Role of rapidity choice for the Balitsky-Kovchegov equation

high-E QCD → (CGC, CDM) → DIS & other observables

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electron-proton scattering

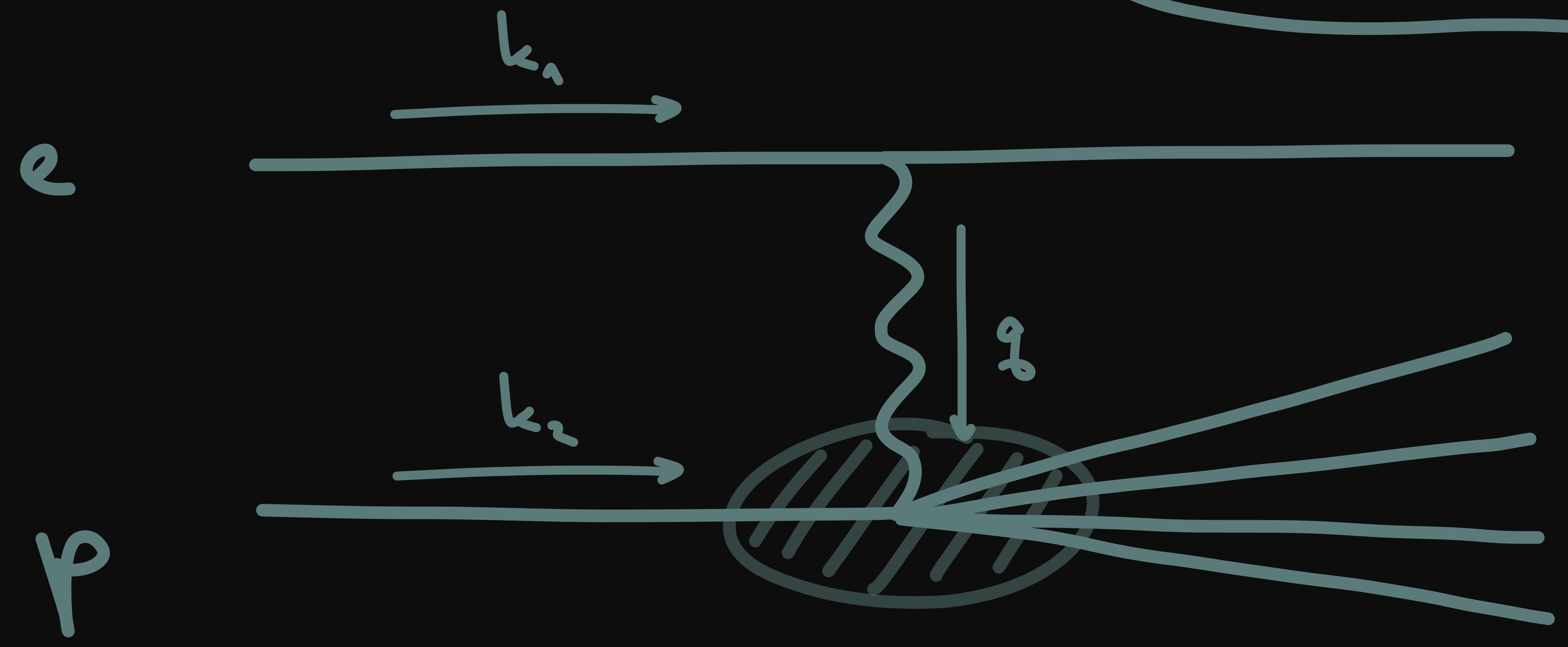
deep inelastic

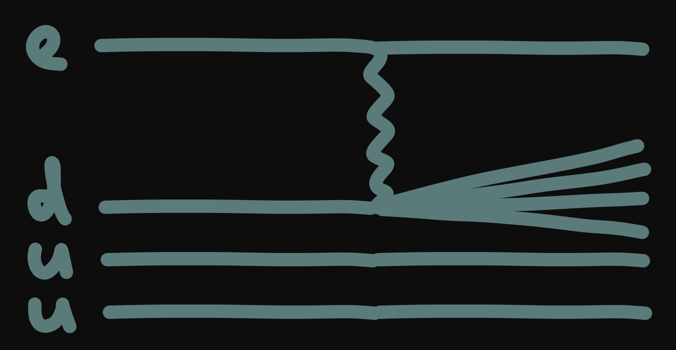
$y := 1 - \frac{E_3}{E_1}$

$$\frac{d^2\sigma}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2F_1(x,Q^2)\right]$$

