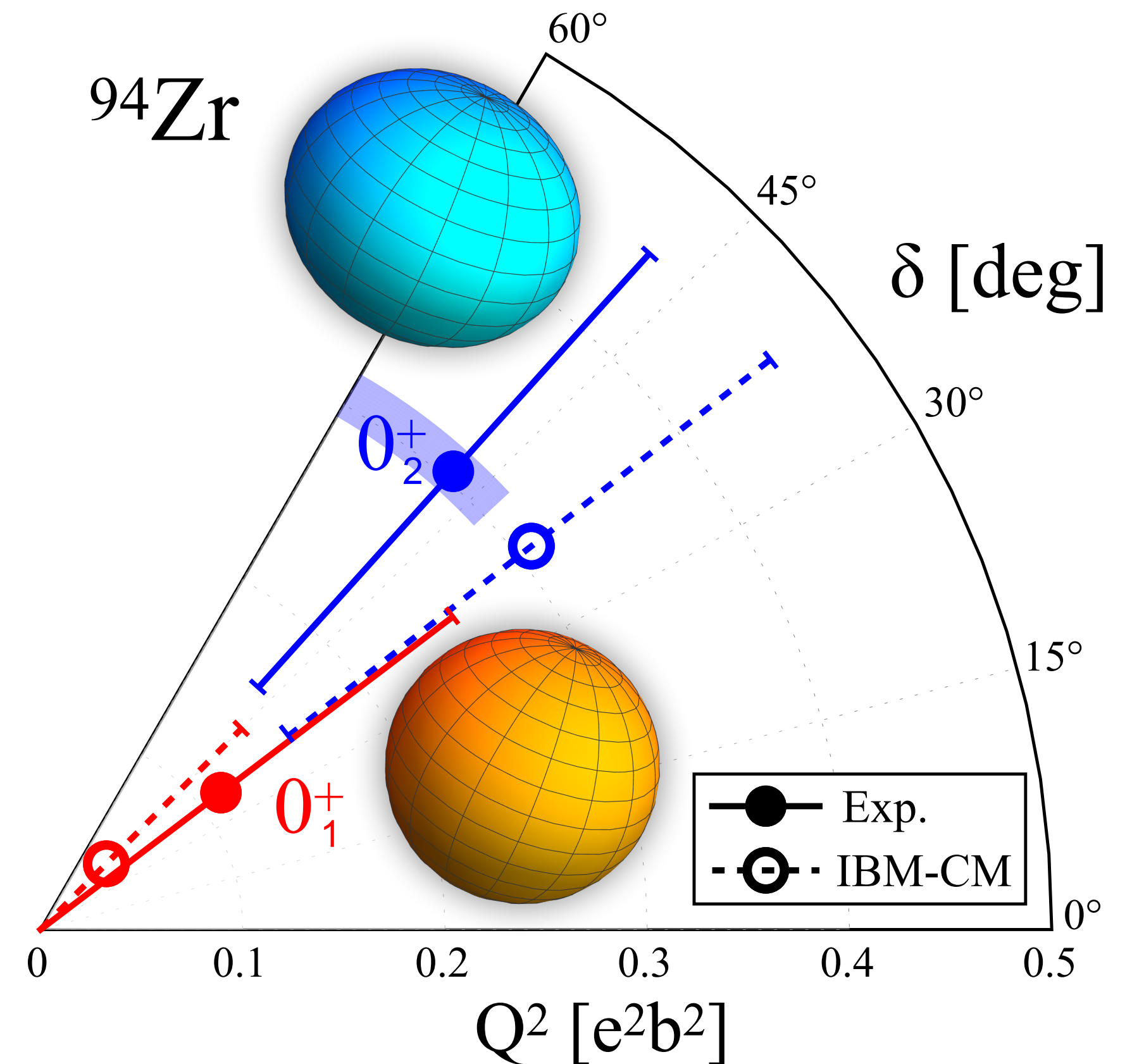


# Spherical-Oblate Shape Coexistence in $^{94}\text{Zr}$ and the SPIDER Coulomb-Excitation Campaign at LNL



# Low-Energy Coulomb Excitation (aka Coulex)

Low-Energy  
Coulomb  
Excitation

SPIDER with  
GALILEO and  
AGATA

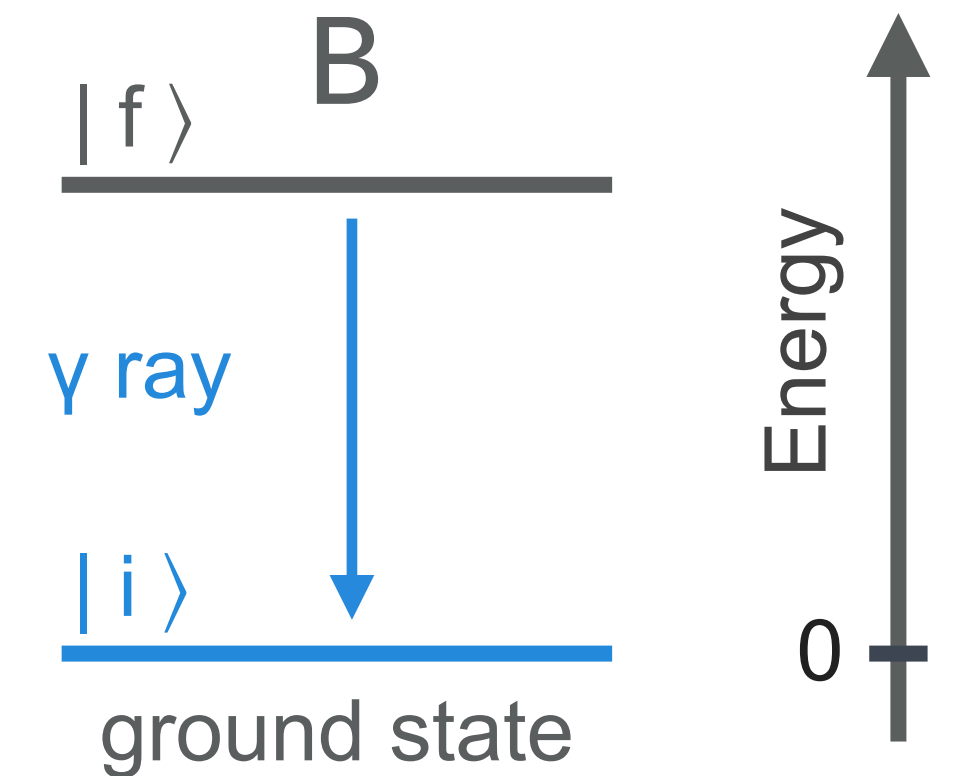
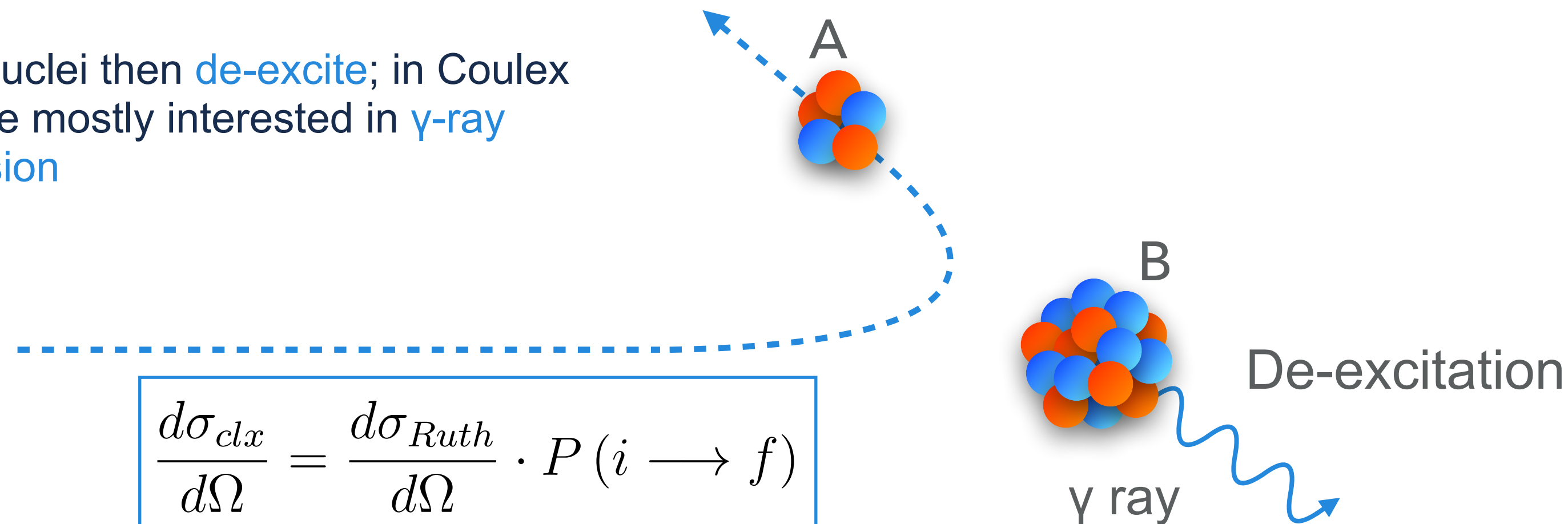
Excited  $0^+$  States  
in Even-Even Mid-  
Mass Nuclei

