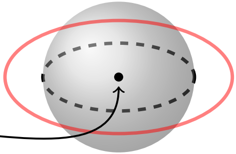

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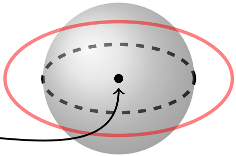

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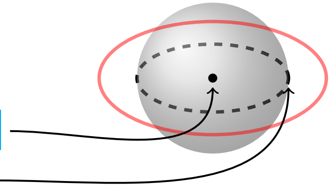

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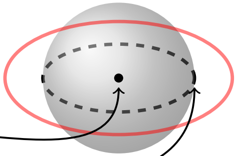

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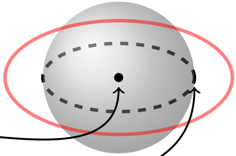

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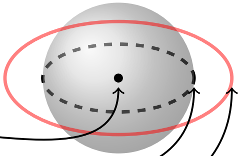

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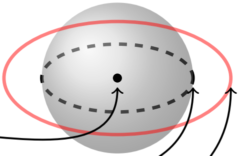

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- **Static and spherically symmetric metric**

We consider the following areal coordinates (t, r, θ, ϕ), the metric is given by

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -A(r)dt^2 + B(r)\underbrace{dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)}_{\textit{Spherically symmetric}} \qquad (2)$$

weher $g_{tt} = -A(r)$ and $g_{rr} = B(r)$ do not depend on t, this corresponding to the stationarity spacetime

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Black hole in a cavity

The Q represent the charge of the black hole and $r+$ is the horizon radius of RN black hole.

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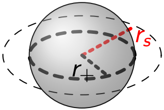

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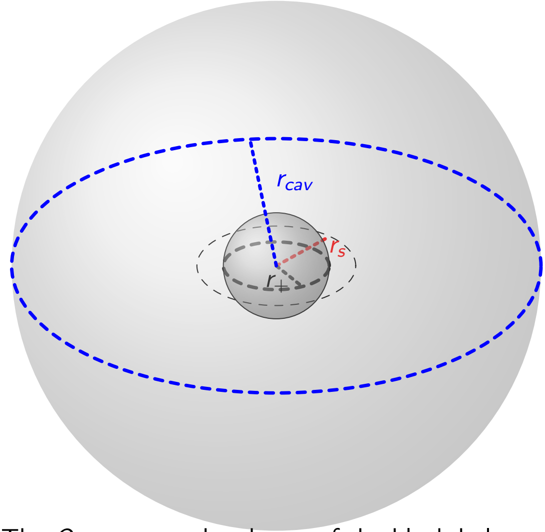

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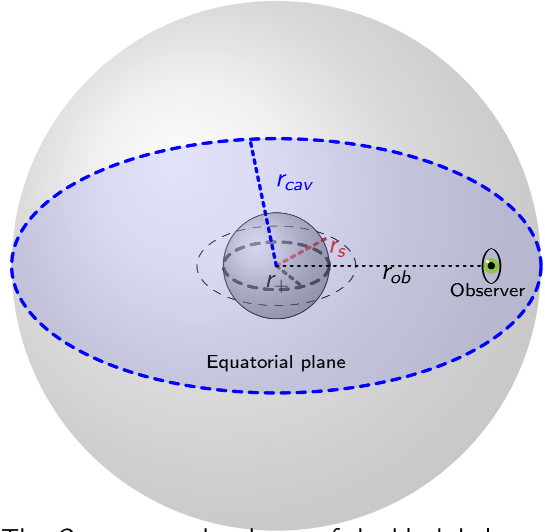