$Q^2 := -q^2$

Bjorken x



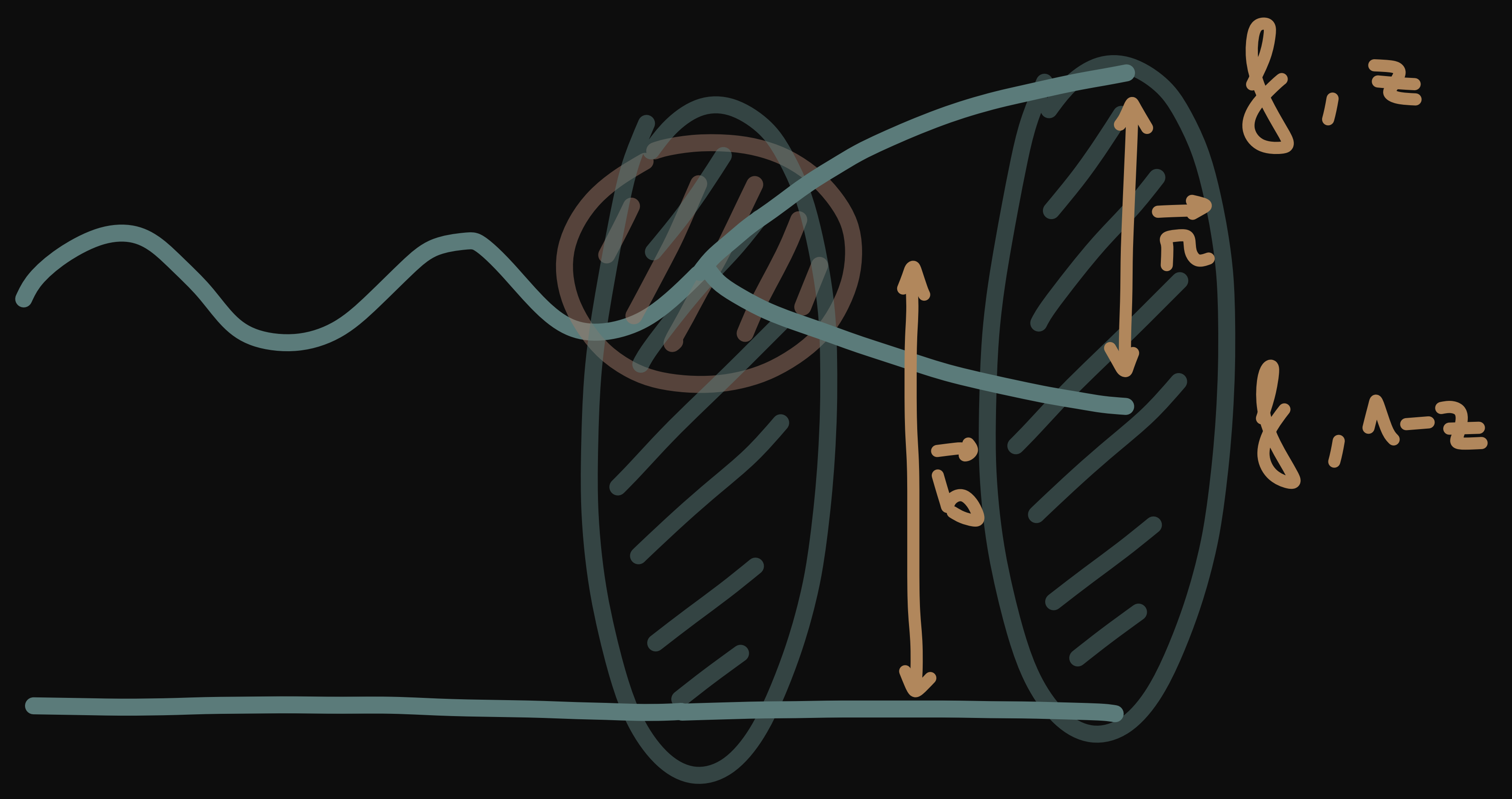


$\sim p_d^+ = x\, p_p^+$

dipole scattering amplitude

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}\left(\sigma_L^{\gamma^* p}(x, Q^2) + \sigma_T^{\gamma^* p}(x, Q^2)\right)$$

$$F_L(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}\sigma_L^{\gamma^* p}(x, Q^2)$$



$$\sigma_{L,T}^{\gamma^* p}(x, Q^2) = \sum_f \int d^2\vec{r} \int_0^1 dz \, |\psi_{T,L}^{(f)}(\vec{r}, Q^2, z)|^2 \, 2\int d^2\vec{b} \, N(\vec{r}, \vec{b}, \tilde{x}_f(x))$$

LO Balitsky-Kovchegov equation

dipole scattering amplitude

$$\frac{\partial N(\vec{r}, \vec{b}, Y)}{\partial Y} = \int d^2\vec{r}_1 K(\vec{r}, \vec{r}_1, \vec{r}_2)\ [N(\vec{r}_1, \vec{b}_1, Y) + N(\vec{r}_2, \vec{b}_2, Y) - N(\vec{r}, b, Y) - N(\vec{r}_1, \vec{b}_1, Y)N(\vec{r}_2, \vec{b}_2, Y)]$$

kernel

$Y = \ln \frac{x_0}{x}$

LO Balitsky-Kovchegov equation

$$\frac{\partial N(\vec{r}, \vec{b}, Y)}{\partial Y} = \int d^2\vec{r}_1 K(\vec{r}, \vec{r}_1, \vec{r}_2)\ [N(\vec{r}_1, \vec{b}_1, Y) + N(\vec{r}_2, \vec{b}_2, Y) - N(\vec{r}, b, Y) - N(\vec{r}_1, \vec{b}_1, Y)N(\vec{r}_2, \vec{b}_2, Y)]$$

$$K_{lo} = \frac{\alpha_s N_C}{2\pi}\frac{r^2}{r_1^2 r_2^2}$$

$$K_{rc} = \frac{\alpha_s(r) N_C}{2\pi}[\frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_1^2}(\frac{\alpha_s(r_1)}{\alpha_s(r_2)} - 1) + \frac{1}{r_2^2}(\frac{\alpha_s(r_2)}{\alpha_s(r_1)} - 1)]$$

$$K_{ci} = \frac{\bar{\alpha}_s}{2\pi}\frac{r^2}{r_1^2 r_2^2}\frac{r^2}{\min\{r_1^2, r_2^2\}}^{\pm\bar{\alpha}_s A_1}$$

