

# Sub-barrier Fusion Excitation Functions of Heavy-Ion Systems

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The Scrovegni Chapel by Giotto, Padova



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## Question

Which kind of information can we extract from the heavy-ion fusion cross sections below the Coulomb barrier ?

We are going to see that

the **slopes** of experimental excitation functions **throw light** on the dynamics of **quantum tunnelling** in heavy-ion sub-barrier fusion, which is strongly connected to the intrinsic degrees of freedom of the colliding nuclei, i.e. their **nuclear structure** and the **dynamics** of inelastic scattering and quasi-elastic transfer reactions



## 50 years ago ... the Wong formula

The **Wong formula** at sub-barrier energies reduces to

$$E\sigma_{fus}(E) = \frac{\hbar\omega R_b^2}{2} \exp\left[\frac{2\pi}{\hbar\omega}(E - V_b)\right]$$

its **derivative** with respect to the energy is

$$\frac{d(E\sigma_{fus})}{dE} = \frac{\hbar\omega R_b^2}{2} \exp\left[\frac{2\pi}{\hbar\omega}(E - V_b)\right] \frac{2\pi}{\hbar\omega} = \frac{2\pi}{\hbar\omega} E\sigma_{fus}$$

The derivative varies **linearly** with  $E\sigma_{fus}$  with a slope

$$\frac{1}{E\sigma_{fus}} \frac{d(E\sigma_{fus})}{dE} = \frac{2\pi}{\hbar\omega} = \frac{d\ln(E\sigma_{fus})}{dE}$$

given by the logarithmic derivative and proportional to the (parabolic) **barrier width**

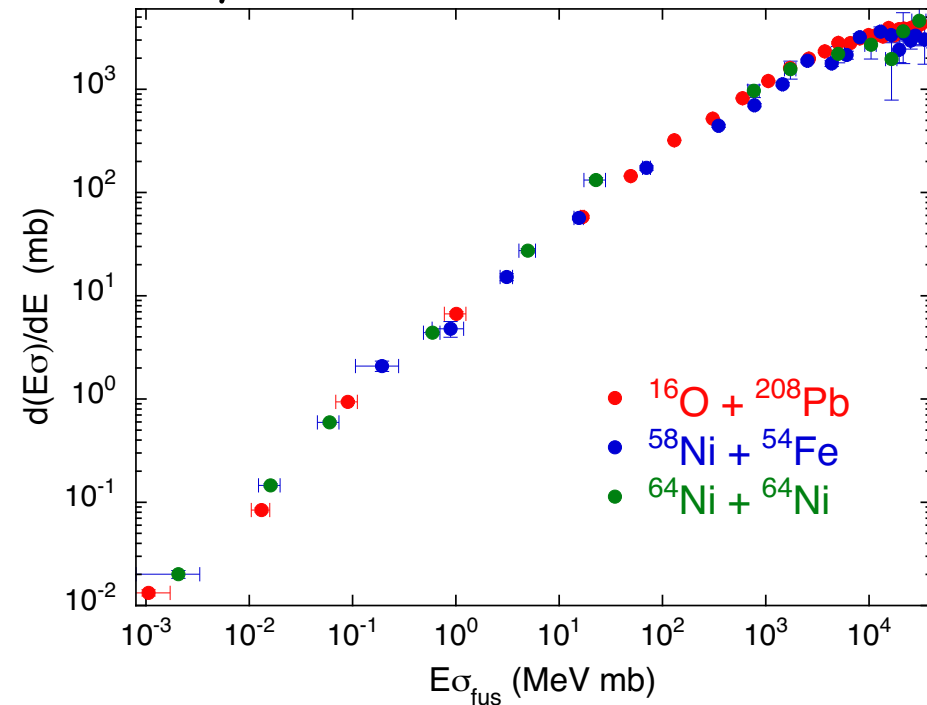
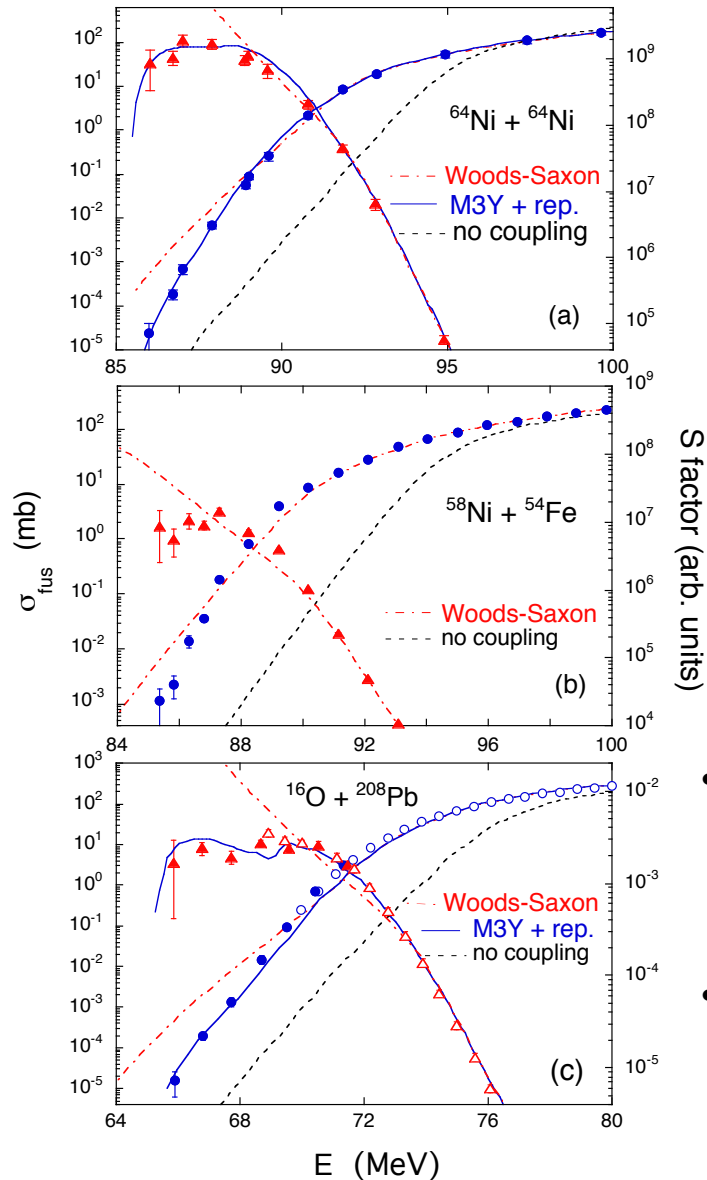


