Calorimetry

Introduction and outlook on future projects

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Content of the lecture

1) Interactions of particle with matter and basics of electromagnetic showers

2) Interactions of hadrons and hadronic showers

3) Particle detection

4) Calorimeter concepts (not only) for future projects

...

Literature

R. Wigmans: Calorimetry
D. Wegener: Detektoren in der Teilchenphysik - Lecture Uni Dortmund
W.R. Leo: Techniques for Nuclear and Particle Phyisics Experiment
C. Grupen: Teilchendetektoren
Sitar et al.: Ionization measuremens in High Energy Physics
arxiv:1412.2653, 1507.05893, 1602.08578

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Lots of Material 'stolen' from presentations and articles found on the Web.

If you find that I have used your material without a citation please write me and I will include the reference

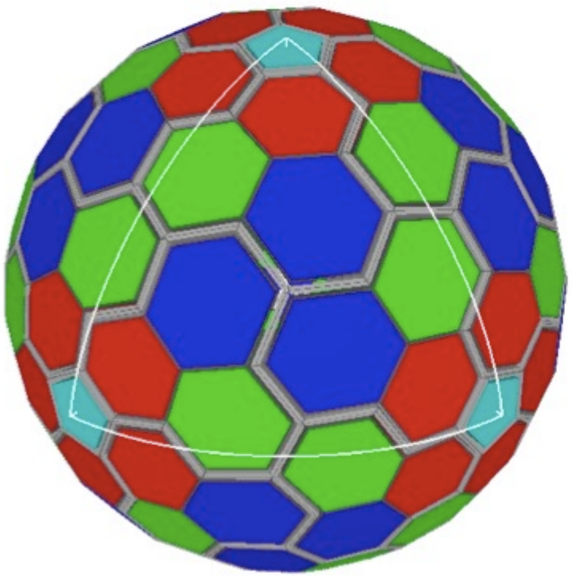
Thanks to Hengne Li for producing several figures for this lecture

Chapter 1

Interactions of particles with matter
and
basics of electromagnetic showers

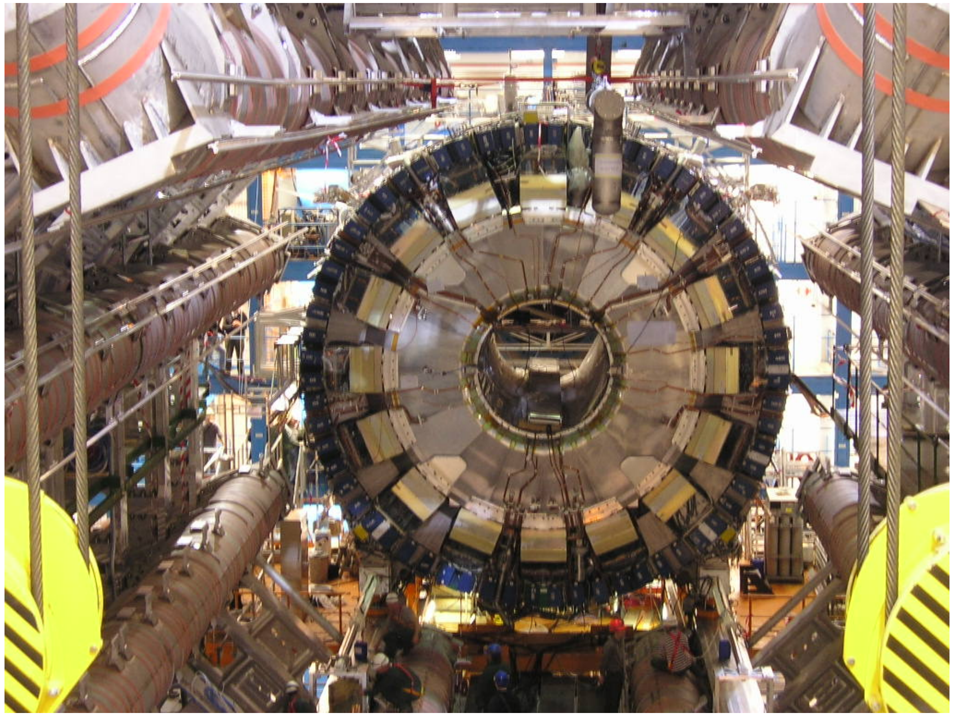
Calorimeters in nuclear and particle physics

