Influence of the laser field on electron muon neutrino processus

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Plan

Introduction

- The considerable and rapid progress made by laser technology since its invention in the 1960s has opened up a new field of theoretical and experimental studies to explore the interactions of the laser field with matter at high intensities. .
- In general, the study of (anti)muon neutrino scattering on electrons has offered important results for understanding the electroweak sector of the Standard Model .
- The purpose of this presentation is mainly to reveal the effect of a strong electromagnetic field on the scattering process $e^-(p_i) + \nu_\mu(k_i) \longrightarrow e^-(p_f) + \nu_\mu(k_f)$, and in particular on its calculated DCS.

Theory

- We consider the scattering of an electron muon-neutrino in a Circularly Polarized EM :

$$e^-(p_i) + \nu_\mu(k_i) \longrightarrow e^-(p_f) + \nu_\mu(k_f), \tag{1}$$

- The corresponding Feynman diagram is :

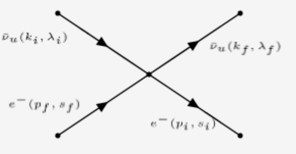

Figure – Lowest Feynman diagram of muon electron-neutrino scattering.

- We assume that this scattering occurs in the presence of a monochromatic laser field circularly polarized

$$A^\mu(x) = |\mathbf{a}|\Big[\eta_1^\mu \cos(\Phi) + \eta_2^\mu \sin(\Phi)\Big], \qquad \Phi = (k.x), \tag{2}$$

Theory

- where : $|\mathbf{a}| = \mathcal{E}_0/\omega,$ $\eta_1^\mu = (0,1,0,0),$ $\eta_2^\mu = (0,0,1,0),$
- Which implies :

$$\eta_1^\mu.\eta_2^\mu = 0 \qquad et \qquad \eta_1^2 = \eta_2^2 = -1,$$

- Transition matrix element :

$$S_{fi} = \frac{-iG}{\sqrt{2}} \int d^4x \left[\overline{\psi}^{f}_{\nu_\mu}(x)\gamma^\mu(1-\gamma_5)\psi^{i}_{\nu_\mu}(x)\right]\left[\overline{\psi}^{f}_{e^-}(x)\gamma_\mu(g_V - g_A\gamma_5)\psi^{i}_{e^-}(x)\right]. \quad (3)$$

Where $G = (1.16637 \pm 0.00002) \times 10^{-11} MeV^{-2}$ is the constant of *Fermi* and g_V, g_A are, respectively, the vector and axial-vector coupling constants .

- Wave functions of outgoing and incoming muon neutrinos :

$$\begin{aligned} \psi^{i}_{\nu_\mu}(x) &= \frac{1}{\sqrt{2E_iV}} u_{\nu_\mu}(k_i,\lambda_i)\exp(-ik_i.x), \\ \psi^{f}_{\nu_\mu}(x) &= \frac{1}{\sqrt{2E_fV}} u_{\nu_\mu}(k_f,\lambda_f)\exp(-ik_f.x), \end{aligned} \quad (4)$$

Theory

- Wave functions of Dirac Volkov :

$$\psi_{i,f}(x) = \left[1 + \frac{e\not{k}\not{A}}{2(k.p_{i,f})}\right] \frac{u(p_{i,f}, s_{i,f})}{\sqrt{2Q_{i,f}V}} \times e^{iS(q_{i,f},x)}, \qquad (5)$$

- Where :

$$S(q_{i,f}, x) = -q_{i,f}.x - \frac{e|\mathbf{a}|(\eta_1.p_{i,f})}{k.p_{i,f}} \sin(\phi) + \frac{e|\mathbf{a}|(\eta_2.p_{i,f})}{k.p_{i,f}} \cos(\phi). \qquad (6)$$

- With :

$$q_{i,f} = p_{i,f} + \frac{e^2|\mathbf{a}|^2}{2(k.p_{i,f})}k, \qquad m_*^2 = m^2 + e^2|\mathbf{a}|^2, \qquad (7)$$