C.Y. Wong, PRL 31, 766 (1973).



## The representation $d(E\sigma)/dE$ vs $E\sigma$

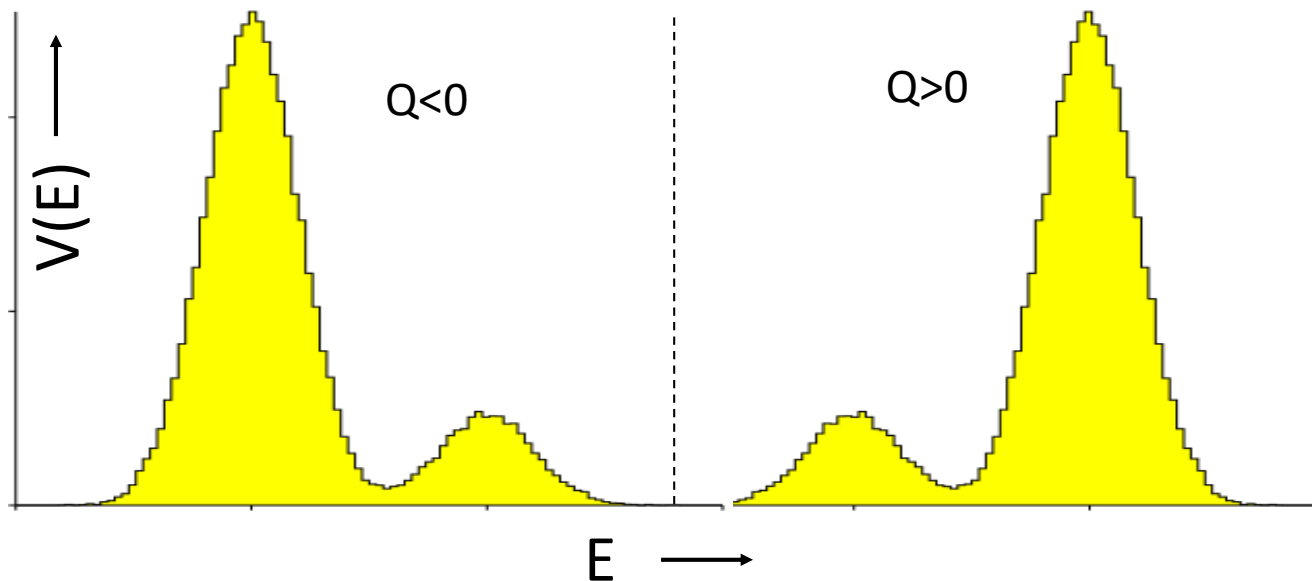
- we remove the effect of the varying Coulomb barrier when comparing different systems



- The behaviour of these **three systems** is very similar, even if they strongly differ in mass asymmetry and nuclear structure
- A good starting point to look at **other cases** where inelastic excitations and/or nucleon transfer channels influence the sub-barrier fusion cross sections

## From the Coupled-Channels model ...

Schematic barrier distributions predicted by the CC model for coupling to one channel with  $Q < 0$  (left) and  $Q > 0$  (right)

