On 6D N=(1,0) Supergravity

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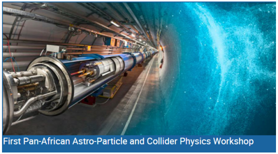


Plan

Motivations

Goal : Build a quantum gravity theory in 4D.

Problem : We have a very limited knowledge of the type of constraints imposed by UV completion in 4D.

Current Works : Find another space to know more about types of constraints.

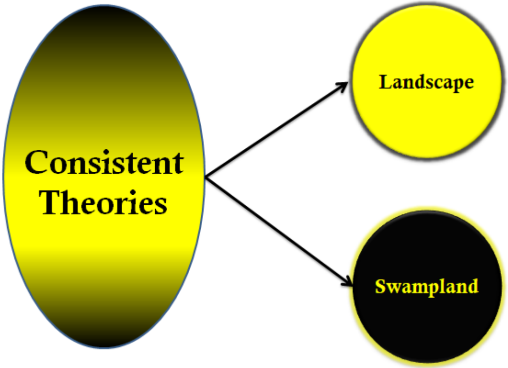
Motivations

Why 6D N=(1,0) supergravity space ?

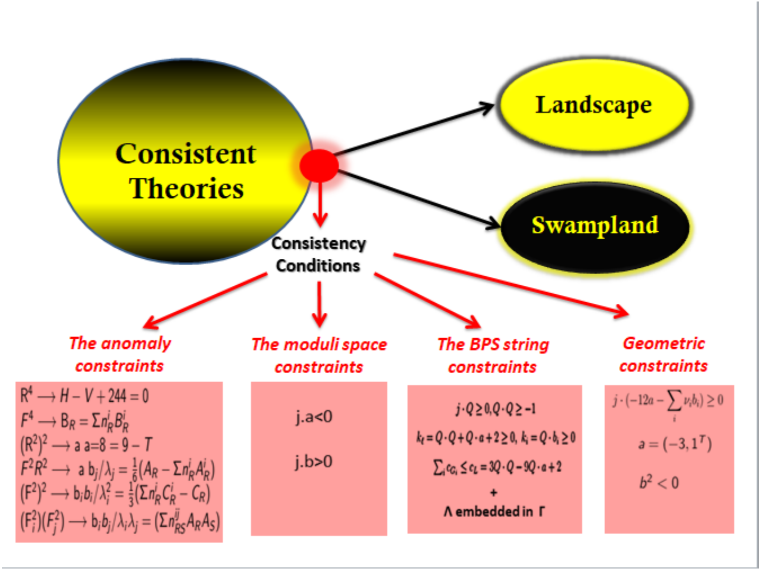
- **Successful Landscape analysis** : Strong constraints.
- **F-theory realizations** : 6D supergravity models are naturally realized within F-theoty compactified on three-folds.
- **8 supersymmetries** : We want to expand the work for 8 supersymmetries but 4D $\mathsf{N} = 2$.

Introduction

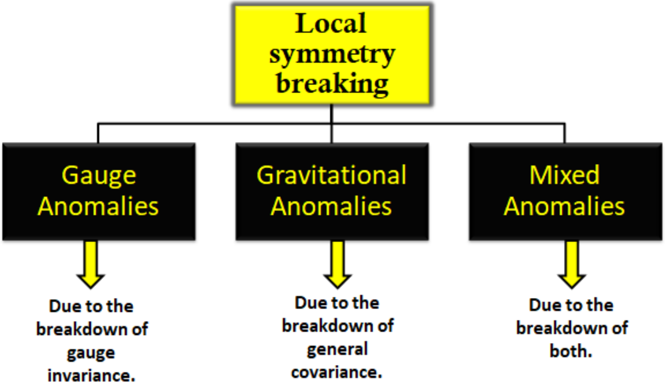
Consistent quantum field theories when coupled to gravity can be categorized as :



what are those constraints and conditions that could make a theory consistent or not ?