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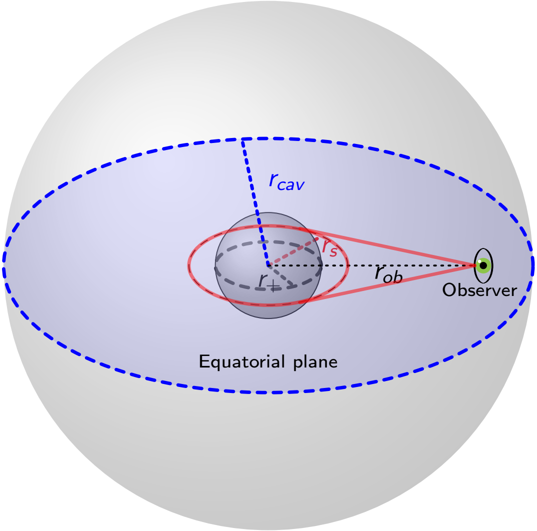

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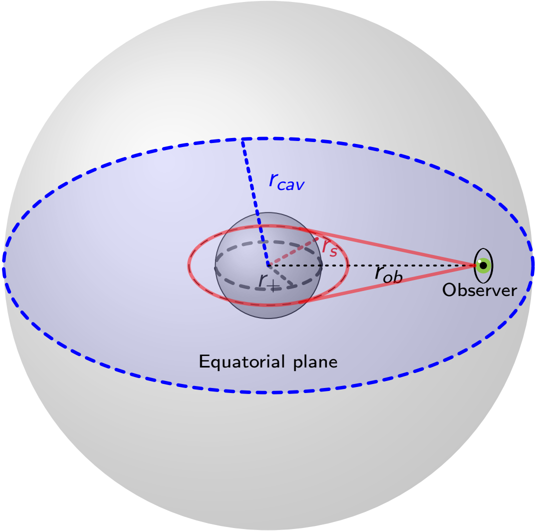

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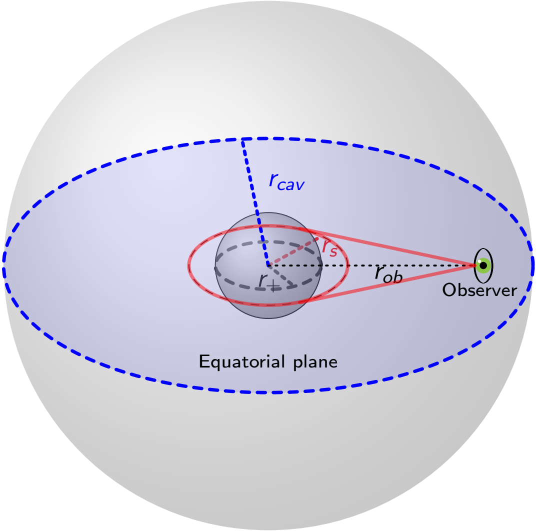


Let us assume that

$$\begin{cases} r_{cav} > r_+ \\ r_{cav} > r_{ob} \end{cases}$$

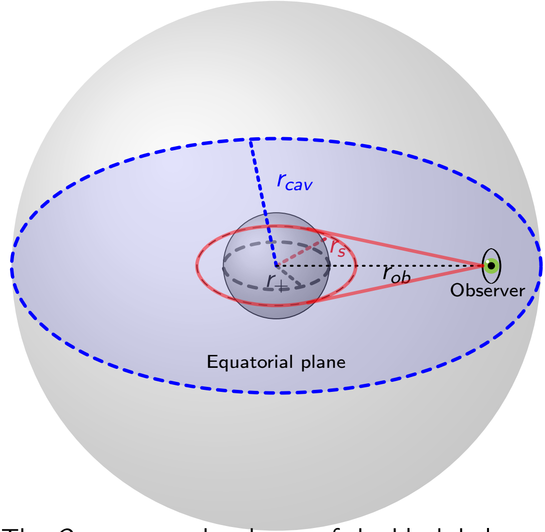
In four dimensions the metric of RN (Reissner-Nordström) black hole in a cavity yields

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{3}$$

where the function metric $f(r)$ is expressed as

$$f(r) = (1 - \frac{r_+}{r})(1 - \frac{Q^2}{r_+ r}) \tag{4}$$

Black hole in a cavity



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Shadow of black hole in a cavity