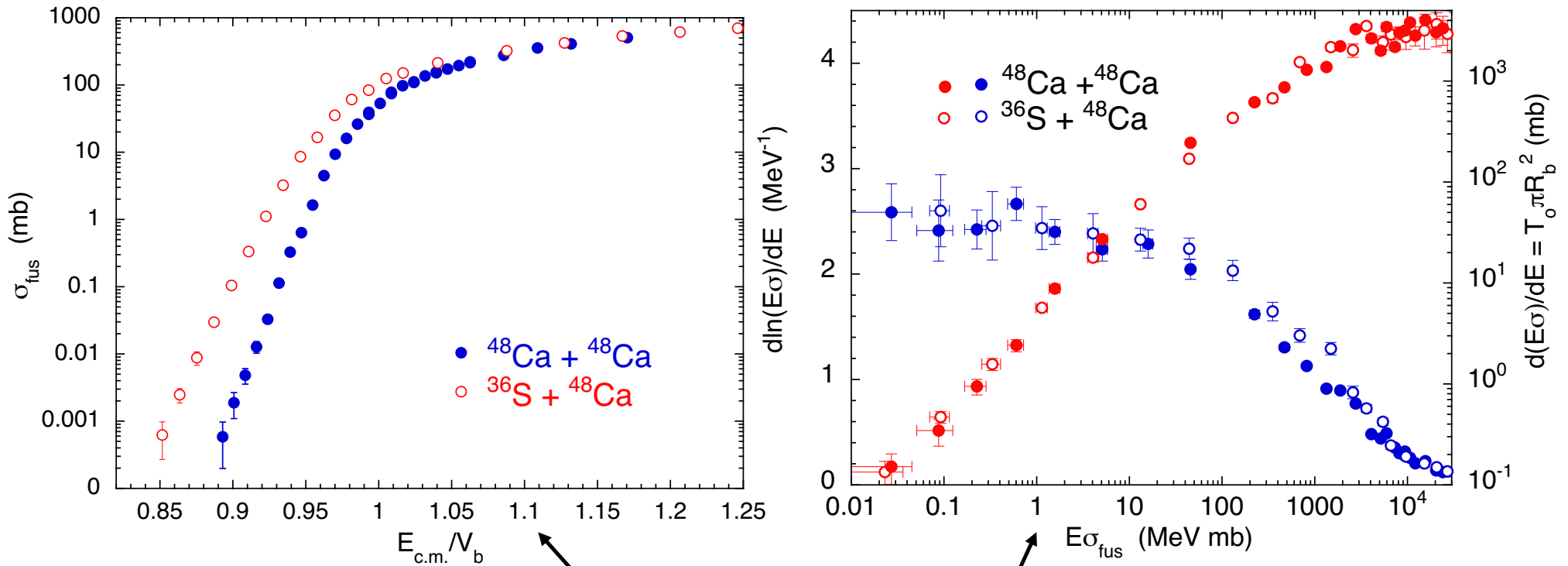


- The transmission coefficient, plotted vs energy, is smoother for couplings to  $Q > 0$  channels, with respect to  $Q < 0$  channels
- $d(E\sigma)/dE$  will be correspondingly smaller (larger)



C.H. Dasso, S. Landowne and A. Winther, NPA 405, 381 (1983)

