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# NLO Scattering in $\phi^4$ Theory Finite System Size Correction

#### J. F. Du Plessis<sup>1</sup> W. A. Horowitz<sup>2</sup>

<sup>1</sup>Department of Physics Stellenbosch University 23787295@sun.ac.za

<sup>2</sup>Department of Physics University of Cape Town

Based on W.A. Horowitz and J.F. Du Plessis, arXiv:2203.01259, W.A. Horowitz and J.F. Du Plessis, in Preparation Pan-African Astro-Particle and Collider Physics Workshop



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## Heavy Ion Collisions



• Heavy ions (such as Pb+Pb) are accelerated to nearly the speed of light.



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## Heavy Ion Collisions



- Heavy ions (such as Pb+Pb) are accelerated to nearly the speed of light.
- $\gamma \gg 10$  Lorentz factors; 'pancake nuclei'



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# Heavy Ion Collisions



- Heavy ions (such as Pb+Pb) are accelerated to nearly the speed of light.
- $\gamma \gg 10$  Lorentz factors; 'pancake nuclei'
- Collision with centers offset by impact parameter b

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# Heavy Ion Collisions



- Heavy ions (such as Pb+Pb) are accelerated to nearly the speed of light.
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- Effective finite system size on the order of nucleus radii  $\sim 5\times 10^{-15} {\rm m}$



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# Heavy Ion Collisions



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#### Experimental Quark Gluon Plasma



• Present in first  $\sim$  0.000001 seconds after Big Bang



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## Experimental Quark Gluon Plasma



- Present in first  $\sim$  0.000001 seconds after Big Bang
- Free color charge, hence 'plasma'

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## Experimental Quark Gluon Plasma



- Present in first  $\sim$  0.000001 seconds after Big Bang
- Free color charge, hence 'plasma'
- Seems to behave like a liquid (highly correlated), not a gas



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#### What is this new phase in QCD?

• It is unclear what happens in QCD just above the transition temperature T = 180 MeV



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#### What is this new phase in QCD?

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#### What is this new phase in QCD?

- It is unclear what happens in QCD just above the transition temperature T = 180 MeV
- For 'small' (experimentally accessible temperatures  $T \sim 350 \text{MeV}$ ) the phase appears to be well described by (strongly coupled) relativistic hydrodynamics under certain assumptions, and crucially dependent on lattice QCD calculations



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## Lattice QCD



• Lattice QCD is a non-peturbative approach to QCD



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#### Lattice QCD



- Lattice QCD is a non-peturbative approach to QCD
- When performing computer Lattice QCD simulations, it is essentially a 'black-box'



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#### Lattice QCD



- Lattice QCD is a non-peturbative approach to QCD
- When performing computer Lattice QCD simulations, it is essentially a 'black-box'
- It is often then unclear what the effect of certain assumptions are on the result of a computation



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#### The role of Lattice QCD in viscosity calculations



• Using Lattice QCD with various assumptions (including infinite system size) we can calculate the trace anomaly in these heavy ion collisions



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#### The role of Lattice QCD in viscosity calculations



- Using Lattice QCD with various assumptions (including infinite system size) we can calculate the trace anomaly in these heavy ion collisions
- Using this trace anomaly, one can calculate the equation of state of the relativistic hydrodynamic system one imagines to describe the system



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### The role of Lattice QCD in viscosity calculations



- Using Lattice QCD with various assumptions (including infinite system size) we can calculate the trace anomaly in these heavy ion collisions
- Using this trace anomaly, one can calculate the equation of state of the relativistic hydrodynamic system one imagines to describe the system
- One then arrives at a very low value of viscosity, as a consequence of the equation of state



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# The Problem



• How do various parameters that differ from the ideal simple case in the Lattice calculation impact this trace anomaly?



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# The Problem



- How do various parameters that differ from the ideal simple case in the Lattice calculation impact this trace anomaly?
- Any effect that have an impact on the 'effective' coupling will also affect the trace anomaly

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# The Problem



- How do various parameters that differ from the ideal simple case in the Lattice calculation impact this trace anomaly?
- Any effect that have an impact on the 'effective' coupling will also affect the trace anomaly
- How large then is each effect on the trace anomaly?



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#### Where we fit in



• An obvious place to check for deviations from the ideal case is in the finite size of the system present in experiments



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#### Where we fit in



- An obvious place to check for deviations from the ideal case is in the finite size of the system present in experiments
- Such finite size systems introduce non-trivial topology, as well as a discretization of momentum modes.



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## Where we fit in



- An obvious place to check for deviations from the ideal case is in the finite size of the system present in experiments
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- The finite system size requires new mathematical methods to properly tackle

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- The finite size calculation in QCD is highly non-trivial



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## Where we fit in



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- The finite size calculation in QCD is highly non-trivial
- First develop the necessary mathematical tools and intuition in a simple case



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From integrals to sums



• With periodic boundary conditions, our momentum space goes from  $\mathbb{R}^3$  to some 3 dimensional lattice  $\Lambda$ , where the lattice spacings in each direction is the inverse of the length scale of that finite dimension



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From integrals to sums



- With periodic boundary conditions, our momentum space goes from  $\mathbb{R}^3$  to some 3 dimensional lattice  $\Lambda$ , where the lattice spacings in each direction is the inverse of the length scale of that finite dimension
- In such discrete momentum spaces, standard techniques, such as dimensional regularization, fails or becomes unnecessarily difficult

