NLO Scattering in ϕ^4 Theory Finite System Size Correction

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Based on W.A. Horowitz and J.F. Du Plessis, arXiv:2203.01259,
W.A. Horowitz and J.F. Du Plessis, in Preparation
Pan-African Astro-Particle and Collider Physics Workshop

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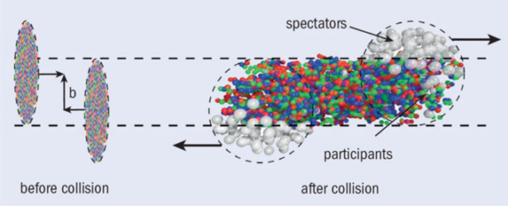
Heavy Ion Collisions



Heavy ions (such as Pb+Pb) are accelerated to nearly the speed of light.

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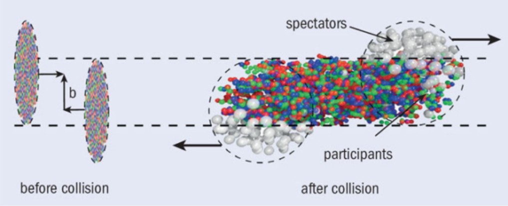

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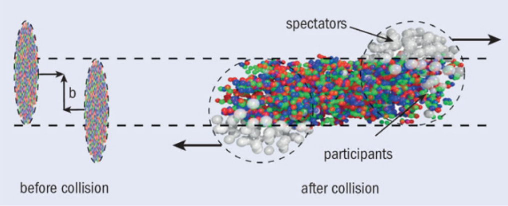

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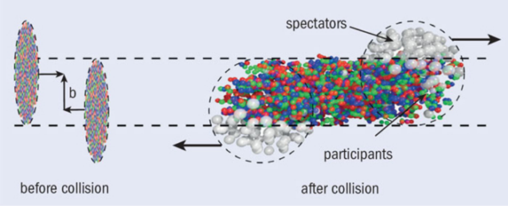

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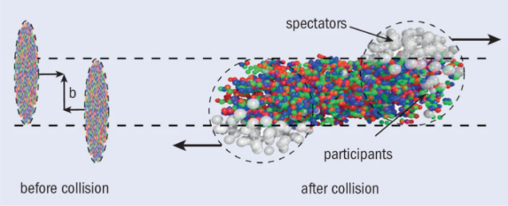

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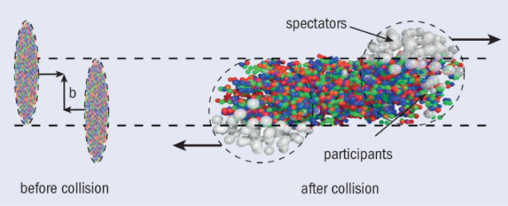

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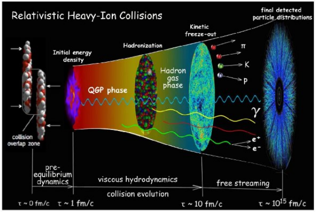
Highly non-trivial situation to describe

Experimental Quark Gluon Plasma



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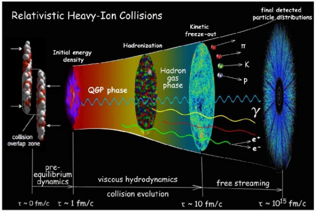

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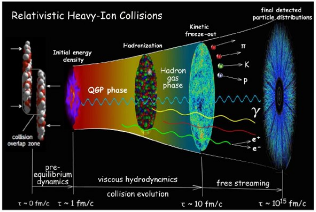

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Seems to behave like a liquid (highly correlated), not a gas

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What is this new phase in QCD?

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For 'small' (experimentally accessible temperatures T 350MeV) the phase appears to be well described by (strongly coupled) relativistic hydrodynamics under certain assumptions, and crucially dependent on lattice QCD calculations

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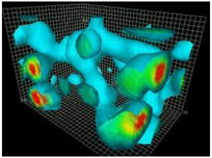
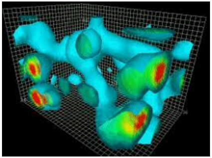
Lattice QCD



Lattice QCD is a non-peturbative approach to QCD

Lattice QCD



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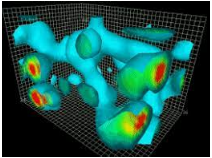

Lattice QCD is a non-peturbative approach to QCD

When performing computer Lattice QCD simulations, it is essentially a 'black-box'

It is often then unclear what the effect of certain assumptions are on the result of a computation

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The role of Lattice QCD in viscosity calculations

Using Lattice QCD with various assumptions (including infinite system size) we can calculate the trace anomaly in these heavy ion collisions

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Using this trace anomaly, one can calculate the equation of state of the relativistic hydrodynamic system one imagines to describe the system

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- Using Lattice QCD with various assumptions (including in nite system size) we can calculate the trace anomaly in these heavy ion collisions
- Using this trace anomaly, one can calculate the equation of state of the relativistic hydrodynamic system one imagines to describe the system
- One then arrives at a very low value of viscosity, as a consequence of the equation of state

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The Problem

How do various parameters that di er from the ideal simple case in the Lattice calculation impact this trace anomaly?