The massless particle equations of motion can be obtained by employing the Hamilton-Jacobi method for a photon in the black hole spacetime as

$$\frac{\partial S}{\partial \tau} = -\frac{1}{2} g^{ij} p_i p_j, \tag{5}$$

where S and τ are the Jacobi action and the affine parameter respectively. By the help of Hamiltonian equation

$$H = \frac{1}{2} g^{ij} p_i p_j = 0. \tag{6}$$

the Hamiltonian equation of photon on the equatorial plane $\theta = \frac{\pi}{2}$ given by

$$(rf(r)p_r)^2 - r^2 p_t^2 + f(r) p_t^2 = 0 \tag{7}$$

where $E = -p_t$ and $L = p_\phi$ are the conserved total energy and the conserved angular momentum of the photon, respectively

Using the Hamiltonian formalism, the equations of motion can be formulated as

$$\begin{aligned}
\frac{dt}{d\tau} &= \frac{E}{f(r)}, \\
\frac{dr}{d\tau} &= \pm\sqrt{f(r)\left(\frac{E^2}{f(r)} - \frac{L^2}{r^2}\right)}, \\
\frac{d\phi}{d\tau} &= -\frac{L}{r^2}.
\end{aligned} \tag{8}$$

one exploits the radial equation of motion taking the following form

$$\left(\frac{dr}{d\tau}\right)^2 + V_{eff}(r) = 0, \tag{9}$$

where $V_{eff}(r)$ indicates the effective potential for a radial particle motion. In particular, it is given by

$$V_{eff} = f(r)\left(\frac{L^2}{r^2} - \frac{E^2}{f(r)}\right). \tag{10}$$

The orbit equation for the photon is obtained by considering the equation

$$\frac{dr}{d\phi} = \pm r\sqrt{f(r)\left[\frac{r^2 f(R)}{R^2 f(r)} - 1\right]}. \tag{11}$$

We consider a light ray and transmitting into the past with an angle α In this way, one has

$$\cot\alpha_{ob} = \frac{\sqrt{g_{rr}}}{\sqrt{g_{\phi\phi}}}\frac{dr}{d\phi}\Big|_{r=r_{ob}} = \frac{1}{r\sqrt{f(r)}}\frac{dr}{d\phi}\Big|_{r=r_{ob}}. \tag{12}$$

Exploiting Eq.(**??**), one can get

$$\sin^2\alpha_{ob} = \frac{f(r_{ob})R^2}{r_{ob}^2 f(R)}. \tag{13}$$

Precisely, the shadow radius of the black hole observed by a static observer placed at r_{ob} has been found to be

$$r_{sh} = r_{ob}\sin\alpha_{ob} = R\sqrt{\frac{f(r_{ob})}{f(R)}}\Bigg|_{R=r_s} . \tag{14}$$

solving this equation

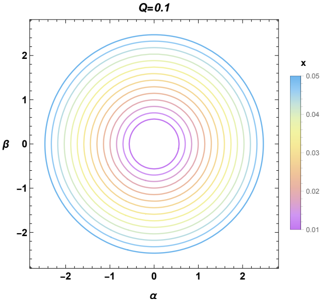
$$r_s f'(r_s) - 2f(r_s) = 0 \tag{15}$$

we get the photon sphere radius

$$r_s = \frac{3Q^2 + 3r_+^2 + \sqrt{9Q^4 - 14Q^2 r_+^2 + 9r+^4}}{4r_+} \tag{16}$$

the apparent shape of the shadow is obtained by using the celestial coordinates α and β

$$\begin{aligned} \alpha &= \lim_{r_{ob} \longrightarrow \infty} \left(-r_{ob}^2 \sin\theta_{ob} \frac{d\phi}{dr}\Big|_{(r_{ob},\theta_{ob})} \right), \\ \beta &= \lim_{r_{ob} \longrightarrow \infty} \left(r_{ob}^2 \frac{d\theta}{dr}\Big|_{(r_{ob},\theta_{ob})} \right). \end{aligned} \tag{17}$$