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From integrals to sums



- With periodic boundary conditions, our momentum space goes from  $\mathbb{R}^3$  to some 3 dimensional lattice  $\Lambda$ , where the lattice spacings in each direction is the inverse of the length scale of that finite dimension
- In such discrete momentum spaces, standard techniques, such as dimensional regularization, fails or becomes unnecessarily difficult
- We have therefore had to utilize alternative, less known, techniques such as denominator regularization



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# Derived expressions

#### • Various results were found (see arXiv:2203.01259)



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# Derived expressions

- Various results were found (see arXiv:2203.01259)
- Two very general generalizations that will certainly be useful elsewhere:



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# Derived expressions

- Various results were found (see arXiv:2203.01259)
- Two very general generalizations that will certainly be useful elsewhere:
- Analytic Continuation of the Generalized Epstein Zeta Function

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#### Generalized Epstein Zeta Function



• In  $2 \rightarrow 2$  NLO scattering we encounter

$$V(p^2) \propto \int_0^1 dx \sum_{\vec{k} \in \mathbb{Z}^3} \frac{\mu^{2\epsilon}}{\left(\sum_{i=1}^3 \left(\frac{k^i}{L_i} + x p^i\right)^2 + \Delta^2\right)^{\frac{3}{2} + \epsilon}}$$



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# Generalized Epstein Zeta Function



• In 2  $\rightarrow$  2 NLO scattering we encounter

$$V(p^2) \propto \int_0^1 dx \sum_{\vec{k} \in \mathbb{Z}^3} rac{\mu^{2\epsilon}}{\left(\sum_{i=1}^3 \left(rac{k^i}{L_i} + x p^i\right)^2 + \Delta^2
ight)^{rac{3}{2} + \epsilon}}$$

• Which we can renormalize to find the amplitude takes the form

$$\mathcal{M} = -\lambda \left[ 1 + \lambda ig( \overline{V}(s) + \overline{V}(t) + \overline{V}(u) ig) 
ight]$$



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#### Derived expressions continued

The prevalence of the sinc function  $sinc(x) = \frac{sin(x)}{x}$  in these finite size systems' descriptions, leads one to seek a generalization to a formula originally proposed by Ramanujan and formalized by Hardy

$$\sum_{0 \le n < x} \frac{r_2(n)}{\sqrt{x - n}} = 2\pi\sqrt{x} + \sum_{n=1}^{\infty} \frac{r_2(n)}{\sqrt{n}} \sin(2\pi\sqrt{nx})$$

which we find to be

$$\sum_{\vec{k}\in\mathbb{Z}^m}\operatorname{sinc}(2\pi R\|\vec{k}\|) = \frac{1}{2R\pi^{\frac{m-1}{2}}\Gamma(\frac{3-m}{2})}\sum_{l=0}^{\lfloor R^2 \rfloor} \frac{r_m(l)}{\sqrt{R^2 - l}}$$

Which enabled us to show that unitarity holds in the finite size system we are describing, for n = 1, 2, 3 finite dimensions.



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Corr.; 0 = m/6

 Corr.; θ = π/3  $Corr.; \theta = \pi/2$ 

Corr.:  $\theta = m/6$ 

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..... Corr.; 8 = m/2

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#### Size of corrections

We find corrections on the same order of magnitude as the infinite volume calculation



FIG. 2. (Top) A plot of the contributions to  $\overline{V}(p^2, \{L_i\}; \mu)$ for the s channel as a function of  $p \equiv |\vec{p}|$ . The real part of the infinite volume contribution is in red; the imaginary part of the infinite volume contribution is in orange. The real part of the finite length correction is in blue; the imaginary part of the finite length correction is in blue-gray. We take  $\mu = 1$ GeV, m = 0.5 GeV, and  $L = 1/\sqrt{3}$  GeV<sup>-1</sup>. (Bottom) The same comparison but as a function of L for fixed  $|\vec{p}| = 1$  GeV.

FIG. 3. (Top) A plot of the contributions to  $\overline{V}(p^2, \{L_i\}; \mu)$ for the t channel as a function of  $p \equiv |\vec{p}|$ , where scattering is into the non-compact direction. The results are shown for three values of the scattering angle  $\theta$ ,  $\theta = \pi/6$ ,  $\pi/3$ , and  $\pi/2$ . for solid, dashed, and dotted curves, respectively. The infinite volume contribution is in red: the finite length correction is in blue. We take  $\mu = 1$  GeV, m = 0.5 GeV, and  $L = 1/\sqrt{3}$ GeV<sup>-1</sup>. (Bottom) The same comparison but as a function of L for fixed  $|\vec{p}| = 1$  GeV.

p (GeV)

 $M : \theta = m \theta$ 

1.0 1.5 L (GeV-1)

Inf. V: θ = π/3 - Corr.: θ = π/3 lef  $V: \theta = \pi/2$ 



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## What remains?

We can see that there is a possibility of reasonable size corrections to calculations performed in finite sized systems. This is an important step of a much longer journey. Some of what remains to be done:

- Thermal Field Theory of finite size  $\phi^4$  theory
- Critical exponents of finite size  $\phi^4$  theory
- Find closed form results for many numerically difficult results already found
- Generalize to more complicated systems, eventually such as QCD itself

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# The End

Thank you for making it to the end of my talk! The first paper is available as a preprint "Finite System Size Correction to NLO Scattering in  $\phi^4$  Theory" at arXiv:2203.01259. Look out for our follow up paper going into the numerics of our results, expected to be on arXiv soon.

Feel free to contact me with any further questions:

23787295@sun.ac.za