LO Balitsky-Kovchegov equation

projectile rapidity $Y = \eta + \ln \frac{Q^2}{Q_0^2}$

$$\frac{\partial N(\vec{r}, \vec{b}, Y)}{\partial Y} = \int d^2\vec{r}_1 K(\vec{r}, \vec{r}_1, \vec{r}_2)\ [N(\vec{r}_1, \vec{b}_1, Y) + N(\vec{r}_2, \vec{b}_2, Y) - N(\vec{r}, b, Y) - N(\vec{r}_1, \vec{b}_1, Y) N(\vec{r}_2, \vec{b}_2, Y)]$$

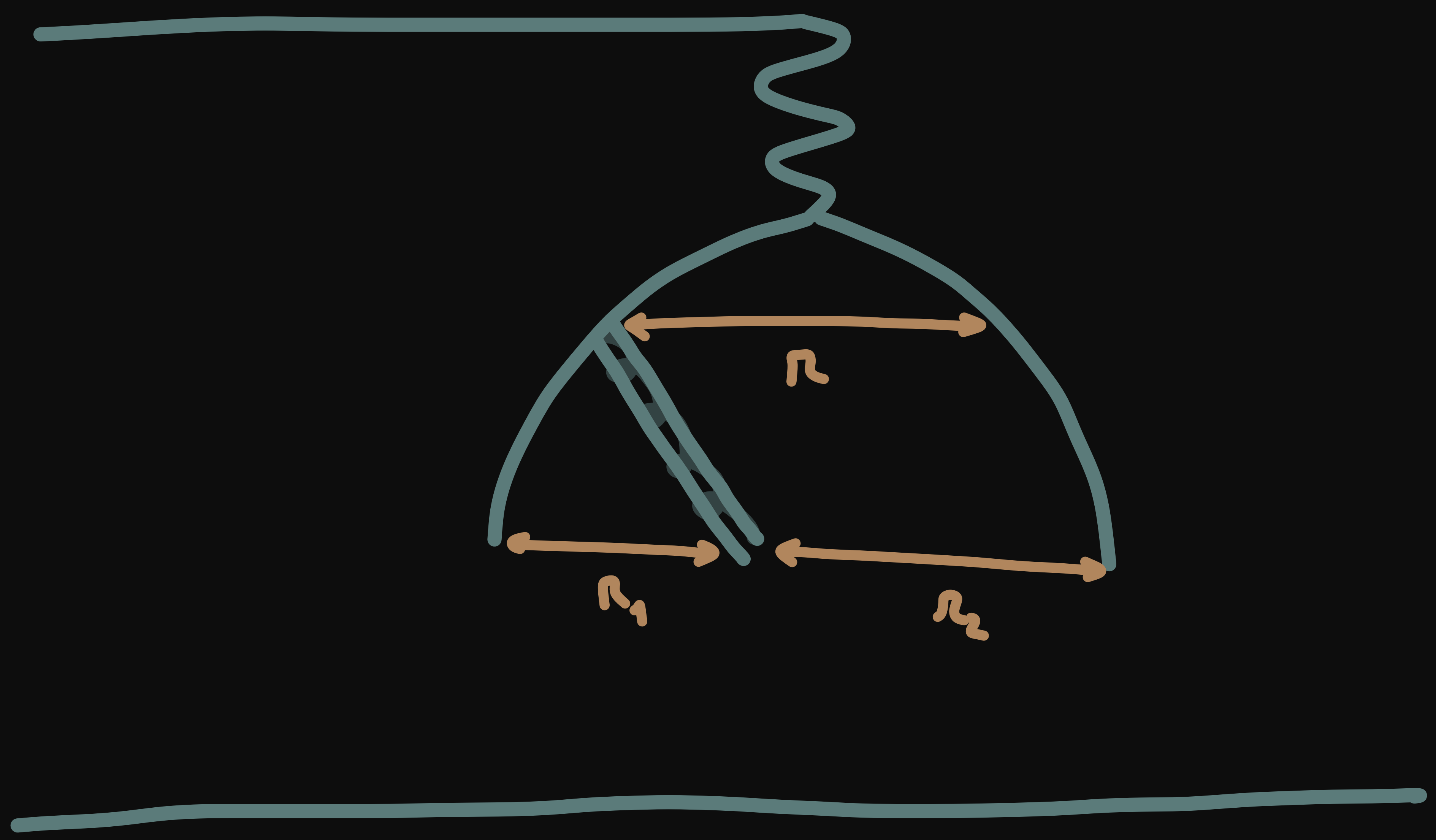
$$\frac{\partial N(\vec{r}, \vec{b}, \eta)}{\partial \eta} = \int d^2\vec{r}_1 K(\vec{r}, \vec{r}_1, \vec{r}_2)\ [N(\vec{r}_1, \vec{b}_1, \eta_1) + N(\vec{r}_2, \vec{b}_2, \eta_2) - N(\vec{r}, b, \eta) - N(\vec{r}_1, \vec{b}_1, \eta_1) N(\vec{r}_2, \vec{b}_2, \eta_2)]$$