Consistent Theories
Landscape
Swampland
Consistency Conditions
The anomaly constraints
$R^4 \longrightarrow H - V + 244 = 0$
$F^4 \longrightarrow B_R = \Sigma n^i_R B^i_R$
$(R^2)^2 \longrightarrow a\,a{=}8 = 9 - T$
$F^2R^2 \longrightarrow a\,b_j/\lambda_j = \frac{1}{6}(A_R - \Sigma n^i_R A^i_R)$
$(F^2)^2 \longrightarrow b_i b_i/\lambda_i^2 = \frac{1}{3}(\Sigma n^i_R C^i_R - C_R)$
$(F^2_i)(F^2_j) \longrightarrow b_i b_j/\lambda_i\lambda_j = (\Sigma n^{ij}_{RS} A_R A_S)$
The moduli space constraints
j.a<0
j.b>0
The BPS string constraints
$j \cdot Q \geq 0, Q \cdot Q \geq -1$
$k_l = Q \cdot Q + Q \cdot a + 2 \geq 0,\ k_i = Q \cdot b_i \geq 0$
$\sum_i c_{G_i} \leq c_L = 3Q \cdot Q - 9Q \cdot a + 2$
+
Λ embedded in Γ
Geometric constraints
$j \cdot (-12a - \sum_i \nu_i b_i) \geq 0$
$a = (-3, 1^T)$
$b^2 < 0$

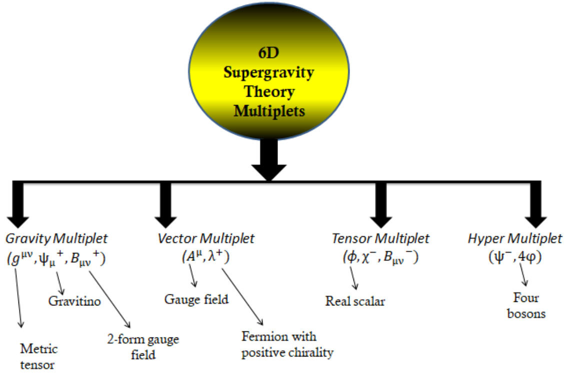
The anomaly constraints



To investigate whether the theory has anomalies or not, we need to express the anomaly polynomial.

Anomalies can be cancelled by the GreenSchwarz-Sagnotti mechanism if the anomaly polynomial I_8 factorizes as : $I_8(R,F) = \frac{1}{2}\Omega_{\alpha\beta}X_4^\alpha X_4^\beta$
$X_4^\alpha = \frac{1}{2}a^\alpha trR^2 + \Sigma b_i^\alpha \frac{2}{\lambda_i} trF_i^2$

So our goal is to express the anomaly polynomial in 6d dimensions and to factorize it so it can be cancelled.



Anomalies

Gauge Anomaly This type of anomaly in 6d is due to the antichiral spin fermion in the hypermultiplet and to the chiral spin fermion in the vector multiplet :

$$I_8^{\text{gauge}, \frac{1}{2}} = -\frac{1}{24(2\pi)^3} Tr_R F^4 + \frac{1}{24(2\pi)^3} \Sigma n_R^i Tr_R F^4 + \frac{1}{4(2\pi)^3} \Sigma n_R^i n_S^i Tr_R F^2 Tr_S F^2$$

R/S are representations of the considered group. n_R^i is the number of the hypermultiplet in the representation R.

Gravitational Anomaly They are contributions from the gravitino, the chiral and the anti-chiral fermions in the vector, tensor and hype rmultiplets. Adding also the contribution from the supergravity 3-form field :
The total gravitational anomaly is :

$$I_8^{\text{grav}} = \frac{1}{24(2\pi)^3}\left(\frac{1}{240}(H - V + 29T - 273) trR^4 + \frac{1}{192}(H - V - 7T + 51)(trR^2)^2\right)$$

