

# $1^+ T_{cc}$ -like states from Laplace sum rules and Double ratio of sum rules<sup>1</sup>

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In collaboration with R. M. Albuquerque and S. Narison



**1<sup>st</sup> Pan-African Astro Particle &  
Collider Physics Workshop**



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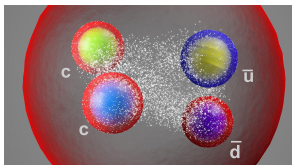
1. R. Albuquerque, S. Narison and D. Rabetiarivony, arXiv:2201.13449 [hep-ph] (will be published in Nuclear Physics A)

# Motivation

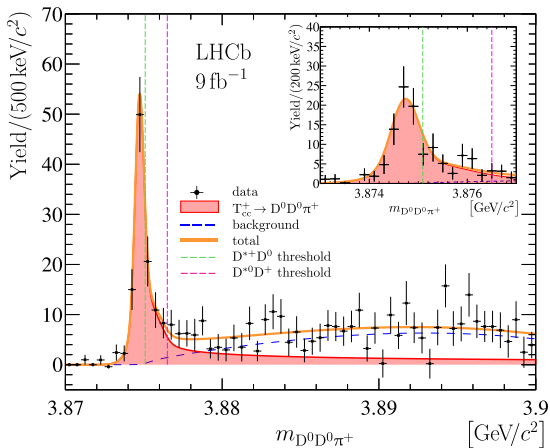
arXiv : 2109.01056 [hep-ex]

Observation of doubly charmed

tetraquark :  $T_{cc}^+$



Mass around 3875 MeV



## Two-point function

Evaluation of two point function  $\rightsquigarrow$  Hadron parameters

$$\Pi_{\mathcal{H}}^{\mu\nu}(q^2) = i \int d^4x \, e^{-iqx} \langle 0 | \mathcal{T} \mathcal{O}_{\mathcal{H}}^{\mu}(x) (\mathcal{O}_{\mathcal{H}}^{\nu}(0))^{\dagger} | 0 \rangle$$

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## Currents

$$\begin{aligned} \mathcal{O}_{\chi_c(3872)} &= \epsilon_{ijk} \epsilon_{mnk} \left[ (q_i^T C \gamma_5 c_j) (\bar{c}_m \gamma^{\mu} C \bar{q}_n^T) + (q_i^T C \gamma^{\mu} c_j) (\bar{c}_m \gamma_5 C \bar{q}_n^T) \right] \\ \mathcal{O}_{T_{cc\bar{u}\bar{d}}^{1+}} &= \frac{1}{\sqrt{2}} \epsilon_{ijk} \epsilon_{mnk} (c_i^T C \gamma^{\mu} c_j) \left[ (\bar{u}_m \gamma_5 C \bar{d}_n^T) - (\bar{d}_m \gamma_5 C \bar{u}_n^T) \right] \\ \mathcal{O}_{T_{cc\bar{u}\bar{s}}^{1+}} &= \epsilon_{ijk} \epsilon_{mnk} (c_i C \gamma^{\mu} c_j^T) (\bar{u}_m \gamma_5 C \bar{s}_n^T) \end{aligned}$$

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- ♣ QCD side : quark and gluon fields, OPE  $\Rightarrow$  Dispersion relation
- ♣ Phenomenological side : Hadron parameters  $\Rightarrow$  Dispersion relation
- ♣ Quark Hadron duality principle  
 QCD side  $\simeq$  PHEN side
- ♣ Inverse Laplace Transform

# Mass Extraction

Finite Energy Inverse Laplace Transform Sum Rule :

$$\mathcal{L}_n^c|_{\mathcal{H}}(\tau, \mu) = \int_{(2M_c+m_q+m_s)^2}^{\tau_c} dt t^n e^{-t\tau} \frac{1}{\pi} \text{Im} \Pi_{\mathcal{H}}(t, \mu)$$

Ansatz :

$$\frac{1}{\pi} \text{Im} \Pi_{\mathcal{H}}(t) = 2f_{\mathcal{H}}^2 M_{\mathcal{H}}^8 \delta(t - M_{\mathcal{H}}^2) + \frac{1}{\pi} \text{Im} \Pi_{\mathcal{H}}^{\text{QCD}}(t) \theta(t - t_c)$$

$$M_{\mathcal{H}}^2 = \mathcal{R}_{\mathcal{H}}^c(\tau_0) = \frac{\mathcal{L}_1^c|_{\mathcal{H}}}{\mathcal{L}_0^c|_{\mathcal{H}}};$$

$$r_{\mathcal{H}'/\mathcal{H}}(\tau_0) \equiv \sqrt{\frac{\mathcal{R}_{\mathcal{H}'}^c}{\mathcal{R}_{\mathcal{H}}^c}} = \frac{M_{\mathcal{H}'}}{M_{\mathcal{H}}}$$

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⊛ We use **stability criteria** with respect to  $(\tau, t_c, \mu)$  to extract the lowest ground state mass and coupling of hadron  $\mathcal{H}$  [S.