- After some manipulation, we find :

$$S_{fi} = \frac{-iG}{\sqrt{32E_iE_fQ_iQ_fV^4}} \int dx\ e^{i(k_f-k_i).x}\ e^{i(S(q_i,x)-S(q_f,x))}$$
$$\left[\bar{u}(p_f, s_f)\Big(1 + C(p_f)\not{A}\not{k}\Big) \times \gamma_\mu(g_V - g_A\gamma_5)\Big(1 + C(p_i)\not{k}\not{A}\Big)\right. \qquad (8)$$
$$\left. u(p_i, s_i)\right]\left[\bar{u}_{\nu_\mu}(k_f, \lambda_f)\gamma^\mu(1-\gamma^5)u_{\nu_\mu}(k_i, \lambda_i)\right],$$

Where

$$C(p_i) = \frac{e}{2(k.p_i)} \quad et \quad C(p_f) = \frac{e}{2(k.p_f)} \qquad (9)$$

Theory

- :Exponential Term

$$e^{i(S(q,x)-S(q_1,x))} = e^{i(q_f-q_i).x} \; e^{-iz\sin(k.x-\varphi)}, \tag{10}$$

where :

$$z = e|\mathbf{a}|\sqrt{\left(\frac{\eta_1.p_i}{k.p_i}-\frac{\eta_1.p_f}{k.p_f}\right)^2+\left(\frac{\eta_2.p_i}{k.p_i}-\frac{\eta_2.p_f}{k.p_f}\right)^2}, \tag{11}$$

and

$$\varphi = \arctan\left[\frac{(\eta_2.p_i)(k.p_f)-(\eta_2.p_f)(k.p_i)}{(\eta_1.p_i)(k.p_f)-(\eta_1.p_f)(k.p_i)}\right]. \tag{12}$$

Thus :

$$\begin{aligned} S_{fi} =& \frac{-iG}{\sqrt{32E_iE_fQ_iQ_fV^4}}\int dx\; e^{i(k_f+q_f-k_i-q_i).x}\; e^{-iz\sin(k.x-\varphi)}\Big[\bar{u}(p_f,s_f)(\Delta^0_\mu \\ &+\Delta^1_\mu\cos(\phi)+\Delta^2_\mu\sin(\phi))\,u(p_i,s_i)\Big]\Big[\bar{u}_{\nu_\mu}(k_f,\lambda_f)\gamma^\mu(1-\gamma^5)u_{\nu_\mu}(k_i,\lambda_i)\Big], \end{aligned} \tag{13}$$

with :

$$\begin{aligned} \Delta^0_\mu &= \gamma_\mu\;(g_V-g_A\gamma_5)+2\;C(p_i)\;C(p_f)\;|\mathbf{a}|^2\;k_\mu\;\not{k}\;(g_V-g_A\gamma_5), \\ \Delta^1_\mu &= C(p_i)|\mathbf{a}|\gamma_\mu\;(g_V-g_A\gamma_5)\not{k}\;\not{\eta}_1+C(p_f)|\mathbf{a}|\;\not{\eta}_1\;\not{k}\;\gamma_\mu\;(g_V-g_A\gamma_5), \\ \Delta^2_\mu &= C(p_i)|\mathbf{a}|\gamma_\mu\;(g_V-g_A\gamma_5)\not{k}\;\not{\eta}_2+C(p_f)|\mathbf{a}|\;\not{\eta}_2\;\not{k}\;\gamma_\mu\;(g_V-g_A\gamma_5). \end{aligned} \tag{14}$$

Theory

- Ordinary transformation of the Bessel function gives :

$$\left\{\begin{array}{c}1\\ \cos(k.x)\\ \sin(k.x)\end{array}\right\} e^{-iz\sin(k.x-\varphi)} = \sum_{n=-\infty}^{+\infty} e^{-in(k.x)} \left\{\begin{array}{c} J_n(z)e^{in\varphi}\\ \frac{1}{2}\left\{J_{n+1}(z)e^{i(n+1)\varphi}+J_{n-1}(z)e^{i(n-1)\varphi}\right\}\\ \frac{1}{2i}\left\{J_{n+1}(z)e^{i(n+1)\varphi}-J_{n-1}(z)e^{i(n-1)\varphi}\right\}\end{array}\right\},$$

$$= \sum_{n=-\infty}^{+\infty} e^{-in(k.x)} \left\{\begin{array}{c} b_n(z)\\ b_{1n}(z)\\ b_{2n}(z)\end{array}\right\}, \quad (15)$$