4π 'Germanium Ball' of AGATA Experiment



1m

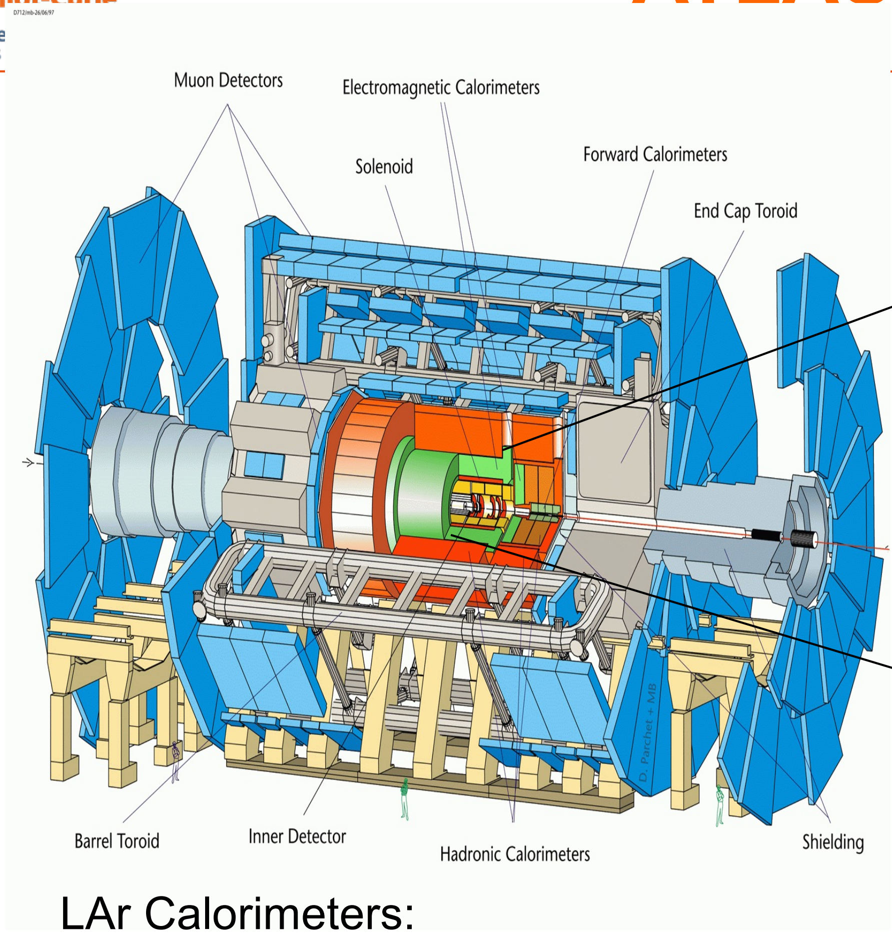
ATLAS TileCal Barrel Calorimeter

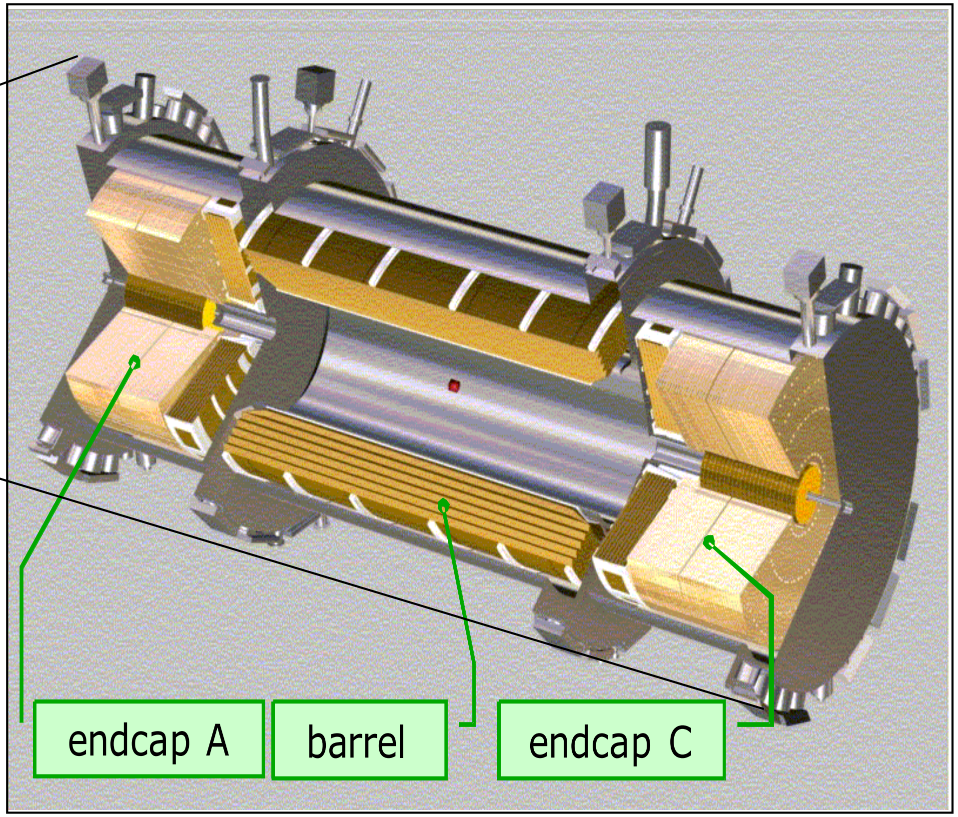


~10m

Calorimeters are employed in 'table top' experiments and in huge experimental apparatuus

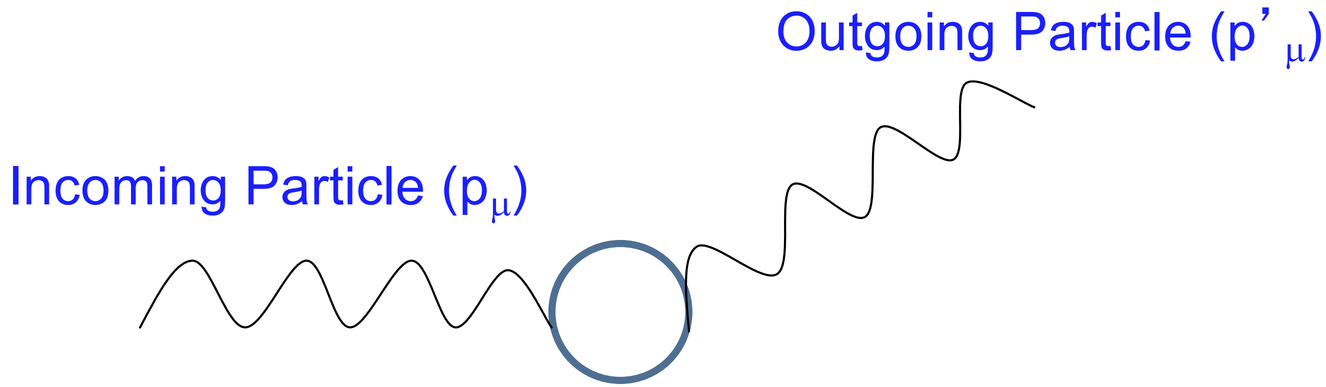
ATLAS Calorimeter





- LAr Calorimeters:
  - em Barrel : ($|\eta|<1.475$) [Pb-LAr]
  - em End-caps : $1.4<|\eta|<3.2$ [Pb-LAr]
  - Hadronic End-cap: $1.5<|\eta|<3.2$ [Cu-LAr]
  - Forward Calorimeter: $3.2<|\eta|<4.9$ [Cu,W-LAr]
- ~190K readout channels
- Hadronic Barrel: Scintillating Tile/Fe calorimeter

Interactions of particles with matter



Scattering Center:
Nucleus or Atomic Shell

Detection Process is based on Scattering of particles while passing detector material

Energy loss of incoming particle: $\Delta E = p_0 - p'_0$

‘Real’ Bethe Bloch Formula

After consistent quantum mechanical calculation
Valid for particles with $m_0 >> m_e$

$\Delta E = 0$: Rutherford Scattering

$\Delta E \neq 0$: Leads to Bethe-Bloch Formula

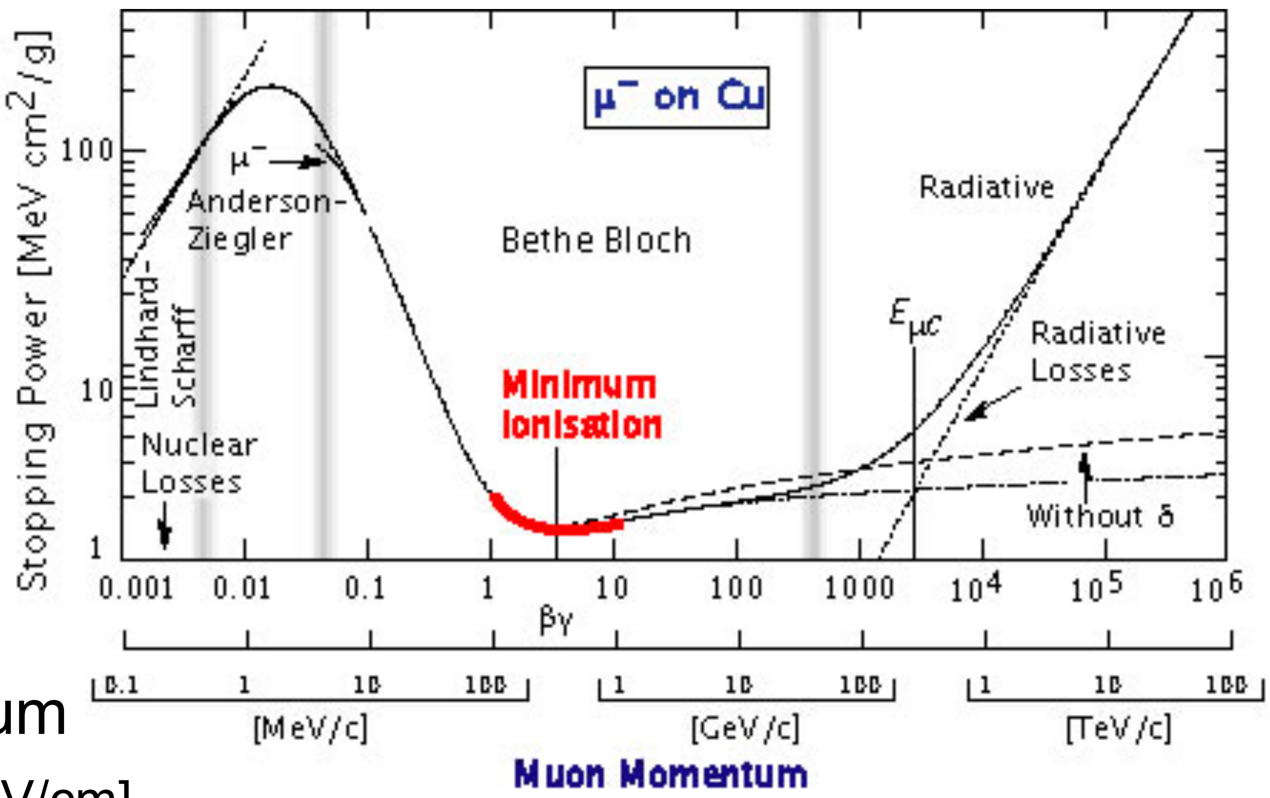
$$\frac{dE}{dx} = 4\pi N_A r_e^2 m_e c^2 z^2 \frac{Z}{A}\frac{1}{\beta}\left[\frac{1}{2}\ln\left(\frac{2 m_e c^2 \gamma^2 \beta^2 T_{max}}{I^2}\right) - \beta^2 - \frac{\delta}{2}\right]$$

