Asymptotic Grand Unification

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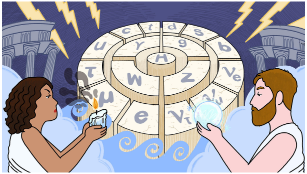
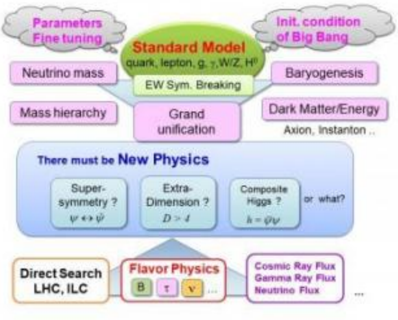
BSM

- The SM of particle physics has been a very successful model in describing most of the particle phenomenology known so far.
  - Do we need any new physics?



- The SM is believed to be only an effective low energy theory for several reasons:
  - The Hierarchy Problem (The Naturalness Problem)
  - Gravity
  - Dark Matter
  - The cosmological constant problem
  - Flavour problem
  - Gauge coupling unification

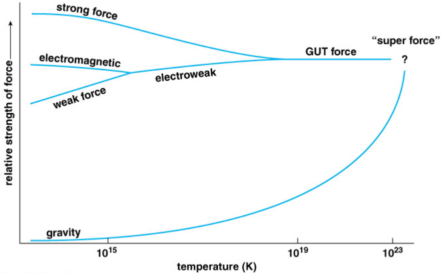
GUT



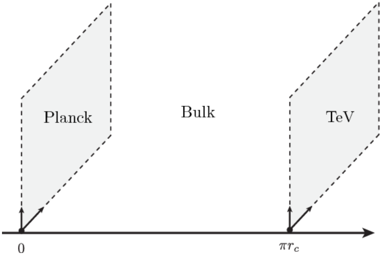
- The idea of GUT is to reduce:
  - All the gauge interactions to one single gauge group.
  - All the fermionic multiplets into one or two different representations for each generation of matter.
- Theories of grand unification continue to play an important role as guiding principle when searching for extensions of the SM.

GUT

- GUT have the ambition of unifying all the forces (except gravity) into a unique simple gauge group.
- In GUT model building, to assume that the unification of gauge couplings occurs at a specific high scale, where the low energy couplings meet via the renormalization group running.
- We consider a unification, where the couplings unify asymptotically. In the models where a compact extra dimension becomes relevant at scales higher than the EW scale and where the gauge symmetry in the bulk is unified.



SO(10) aGUT

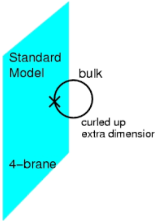


- In this model, we consider a minimal SO(10) grand unified model in five dimensions.
- The extra dimension is compactified on an $S^1/Z_2 \times Z_2'$ orbifold of radius R.
- The gauge breaking is achieved by use of two parities, defined by two matrices P_0 and P_1.
- This parities is correspond to a mirror symmetry around the fixed points $y = 0$ and $y = \pi R/2$.

SO(10) aGUT

- We choose the P_0 and P_1 matrices to be:

$$P_0 = \text{diag}(-1,-1,-1,+1,+1) \otimes \text{diag}(+1,+1),$$
$$P_1 = \text{diag}(+1,+1,+1,+1,+1) \otimes \text{diag}(+1,+1).$$



- The SO(10) gauge symmetry is broken down to Pati-Salam $(SU(4)_C \times SU(2)_L \times SU(2)_R)$ by a $Z_2 \times Z_2'$ orbifold twisting which generates two inequivalent fixed points.
  - One is the preserved SO(10) symmetric fixed point **(we call this the visible brane)**.
  - The other has only a Pati-Salam symmetry **(we refer to as the PS hidden brane)**.

SO(10) aGUT

- The breaking is satisfied if there is scalar in **16** or **126** representations of $SO(10)$,
- The breaking to the $SU(5)$ is done by the ordinary Higgs mechanism on the brane (we refer to this as brane breaking), where the VEV of the Higgs field along the right handed neutrino direction.

$$\mathbf{16} \Rightarrow \mathbf{10} + \overline{\mathbf{5}} + \mathbf{1}$$
$$\mathbf{126} \Rightarrow \mathbf{50} + \mathbf{45} + \overline{\mathbf{15}} + \mathbf{10} + \overline{\mathbf{5}} + \mathbf{1}$$

- Both **16** and **126** representations contain a singlet under $SU(5)$, we choose the adjoint scalar in **16**.

