Microscopic analysis of proton-nucleus scattering data at energies from 200 to 1000 MeV

V.K.Lukyanov[1], E.V.Zemlyanaya[1], K.V.Lukyanov[1], I.A.M.Abdul-Magead[2]

[1] Joint Institute for Nuclear Research, Dubna 141980, Russia
[2] Cairo University, Giza, Egypt

The African Nuclear Physics Conference
Kruger Region, South Africa, 2023

Introduction

The proton-nucleus scattering is the traditional topic of investigations to obtain information on a structure of nuclei, including the nuclear mean-field optical potential (OP), nuclear radius, the nuclear density distribution. In this connection, the problems are considered when constructing the models of nuclear structure as the multi-particle systems.

At comparably **low energies** of incident protons about 20-50 MeV, one can construct the respective OP with accounting for the direct[1] and exchanged[2] contributions. In these studies the result was reduced to the non-local potential, and then the analysis was made of elastic scattering of protons and neutrons on the nuclei ^{40,48}Ca, ^{90}Zr, ^{208}Pb. In this case and many other constructions of nuclear potentials, the estimations are made only of the real parts of potentials while their imaginary parts as usually are introduced in the phenomenological way.

In the case of **relativistic energies**, calculations of cross sections are standardly based on the HEA theory, without explicit construction of OP.

Our studies are aimed onto analysis of the data at energies about **200-1000 MeV**, where the re-scattering processes and the non-locality effects do not play a decisive role but relativistic effects should be taken into account.

[1] Doan Thi Loan et al., Phys.Rev. C **92** 034304 (2015)

[2] Doan Thi Loan et al., J.Phys G **47** 035106 (2020)

Our approach

We follow the theoretical approach previously developed and used for the pion-nucleus scattering analysis at the (3,3) resonance energy region[3]. It is based on the microscopic, HEA-based 3-parameter model of OP and on numerical solution of the respective relativistic equation for calculation of observables.

This folding model[4] of OP depends on the density distribution function of nucleons in a nucleus and on the elementary nucleon-nucleon amplitude of scattering of an incident proton on the bounded nuclear ("in-medium") nucleon which itself depends on three parameters:

- the total nucleon-nucleon scattering cross section,
- the ratio of real to imaginary parts of the scattering amplitude at forward angles,
- the slope parameter.

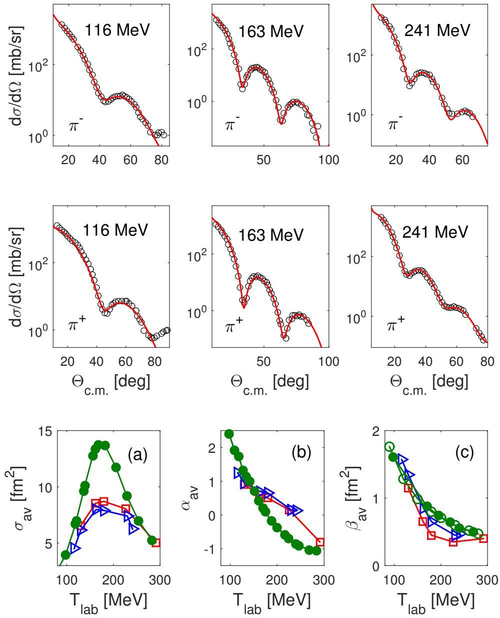
These three "in-medium" parameters of the NN scattering amplitude are not known. They are adjusted to the experimental data on elastic proton-nucleus scattering and compared with the "free" ones known from analysis of proton-nucleon scattering data. Such analysis allows one to estimate effect on nuclear matter on the NN scattering amplitude.

---

[3] V.K.Lukyanov et al. Nucl. Phys A **1010** (2021) 122190

[4] V.K.Lukyanov et al., Phys. Atom. Nucl. **69** (2006) 240

$\pi^{\pm}+{}^{40}$Ca scattering & "inmedium" effect



Microscopic OP: basic formulae (1/2)

An expression for the OP in the case of comparably high energies of incident protons $E >> U$ can be written as follows:

$$U_{opt}(r) = -\frac{(\hbar c)\beta_c}{(2\pi)^2 k} \int e^{-i\mathbf{qr}} \rho(q)\, F_N(q)\, d^3 q. \qquad (1)$$

Here ρ is the density distribution function, $\beta_c = v_{c.m.}/c = k_{lab}/[E_{lab} + m^2/M_A]$ is the ratio of the proton velocity at the c.m. system to the light velocity [1], which is expressed through its total energy in the lab system $E_{lab} = (k_{lab}^2 + m^2)^{1/2} = T_{lab} + m$, where k_{lab} is the moment of the relative motion, T_{lab} is the proton kinetic energy at the lab system, and M_A is the target nucleus mass.