- z - Charge of incoming particle
- β, γ - Velocity, gamma-factor of incoming particle
- Z, A - Nuclear charge and mass of absorber
- r_e, m_e - Classical electron radius and electron mass
- N_A - Avogadro’s Number = 6.022×10^{23} Mol^{-1}
- I - Ionisation Constant, characterizes Material typical values 15 eV
- δ – Fermi’s density correction
- T_{max} - maximal transferrable energy

Discussion of Bethe Bloch Formula II

Minimal Ionizing Particles (MIPS)

dE/dx passes
Broad minimum @
$\beta\gamma \approx 4$



Contributions from energy losses start to dominate kinematic dependency of cross sections

typical values in Minimum

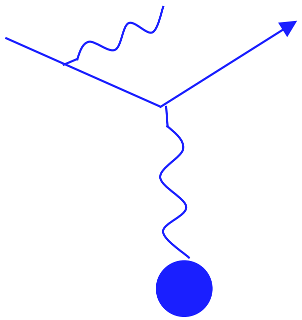
	[MeV/(g/cm^2)]	[MeV/cm]
Lead	1.13	20.66
Steel	1.51	11.65
O2	1.82	$2.6\cdot10^{-3}$

Role of Minimal Ionizing Particles ?

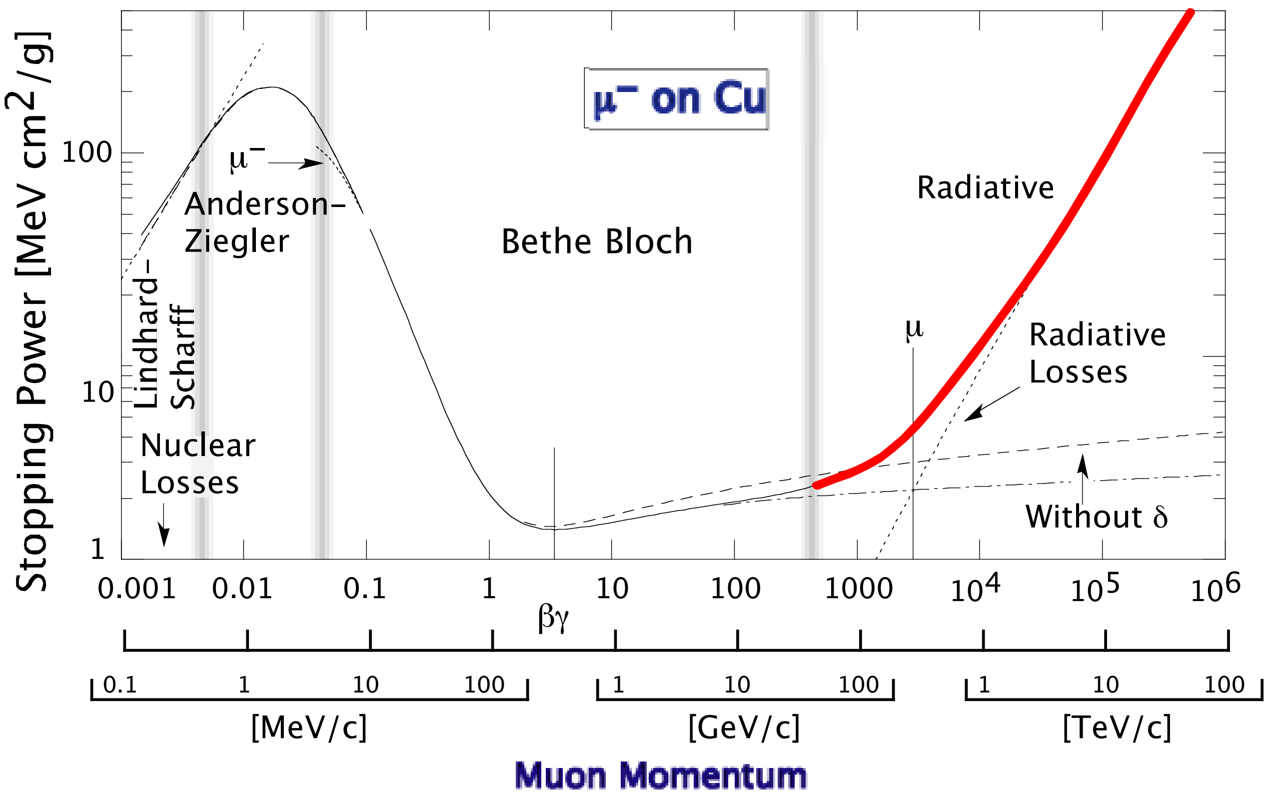
Discussion of Bethe Bloch Formula V

Radiative losses - Not included in Bethe-Bloch Formula

Particles interact with Coulomb Field Of <u>nuclei</u> of Absorber atoms



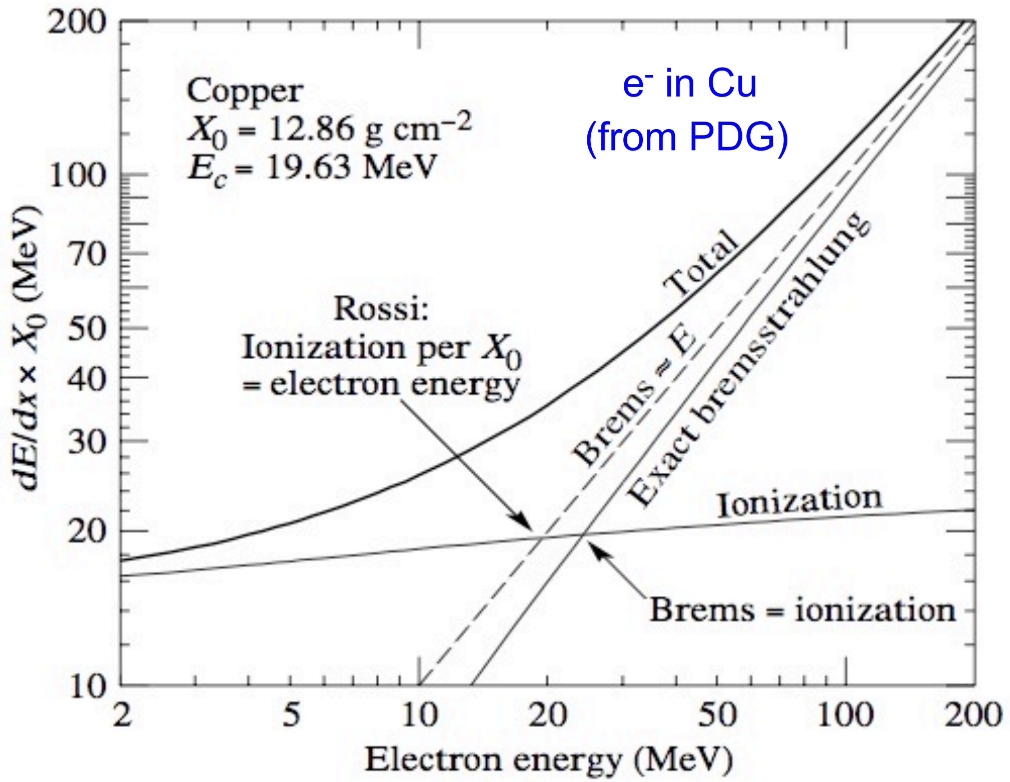
Energy loss due to Bremsstrahlung



Important for e.g. muons with E > 100 GeV
Dominant energy loss process for electrons (and positrons) $(m_\mu/m_e)^2 \sim 40000$

Critical Energy

Point where $\Delta E_{Ion} = \Delta E_{Brems}$



$$\varepsilon_c = \frac{610\ \text{MeV}}{Z + 1.24}$$

For materials in solid and liquid phase

$$\varepsilon_c = \frac{710\ \text{MeV}}{Z + 0.92}$$

For gases

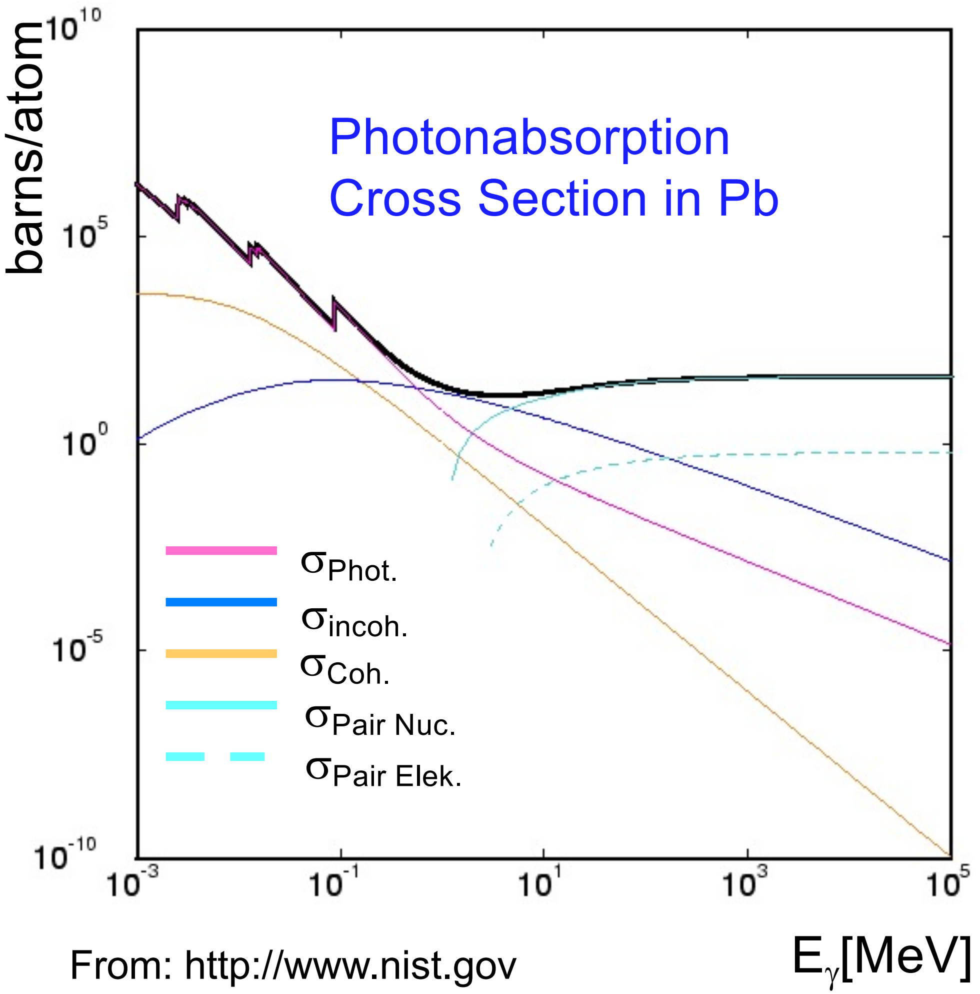
Emperical values
Based on fits to data

ε_c is a characteristic parameter of material

e.g. ε_c for Uranium 6.75 MeV

Overview on photon absorption cross section