SO(10) aGUT

- The 15 fermion fields of each SM generation and an extra right-handed neutrino field fit into a **16** and $\overline{\mathbf{16}}$ dimensional spinor representation field of $SO(10)$

$$\mathbf{16} = (t_L, \nu_L, b_L, \tau_L, B_L, \mathcal{T}_L, T_L, \mathcal{N}_L, T_R, \mathcal{N}_R, B_R, \mathcal{T}_R, b_R, \tau_R, t_R, \nu_R)$$
$$\overline{\mathbf{16}} = (T_L^c, \mathcal{N}_L^c, B_L^c, \mathcal{T}_L^c, b_L^c, \tau_L^c, t_L^c, \nu_L^c, t_R^c, \nu_R^c, b_R^c, \tau_R^c, B_R^c, \mathcal{T}_R^c, T_R^c, \mathcal{N}_R^c)$$

- We can write down the most general Lagrangian as follows:

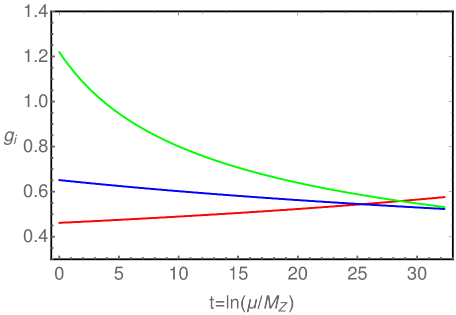
$$\mathcal{L}_{\mathrm{SO(10)}} = -\frac{1}{4} F_{MN}^{(a)} F^{(a)MN} - \frac{1}{2\xi}\left(\partial_\mu A^\mu - \xi \partial_5 A_y\right)^2 + i\overline{\mathbf{16}}\not{D}\mathbf{16}$$
$$+ \sum_{a,b} Y_{10}^{ab}\, \overline{\mathbf{16}}_a\, \Gamma^i\, \mathbf{16}_b H_{10}^i + |D_M H_{10}|^2 - V(H_{10}),$$

Running of the gauge couplings

- We have in the SM the numerical coefficients

$$b_i = \left(\frac{41}{10}, -\frac{19}{6}, -7\right),$$

- These equations will be used for the running of the gauge couplings between the EW scale and the compactification scale.



Running of the gauge couplings

- Above the compactification scale, the couplings behave as a unified version and will all share the same gauge coefficient being :

$$c_{SO(10)} = -\frac{11}{3}C_2(G)+\frac{1}{6}C_2(G)+\frac{4}{3}2n_g(T_R(16))+\frac{1}{3}n_H^{10}T_R(10)$$

- where each term refers to a different field (Gauge 4-components + gauge 5th component + fermion + scalar),
  - n_g being the number of fermion generation in the bulk.
  - n_H^{10} being the number of 10 Higgs in the theory.
- In $SO(10)$, we have:
  - $C_2(G) = N - 2 = 8$.
  - $T_R(16) = 2$.
  - $T_R(10) = 1$.

Running of the gauge couplings

- For one 10-Higgs and three bulk fermions families, we obtain :

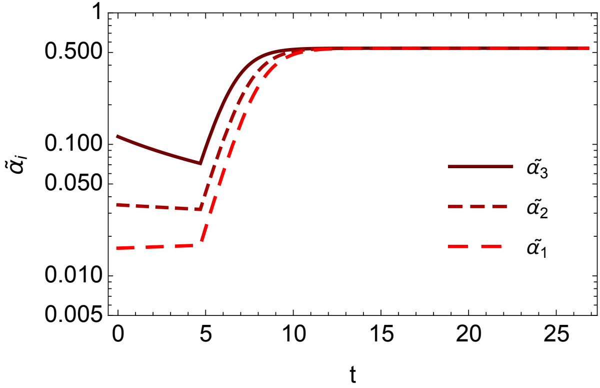
$$c_{SO(10)} = -\frac{35}{3}$$

- The RGE becomes then :

$$2\pi\frac{d\alpha_1}{dt} = \frac{41}{10}\alpha_1^2 + (S(t)-1)\,c_{SO(10)}\alpha_1^2\,,$$
$$2\pi\frac{d\alpha_2}{dt} = -\frac{19}{6}\alpha_2^2 + (S(t)-1)\,c_{SO(10)}\alpha_2^2\,,$$
$$2\pi\frac{d\alpha_3}{dt} = -7\alpha_3^2 + (S(t)-1)\,c_{SO(10)}\alpha_3^2\,.$$

- $t = \ln\left(\frac{\mu}{M_Z}\right)$ and contains the energy scale paameter μ. We chose to use the Z mass as a reference scale, so that for $\mu = M_Z$ we have $t = 0$ and we can fix the initial conditions of the running.