target rapidity $\eta = \ln \frac{x_0}{x}$

$\eta_j = \eta - \max\{0, 2 \log \frac{r}{r_j}\}$

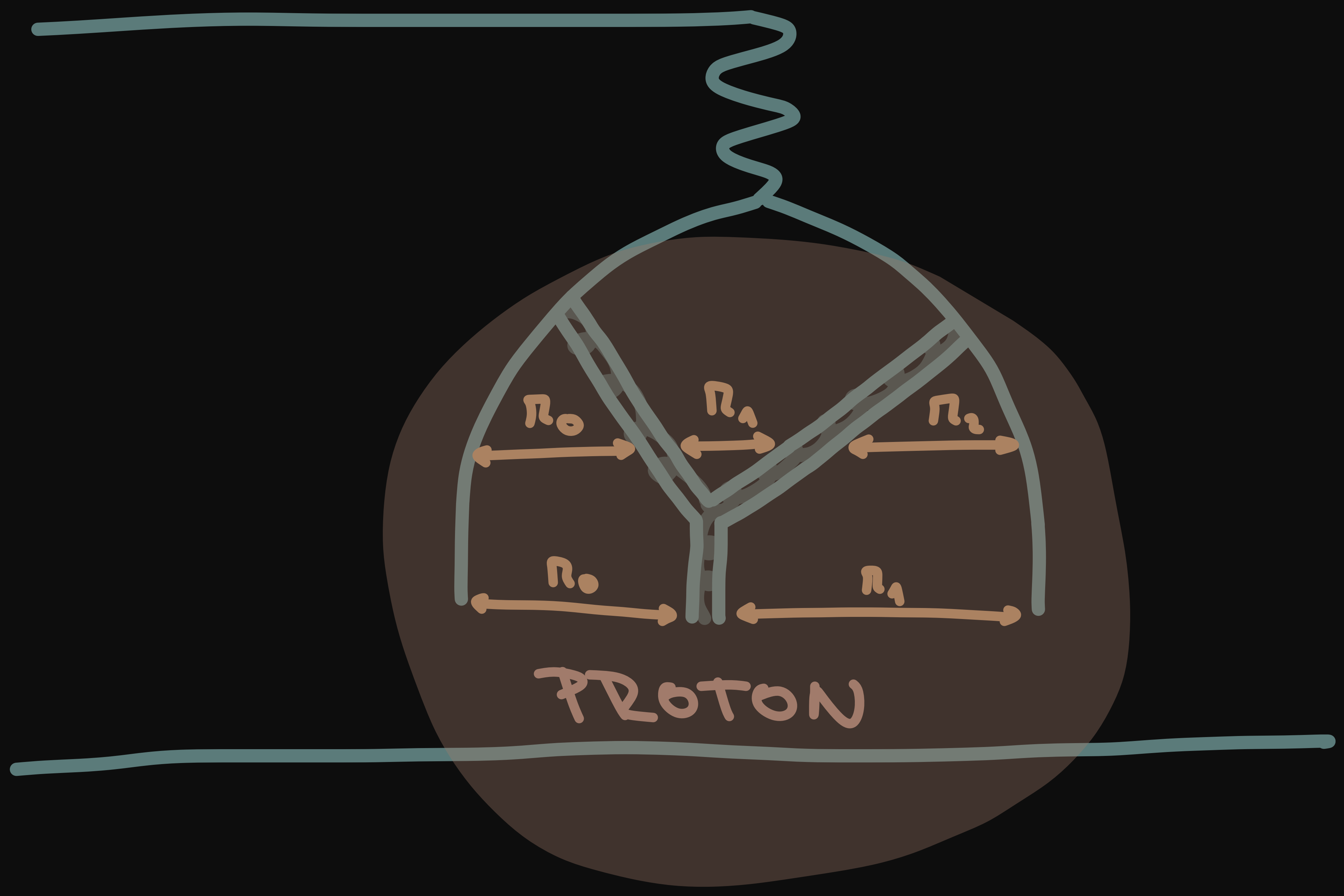
LO Balitsky-Kovchegov equation

$$\frac{\partial N(\vec{r}, \vec{b}, Y)}{\partial Y} = \int d^2\vec{r}_1 K(\vec{r}, \vec{r}_1, \vec{r}_2)\ [N(\vec{r}_1, \vec{b}_1, Y) + N(\vec{r}_2, \vec{b}_2, Y) - N(\vec{r}, b, Y) - N(\vec{r}_1, \vec{b}_1, Y)N(\vec{r}_2, \vec{b}_2, Y)]$$



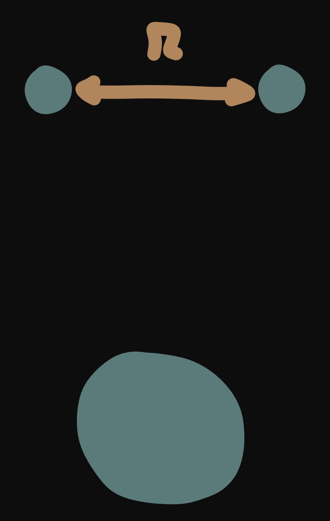
LO Balitsky-Kovchegov equation

$$\frac{\partial N(\vec{r}, \vec{b}, Y)}{\partial Y} = \int d^2\vec{r}_1 K(\vec{r}, \vec{r}_1, \vec{r}_2)\ [N(\vec{r}_1, \vec{b}_1, Y) + N(\vec{r}_2, \vec{b}_2, Y) - N(\vec{r}, b, Y) - N(\vec{r}_1, \vec{b}_1, Y)N(\vec{r}_2, \vec{b}_2, Y)]$$