General: Beer's Absorption Law: $I = I_0 e^{-\mu x}$, μ = Absorptioncoefficient



From: http://www.nist.gov

Different processes for different energies:

1) Photoelectric Effect
   $E \leq 500$ keV
2) Compton Scattering
   500 keV < E < 5 MeV
3) Pair Production
   $E > 2m_e$

Electromagnetic showers - Basic considerations

Above all relevant electromagnetic interactions were introduced

For electrons/positrons

a) Ionization
b) Bremsstrahlung
(Annihilation for Positrons)

For Photons

a) Photoelectric Effect
b) Compton Scattering
c) Pair Production

- A primary particle looses energy due to one of these interactions in a given absorber material

- The produced secondaries in turn do interact with the material and produce themselves further particles

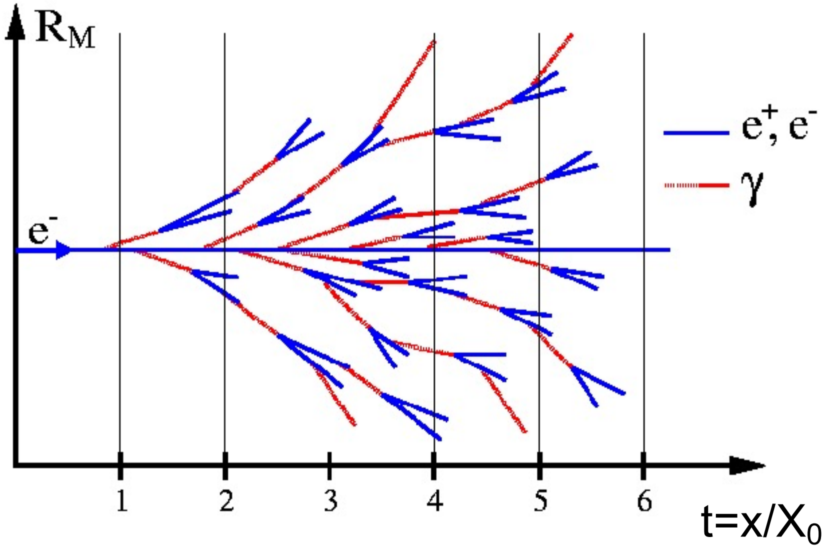
Development of an electromagnetic cascade

High energetic particles

Energy loss of electron by Bremsstrahlung:
Photons convert into e^+e^--Pairs

$$\frac{dE}{dx} = -\frac{E_e}{X_0} \Rightarrow E = E_0 e^{-x/X_0}$$

Simple shower model
(see Longo for detailed discussion)



- Energy loss after X_0: $E_1 = E_0/2$
- Photons -> materialize after X_0
  $E_{\pm} = E_1/2$

Number of particles after t: $N(t) = 2^t$
Each Particle has energy

$$E = \frac{E_0}{N(t)} = E_0 2^{-t} \Rightarrow t = \ln\left(\frac{E_0}{E}\right)/\ln 2$$

Shower continues until particles reach critical energy (see p. 16) where $t_{max} = \ln(E_0/\varepsilon_c)$

For high energetic particles shower contain thousands of particles

Shower Maximum increases logarithmically with Energy of primary particle
(Important for detector design !!!)