Then, for the nucleon-nucleon amplitude of scattering $F_N(q)$ one uses the expression

$$F_N(q) = \frac{k}{4\pi}(i+\alpha)\,\sigma\, f_N(q), \quad f_N(q) = e^{-\beta q^2/2}. \qquad (2)$$

One should underline that, in the case of the pA scattering, this amplitude describes the proton scattering on the bounded (not free!) nucleons.

---

[1] In eq.(1) we use the units MeV and fm, which follows to $\hbar c = 197.327$ MeV fm. In the other cases, the natural system of units is used where $\hbar$=c=1, and thus E, T, k, m have the same dimensions [MeV].

Microscopic OP: basic formulae (2/2)

After substituting F_N into (1) and integrating over the angular variables, one can get the OP in the form

$$U_{opt}(r) = V(r) + iW(r) = -\frac{(\hbar c)\beta_c}{(2\pi)^2}\sigma(\alpha + i)\int j_0(qr)\rho(q)f_N(q)q^2 dq. \quad (3)$$

In calculations, we used the nuclear density distribution in the form of the symmetrized Fermi function

$$\rho(r) = \rho_0 \frac{\sinh(R/a)}{\cosh(R/a) + \cosh(r/a)}, \ \rho_0 = \frac{4A}{5\pi R^3}\left[1 + \left(\frac{\pi a}{R}\right)^2\right]^{-1} \quad (4)$$

with the corresponding Fourier transformation

$$\rho_{SF}(q) = -\rho_0 \frac{4\pi^2 aR}{q} \frac{\cos qR}{\sinh(\pi aq)}\left[1 - \left(\frac{\pi a}{R}\right)\coth(\pi aq)\tan qR\right], \ R \geq \pi a. \quad (5)$$

Here the transfer momentum

$$q = 2k\sin(\theta/2)\left(1 + \frac{2E}{M_A}\sin^2(\theta/2)\right), \quad (6)$$

where θ is the incident proton angle of scattering.

Average pN-amplitude parameters

As mentioned above, we compare calculated "in-medium" parameters (when proton scatters on bounden nuclear nucleon) with "free" parameters of amplitude of proton scattering of free nucleons. In our study, we do not distinguish the cases when the incident proton scatters on the proton or on the neutron inside the target nucleus. Therefore we compare our calculations of "in-medium" proton-nucleon cross sections with the averaged " "free" cross sections

$$\sigma = \frac{\sigma_{pp} \cdot Z + \sigma_{pn} \cdot N}{Z + N}. \quad (7)$$

where indexes "pp" and "pn" mean, respectively, the experimental proton-proton and proton-neutron cross sections, Z is number of protons and N is number of neutrons in the target nucleus.

The same expressions we use for the other experimentally known "free" parameters α and β in the OP (3).

Also, we use in our comparisons the expressions for the energy dependence of the elementary cross sections σ_{pp} and σ_{pn} obtained in [1].

$$\sigma_{pp} = 19.6 + 4253/E - 375/\sqrt{E} + 3.86 \cdot 10^{-2} E \ mb \quad (8)$$

and

$$\sigma_{pn} = 89.4 - 2025/\sqrt{E} + 19108/E - 43535/E^2 \ mb. \quad (9)$$

[1] C.A. Bertulani and C.D. C.De Conti, Phys.Rev. C **81** 064603 (2010)

Relativistic equation

Cross sections of the proton-nucleus scatterins are calculated via numerical solving the relativistic Klein-Gordon wave equation in the form where the terms of the order $(u/E)^2 << 1$ are neglected.

$$(\nabla + k^2)\,\psi(\vec{r}) = 2\mu\, U_{eff}(r)\,\psi(\vec{r}). \qquad (10)$$

Here the relativization is taken into account by introducing the effective mass $\bar{m}$ in the effective potential

$$U_{eff}(r) = \gamma^{(r)} \cdot U(r), \quad U(r) = U_{opt}(r) + U_c(r), \quad \gamma^{(r)} = \frac{\bar{\mu}}{\mu}, \qquad (11)$$

where U_{opt} is the folding OP (3), μ and $\bar{\mu}$ are the reduced masses of the form:

$$\mu = \frac{mM}{m+M}, \quad \bar{\mu} = \frac{\bar{m}M}{\bar{m}+M}, \quad \bar{m} = \sqrt{k^2 + m^2}. \qquad (12)$$