- Matrix element becomes :

$$S_{fi} = \frac{-iG}{\sqrt{32E_iE_fQ_iQ_fV^4}} \sum_{n=-\infty}^{+\infty} (2\pi)^4\delta^4(q_f+k_f-q_i-k_i-nk)\ M_{fi}^n. \quad (16)$$

where :

$$M_{fi}^n = \left[\bar{u}(p_f,s_f)\Gamma_\mu^n u(p_i,s_i)\right]\left[\bar{u}_{\nu_\mu}(k_f,t_f)\gamma^\mu(1-\gamma^5)u_{\nu_\mu}(k_i,t_i)\right], \quad (17)$$

with :

$$\Gamma_\mu^n = \Delta_{0\mu}b_n(z)+\Delta_{1\mu}b_{1n}(z)+\Delta_{2\mu}b_{2n}(z). \quad (18)$$

- Differential cross section :

$$\frac{d\overline{\sigma}}{d\Omega} = \sum_{n=-\infty}^{+\infty} \frac{G^2}{64Q_fQ_iE_fE_i}\frac{|\mathbf{q}_f|^2 d|\mathbf{q}_f|}{(2\pi)^2|J_{inc}|V}\delta^0(Q_f+E_f-Q_i-E_i-n\omega)\sum_{t_{i,f},s_{i,f}}|M_{fi}^n|^2\Big|_{\mathbf{q}_f+\mathbf{k}_f-\mathbf{q}_i-\mathbf{k}_i-n\mathbf{k}=\mathbf{0}}. \quad (19)$$

Theory

- Flux of incident particles :

$$\frac{1}{|J_{inc}|} = \frac{E_i^{\mu^-} Q_i V}{m_\mu |\mathbf{q}_i|}. \quad (20)$$

We use the following relations :

$$\begin{aligned} &[(2\pi)^4\delta^4(p_f + k_f - p_i - k_i)]^2 = VT(2\pi)^4\delta^4(p_f + k_f - p_i - k_i), \\ &\delta^4(p_f + k_f - p_i - k_i) = \delta^0(p_f^0 + E_f - p_i^0 - E_i)\delta^3(\mathbf{p}_f + \mathbf{k}_f - \mathbf{p}_i - \mathbf{k}_i), \\ &d^3p_f = |\mathbf{p}_f|^2 d|\mathbf{p}_f| d\Omega, \qquad \int dx\, f(x)\delta(g(x)) = \frac{f(x)}{|g'(x)|}\Big|_{g(x)=0.}. \end{aligned} \quad (21)$$

- Differential cross section :

$$\left(\frac{d\overline{\sigma}}{d\Omega}\right)^{with\ laser} = \sum_{n=-\infty}^{+\infty} \frac{d\overline{\sigma}^n}{d\Omega} = \sum_{n=-\infty}^{+\infty} \frac{G^2}{64 Q_f\ E_f} \frac{|\mathbf{q}_f|^2}{(2\pi)^2 |(k_i.q_i)} \frac{|\overline{M}_{fi}|^2}{|g'(|\mathbf{q}_f|)|}, \quad (22)$$

Where :

$$g'(|\mathbf{q}_f|) = \frac{|\mathbf{q}_f|}{\sqrt{|\mathbf{q}_f|^2 + m_*^2}} + \frac{|\mathbf{q}_f| + E_i\cos(\theta_f) - n\omega\ \cos(\theta_f) - |\mathbf{q}_i| F(\phi_i, \phi_f, \theta_i, \theta_f)}{E_f}, \quad (23)$$

With :

$$\begin{aligned} E_f =& \Big[|\mathbf{q}_i|^2 + |\mathbf{q}_f|^2 - 2|\mathbf{q}_f|\Big(|\mathbf{q}_i| F(\phi_i, \phi_f, \theta_i, \theta_f) - E_i\cos(\theta_f) + n\omega\cos(\theta_f)\Big) \\ &+ 2|\mathbf{q}_i|\Big(n\omega\cos(\theta_i) - E_i\cos(\theta_i)\Big) + E_i^2 + (n\omega)^2 - 2 * E_i\ n\omega\Big]^{1/2}, \end{aligned} \quad (24)$$

- Spinorial part :

$$|\overline{M}_{fi}|^2 = Tr\big[(\not{p}_f + m)\Lambda_\mu^n(\not{p}_i + m)\bar{\Lambda}_\nu^n\big]\ Tr\big[\not{k}_f\gamma^\mu(1-\gamma^5)\not{k}_i\gamma^\nu(1-\gamma^5)\big], \quad (25)$$