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Any e ect that have an impact on the `e ective' coupling will also a ect the trace anomaly

How large then is each e ect on the trace anomaly?

Where we fit in

An obvious place to check for deviations from the ideal case is in the finite size of the system present in experiments

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- The nite size calculation in QCD is highly non-trivial
- First develop the necessary mathematical tools and intuition in a simple case
- Consider periodic boundary conditions and theory

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From integrals to sums

$$\int d^3p \quad \sum$$

With periodic boundary conditions, our momentum space goes from R^3 to some 3 dimensional lattice , where the lattice spacings in each direction is the inverse of the length scale of that nite dimension

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In such discrete momentum spaces, standard techniques, such as dimensional regularization, fails or becomes unnecessarily di cult

We have therefore had to utilize alternative, less known, techniques such as denominator regularization

Derived expressions

Various results were found (see arXiv:2203.01259)

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Two very general generalizations that will certainly be useful elsewhere:

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Analytic Continuation of the Generalized Epstein Zeta Function

$$\underset{n 2 Z^p}{X} (a_i^2 n_i^2 + b_i n_i + c \quad i") \quad s$$

$$= \frac{1}{a_1 \quad a_p} \frac{1}{(s)} \quad p{=}2 \quad s \quad \frac{p}{2} \quad c \quad X \frac{b_i^2}{4a_i^2} \quad i" \quad \frac{p}{2} \quad s$$

$$+ 2 \quad s \underset{m 2 Z^p}{X} {}^0 e^{\,2 \, i \, P \frac{m_i b_i}{2a_i^2}} \quad \frac{c \quad P \frac{b_i^2}{4a_i^2} \quad i"}{P \frac{m_i^2}{a_i^2}} \quad \frac{p}{4} \quad \frac{s}{2}$$

$$K_{s \quad \frac{p}{2}} \quad 2 \quad s \overline{(c \quad X \frac{b_i^2}{4a_i^2} \quad i") \quad X \frac{m_i^2}{a_i^2}}$$

Generalized Epstein Zeta Function

In $2 \to 2$ NLO scattering we encounter

$$V(p^2) \propto \int_0^1 dx \sum_{\vec{k} \in \mathbb{Z}^3} \frac{2}{\left[\sum_{i=1}^{3} \left(\frac{k^i}{L_i} + x p^i \right)^2 + \;^2 \right]^{\frac{3}{2}+}}$$

Generalized Epstein Zeta Function

In 2 ! 2 NLO scattering we encounter

$$V(p^2) / \int_0^1 dx \sum_{\vec{k} 2 Z^3} \frac{2}{\left[\sum_{i=1}^{3} \left(\frac{k^i}{L_i} + x p^i\right)^2 + \quad ^2\right]^{\frac{3}{2}+}}$$

Which we can renormalize to nd the amplitude takes the form

$$M = 1 + \overline{V}(s) + \overline{V}(t) + \overline{V}(u)$$

Derived expressions continued

The prevalence of the sinc function $\mathrm{sinc}(x) = \frac{\sin(x)}{x}$ in these finite size systems' descriptions, leads one to seek a generalization to a formula originally proposed by Ramanujan and formalized by Hardy

$$\sum_{0 \le n < x} \frac{r_2(n)}{\sqrt{x}\sqrt{n}} = 2\pi\sqrt{x} + \sum_{n=1}^{\infty} \frac{r_2(n)}{\sqrt{n}} \sin(2\pi\sqrt{nx})$$

which we find to be

$$\sum_{\vec{k} \in Z^m} \mathrm{sinc}(2\pi R\|\vec{k}\|) = \frac{1}{2R\pi^{\frac{m-1}{2}}\Gamma(\frac{3-m}{2})} \sum_{l=0}^{\lfloor R^2 \rfloor} \frac{r_m(l)}{\sqrt{R^2 - l}^{\,m-1}}$$

Which enabled us to show that unitarity holds in the finite size system we are describing, for $m = 1; 2; 3$ finite dimensions.

Size of corrections

We find corrections on the same order of magnitude as the infinite volume calculation

What remains?

We can see that there is a possibility of reasonable size corrections to calculations performed in nite sized systems. This is an important step of a much longer journey. Some of what remains to be done:

- Thermal Field Theory of nite size ^{4} theory
- Critical exponents of nite size ^{4} theory
- Find closed form results for many numerically di cult results already found
- Generalize to more complicated systems, eventually such as QCD itself

The End

Thank you for making it to the end of my talk!
The first paper is available as a preprint "Finite System Size Correction to NLO Scattering in ^{4} Theory" at arXiv:2203.01259.
Look out for our follow up paper going into the numerics of our results, expected to be on arXiv soon.
Feel free to contact me with any further questions:
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