Scattering amplitudes and its soft decomposition

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Introduction

- Many progress has been made to unravel the mathematical structure of scattering amplitudes in gauge theories and gravity.
- These mathematical structures are often hidden in the complexity of the traditional Feynman diagram expansion.
- Powerful tools beyond Feynman diagrams have been largely developed: BCFW recursion, MHV rules, generalized unitarity cut, . . .

Introduction

- The Britto-Cachazo-Feng-Witten recursion relations were developed allowing any tree-level helicity amplitude of gluons to be calculated using a simple on-shell recursion relation.
- The on-shell recursion relation links higher point amplitude to the lower points, starting from the three points amplitudes.
- Central to this construction have been unitarity, locality and the transformation properties under Wigner's "little" group, a subgroup of the Lorentz group.

Objective

- **Goal:** The main objective of this work is to investigate the possibility of constructing any tree-level helicity amplitude of gluons from them asymptotic symmetries.
- **Expectation:** Such construction might leads to a new type of recursion that connects the lower points amplitudes to the higher, similar to the BCFW recursion.
- **Observation:** it is possible to connect an *n*-point amplitude to an $(n-1)$-point through the soft theorem of Weinberg:

$$\lim_{p\to 0} A_n(p) = S(p) A_{n-1}(0),$$

$S(p)$ is known as the soft operator. (universal)

Soft operator

- The Weinberg soft factorization is known to be universal, and we already showed in [arxiv:2002.02120] that

$$[H(p_i), S(p_j)] = \delta_{ij} h_j S(p_j),$$

- $H(p_i)$ helicity operator of the i-th particle:

$$H(p_i) = -\frac{1}{2}\left(\lambda_i^a \frac{\partial}{\partial \lambda_i^a} - \bar{\lambda}_i^{\dot{a}} \frac{\partial}{\partial \bar{\lambda}_i^{\dot{a}}}\right)$$

- $S(p_j)$ soft operator of the the i-th particle of helicity h_j: composition of the Weinberg soft factor and a soft momentum deformation.

Spinor helicity formalism

- Spinor variables λ_a and $\bar{\lambda}_{\dot{a}}$:

$$p_\mu \longrightarrow \sigma^\mu_{a\dot{a}} p_\mu = \lambda_a \bar{\lambda}_{\dot{a}}$$

- Invariant product

$$\langle 12 \rangle = \varepsilon^{ab} \lambda_a^{(1)} \lambda_b^{(2)} \quad \text{and} \quad [12] = \varepsilon^{\dot{a}\dot{b}} \bar{\lambda}_{\dot{a}}^{(1)} \bar{\lambda}_{\dot{b}}^{(2)}$$

- Lorentz scalar product to spinor products

$$p_1 \cdot p_2 = \frac{1}{2} \langle 12 \rangle [12]$$

- Other invariant variables

$$S_{12\cdots n} = (p_1 + p_2 + \cdots + p_n)^2 \quad \text{and} \quad [i(j)k\rangle = [ij] \langle jk \rangle$$

Inverse soft limit

- The soft factorization connect the A_n amplitude to the A_{n-1} amplitude in one direction:

$$A_n \xrightarrow{\text{soft limit}} A_{n-1}$$

- Inverting the soft limit will not always lead to the original amplitude

$$A_{n-1} \xrightarrow{\text{inverse soft limit}} A_n - R_n$$

where R_n is the missing part of the amplitude,

$$R_n \xrightarrow{\text{soft limit}} 0$$

NMHV Amplitude: $A_6(1^-2^-3^-4^+5^+6^+)$

- Consider the 6-point helicity amplitude of gluons:

$$\frac{1}{[2|3+4|5\rangle}\left(\frac{[4|5+6|1\rangle^3}{[34][23]\langle 56\rangle\langle 61\rangle S_{234}}+\frac{[6|1+2|3\rangle^3}{[61][12]\langle 34\rangle\langle 45\rangle S_{612}}\right)$$

- Taking the limit as the "6" go soft: the second expression tend to zero and the first term will be factorized as

$$\frac{\langle 51\rangle}{\langle 56\rangle\langle 61\rangle}\times\frac{[45]^3}{[12][23][34][51]}=S(6^+)\times A_5(1^-2^-3^-4^+5^+)$$

- Inverting the soft limit will leads only to the first term

$$R_6^{[6]}=\frac{1}{[2|3+4|5\rangle}\frac{[6|1+2|3\rangle^3}{[61][12]\langle 34\rangle\langle 45\rangle S_{612}}$$