Scaling variables I – Radiation Length

(Not only) For Bremsstrahlung: $$-\left(\frac{dE}{dx}\right)_{rad} = N\int_0^{E_{\gamma 0}} E_\gamma \frac{d\sigma_{rad}}{dE_\gamma} dE_\gamma$$

Spectrum of radiated photons: $$\frac{d\sigma_{rad}}{dE_\gamma} \sim \frac{1}{E_\gamma} f(Z)$$

Define: $$\Phi_{rad} = \frac{1}{E_0} N\int_0^{E_{\gamma 0}} E_\gamma \frac{d\sigma_{rad}}{dE_\gamma} dE_\gamma \Rightarrow -\frac{dE}{dx} = NE_0\Phi_{rad}$$

Φ_{rad} is independent of the energy of the radiated photon and only a function of the the material

Radiation Length X_0: Distance after which a particle has lost 1/e due to radiation

$$E = E_0 \exp\left(-\frac{x}{X_0}\right)$$ With $X_0 = 1/N\Phi_{rad} \Rightarrow X_0 = \frac{A}{4\alpha N_A Z(Z+1) r_e^2 \ln(183 Z^{-1/3})} [g/cm^2]$

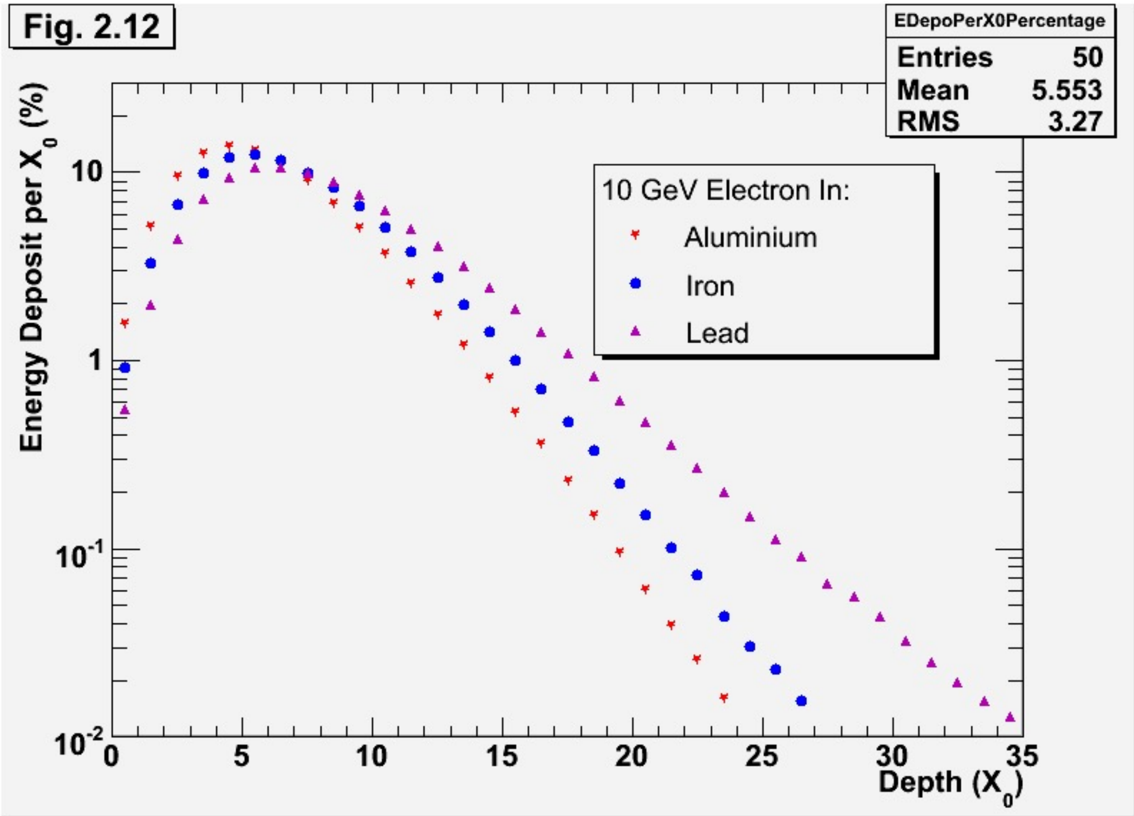
Absorption Coefficient Photons

$$\mu(E >> m_e c^2) = \frac{28}{9} n Z^2 \alpha r_e^2 \ln\frac{183}{Z^{1/3}} = \frac{7}{9} X_0^{-1}$$

i.e. Photons travel a longer distance before they interact

Some values:
$X_{0,Air}$=30 420 cm
$X_{0,Al}$ = 8.9 cm
$X_{0,Pb}$ = 0.56 cm

Longitudinal shower profile



Shape of shower profiles
Material independent
=
Energy loss
Independent of material

.. as function of X_0

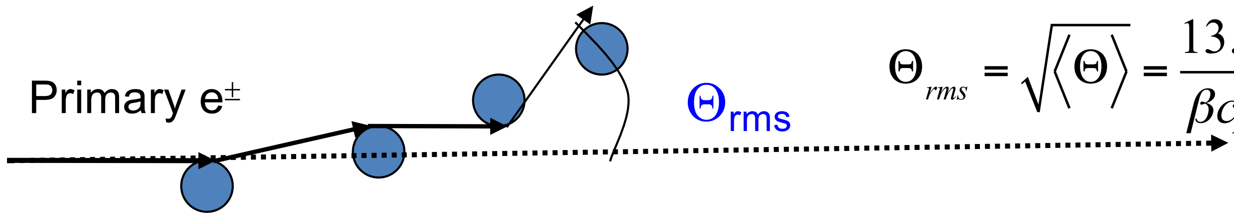
Differences:
- Pair production by emitted photons increases with Z and extends to much lower energies
- Critical energy is Z-dependent e.g. 43 MeV for Al 7 MeV for Pb
- X_0 looses meaning at low energies

Scaling variables II – Molière Radius

2 Effects

1) Electrons (and positrons) undergo multiple scattering in the Coulomb field of absorber nuclei

Θ_{mean} = 0 with standard deviation



$$\Theta_{rms} = \sqrt{\langle\Theta\rangle} = \frac{13.6}{\beta c p}\sqrt{\frac{x}{X_0}}$$

For projected Scattering angle distribution

2) Isotropic production of secondaries by Photoelectric effect and Compton Scattering