The Zr Isotopic  
Chain and QPTs

Coulex  
Experiment  
on  $^{94}\text{Zr}$

Model-  
Independent  
Determination of  
Shapes in  $^{94}\text{Zr}$

- ▶ Inelastic scattering between two interacting nuclei, in the “purely” electromagnetic regime
- ▶ The time-dependent electromagnetic field between the two nuclei can induce excitations
- ▶ The nuclei then de-excite; in Coulex we are mostly interested in  $\gamma$ -ray emission



$$\frac{d\sigma_{clx}}{d\Omega} = \frac{d\sigma_{Ruth}}{d\Omega} \cdot P(i \rightarrow f)$$

- ▶ Example: first  $2^+$  state in an even-even target nucleus

$$P(0_1^+ \rightarrow 2_1^+) = F(\theta, E_P) \sim |\langle 2^+ || E2 || 0^+ \rangle|^2 \left[ 1 + 1.32 \frac{A_P}{Z_T} \frac{\Delta E}{\left(1 + \frac{A_P}{A_T}\right)} \sim \langle 2^+ || E2 || 2^+ \rangle Q_s(2^+) K(\theta, E_P) \right]$$

Access to: transition probabilities, spectroscopic quadrupole moments

# Quadrupole Sum Rules & Rotational Invariants

Low-Energy  
Coulomb  
Excitation

SPIDER with  
GALILEO and  
AGATA

Excited  $0^+$  States  
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Mass Nuclei

The Zr Isotopic  
Chain and QPTs

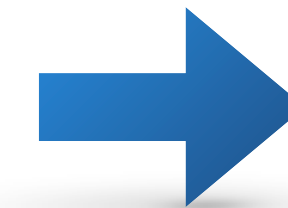
Coulex  
Experiment  
on  $^{94}\text{Zr}$

Model-  
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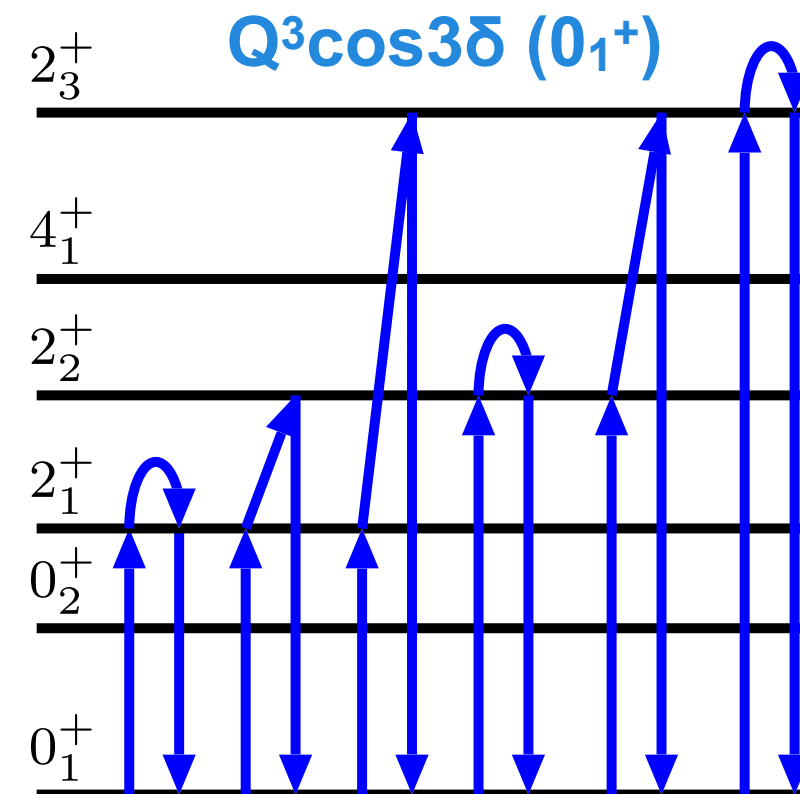
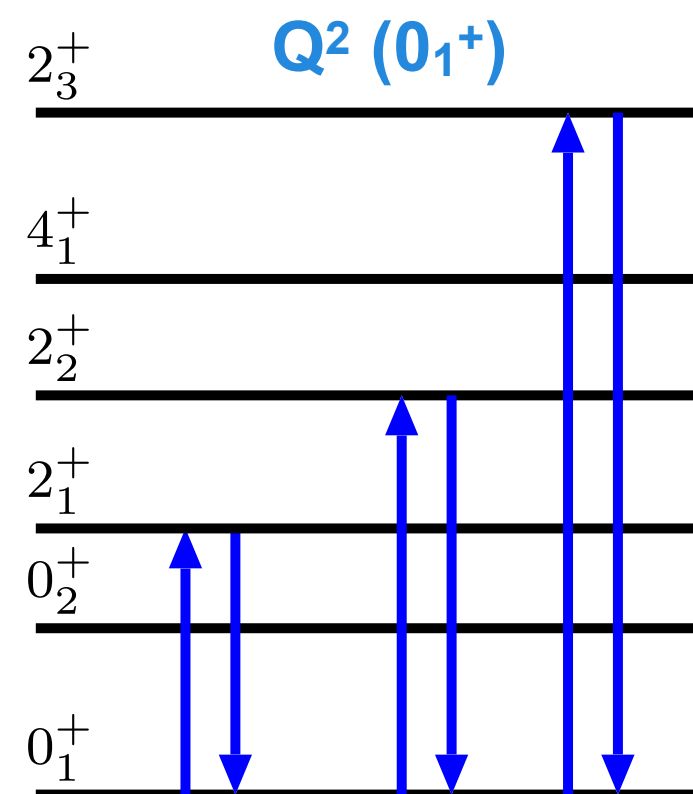
- Unique feature of Coulex  $\Rightarrow$  Possible to get **relative signs** of transitional matrix elements, together with **spectroscopic quadrupole moments** of short-living excited states with their sign
- Quadrupole Sum Rules  $\Rightarrow$   **$(\beta, \gamma)$  deformation parameters** for g.s. and excited states in a model-independent way

$$\langle i | Q^2 | i \rangle = \frac{\sqrt{5}}{\sqrt{2I_i + 1}} \sum_t \langle i || E2 || t \rangle \langle t || E2 || i \rangle \begin{Bmatrix} 2 & 2 & 0 \\ I_i & I_i & I_t \end{Bmatrix}$$