## Two reference cases

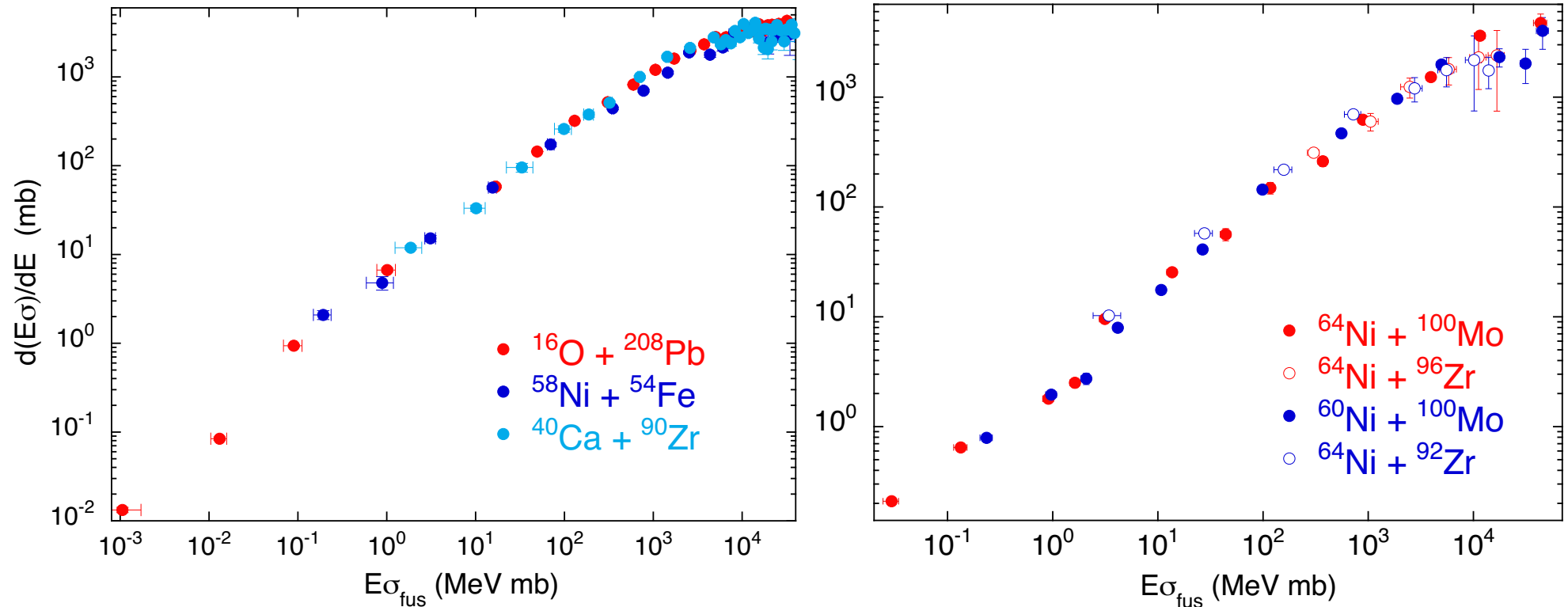


- Experimental **fusion** excitation functions,  $d[\ln(E\sigma)]/dE$  and  $d(E\sigma)/dE$  vs  $\sigma E$  the corresponding barriers
- The two barriers have approximately the same width
- N.B. in both cases, the measured barrier distributions are dominated by a single strong peak



A.M. Stefanini, et al., PLB 679 (2009) 95; PRC 78 (2008) 044607

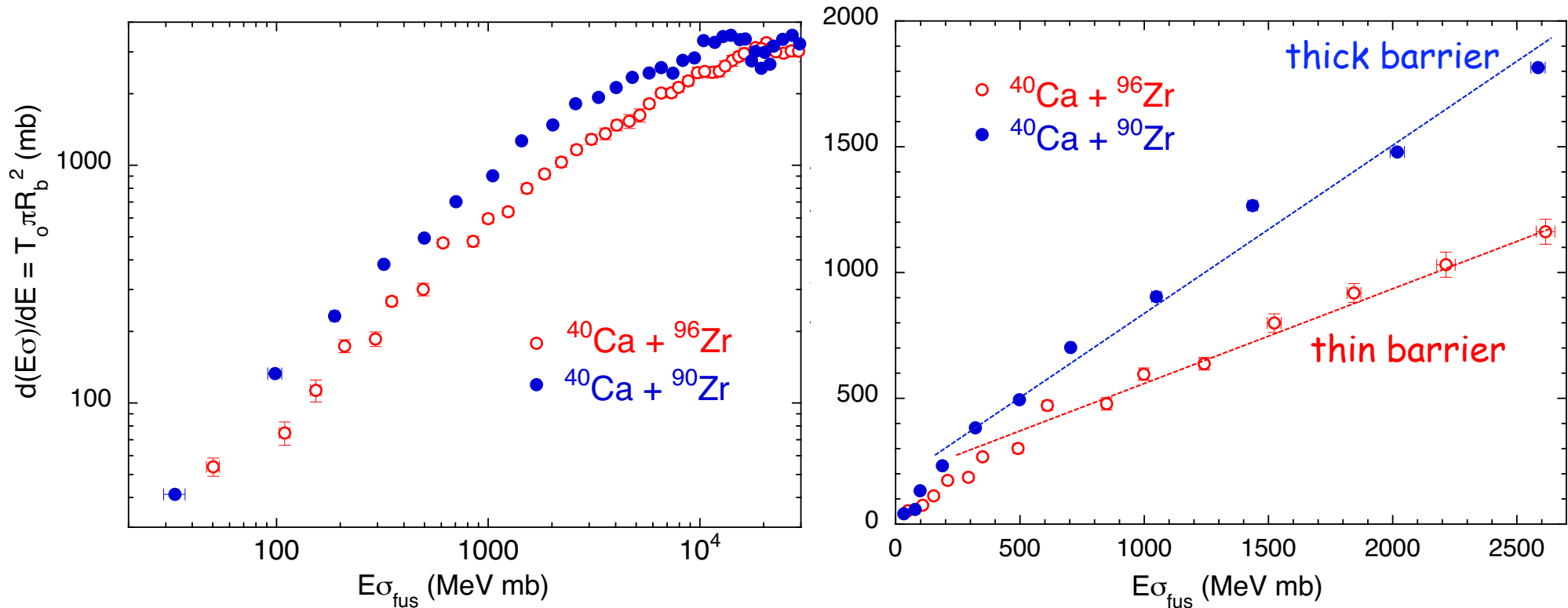
## Couplings to inelastic channels



- $d(E\sigma)/dE$  does not essentially change when comparing systems where couplings to **inelastic excitations** are dominant