Low energetic component of shower

Expect different behaviour in region 'far' away from shower axis

Transversal extension of shower characterized by Molière Radius

$$\rho_M = mc^2\sqrt{4\pi/\alpha}\,\frac{X_0}{\varepsilon_C} = 21.2MeV\,\frac{X_0}{\varepsilon_c}$$

90% of shower energy is contained within ρ_M

Weak/no Z dependence

Typical Values for ε_c, ρ_M:

	ε_c[MeV]	R_M [cm]
Pb	7.2	1.6
NaJ	12.5	4.4
Air	87	7400

Energy resolution – The master formula

$$N^{total} \propto \frac{E_0}{E_c}$$ total number of track segments

$$\frac{\sigma(E)}{E} \propto \frac{\sigma(N)}{N} \propto \frac{1}{\sqrt{N}} \propto \frac{1}{\sqrt{E_0}}$$ holds also for hadron calorimeters

Also spatial and angular resolution scale like $1/\sqrt{E}$

Relative energy resolution of a calorimeter improves with E_0

More general:

$$\frac{\sigma(E)}{E} = \frac{a}{\sqrt{E}} \oplus b \oplus \frac{c}{E}$$

Stochastic term

Constant term

Inhomogenities
Bad cell inter-calibration
Non-linearities

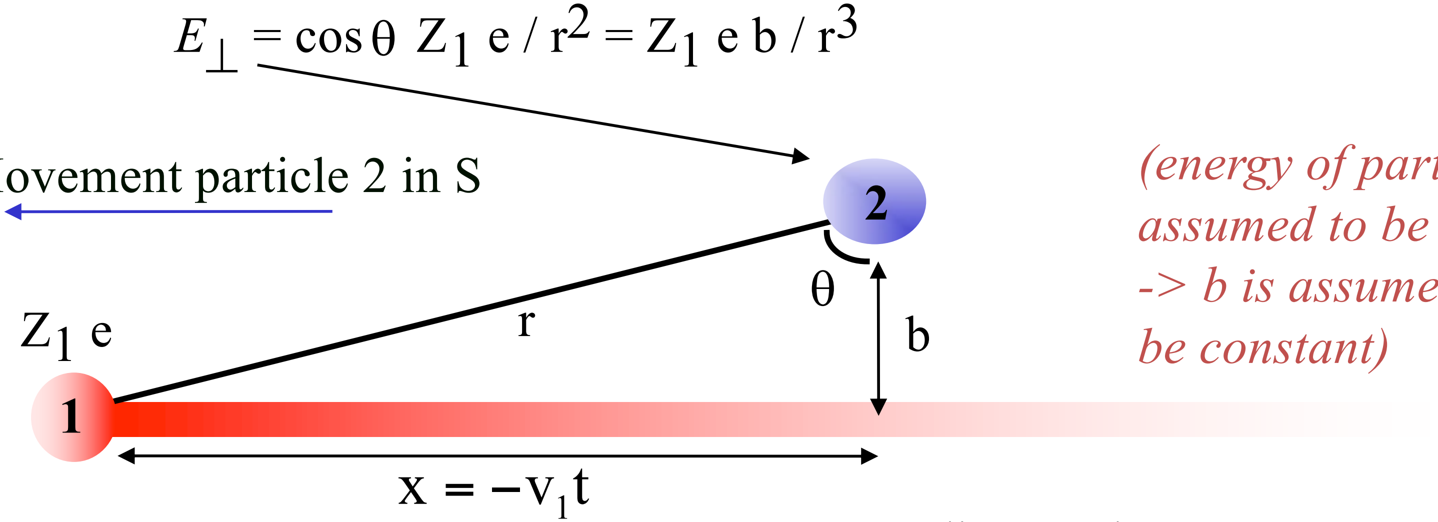
Noise term

Electronic noise
radioactivity
pile up

Backup

Energy loss due to collisions

à la Leo: Partially taken from lecture at NIKHEF by unknown author



(energy of particle is assumed to be high, -> b is assumed to be constant)

t = 0: distance between 1 and 2 is minimal

In the laboratory system S' particle 2 is in rest, so there is no effect from the (time dependent) magnetic field caused by particle 1.

In S': $E'_\perp = \gamma E_\perp = \gamma Z_1 e b / r^3$

time in S' !

$$r^2 = b^2 + x^2 = b^2 + v_1^2 t^2 = b^2 + \gamma^2 v_1^2 t'^2$$

electric field strength

$$E_\perp = \gamma Z_1 e b / \left(b^2 + \gamma^2 v_1^2 t'^2\right)^{3/2}$$

Energy loss due to collisions cont'd

$$\Delta p_\perp = \int F_\perp \, dt = \int_{-\infty}^{\infty} \frac{Z_2 e \gamma Z_1 e \, dt}{\left(b^2 + \gamma^2 v_1^2 t^2\right)^{\frac{3}{2}}}$$

$$\Delta p = \Delta p_\perp = Z_2 e \gamma Z_1 e \left[\frac{t}{b^2 \left(b^2 + \gamma^2 v_1^2 t^2\right)^{\frac{1}{2}}} \right]_{-\infty}^{\infty} = \frac{2 Z_1 Z_2 e^2}{v_1 b}$$

$$\Delta E = \frac{(\Delta p)^2}{2m} = \frac{2 Z_1^2 Z_2^2 e^4}{b^2 v_1^2 m}$$

(Particle 2 is in rest -> non-relativistic calculation of energy loss possible)

Interactions with electron: $m = m_e$, $Z_2 = 1$

Interactions with nucleus with mass number A and atomic number Z: $m = Am_p \approx 2Zm_p$, $Z_2 = Z$

$$\frac{\Delta E(e)}{\Delta E(nucleus)} = \frac{2 Z_1^2 e^4}{b^2 v_1^2 m_e} \bigg/ \frac{2 Z_1^2 Z^2 e^4}{b^2 v_1^2 2Z m_p} = \frac{2 m_p}{Z m_e} \approx 4000 / Z$$

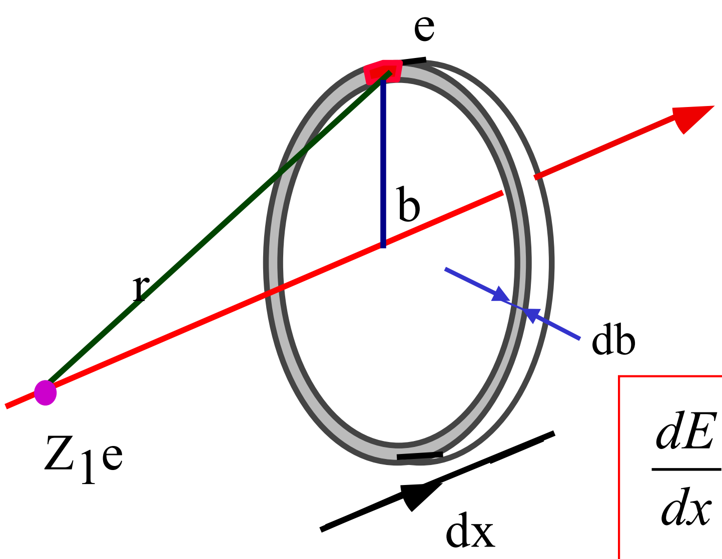
-> Energy loss due to collisions is dominated by interactions with the electrons

(NB: we are comparing interactions with 1 electron to interactions with the nucleus of an atom, a non-ionized atom has Z electrons)