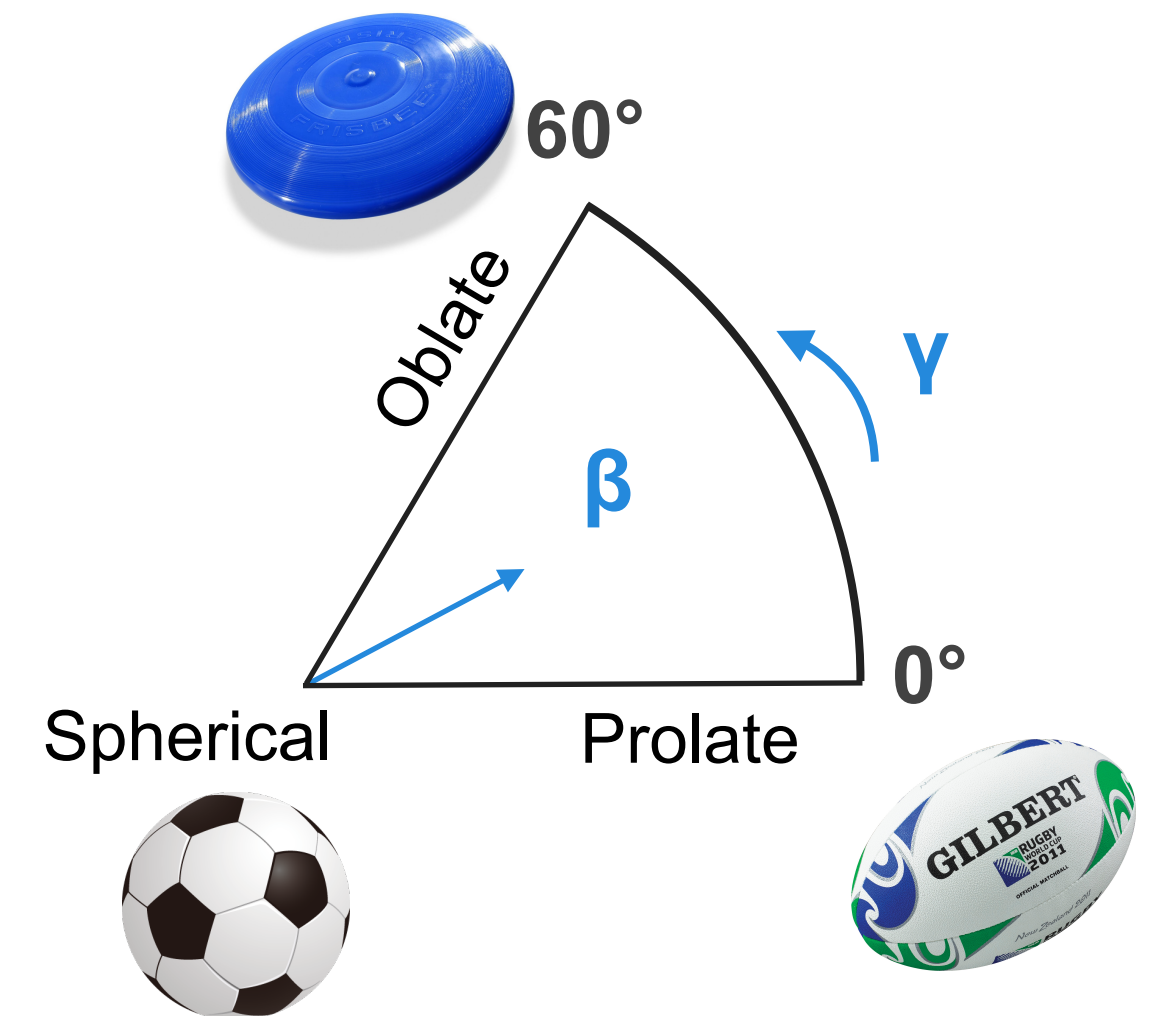
$$\langle i | Q^3 \cos(3\delta) | i \rangle = -\frac{\sqrt{35}}{\sqrt{2}} \frac{1}{2I_i + 1} \sum_{tu} \langle i || E2 || t \rangle \langle t || E2 || u \rangle \langle u || E2 || i \rangle \begin{Bmatrix} 2 & 2 & 2 \\ I_i & I_t & I_u \end{Bmatrix}$$



**$(Q^2, \delta)$   
equivalent to  
 $(\beta, \gamma)$**



*K. Kumar, Phys. Rev. Lett. 28 (1972) 249 & D. Cline, Annu. Rev. Nucl. Part. Sci. 36 (1986) 683*





Quadrupole

Hot topic not only in low-energy nuclear physics  $\Rightarrow$  New studies at high-energy physics facilities, highly complementary to Coulex

Invariants

- Unique features of quadrupole moments of nuclei
- Quadrupole moments

Low-Energy Coulomb Excitation

SPIDER with GALILEO and AGATA

Excited  $0^+$  States in Even-Even Mid-Mass Nuclei

The Zr Isotopic Chain and QPTs

Coulex Experiment on  $^{94}\text{Zr}$

Model-Independent Determination of Shapes in  $^{94}\text{Zr}$

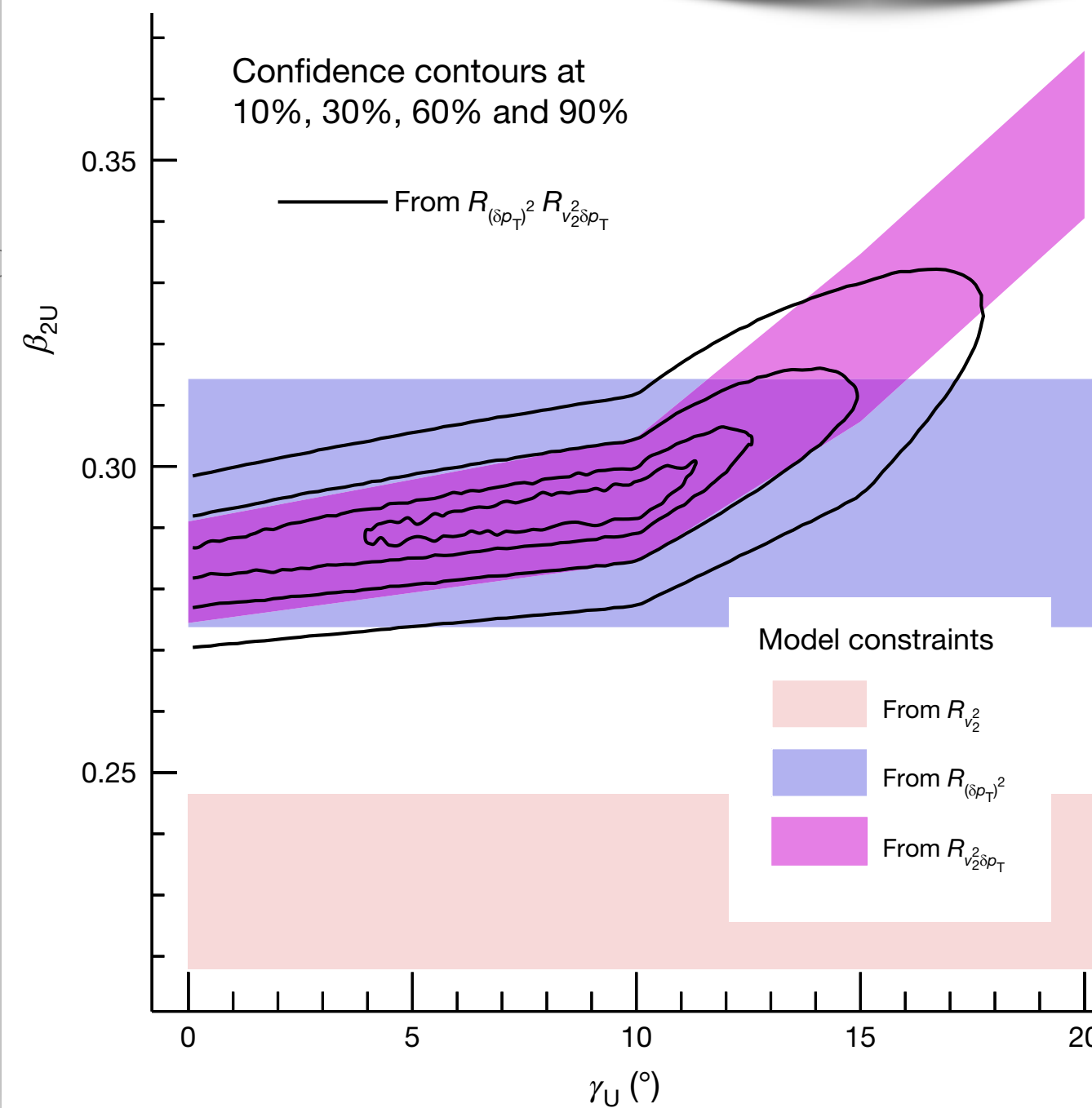
**nature**

Article | [Open access](#) | Published: 06 November 2024

## Imaging shapes of atomic nuclei in high-energy nuclear collisions

[STAR Collaboration](#)

**RHIC - STAR**



K. Kumar, *Phys. Rev. Lett.* **26** (1972) 249 & D. Gliné, *Annu. Rev. Nucl. Part. Sci.* **30** (1980) 665

PHYSICAL REVIEW LETTERS **133**, 192301 (2024)

## Exploring the Nuclear-Shape Phase Transition in Ultrarelativistic $^{129}\text{Xe} + ^{129}\text{Xe}$ Collisions at the LHC

Shujun Zhao,<sup>1,2,\*</sup> Hao-jie Xu<sup>2,3,†</sup> You Zhou<sup>4,\*</sup> Yu-Xin Liu,<sup>1,5,6,§</sup> and Huichao Song<sup>1,5,6,||</sup>