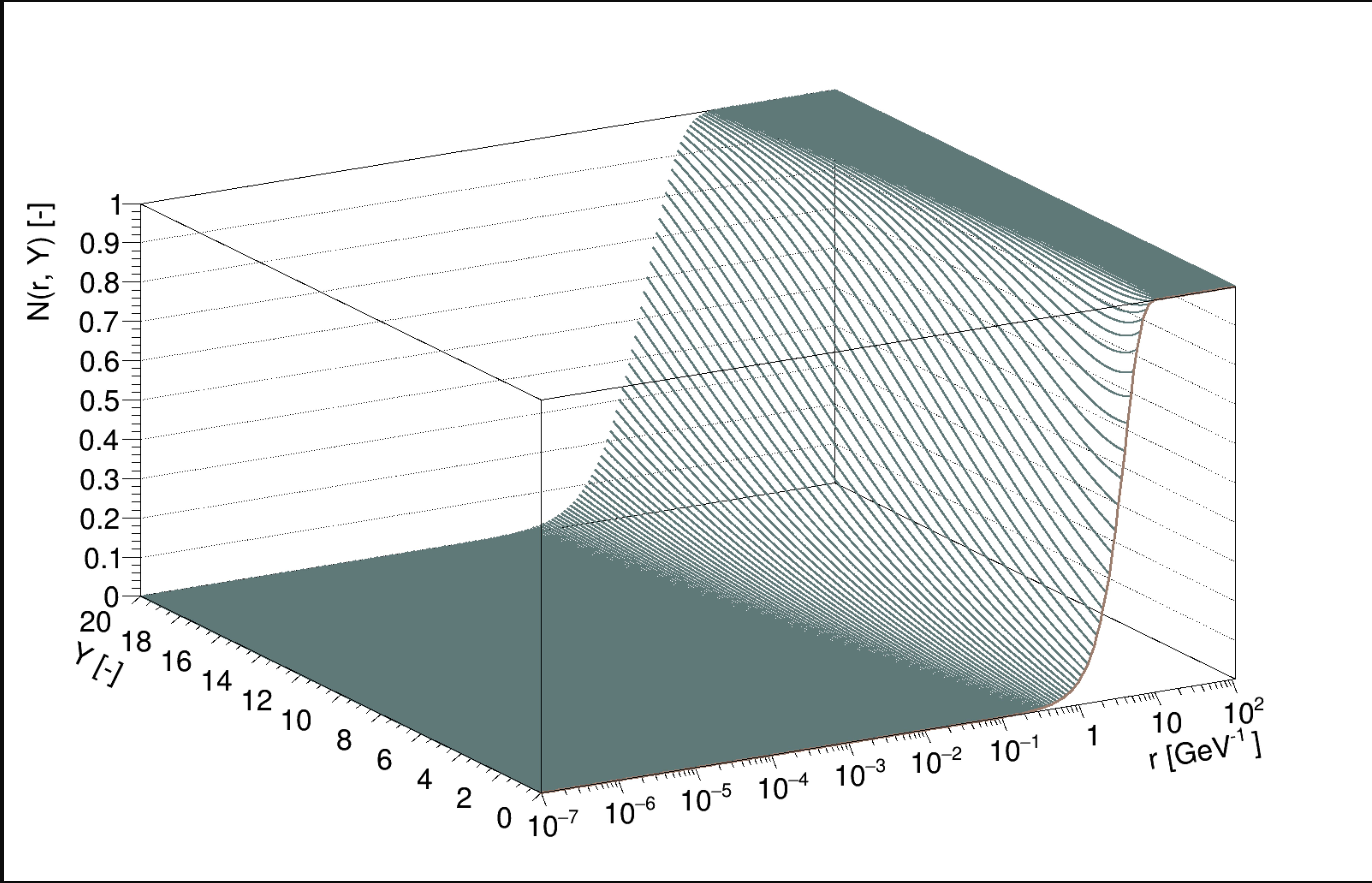


1D solution

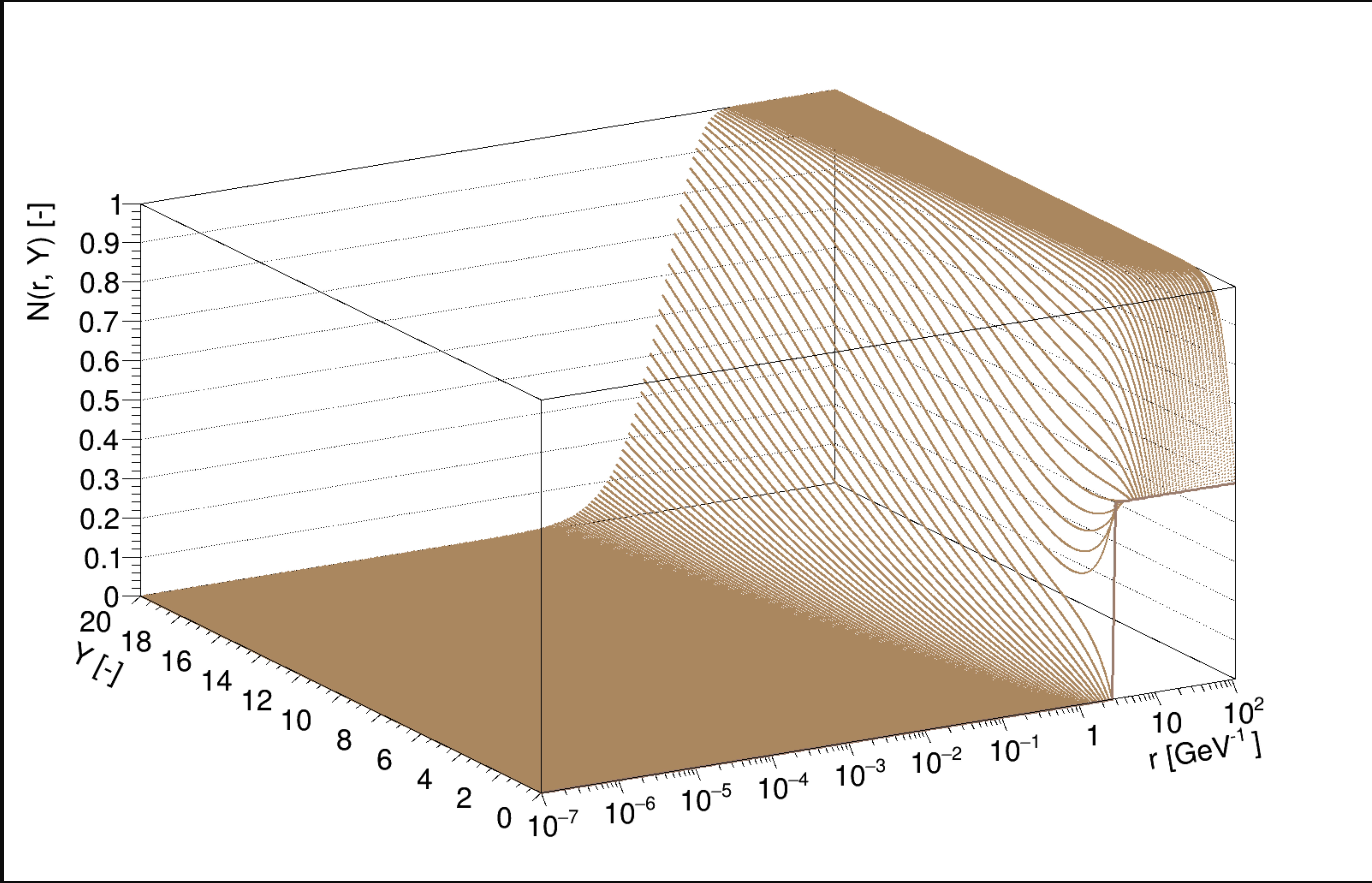
$$2\int d\vec{b}\, N(\underbrace{\vec{r}, \vec{b}}_{\text{4-dim}}, Y) \approx \sigma_0\, N(r, Y)$$



1D solution


N(r, Y) [-]
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Y [-]
20
18
16
14
12
10
8
6
4
2
0
10⁻⁷
10⁻⁶