Running of the gauge couplings



- Running of the gauge couplings using one-loop factors, with $R^{-1} = 10$TeV. The range of t corresponds to the Z mass (t=0) and the reduced 5D Planck mass.

The asymptotic behaviour

- The asymptotic behaviour of the running of the gauge couplings can be easily understood when rewriting

$$2\pi \, \frac{d\,\alpha_i}{d\,t} = b^{\rm SM}\,\alpha_i^2 + (S(t)-1)\; b^{\rm aGUT}\,\alpha_i^2,$$

interm of $\tilde{\alpha}$ at large energies. Keeping the leading terms in $1/\mu R$

$$\frac{d\tilde{\alpha}}{dt} = \tilde{\alpha} + \frac{b^{aGUT}}{2\,\pi}(\tilde{\alpha})^2$$

where

$$\tilde{\alpha}_i(\mu) \sim \frac{\alpha_i(\mu)}{2}\,\mu\,R$$

- The beta function vanishes at:

$$\tilde{\alpha}|_{\rm IV} = 0, \qquad \tilde{\alpha}|_{\rm UV} = -\frac{2\,\pi}{b^{aGUT}}$$

which are the IR and the UV fixed points.

The asymptotic behaviour

- The UV fixed point only exists for $b^{aGUT} < 0$, accordingly the fixed point's existence requires $n_g \leqslant 5$.
- For 6 or more bulk generations the asymptotic unification would fail.
- For 3 bulk generations we find:

$$\tilde{\alpha} = \frac{6\pi}{35}$$

Running of the Yukawa couplings

- Adding all the parts for the bulk top Yukawa beta function above the compactification scale, we obtain the following RGE with respect to $t = \ln(\mu/m_Z)$:

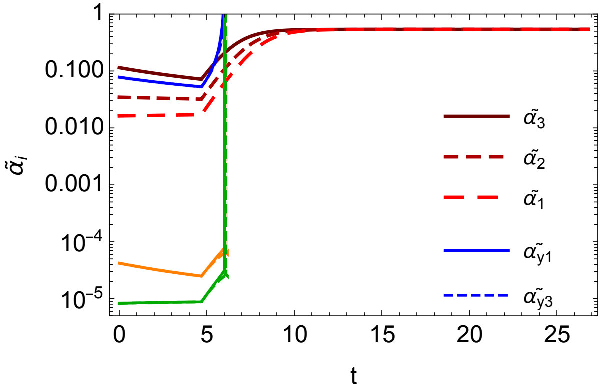
$$16\pi^2 \frac{dY}{dt}\bigg|_{aGUT} = y^3 \left(2 \times 8 + \frac{1}{2} \times 10 + \frac{1}{2} \times 10 + 2 \times 32\right)$$
$$+ y g_5^2 \left(6 \times \left(-\frac{27}{8}\right) - 3 \times \frac{9}{2} + 2 \times \frac{27}{8} + \frac{1}{2} \times \frac{45}{8} + \frac{1}{2} \times \frac{45}{8}\right)$$
$$+ y g_5^2 \xi \left(2 \times \left(-\frac{27}{8}\right) + \frac{45}{8} + \frac{45}{8} + \frac{9}{2} - 2 \times \frac{9}{4} - 2 \times \frac{9}{4}\right),$$

Running of the Yukawa couplings

- We computed the coefficients of the gauge contribution with an unified gauge coupling g_5.
- The sum of the contributions shows that the gauge-parameter ξ vanishes as expected by the gauge-invariance.
- Then, the one loop RGE gives:

$$16\pi^2 \frac{\mathrm{d}Y}{\mathrm{d}t}\bigg|_{\mathrm{aGUT}} = 90y^3 - \frac{171}{8} y g_5^2$$

Running of the Yukawa couplings



- Running of the gauge and Yukawa couplings using one-loop factors, with $R^{-1} = 10\text{TeV}$.

Conclusion

- We explicitly tested the asymptotic grand unification of a minimal 5-dimensional model with SO(10) gauge theory compactified on an $S^1/Z_2 \times Z_2'$ orbifold.
- We considered that all the matter fields propagate in the bulk.
- We shown that the gauge couplings asymptotically run to a unified fixed point in the UV.
- The Yukawa couplings will typically hit a Landau pole before the GUT scale in this class of $SO(10)$ models.