NMHV Amplitude: $A_6(1^-2^-3^-4^+5^+6^+)$

▷ Viewed by the soft particle "6", the $A_6(1^-2^-3^-4^+5^+6^+)$ is decomposed as

$$A_6 = A_6^{[6]} + R_6^{[6]},$$

- $A_6^{[6]}$, is the soft invariant part of the amplitude viewed by the 6-th, such that

$$A_6^{[6]} \xrightarrow{\text{SL}(6)} A_5 \xrightarrow{\text{ISL}(6)} A_6^{[6]}$$

- $R_6^{[6]}$, is the soft part viewed by 6, such that

$$R_6^{[6]} \xrightarrow{\text{SL}_6} 0$$

NMHV Amplitude $A_6(1^-2^-3^-4^+5^+6^+)$

- Taking "1" to be soft: the first term of the amplitude tend to zero and we got

$$\lim_{1\to 0} A_6(1^-2^-3^-4^+5^+6^+) = S(1^-) \times A_5(2^-3^-4^+5^+6^+)$$

- The soft part of the amplitude viewed from "1" is then

$$R_6^{[1]} = \frac{1}{[2|3+4|5\rangle} \frac{[4|5+6|1\rangle^3}{[34][23]\langle 56\rangle \langle 61\rangle S_{234}}$$

Soft structure of amplitudes

▷ Viewed from the *i*-th particle of a given *n*-point amplitude, the amplitude can be decomposed as

- the core of the soft theorem $A_n^{[i]}$: the part that can be recovered by ISL.
- the soft shell of the amplitude $R_n^{[i]}$: the part that is lost from SL.

$$A_n = A_n^{[i]} + R_n^{[i]}.$$

▷ Knowing the "soft shell" $R_n^{[i]}$ of an amplitude, the A_{n-1} amplitudes can be mapped to A_n amplitude through ISL.

Soft structure of amplitudes

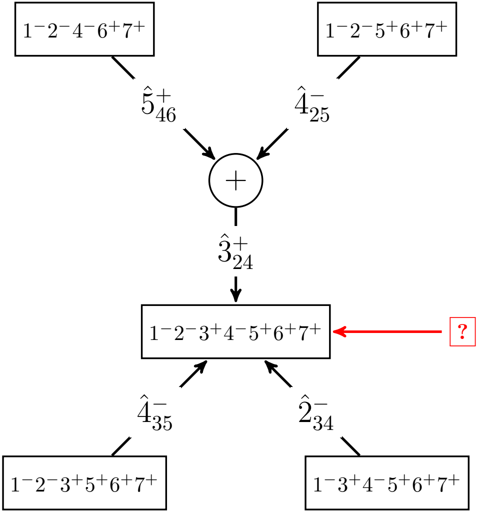
- The soft decomposition of an amplitude is not unique.
- For the 6-point amplitude $A_6(1^-2^-3^-4^+5^+6^+)$, there are 6 different decomposition (one for each particle), such that

$$\begin{cases} A_6^{[1]} = A_6^{[4]} = R_6^{[3]} = R_6^{[6]} \\ A_6^{[3]} = A_6^{[6]} = R_6^{[1]} = R_6^{[4]} \\ A_6^{[2]} = A_6^{[5]} \quad \text{and} \quad R_6^{[2]} = R_6^{[5]} \end{cases}$$

- And it turns out $A_6(1^-2^-3^-4^+5^+6^+)$ can be reconstructed from the A_5 amplitudes through ISL, which is given by

$$A_6 = \mathrm{ISL}_1 A_6(2^-3^-4^+5^+6^+) + \mathrm{ISL}_6 A_6(1^-2^-3^-4^+5^+)$$

Soft structure of amplitudes



Summary

- We aim to construct scattering amplitude from the asymptotic symmetries of the individual particles.
- Weinberg soft theorem can factorize A_{n-1} amplitudes from A_n amplitude.
- Soft operator are used to ISL A_{n-1} to the core $A_n^{[i]}$.
- Knowing the soft shell $R_n^{[i]}$ of amplitudes allow us to map A_{n-1} amplitude to the A_n.
- To develop an algorithm to generate the soft shell $R_n^{[i]}$, we have observed all possible soft decomposition of amplitudes up to A_7 for all possible helicity configuration.