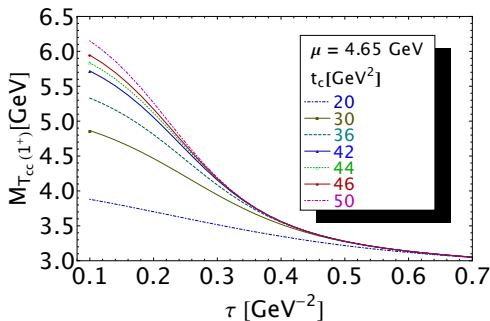
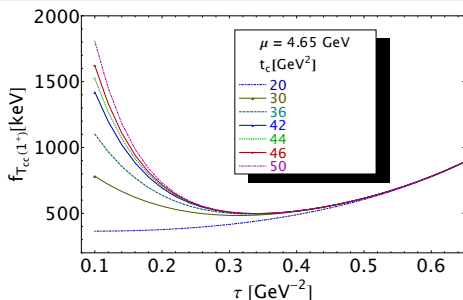
**Narison : QCD21]**

⊛ **OPE convergence** obtained for condensates up to  $d \leq 6$

⊛ To prevent the **violation of factorization** the inclusion of higher dimension condensates is not suggested

⊛ Inclusion of **NLO PT correction** to justify the use of  $\overline{\text{MS}}$  running mass for heavy quark

# $J^P = 1^+ T_{cc\bar{u}\bar{d}}$ and $T_{cc\bar{u}\bar{s}}$ states



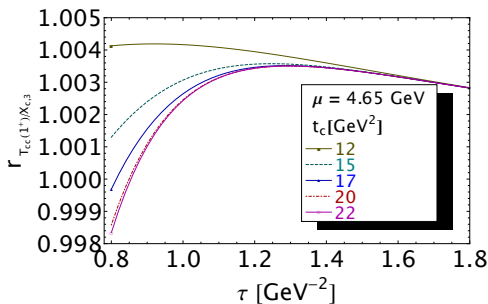
♣ Beginning of  $\tau$ -stability for :  $(\tau, t_c) = (0.31, 30)$  ( $\text{GeV}^{-2}, \text{GeV}^2$ )

♣  $t_c$ -stability reached for :  $(\tau, t_c) = (0.34, 46)$  ( $\text{GeV}^{-2}, \text{GeV}^2$ )

$$\Rightarrow f_{T_{cc}(1^+)} = 491(48) \text{ keV} ; \quad M_{T_{cc}(1^+)} = 3885(123) \text{ MeV}$$



# $J^P = 1^+ T_{cc\bar{u}\bar{d}}$ and $T_{cc\bar{u}\bar{s}}$ states



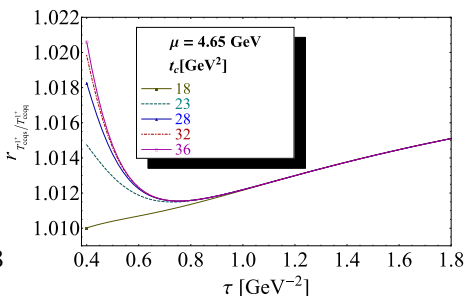
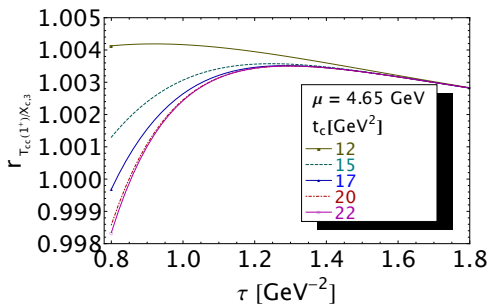
♣ Stability region :

$$(\tau, t_c) = (1.24, 15) \sim (1.30, 20)$$

$$\odot r_{T_{cc}^{1^+}/\chi_c} = 1.0035(10)$$

$$\Rightarrow M_{T_{cc}}(1^+) = 3886(4) \text{ MeV}$$

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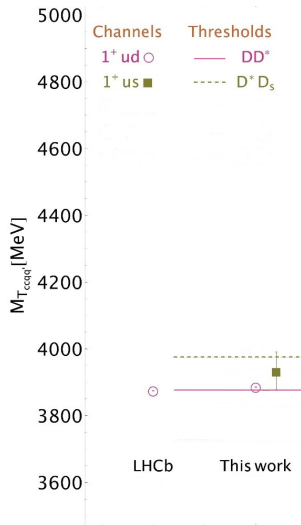
♣ Stability region :

$$(\tau, t_c) = (0.72, 23) \sim (0.74, 32)$$

$$\odot r_{T_{cc\bar{s}\bar{u}}/T_{cc}(1^+)} = 1.0115(13)$$

$$\Rightarrow M_{T_{cc\bar{s}\bar{u}}}(1^+) = 3931(7) \text{ MeV}$$

# Summary



- ⊗ We have systematically evaluated the masses and couplings of  $I(J^P) = 0(1^+)$   $(cc)(\bar{u}\bar{d})$  and  $I(J^P) = \frac{1}{2}(1^+)$   $(cc)(\bar{u}\bar{s})$  tetraquark states using QCD Laplace sum rules to study the recent data from LHCb.
- ⊗ We have included the NLO PT corrections to justify the use of the running heavy quark mass and to improve the LO results in literature.
- ⊗ Our predictions are grouped around the physical threshold.
- ⊗ The mass shift due to SU3 breaking is tiny.