## Couplings to transfer channels



- For  $^{40}\text{Ca} + ^{96}\text{Zr}$  transfer couplings are dominant, and a smaller derivative  $d(E\sigma)/dE$  is observed
- the effective one-dimensional barrier is **thinner simulating** a wider barrier distribution
- the **linear plot** (right) makes even more clear the difference between the two systems

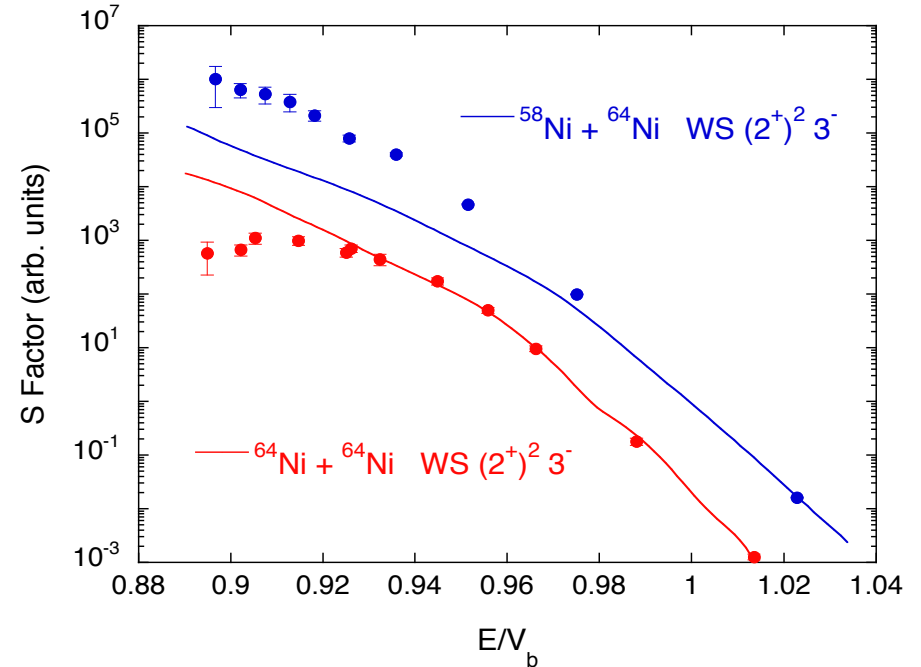
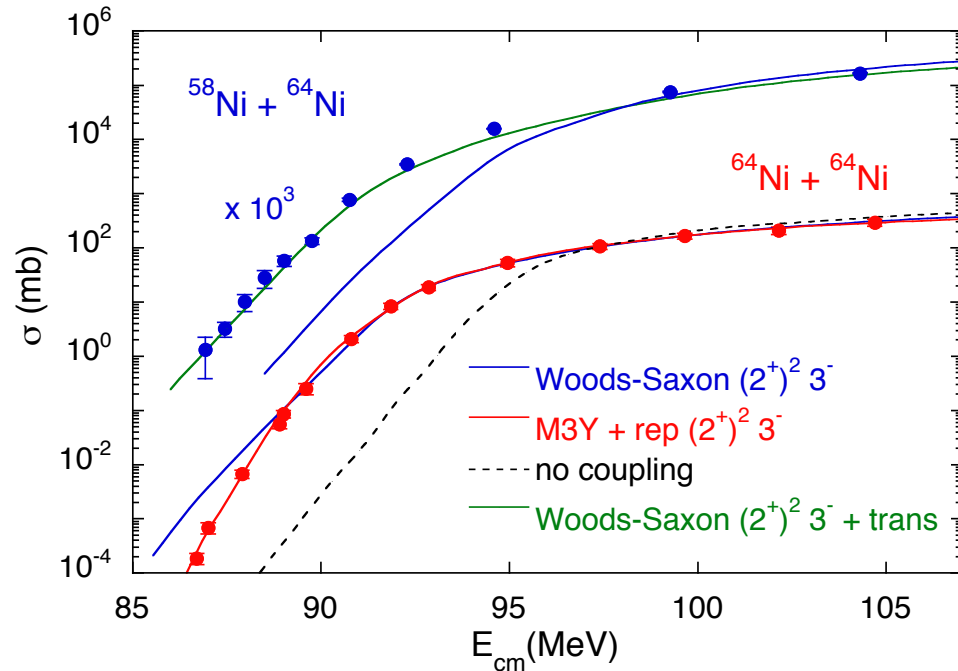


A.M. Stefanini et al., PLB 728 (2014) 639  
H. Timmers et al., NPA 633 (1998) 421