10⁻⁵
10⁻⁴
10⁻³
10⁻²
10⁻¹
1
10
10²
r [GeV⁻¹]

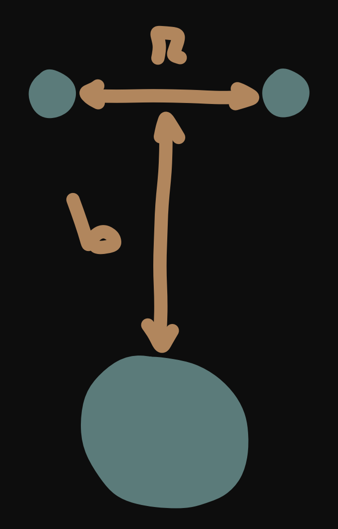
1D solution



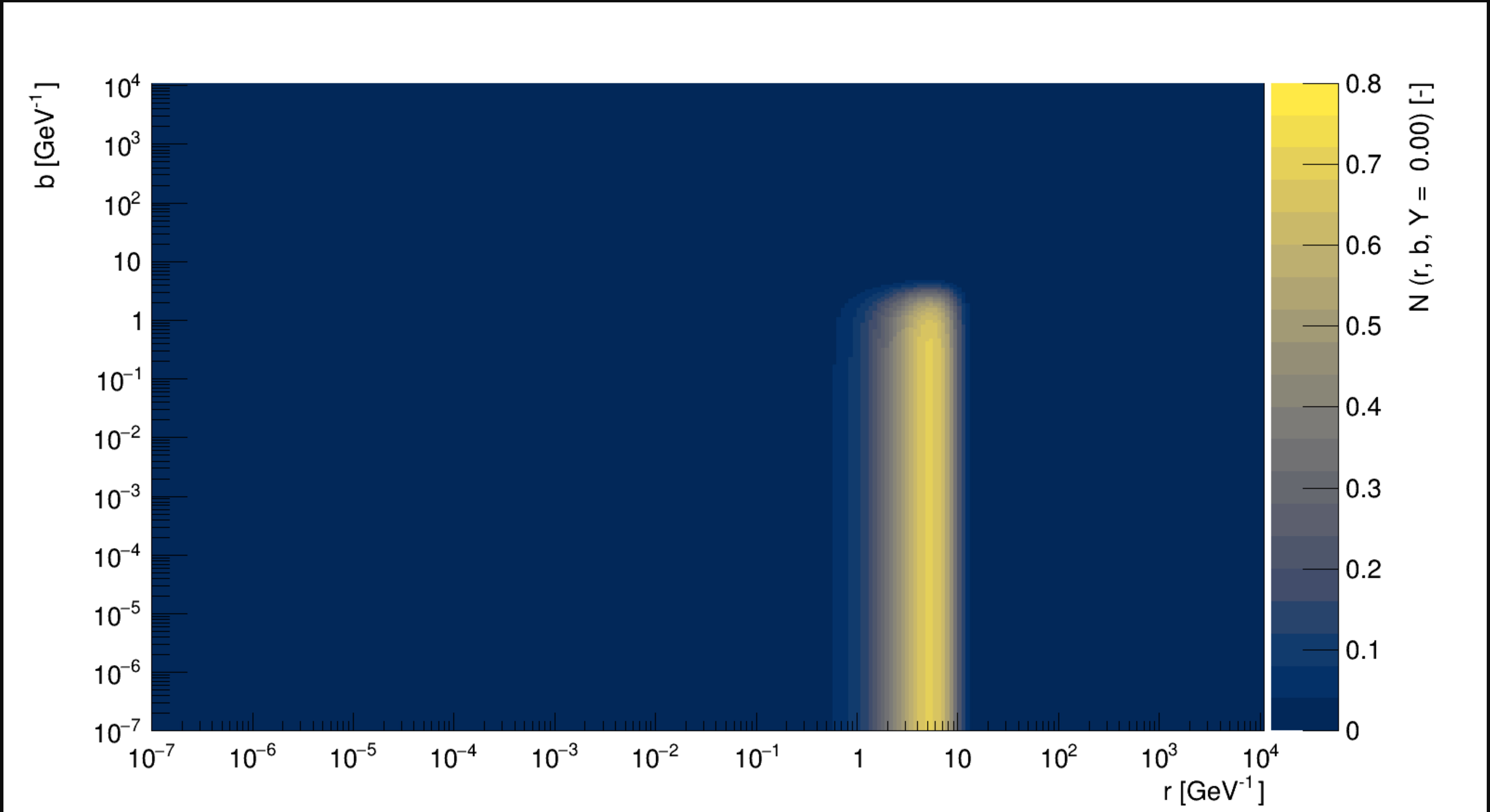
N(r, Y) [-]
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Y [-]
20
18
16
14
12
10
8
6
4
2
0
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10
10^{2}
r [GeV^{-1}]