Mixed Anomaly Due to the chiral and the anti chiral spin fermions in vector and hyper multiplets :

$$I_8^{\text{mixed}, \frac{1}{2}} = \frac{1}{(2\pi)^3}\left(\frac{1}{96} trR^2 Tr_R F^2 - \frac{1}{96} trR^2 \Sigma n_R^i Tr_R F^2\right)$$

The anomaly polynomial

We sum all the anomalies above to get :

$$(2\pi)^3 I_8 = \frac{1}{5760}(H - V + 244)trR^4 + \frac{1}{4608}(H - V + 44)(trR^2)^2 + \frac{1}{96}trR^2(Tr_R F^2 - \Sigma n_R^i Tr_R F^2) - \frac{1}{24}Tr_R F^4 + \frac{1}{24}\Sigma n_R^i Tr_R F^4 + \frac{1}{4}\Sigma n_{RS}^{ij} Tr_R F^2 Tr_S F^2$$

By taking T=1, one tensor multiplet.

However, the term trR^4 cannot be factorized in the said way, thus the term associated with it needs to vanish : H-V+244=0

To transform the trace from R representation to the fundamental representation, we use :

$$Tr_R F^2 = A_R trF^2, \; Tr_R F^4 = B_R trF^4 + C_R (trF^2)^2$$

Finally, we have :

$$I_8 = \Big[(trR^2)^2 + \frac{1}{6} trR^2 (A_R - \Sigma n_R^i A_R^i) trF^2 - \frac{2}{3}(B_R - \Sigma n_R^i B_R^i) trF^4 - \frac{2}{3}(C_R - \Sigma n_R^i C_R^i)(trF^2)^2 + 4\Sigma n_{RS}^{ij} tr_R F^2 tr_S F^2 A_R A_S \Big]$$

The goal is to write is as :

$$I_8(R,F) = \frac{1}{2}\Omega_{\alpha\beta} X_4^\alpha X_4^\beta \text{ where } X_4^\alpha = \frac{1}{2} a^\alpha trR^2 + \Sigma b_i^\alpha \frac{2}{\lambda_i} trF_i^2$$

Thus :

$$I_8(R,F) = \frac{1}{2}\Big[\frac{a^\alpha a^\beta}{4}(trR^2)^2 - 2\frac{\Sigma a^\alpha b_j^\beta}{\lambda_j} trR^2 trF^2 + 4\frac{\Sigma b_i^\alpha \Sigma b_j^\beta}{\lambda_i \lambda_j} trF_i^2 trF_j^2 \Big]$$

The anomaly constraints

The anomaly factorization conditions for gravitational, gauge and mixed anomalies are summarized as follows :

$\mathrm{R}^4 \longrightarrow H - V + 244 = 0 = 273 - 29T$
$F^4 \longrightarrow \mathrm{B}_R = \Sigma n_R^i B_R^i$
$(\mathrm{R}^2)^2 \longrightarrow$ a a$=8 = 9 - T$
$F^2 R^2 \longrightarrow$ a $\mathrm{b}_j/\lambda_j = \frac{1}{6}(A_R - \Sigma n_R^i A_R^i)$
$(\mathrm{F}^2)^2 \longrightarrow \mathrm{b}_i b_i/\lambda_i^2 = \frac{1}{3}(\Sigma n_R^i C_R^i - C_R)$
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The vectors, a^α, b_i^α are constrained to have integer inner products $a \quad a, a \quad b_j, b_i \quad b_i$. We call this the anomaly lattice.

Also...

The 6D theory has gravity/gauge dyonic strings with charges a, b_i. Those charges span the anomaly lattice which is contained in the full string lattice of the 6d theory. The anomaly lattice is required to have a unimodular embedding into a self-dual lattice and this fact provides a constraint on possible theories.

How does this work?