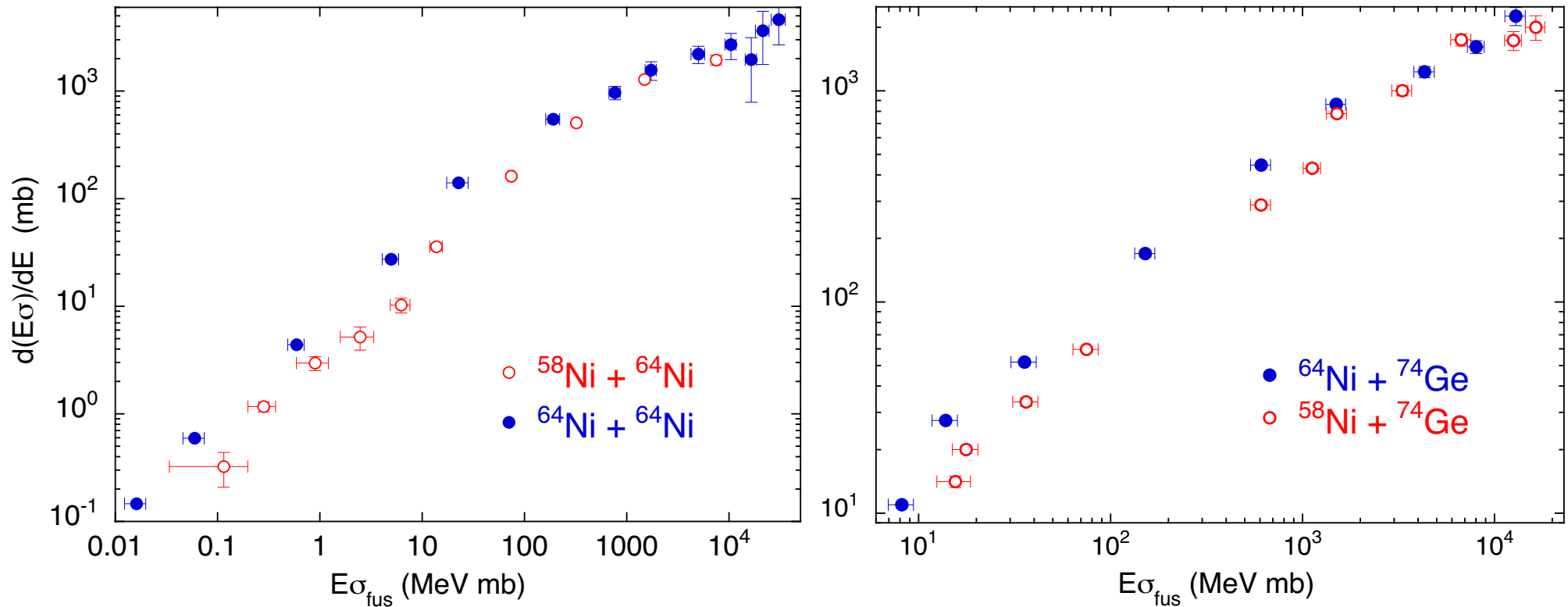
## Fusion cross sections and S-factors of $^{58,64}\text{Ni} + ^{64}\text{Ni}$



- No fusion hindrance has been observed down to about 1 mb in  $^{58}\text{Ni} + ^{64}\text{Ni}$
- The behaviour is very different from  $^{64}\text{Ni} + ^{64}\text{Ni}$
- The difference is due to the existence of  $Q > 0$  transfer channels in  $^{58}\text{Ni} + ^{64}\text{Ni}$ , given the similar low-energy vibrational nature of the two nuclei.