Energy loss due to collisions cont'd

So far interactions with single electrons
but material consists of many electrons and nuclei

N. Bohr: Particle passes through center of thin shell



Number of electrons in shell: $n_e\, 2\pi b\, db\, dx$
with n_e = number of atoms per cm^3

$$\Delta E = \frac{4\pi Z_1^2 e^4 n_e\, db\, dx}{b v_1^2 m}$$

$$\frac{dE}{dx} = 2\pi n_e \left(\frac{2Z_1^2 e^4}{m_e v_1^2}\right) \int_{b_{min}}^{b_{max}} \frac{db}{b} = \frac{4\pi n_e Z_1^2 e^4}{m_e v_1^2} \ln \frac{b_{max}}{b_{min}}$$

Value for b_{max}

Determination of energy loss

Transversal Field Strength: $(b^2 + \gamma^2 v_1^2 t^2)^{3/2}$

'Interaction' Time: $b / v_1\gamma$

Interaction Time < Orbital Frequency ω of Electrons such that binding effects can be neglected

$\Rightarrow b_{max} = \gamma v_1/\omega$

Maximal Energy Transfer for m >> m_e(see above): $2m_e\beta^2\gamma^2$

Using: $\Delta E = \frac{2Z_1^2 e^4}{b^2 v_1^2 m_e}$ it follows: $b_{min} = \frac{Z_1 e^4}{\gamma v_1^2 m_e}$ so:

$$\frac{dE}{dx} = \frac{4\pi n_e Z_1^2 e^4}{m_e v_1^2} \ln \frac{m_e v_1^3 \gamma^2}{Z_1 e^2 \omega}$$

Detailed discussion follows

Discussion of Bethe Bloch Formula I

Describes Energy Loss by Excitation and Ionisation !!

We do not consider lowest energy losses

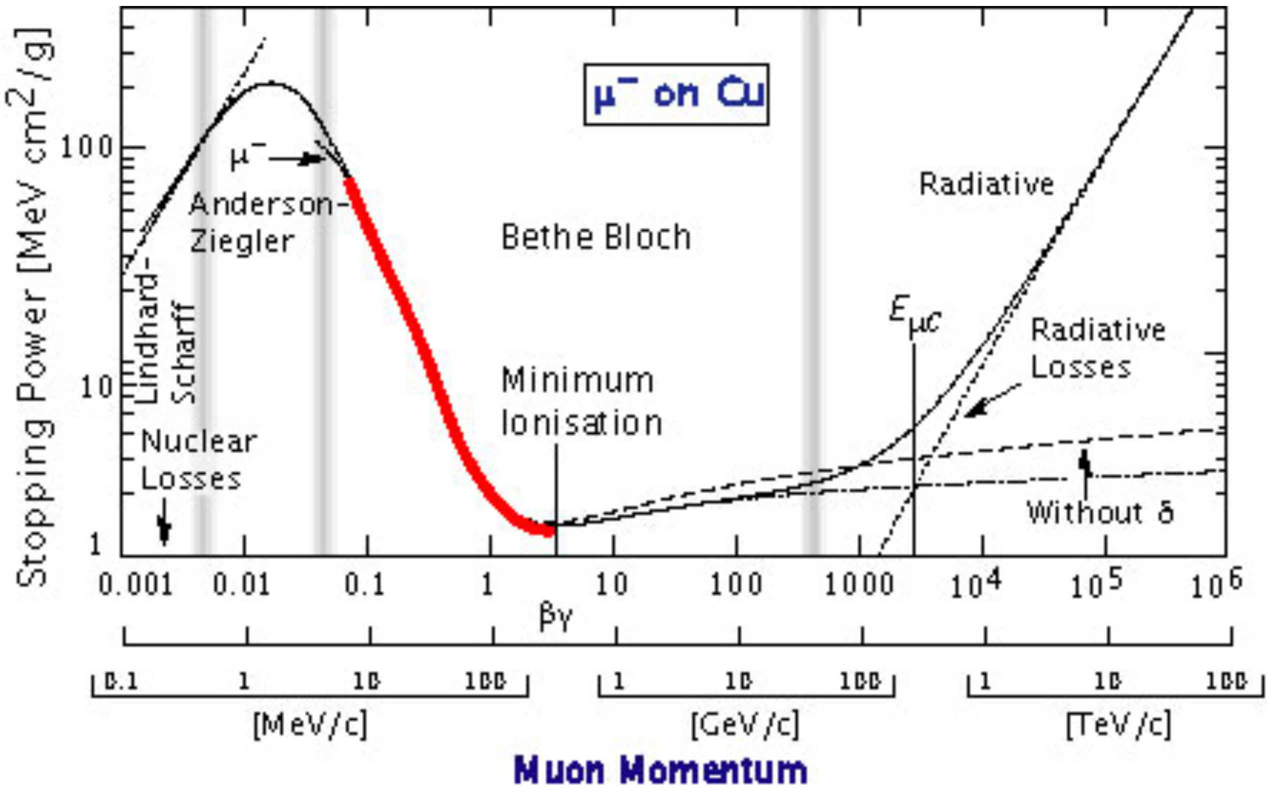
'Kinematic' drop

~ $1/\beta^2$

Scattering Amplitudes:

$$f_i(\theta) \propto 1 \Big/ (\vec{p} - \vec{p}\,')^2, (\vec{p} - \vec{p}\,')^2 \propto v^2$$

Large angle scattering becomes less probable with increasing energy of incoming particle.