Example : 6D supergravity theory with T = 1, gauge group G=SU(4)

The matter content is :

$$1 \times (15) + 40 \times (4) + 10 \times (6)$$

- (15) → Adjoint representation
- (4) → Fundamental representation
- (6) → Antisymmetric representation

Now we need to write the anomaly lattice.

$$a.b=\frac{\lambda}{6}\left(A_{adj}-\sum n_R A_R\right)=\frac{1}{6}(8-(8+40*1+10*2)=-10$$

a.a=9-T=9-1=8

$$\Lambda=\begin{pmatrix} a.a & -a.b \\ -a.b & b.b \end{pmatrix}$$

$$b.b=\frac{\lambda^2}{3}\left(\sum n_R C_R - C_{adj}\right)=\frac{1}{3}(6+40*0+10*3-6)=10$$

Group	Representation	Dimension	A_R	B_R	C_R
$SU(N)$	$\square$	N	1	1	0
	Adjoint	N^2-1	$2N$	$2N$	6
	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\frac{N(N-1)}{2}$	$N-2$	$N-8$	3
	$\square\square$	$\frac{N(N+1)}{2}$	$N+2$	$N+8$	3
	$\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}$	$\frac{N(N-1)(N-2)}{6}$	$\frac{N^2-5N+6}{2}$	$\frac{N^2-17N+54}{2}$	$3N-12$
	$\begin{smallmatrix}\square\square\\\square\end{smallmatrix}$	$\frac{N(N^2-1)}{3}$	N^2-3	N^2-27	$6N$
	$\square\square\square$	$\frac{N(N+1)(N+2)}{6}$	$\frac{N^2+5N+6}{2}$	$\frac{N^2+17N+54}{2}$	$3N+12$

G	$SU(N)$	$SO(N)$	$Sp(N)$	G_2	F_4	E_6	E_7	E_8
λ	1	2	1	2	6	6	12	60

Example : 6D supergravity theory with T = 1, gauge group G=SU(4)

$$\Lambda=\begin{pmatrix} 8 & 10 \\ 10 & 10 \end{pmatrix}$$

The lattice above needs to have a perfect square determinant. Since the determinant is -20, not a perfect square than the lattice does not admit an embedding into any unimodular lattice.

Moduli space

The moduli space of the 6d (1,0) supergravity locally takes the form SO(1,T)/SO(T) which is parametrized by the a vector j with positive norm $j.j > 0$ representing the positivity of the metric on the moduli space.

The next set of constraints are required for the proper sign of the gauge kinetic term and gravitational term.
Actually, The gauge kinetic term is proportional to $-j.b_i trF^2$. Thus we require that that $j.b_i > 0$.
The gravitational term is proportional to $j.atrR^2$, we require consequently that $j.a < 0$.

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For example, if a theory has : $b = \lambda a, \lambda > 0$, it will be inconsistent because we can't find any a, b that verify simultaneously $j.a < 0$ and $j.b > 0$.

BPS string consideration

The existence of the two-form fields B_2 implies the existence of string sources in accordance with the hypothesis that the spectrum of a gravitational theory needs to be complete.

The worldsheet theory on those strings gives rise to 2d SCFT at low energy with charge Q. Next, we need to express the anomaly inflow from 6d to 2d :

$$I_4 = -I_4^{inflow} = \Omega_{\alpha\beta} Q^\alpha (X_4^\alpha + \frac{1}{2} Q^\beta (c(I) - c(R))$$

since :

$$X_4^\alpha = \frac{1}{2} a^\alpha trR^2 + \frac{1}{4} \Sigma b_i^\alpha trF_i^2$$

then :

$$I_4 = \Omega_{\alpha\beta} Q^\alpha (\frac{1}{2} a^\alpha trR^2 + \frac{1}{4} \Sigma b_i^\alpha trF_i^2 + \frac{1}{2} Q^\beta (c(I) - c(R))$$

we have :

$$trR^2 = \frac{-1}{2} p(T) + c(I) + c(R)$$

Then :

$$I_4 = \frac{-Q.a}{4} p(T) + \frac{1}{4} \Sigma Q.b_i^\alpha trF_i^2 - \frac{Q.Q - Q.a}{2} c(R) + \frac{Q.Q + Q.a}{2} c(I)$$