## The cases of $^{58,64}\text{Ni} + ^{64}\text{Ni}$ , $^{74}\text{Ge}$



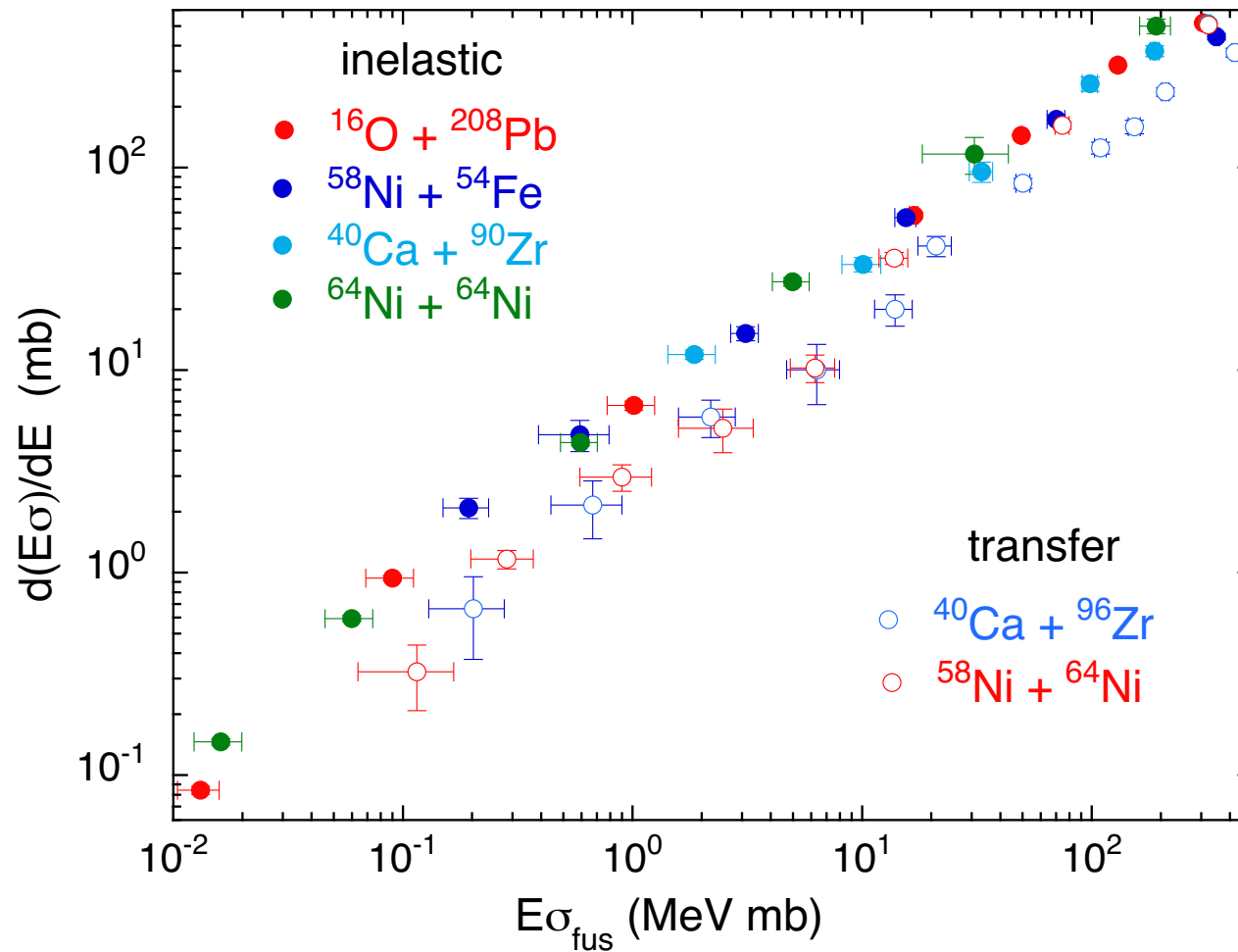
- Nucleon transfer couplings with  $Q > 0$  produce large cross section enhancements in  $^{58}\text{Ni} + ^{64}\text{Ni}$   $\longrightarrow$  a smaller derivative below  $E\sigma \approx 50$  MeV mb
- $^{74}\text{Ge}$  is vibrational, however the influence of strong  $Q > 0$  neutron pick-up couplings in  $^{58}\text{Ni} + ^{74}\text{Ge}$ , produces a less steep slope, even if  $^{58}\text{Ni}$  is more rigid than  $^{64}\text{Ni}$



A.M. Stefanini, et al., PRC 100 044619 (2019)  
 C.L.Jiang, et al., PRL 93, 012701 (2004)  
 M. Beckerman et al., PRC 25, 837 (1982)