<sup>1</sup>School of Physics, Peking University, Beijing 100871, China

<sup>2</sup>School of Science, Huzhou University, Huzhou, Zhejiang 313000, China

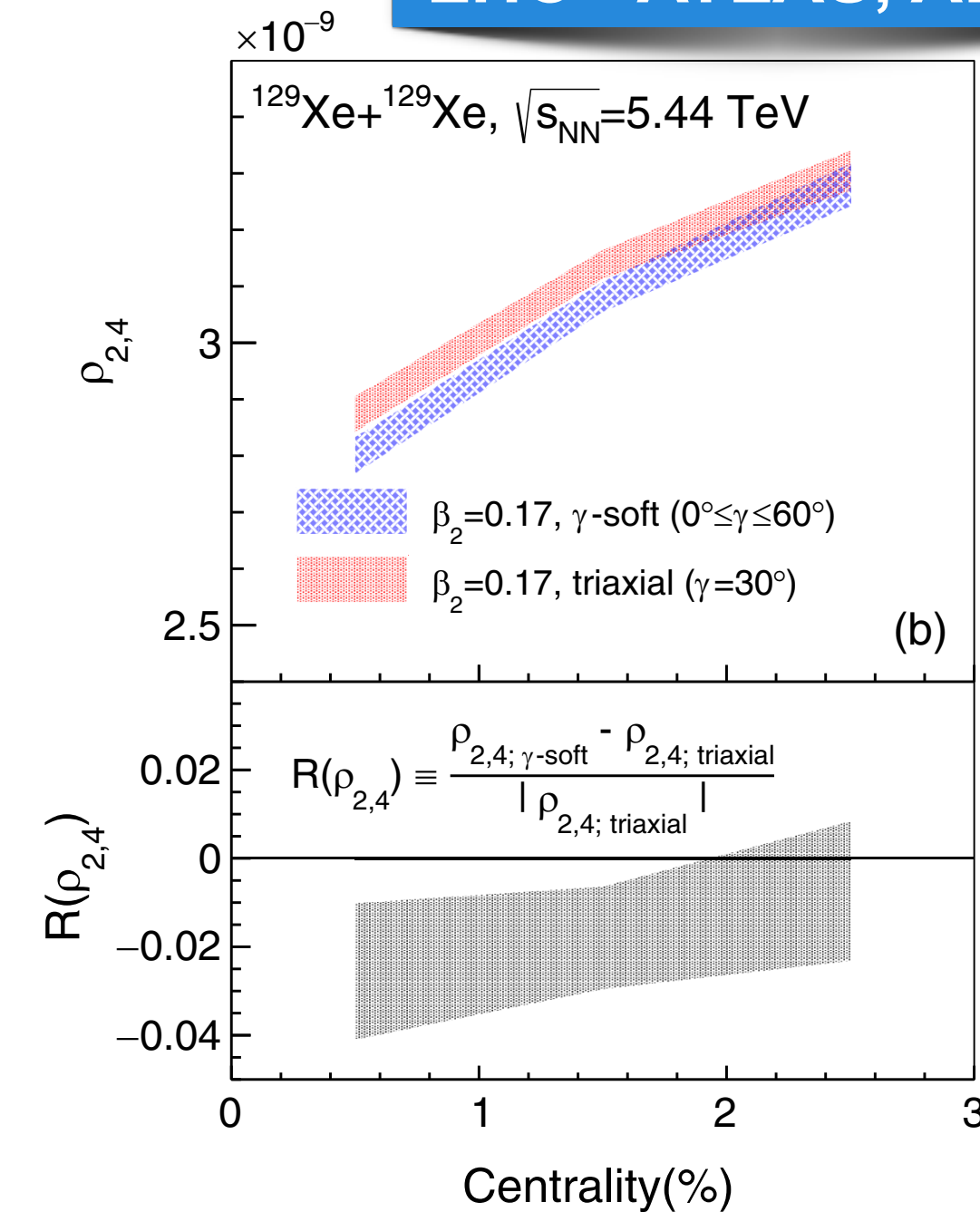
<sup>3</sup>Strong-Coupling Physics International Research Laboratory (SPiRL), Huzhou University, Huzhou, Zhejiang 313000, China

<sup>4</sup>Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17, 2100 Copenhagen, Denmark

<sup>5</sup>Collaborative Innovation Center of Quantum Matter, Beijing 100871, China

<sup>6</sup>Center for High Energy Physics, Peking University, Beijing 100871, China

**LHC - ATLAS, ALICE**



quadrupole

way

$\delta$ )  
lent to  
 $\gamma$ )

$\gamma$   
0°





# Experimental Requirements

Low-Energy  
Coulomb  
Excitation

SPIDER with  
GALILEO and  
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The Zr Isotopic  
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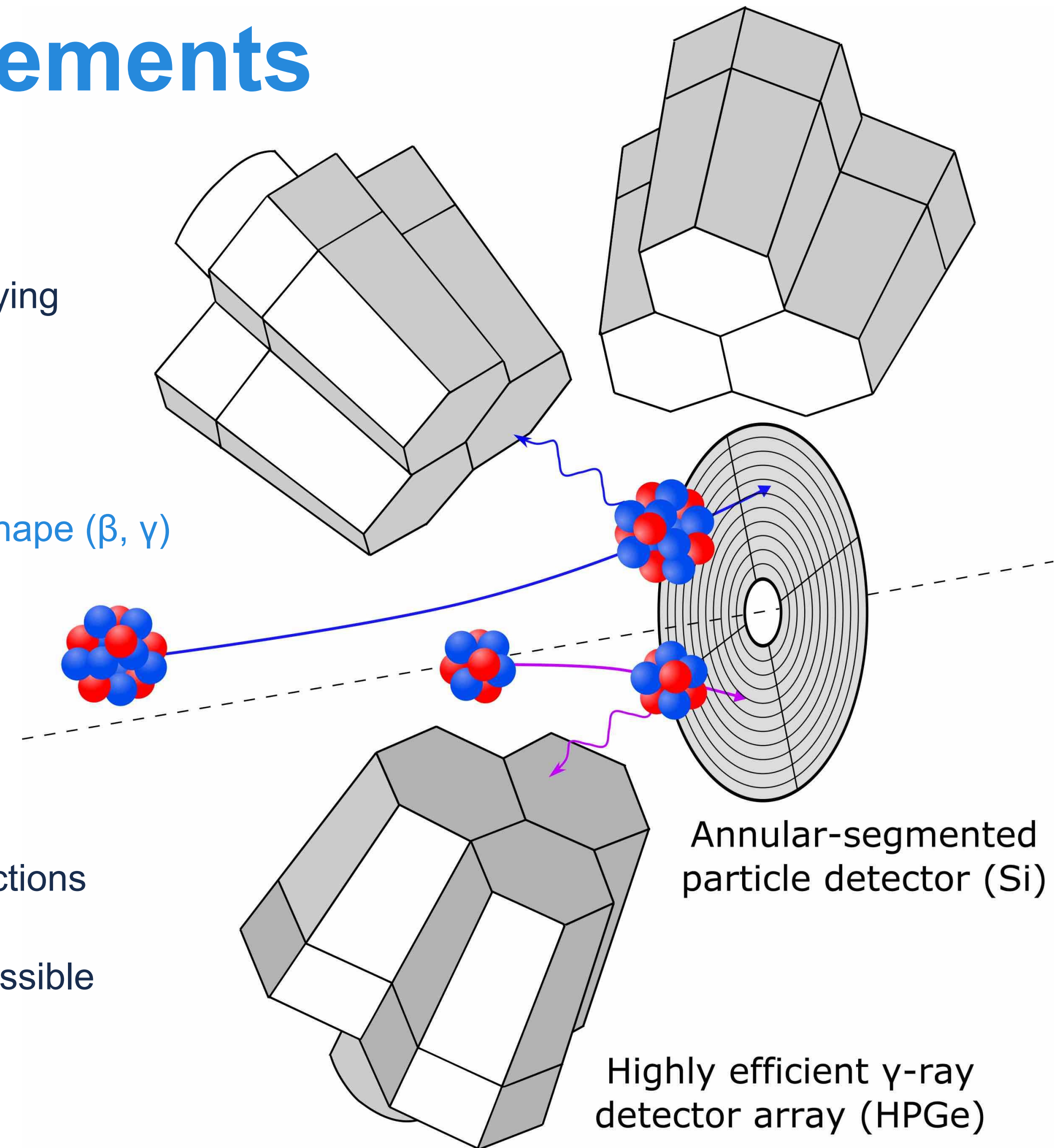
Model-  
Independent  
Determination of  
Shapes in  $^{94}\text{Zr}$