2D solution

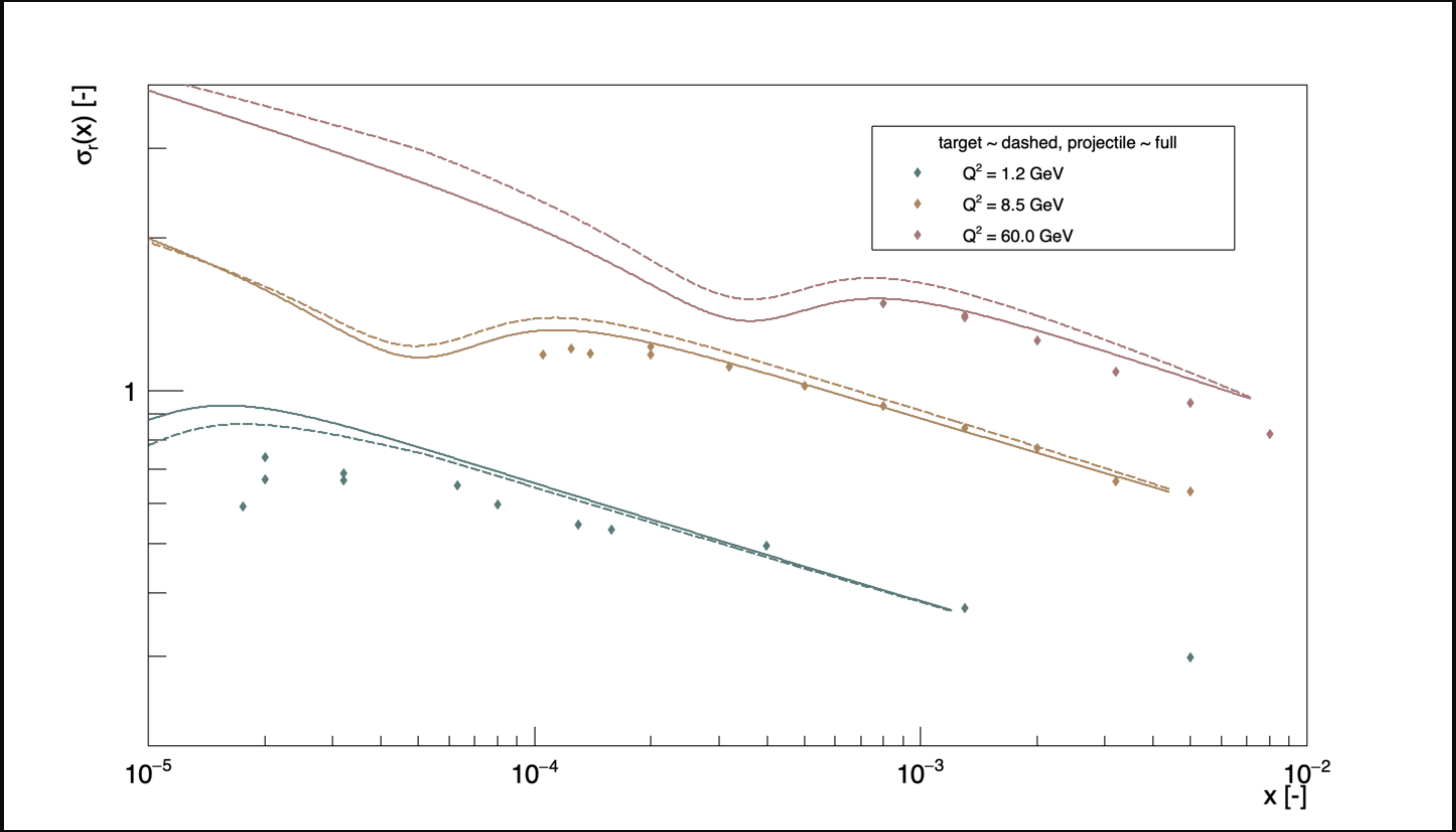
$$2\int d\vec{b}N(\vec{r}, \vec{b}, Y) \approx 4\pi\int dbN(r, b, Y)$$