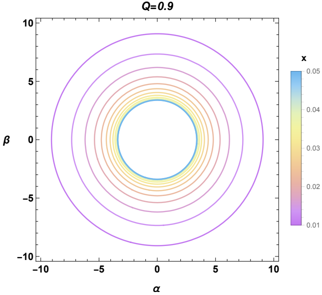


Figure: *Shadows of black holes in a cavity by varying the parameter x for different values of Q. The observer is positioned at $r_{ob} = 15$ and $\theta_{ob} = \frac{\pi}{2}$.*

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Thermodynamic of black hole in a cavity

Corresponding to the Hawking temperature

$$T_H = \frac{f'(r_+)}{4\pi} \tag{18}$$

the temperature of a cavity is defined as

$$T_{cav} = \frac{T_H}{\sqrt{f(r_b)}} \tag{19}$$

we notice that for $\lim_{r_{cav}\to+\infty} f(r_{cav}) = 1$, in this situation the $T_{cav} = T_H$. It is worth noting that the physical range of the event horizon is constrained

$$\frac{Q}{r_{cav}} \leqslant x \leqslant \frac{1}{r_{cav}} \tag{20}$$

However, the temperature as function of Q, x and r_{cav} parameters given by

$$T_{cav} = \frac{1 - \frac{Q^2}{r_{cav}^2 x^2}}{4\pi x r_{cav}\sqrt{f(r_{cav})}} \tag{21}$$

First, we plot the four dimensional temperature as function of x parameter

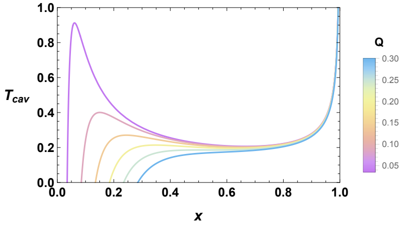

Figure: The temperature in the cavity as function of x by taking $r_{cav} = 1$.

This figure show that the temperature is similar to the temperature graph of RN-AdS black holes. However, the one difference, that is when x approaches 1, the temperature diverges

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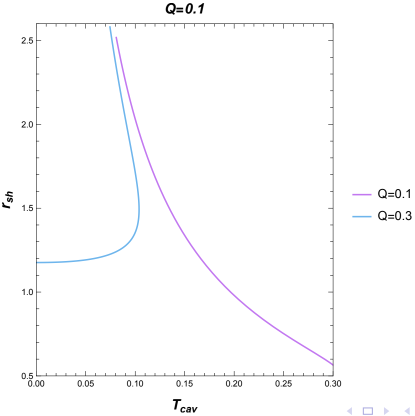
Relation between thermodynamic and shadow of black hole in a cavity

In this part, by the help of two equation corresponding to the temperature and the shadow of black hole in a cavity we plot the diagram shows the relation between them



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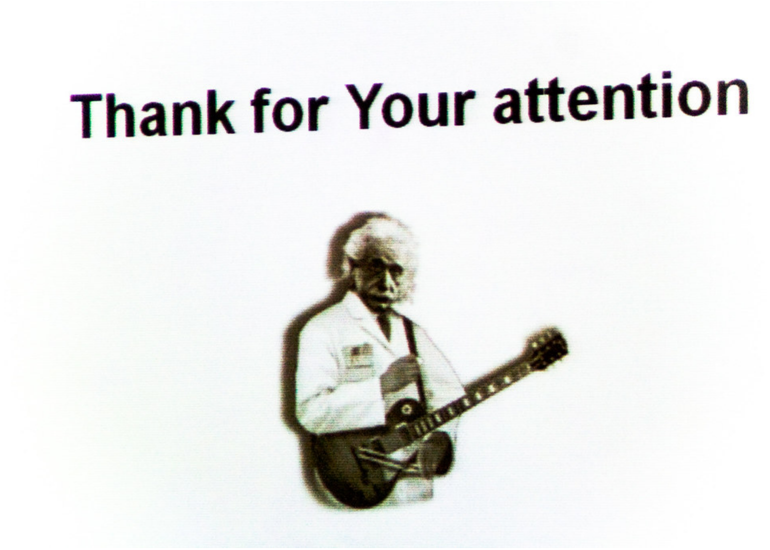
- The charge Q present an interesting effect on shadow and the black hole thermodynamics in a cavity.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