## What can we get?

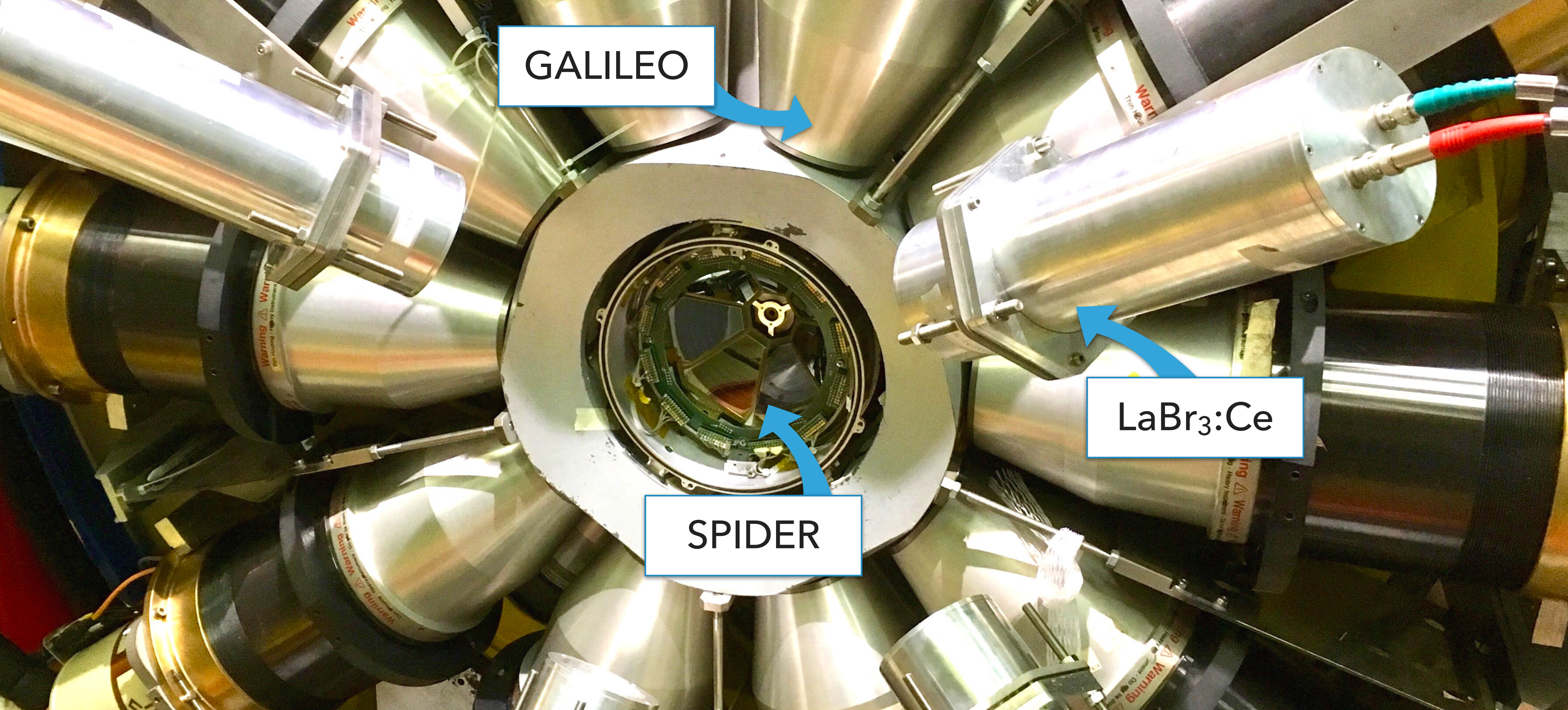
- ▶ Reduced transition probabilities for transitions between low-lying states, mainly  $E2$  and  $E3$  multipolarities
- ▶ Spectroscopic quadrupole moments for excited states
- ▶ Rotational invariants ( $Q, \delta$ )  $\Rightarrow$  Direct access to the nuclear shape ( $\beta, \gamma$ )

## What do we need?

- ▶ Beam energies around 3-6 MeV/A
- ▶ Good technique for weak-intensity beams  $\Rightarrow$  Large cross sections
- ▶ Gamma-ray array and heavy ion detector with as good as possible efficiency, energy or time resolutions, and segmentations