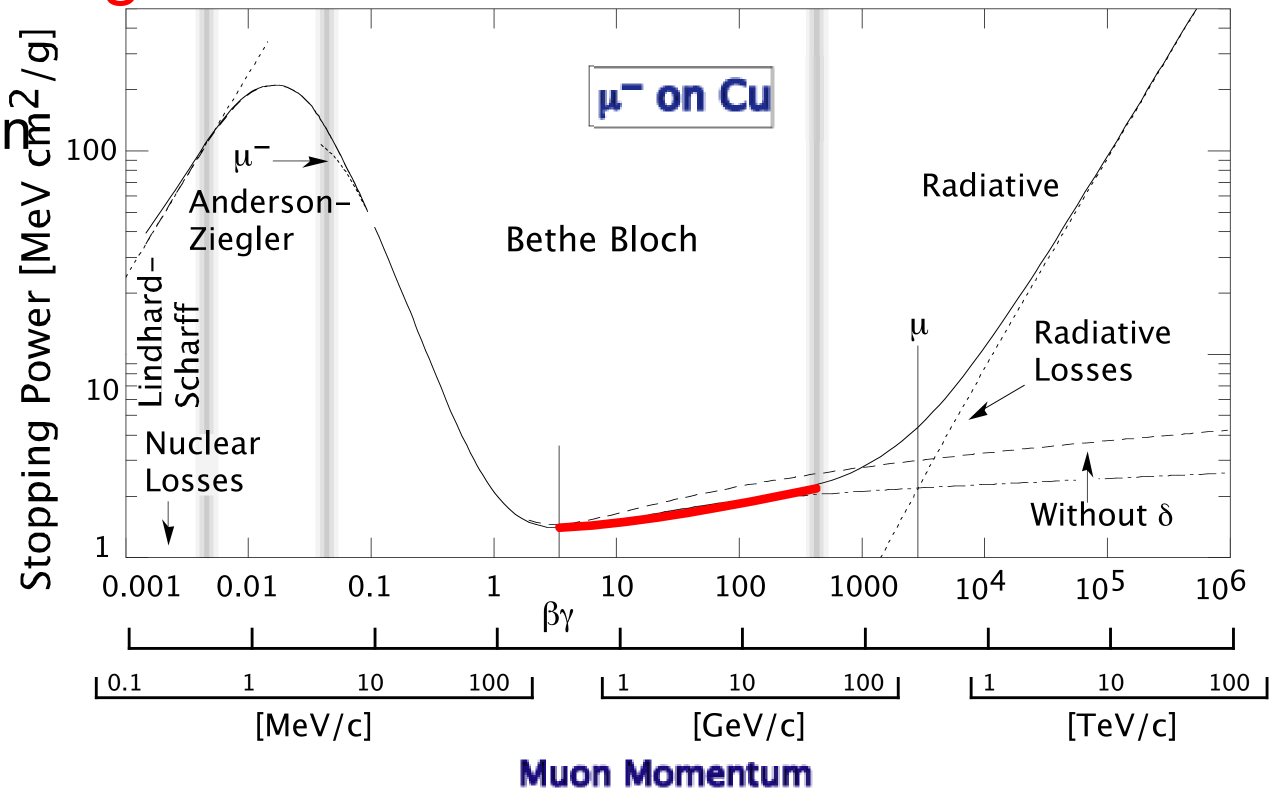


Drop continues until $\beta\gamma \sim 4$

Discussion of Bethe Bloch Formula III

Logarithmic rise

'Visible' Consequence of Excitation and Ionization Interactions.
Dominate over kinematic drop



Interesting question:
Energy distribution of electrons created by Ionization.

δ-Electrons

Non relativistic: $E_1 \approx M$

$$T_{\max} = 4\frac{m}{M_{Abs.}} \bullet \frac{\vec{p}_1^{\,2}}{2M_{Abs.}}$$

Relativistic: $|p_1| \approx M$

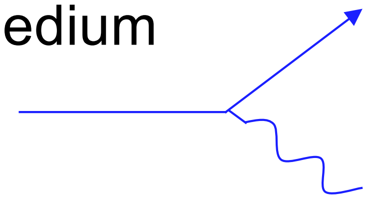
$$T_{\max} \approx \frac{2mc^2\beta^2\gamma^2}{1+2\frac{m\gamma}{M}+\left(\frac{m}{M}\right)^2}$$

In the relativistic case an incoming particle can transfer (nearly) its whole energy to an electron of the Absorber
These δ-electrons themselves can ionize the absorber !

Discussion of Bethe Bloch Formula IV

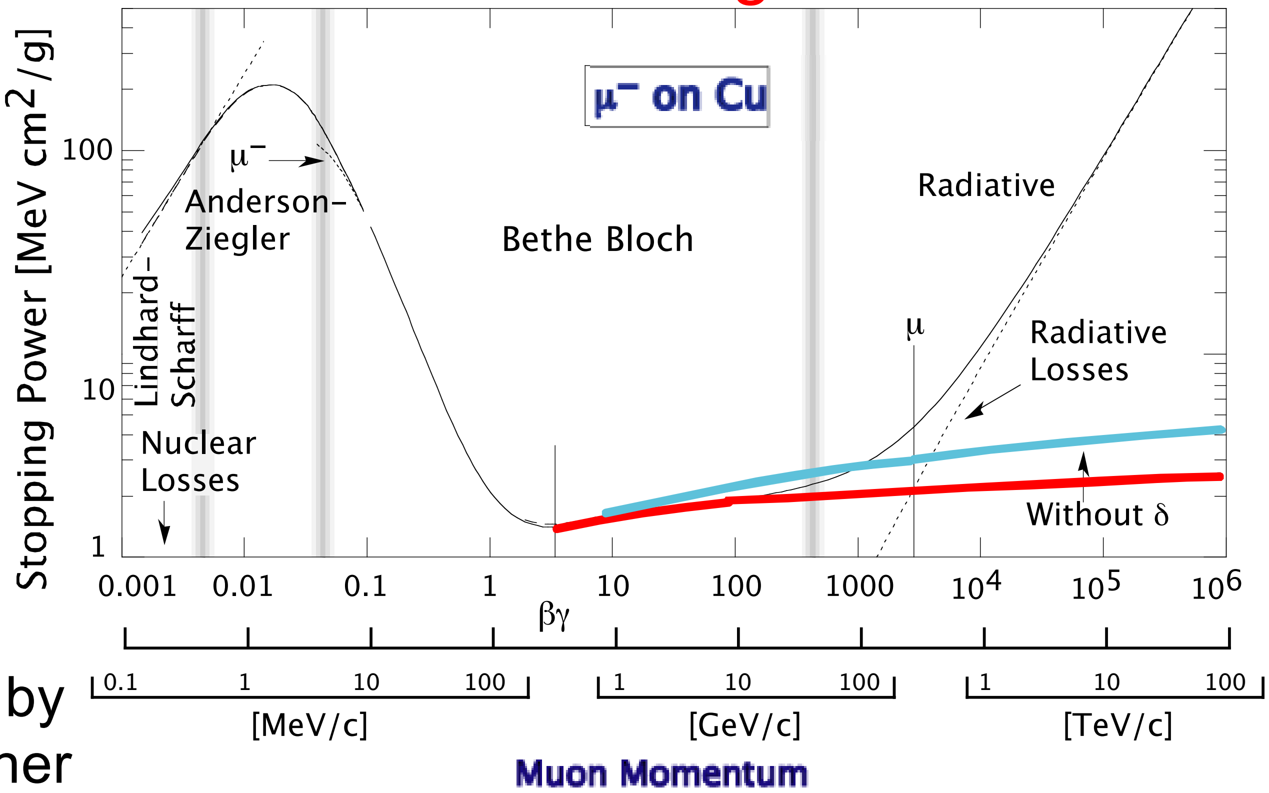
Fermi's Density Correction - 'Tames' logarithmic rise

Interaction with absorber medium

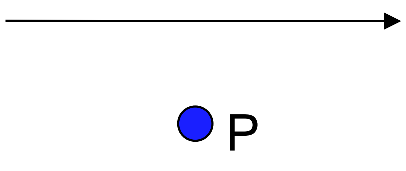


Energy losses not treated as statistically independent processes

⇒ Single $\gamma \rightarrow$ elm. wave



Basic Effect: Amplitude felt by Atom at P is shielded by other Atoms ⇒ Damped wave !!!



P does not contribute to energy loss !

Density Correction more important in Solids than in Gases

Remark: Density Correction tightly coupled to Čerenkov-Effect

Bethe Bloch for ultrarelativistic particles

v ~ c, i.e. electrons and positrons

See e.g. E.A. Uehling 1954, Ann. Nucl. Sci. 4, 315 Sect. 1.1

Results for Electrons:

$$\frac{dE}{dx} = 4\pi N_A r_e^2 m_e c^2 z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln\left(\frac{2 m_e c^2 \gamma^{3/2}}{\sqrt{2} I^2} \right) + \frac{1}{16} - \frac{\delta}{2} \right]$$

Results for Positrons:

$$\frac{dE}{dx} = 4\pi N_A r_e^2 m_e c^2 z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln\left(\frac{2\sqrt{2} m_e c^2 \gamma^{3/2}}{I^2} \right) + \frac{23}{24} - \frac{\delta}{2} \right]$$

These particles are already ultrarelativistic at E ≈100 MeV