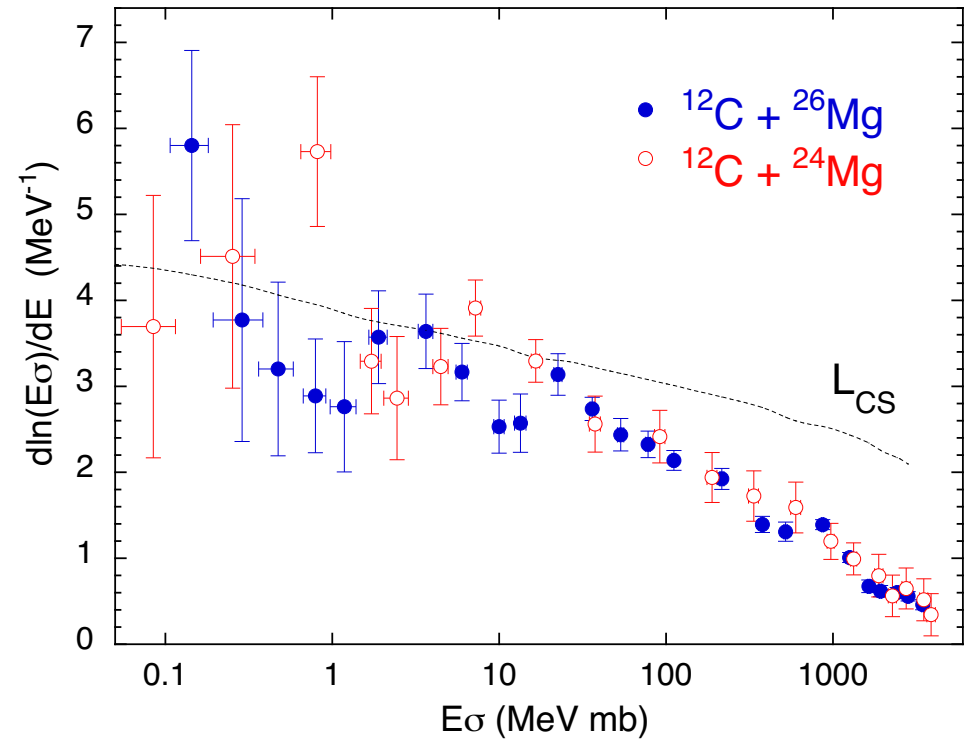
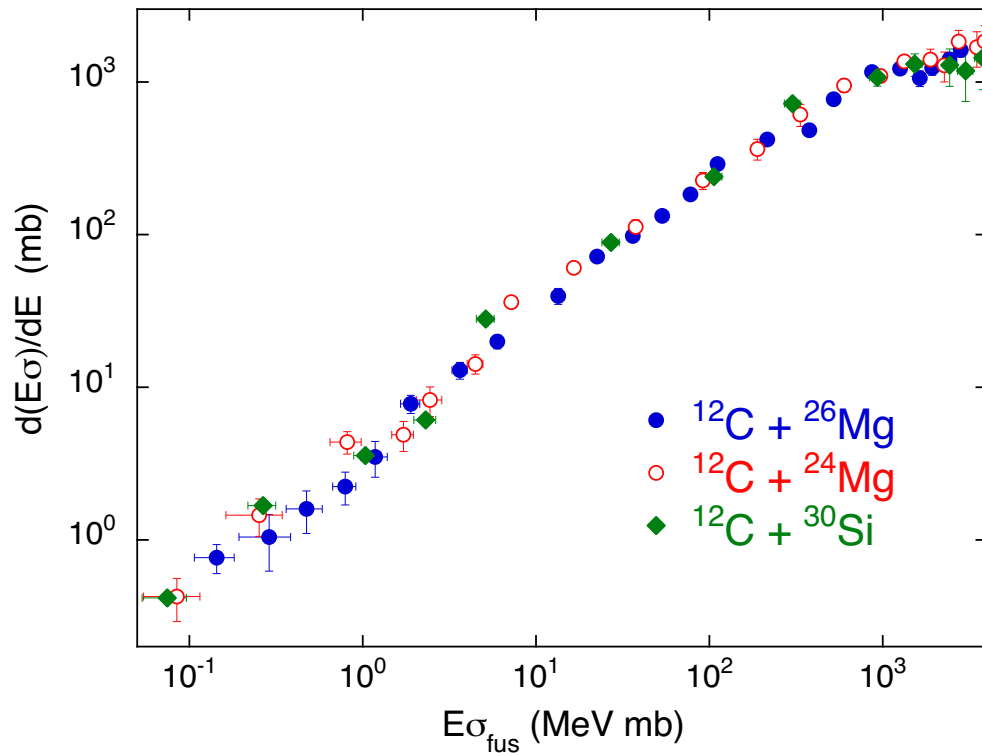
All together now !



- Two well separated **groups** of systems are evident, matching the nature of the dominant couplings



## What about medium-light systems ?



- The behaviour of the three systems (left) is **similar**
- $^{30}\text{Si}$  is **spherical**, while  $^{26,24}\text{Mg}$  have a permanent **prolate** deformation
- one observes various **oscillations** in the logarithmic derivatives (right), even if the experimental errors are **large** for  $^{24}\text{Mg} + ^{12}\text{C}$  (identifying the **hindrance threshold** is not straightforward)



A.M.Stefanini et al., PRC 108, 014602 (2023)

G. Montagnoli et al., PRC 101, 044608 (2020); PRC 97, 024610 (2018)



## Summary and Conclusions

- the **Wong's** formula, in relation to basic concepts of the **CC model**
- the representation of  **$d(E\sigma)/dE$  vs  $\sigma E$**  removes the difference due to the Coulomb barrier height of different systems
- it is sensitive to the **barrier width**
- $d(E\sigma)/dE$  vs  $\sigma E$  does not essentially change when comparing systems where couplings to **inelastic excitations** are dominant
- strong **transfer couplings** with  $Q > 0$  change that slope, as predicted by the **CC model**
- for **medium-light systems** this analysis may be complicated by the presence of cross section oscillations



# Collaboration

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*Thank you for your attention*