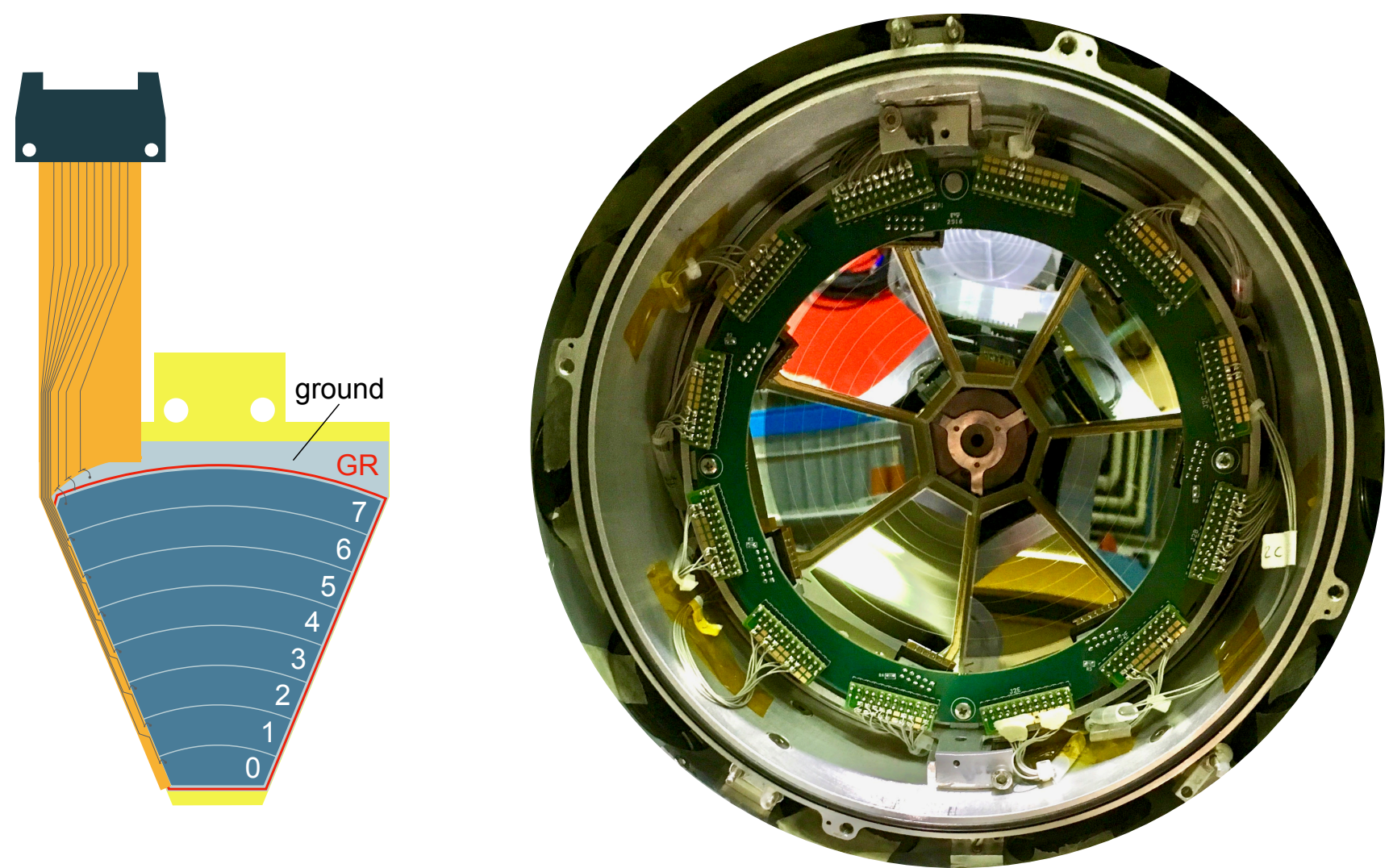


# GALILEO with SPIDER (2016 - 2019)



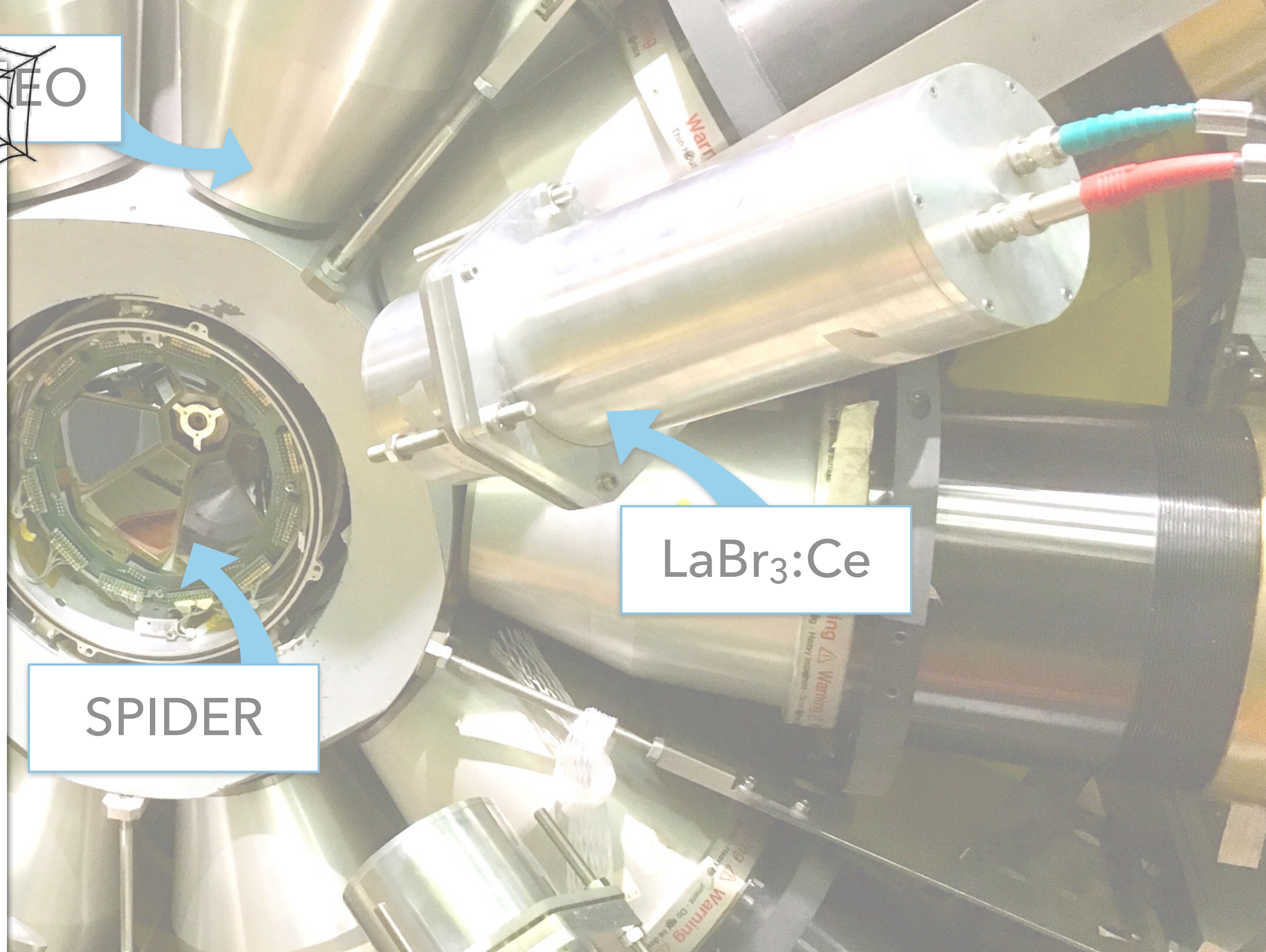
## SPIDER: the Silicon Pie Detector

- ▶ Modular segmented silicon detector, designed for low-energy Coulomb-excitation measurements
- ▶ Independent sectors, 8 strips + guard ring
- ▶ Detector thickness  $\sim 300\text{ }\mu\text{m}$ , dead layers  $\sim 50\text{ nm}$  in the junction (front) side and  $\sim 350\text{ nm}$  in the ohmic (rear) side
- ▶ Cone configuration (7 sectors) at backward angles: 8.5 cm from the target  $\Rightarrow \Delta\Theta = 37.4^\circ$ ,  $\Omega/4\pi = 17.3\%$



M. Rocchini, K. Hadyńska-Klęk, A. Nannini et al., Nuclear Inst. and Methods in Physics Research A 971 (2020) 164030

ANPC, 24-28/11/2025, iThemba LABS (South Africa)