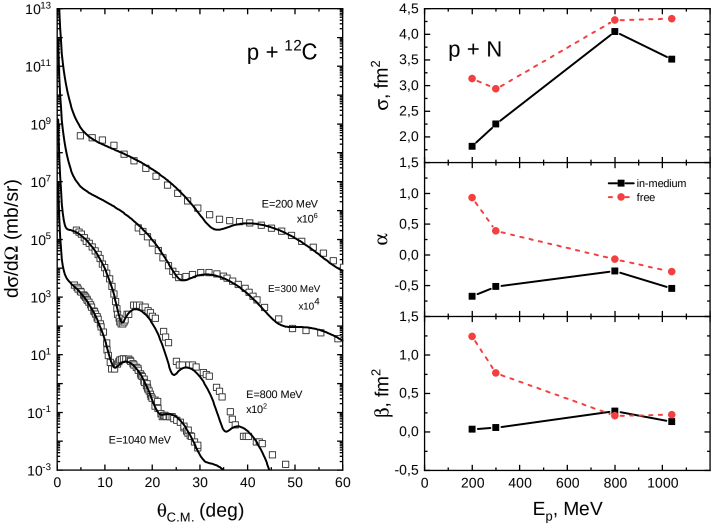
The relativistic momentum k in the center of mass system is equal to

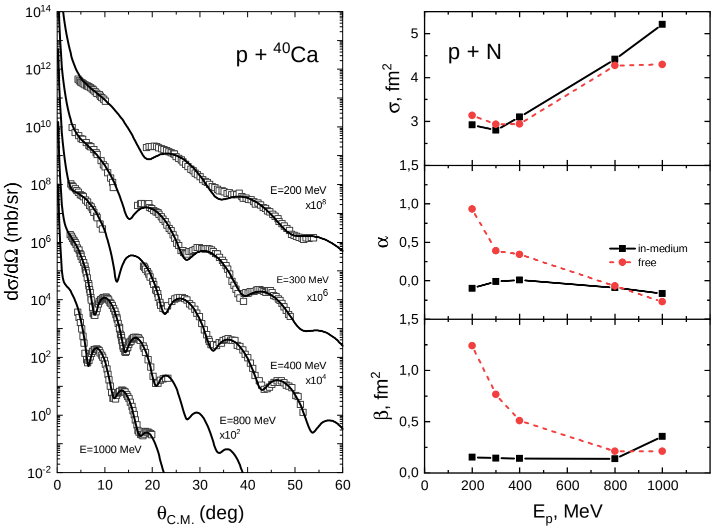
$$k = \frac{M\,k_{lab}}{\sqrt{(m+M)^2 + 2MT_{lab}}} = \frac{M\sqrt{T_{lab}(T_{lab}+2m)}}{\sqrt{(m+M)^2 + 2MT_{lab}}}. \qquad (13)$$

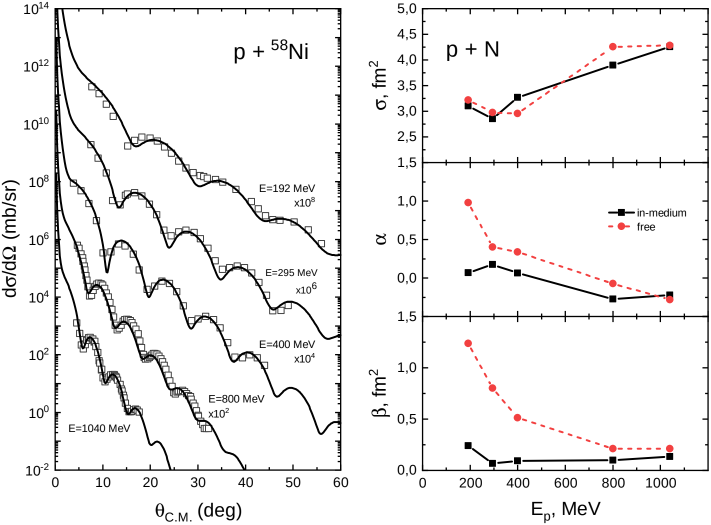
The wave equation in the "non-relativistic" form (10) is solved numerically with a help of the standard computer code DWUCK4, where one takes into account the relativization and distortion effects in scattering of a nucleon in the target nucleus field.