2D solution


b [GeV^{-1}]
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10 10^2 10^3 10^4
r [GeV^{-1}]
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
N (r, b, Y = 0.00) [-]

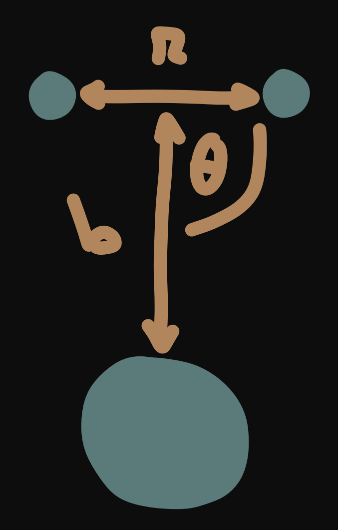
Y vs η



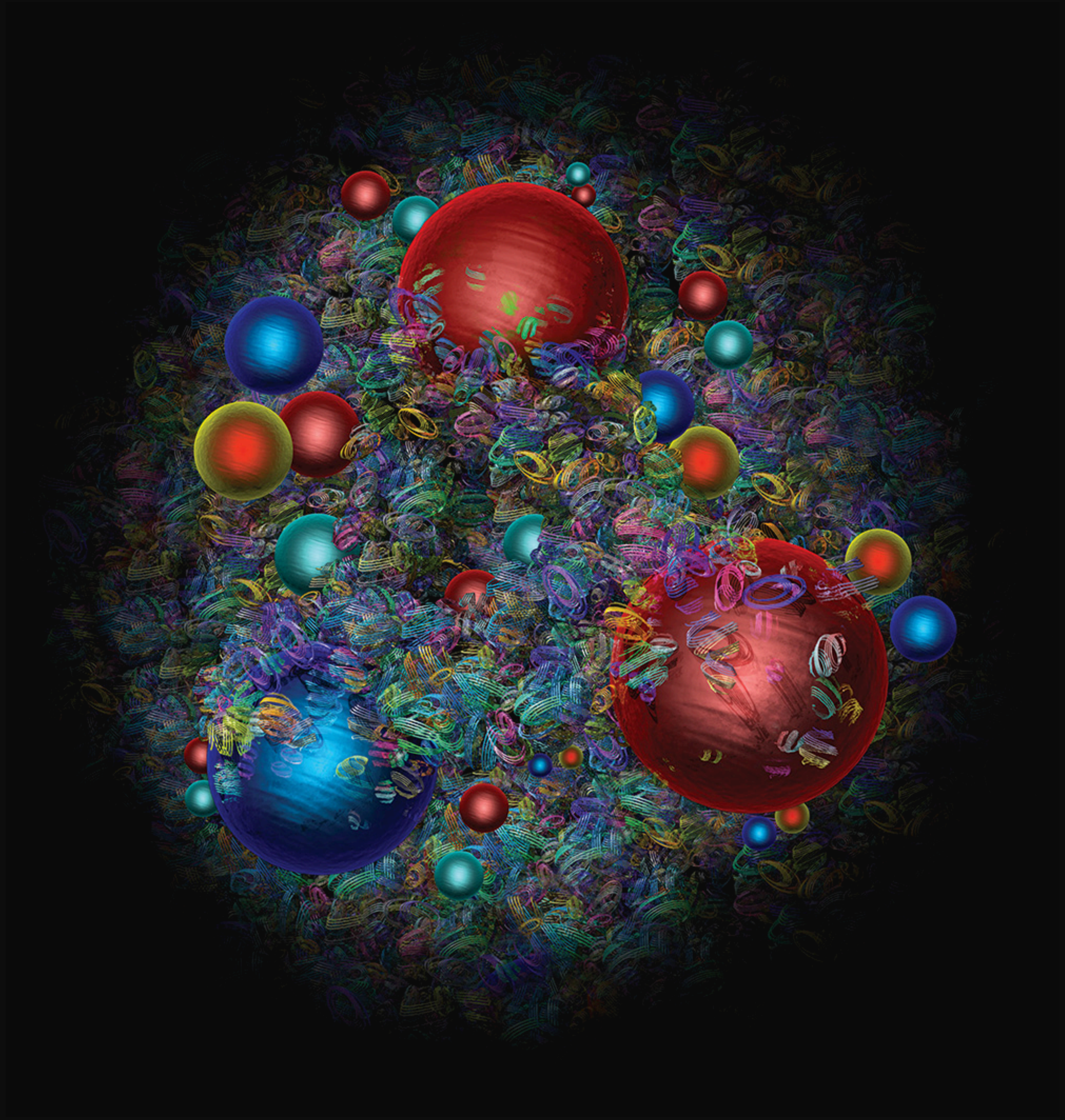
$\sigma_r(x)$ [-]
1
target ~ dashed, projectile ~ full
Q^2 = 1.2 GeV
Q^2 = 8.5 GeV
Q^2 = 60.0 GeV
10^{-5}
10^{-4}
10^{-3}
10^{-2}
x [-]

further steps

- calculate 3D solution
- calculate 4D solution
- implement higher order corrections



thank you for your attention



Daniel Dominguez. “An artist’s impression of the mayhem of quarks and gluons inside the proton.” In: (June 2019). General Photo. url: https: //cds.cern.ch/record/2678286