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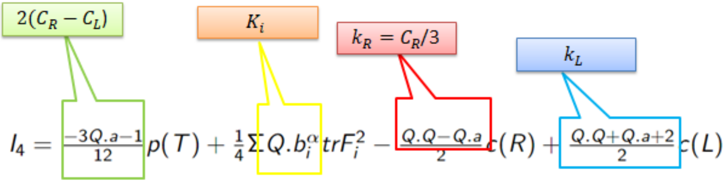
By eliminating the contribution from the center of mass degrees of freedom :

$$I_4 \dashrightarrow I_4 - \left(-\frac{1}{12} p(T) - c(I)\right)$$

Which gives :

$$I_4 = -\frac{3Q.a - 1}{12} p(T) + \frac{1}{4} \Sigma Q.b_i^{\alpha} trF_i^2 - \frac{Q.Q - Q.a}{2} c(R) + \frac{Q.Q + Q.a + 2}{2} c(I)$$



Finally, since the central charges need to be positive :

$$Q.Q \geq -1, k_l = Q.Q + Q.a + 2 \geq 0, k_i = Q.b_i \geq 0$$

A worldsheet theory has non-negative tension only if $Q \cdot j \geq 0$. Only strings with $Q \cdot j \geq 0$ embedded in 6d supergravity theories give rise to unitary 2d SCFTs.

The second condition that rises from SCFT : The central charge of group G is,

$$c_G = \frac{k.dimG}{k+h}$$

where k is the k_i defined by Q.b previously and h is the dual Coxeter number of group G, for SU(N) is equal to N.
The SCFT states that, the current algebra for group G is on the left-moving sector. This tells us that the sum of the group central charges is less than c_L.
Let's take an example

The example is the 6d supergravity with T = 1 and SU(N) gauge group with :

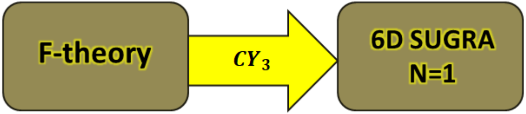
$$\Omega = (1, -1), a = (-3, 1), b = (0, -1), j = (n, 1)$$

Let's derive the consistency conditions for $Q = (q_1, q_2)$.

$$\begin{aligned} Q.Q \geq -1 &= \quad q_1^2 - q_2^2 \geq -1 \\ Q.Q + Q.a + 2 \geq 0 &= \quad q_1^2 - q_2^2 - 3q_1 - q_2 \geq -2 \\ k_i = Q.b_i &= q_2 \geq 0 \\ j.Q &= nq_1 - q_2 \geq 0 \\ c_{Gi} &= \frac{k.dimG}{k+h} \leq c_L = 3Q.Q - 9Q.a + 2 \\ &= \frac{q_2(N^2-1)}{q_2+N} \leq 3(q_1^2 - q_2^2) + 9(3q_1 + q_2) + 2 \end{aligned}$$

Geometric conditions

6D $N = (1, 0)$ supergravity models are 'naturally' realized in the context of F-theory : a large class of 6D supergravity theories are realized by F-theoty compactified on three-folds.



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One of F-theory constraints on low-energy supergravity is the Kodaira condition :

$$-12K_B = \square \; \nu_i \;\; {}_i + Y$$

with,

$$J.Y > 0$$

Where K_B, is a canonical divisor. $_i$, are effective, irreducible curves.
Y, is a residual divisor and J is a Kahler class .

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By constructing the following mapping from 6d sugra to f-theory :

$$\begin{aligned} a &= \quad K_B \\ b &= \quad {}_i \\ j &= \quad J \end{aligned}$$

The Kodaira condition becomes :

$$j(-12a - \square \, \nu_i b_i) \geq 0$$

Additional constraints on both a and b_i could be formulated based on the previous map. Since :

$$-K_B = (3, -1, -1..., -1)$$

Then, we have :

$$a = (-3, 1^T)$$

Also :

$$< 0$$

Then :

$$b^2 < 0$$