Results and Discussions

DCS with and without laser

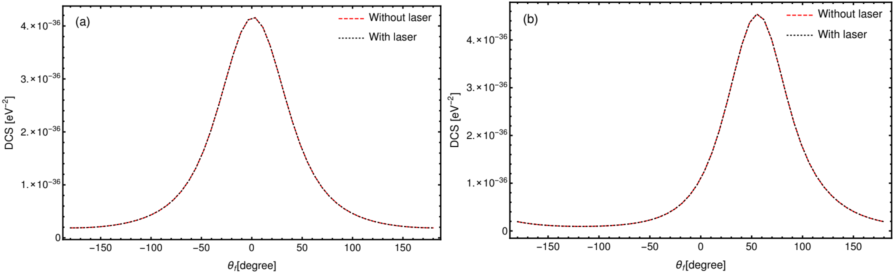

Figure – The variation of the differential cross section with and without laser depending on the scattering angle θ_f with spherical coordinates $\theta_i = 1^\circ, \quad \mathcal{E}_0 = 0 \; et \; n = 0$. (a) et (b) $\theta_i = 45^\circ$.

Results and Discussions

DCS with laser

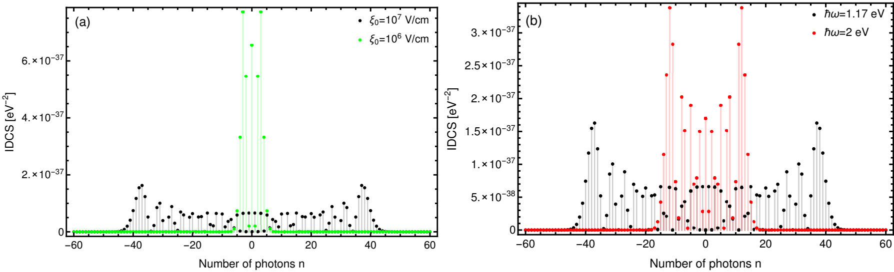

Figure – The behavior of the IDCS, $d\overline{\sigma}^n/d\Omega$, versus the number of photons n. The different parameters are (a) $\hbar\omega = 1.17$ eV, $\theta_i = 1^\circ$ and $\theta_f = 0^\circ$, (b) $\mathcal{E}_0 = 10^7$ V/cm, $\theta_i = 1^\circ$ and $\theta_f = 0^\circ$,

Results and Discussions

SDCS as a function of final angle θ_f

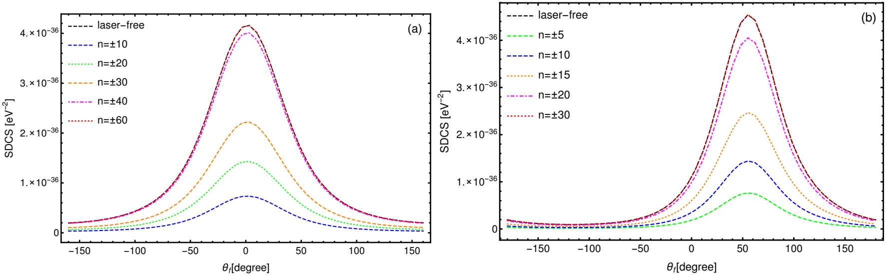

Figure – The variations of the SDCS as a function of the final angle θ_f for different numbers of exchanged photons. The intensity and frequency of the laser field are respectively $\mathcal{E}_0 = 10^7$ V/cm and $\hbar\omega = 1.17$ eV. The initial angle is (a) $\theta_i = 1^\circ$ and (b) $\theta_i = 45^\circ$. The notation $n = \pm N$ means we summed over the range of values $-N \leq n \leq +N$.

Conclusion

- the results obtained show that the laser field has a considerable influence, by its the intensity and its the frequency, on the the DCS as well as on the photon exchange process between the laser and the scattering system.
- the differential cross section The differential cross section of electron muon-neutrino scatteringin the presence of a circularly polarized electromagnetic field can be affected by the laser field as long as the number of exchanged photons is not sufficient to verify the sum rule.

$$\sum_{n=-\text{cutoff}}^{+\text{cutoff}} \frac{d\overline{\sigma}^n}{d\Omega} = \left(\frac{d\overline{\sigma}}{d\Omega}\right)^{laser-free} . \qquad (26)$$


Merci pour
votre attention

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