# SPIDER (2016 - 2019)

Marco Rocchini

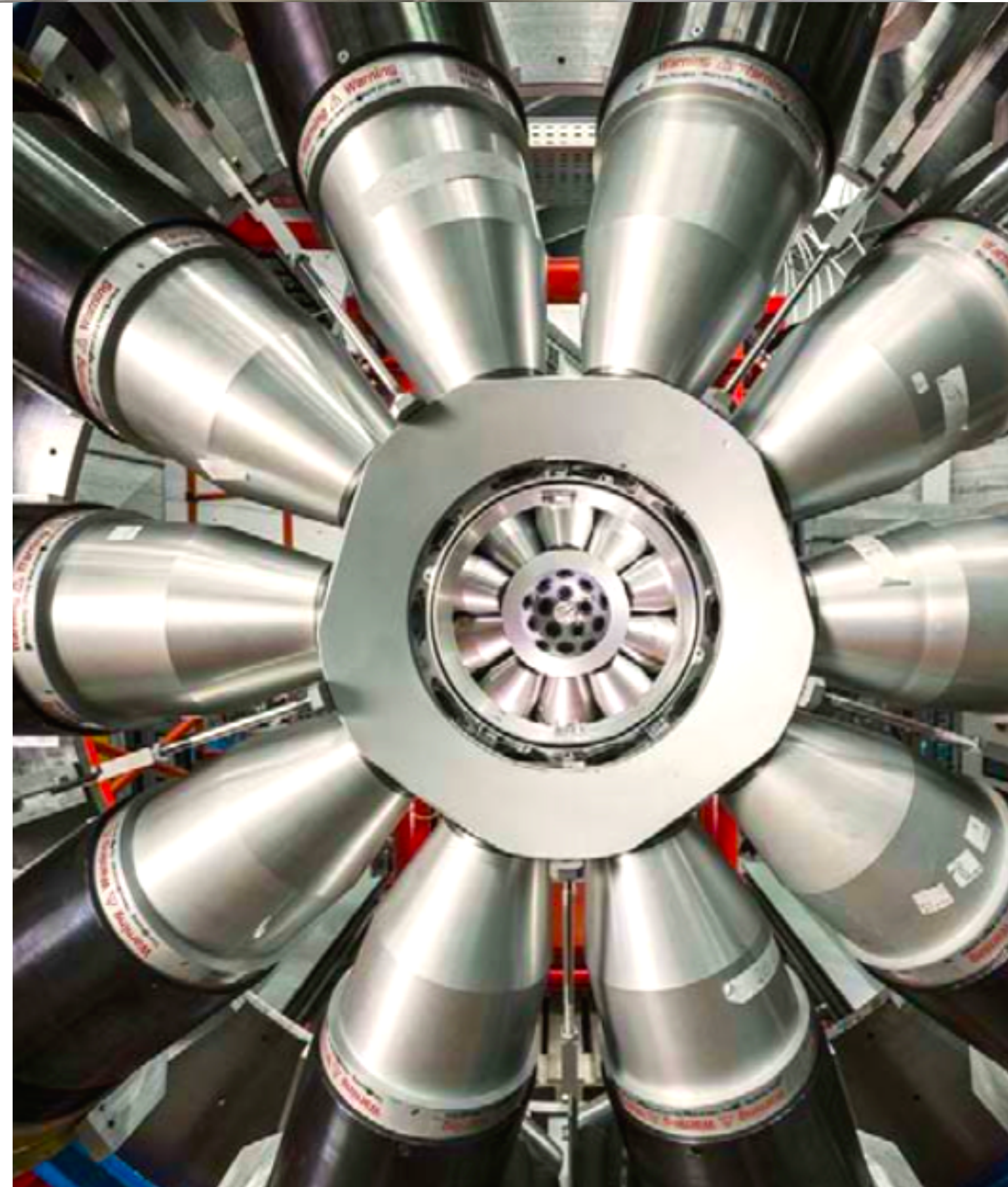


# GALILEO

## GALILEO: the LNL Resident $\gamma$ -Ray Spectrometer

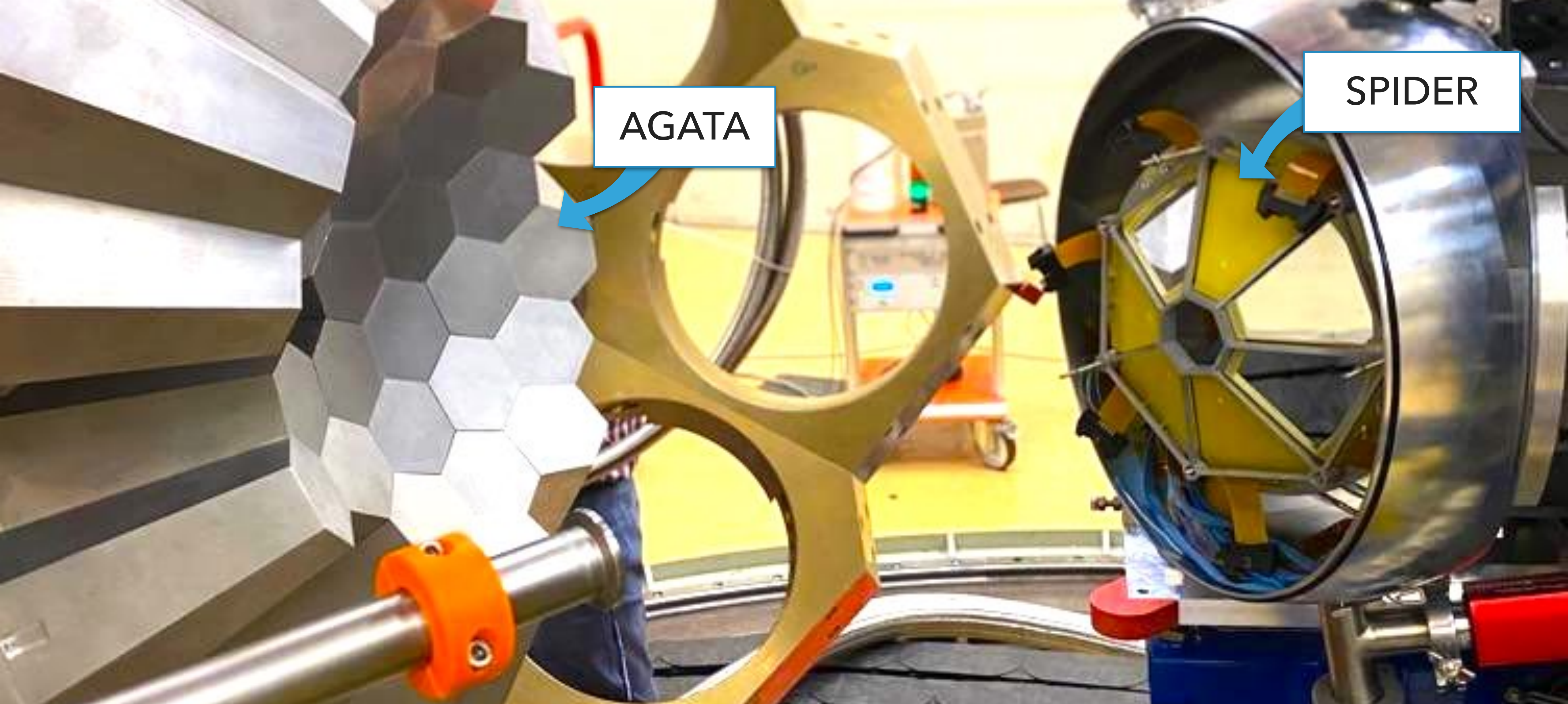
- ▶ 25 HPGe Compton-suppressed detectors (GASP type)
- ▶ FWHM (@1332.5 keV) < 2.4 keV
- ▶ Efficiency (@1332.5 keV) = 2.1%
- ▶ Full digital electronics (takes advantage of the developments made for AGATA)
- ▶ Triggerless DAQ

A. Goasduff, D. Mengoni, F. Recchia, J.J. Valiente-Dobón et al., *Nuclear Inst. and Methods in Physics Research A* 1015 (2021) 165753



# GALILEO





# AGATA with SPIDER (2022 - ongoing)

ANPC, 24-28/11/2025, iThemba LABS (South Africa)

Marco Rocchini



AGATA

SPIDER

## AGATA: the Advanced Gamma Tracking Array

- ▶ Last generation  $\gamma$ -ray spectrometer composed of highly-segmented HPGe detectors
- ▶ Employs advanced PSA and  $\gamma$ -ray tracking methods to avoid Compton-suppressors and guarantee high efficiency

# AGATA with SPID

ANPC, 24-28/11/2025, iThemba LABS (South Africa)



J.J. Valiente-Dobón, R. Menegazzo,  
A. Goasduff et al., Nuclear Inst. and  
Methods in Physics Research A 1049  
(2023) 168040





More than 20 experiments performed with GALILEO and AGATA coupled to SPIDER from A ~ 40 to A ~ 160, with SPs from 3 different continents (other 3 accepted)



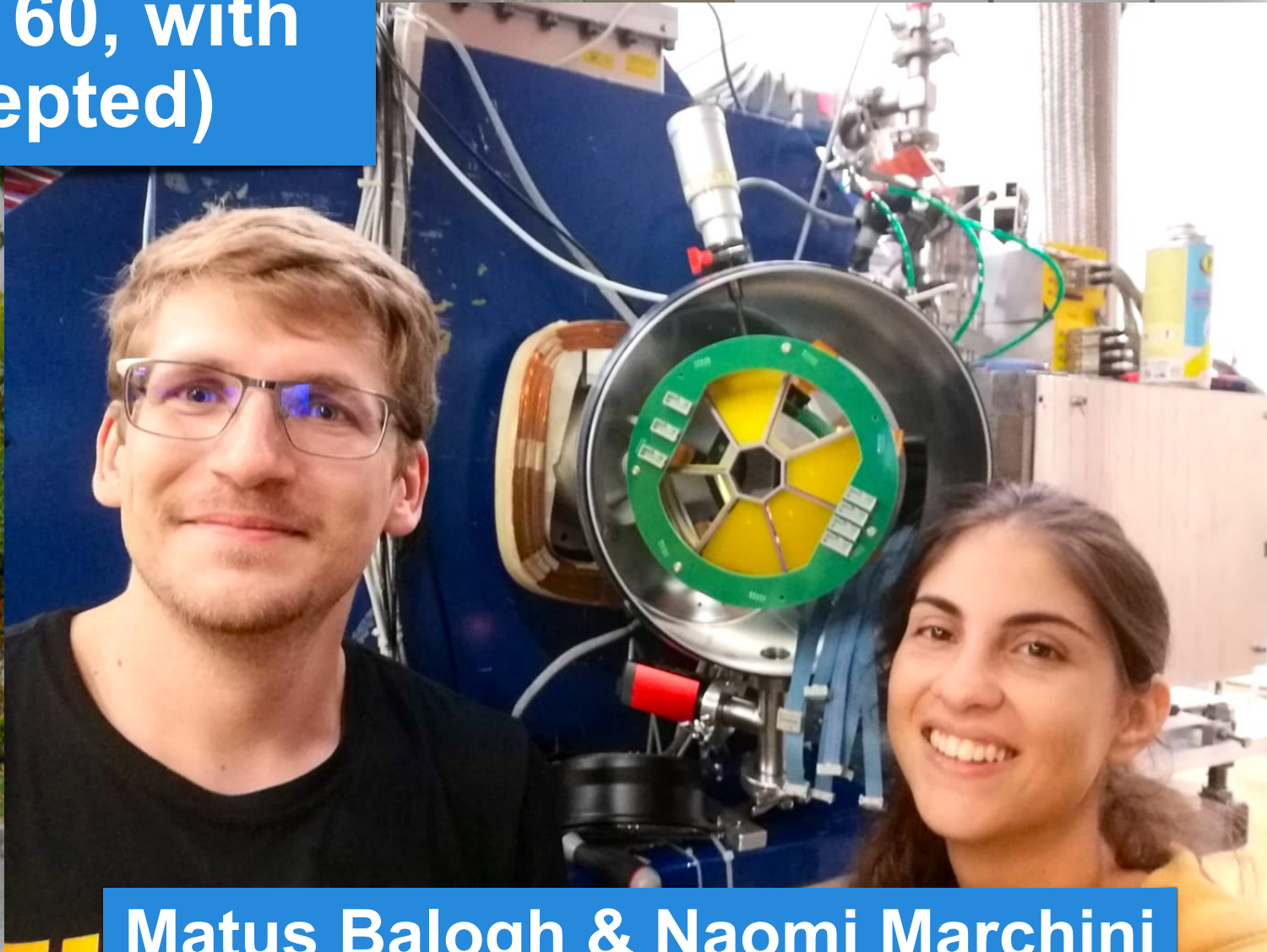
SPIDER

AGATA: the Advanced GAMMA Tracking Array

► Last generation γ-ray

composed of  
1000  
crystals

Advanced  
γ tracking  
solid  
state  
detectors  
high



Matus Balogh & Naomi Marchini  
In charge for SPIDER

Menegazzo,  
Nuclear Inst. and  
Research A 1049

(2023) 168040



# Excited $0^+$ States in Mid-Mass Nuclei

- Low-Energy Coulomb Excitation
- SPIDER with GALILEO and AGATA
- Excited  $0^+$  States in Even-Even Mid-Mass Nuclei
- The Zr Isotopic Chain and QPTs
- Coulex Experiment on  $^{94}\text{Zr}$
- Model-Independent Determination of Shapes in  $^{94}\text{Zr}$