Results

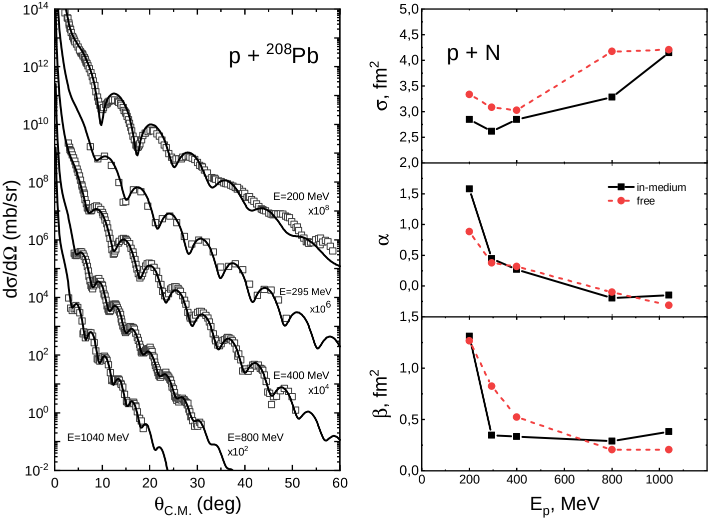
- In calculations of the elastic scattering cross sections, basing on the constructed proton-nucleus OP, and then in comparison of them with the corresponding experimental data one can get the bestfit parameters σ, α, β of the elementary amplitude of scattering of nucleons on the bounded nuclear nucleons – “in-medium” parameters.
- Thus we have a possibility to compare the information on these typical characteristics of the proton scattering on the free nucleons with the same characteristics but for the proton scattering on the bounded nuclear nucleons (“in-medium” effect). It is natural one that the obtained sets of such parameters are different for the different collision energies.
- The corresponding fitted “in-medium” parameters σ, α, β of the proton scattering pN-amplitude, versus energy, are shown by the black squares and solid line while the respective experimental data of the free pN scattering are given by the red circles and dotted lines.


p + 12C
dσ/dΩ (mb/sr)
θC.M. (deg)
E=200 MeV x10^6
E=300 MeV x10^4
E=800 MeV x10^2
E=1040 MeV
p + N
σ, fm^2
α
β, fm^2
Ep, MeV
in-medium
free


p + ^{40}Ca
p + N
$d\sigma/d\Omega$ (mb/sr)
$\theta_{C.M.}$ (deg)
E=200 MeV x10^8
E=300 MeV x10^6
E=400 MeV x10^4
E=800 MeV x10^2
E=1000 MeV
σ, fm^2
α
β, fm^2
in-medium
free
E_p, MeV



p + 58Ni
E=192 MeV
x10^8
E=295 MeV
x10^6
E=400 MeV
x10^4
E=800 MeV
x10^2
E=1040 MeV
dσ/dΩ (mb/sr)
θC.M. (deg)
p + N
σ, fm2
α
β, fm2
in-medium
free
Ep, MeV


p + 208Pb
E=200 MeV
x10^8
E=295 MeV
x10^6
E=400 MeV
x10^4
E=800 MeV
x10^2
E=1040 MeV
dσ/dΩ (mb/sr)
θC.M. (deg)
p + N
σ, fm²
α
β, fm²
in-medium
free
Ep, MeV

Summary

- The HEA-based microscopic model of OP provides good agreement with experimental data on proton-scattering on target nuclei ^{208}Pb, ^{58}Ni, ^{40}Ca and ^{12}C at energies between 200 and 1000 MeV.
- The peculiarity of this OP is that the target nucleon under consideration is not a free, but the bounded nuclear nucleon, and therefore the fitting parameters obtained from proton-nucleus scattering data do not coincide with those obtained from proton scattering on the free nucleon target.
- In general, the $\sigma(E)$ curves for the "in-medium" pN-scattering are close to the corresponding experimental data on "free" pN-scattering. As for the other "in-medium" and "free" characteristics, α and β, they look to be rather in agreement between each other in case of the heavier target nuclei ^{208}Pb and ^{58}Ni while for more light nuclei ^{40}Ca and ^{12}C one sees a noticeable disagreements.
- Such behaviour may be connected with decreasing the nucleon bounding energy in nuclei with increasing the nuclear atomic numbers when the nucleons become more individual in the heavy nuclei.

Next steps

- Effect of spin-orbit potential contribution
- Lower energies
- Inelastic scattering

Acknowledgments

The work is partly supported by the Scientific Cooperation Program “ARE – JINR”

Thank you for your attention!