HITES 2012  
Journal of Physics: Conference Series **403** (2012) 012011  
IOP Publishing  
doi:10.1088/1742-6596/403/1/012011

## Do we understand excited $0^+$ states in nuclei?

**John L Wood<sup>1</sup>**  
School of Physics, Georgia Institute of Technology, Atlanta, Georgia, 30332-0430, USA



## Excited $0^+$ States in Mid-Mass Nuclei

Low-Energy  
Coulomb  
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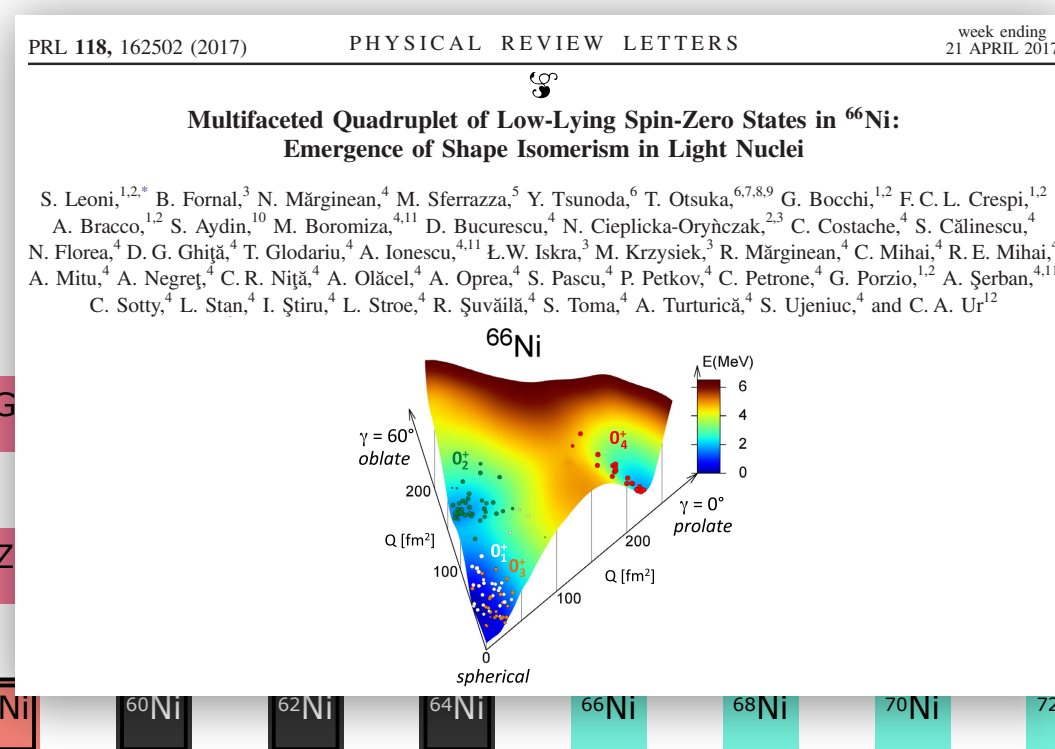
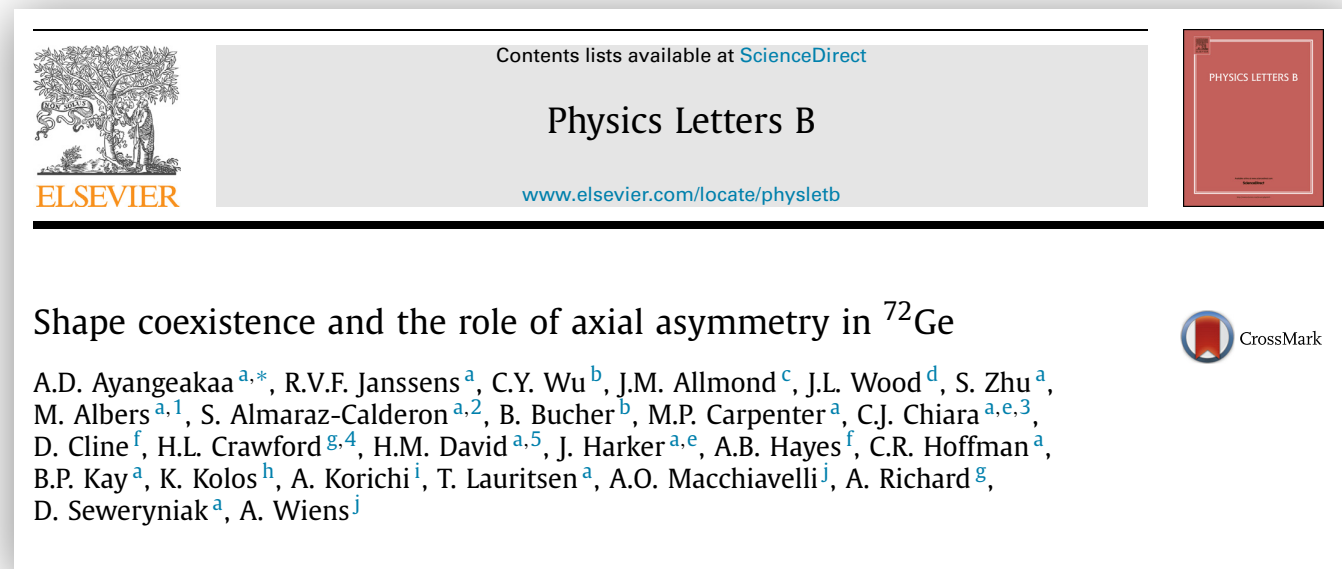
Model-  
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HITES 2012 IOP Publishing  
Journal of Physics: Conference Series **403** (2012) 012011 doi:10.1088/1742-6596/403/1/012011

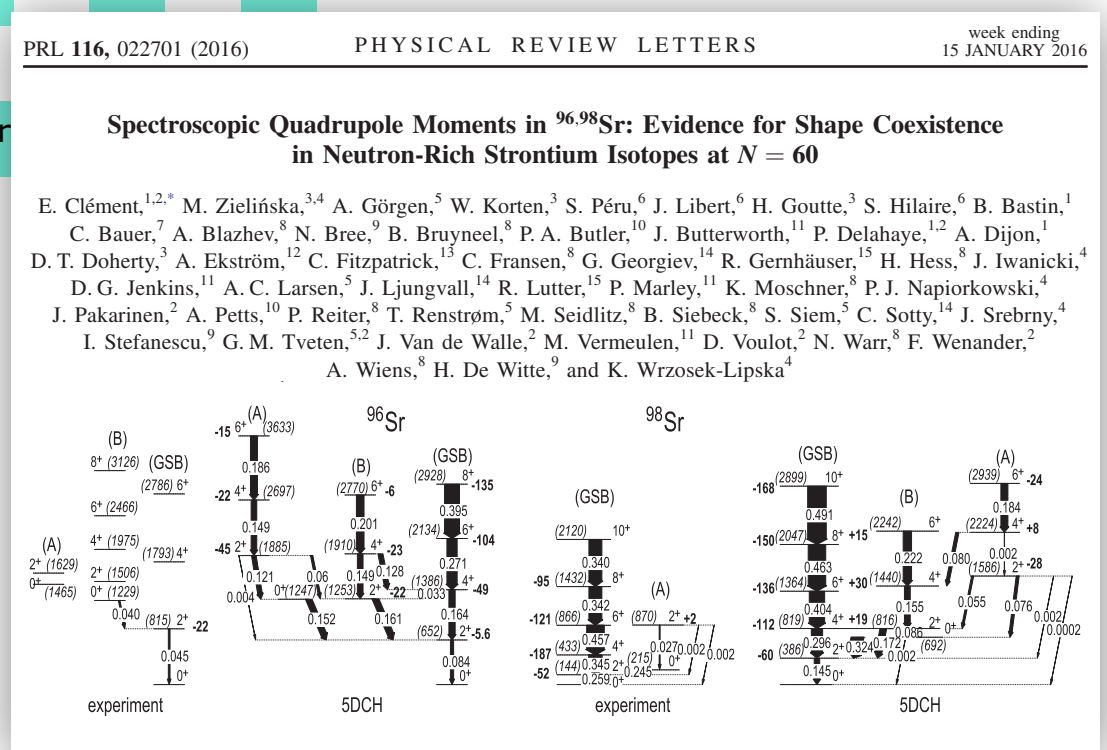
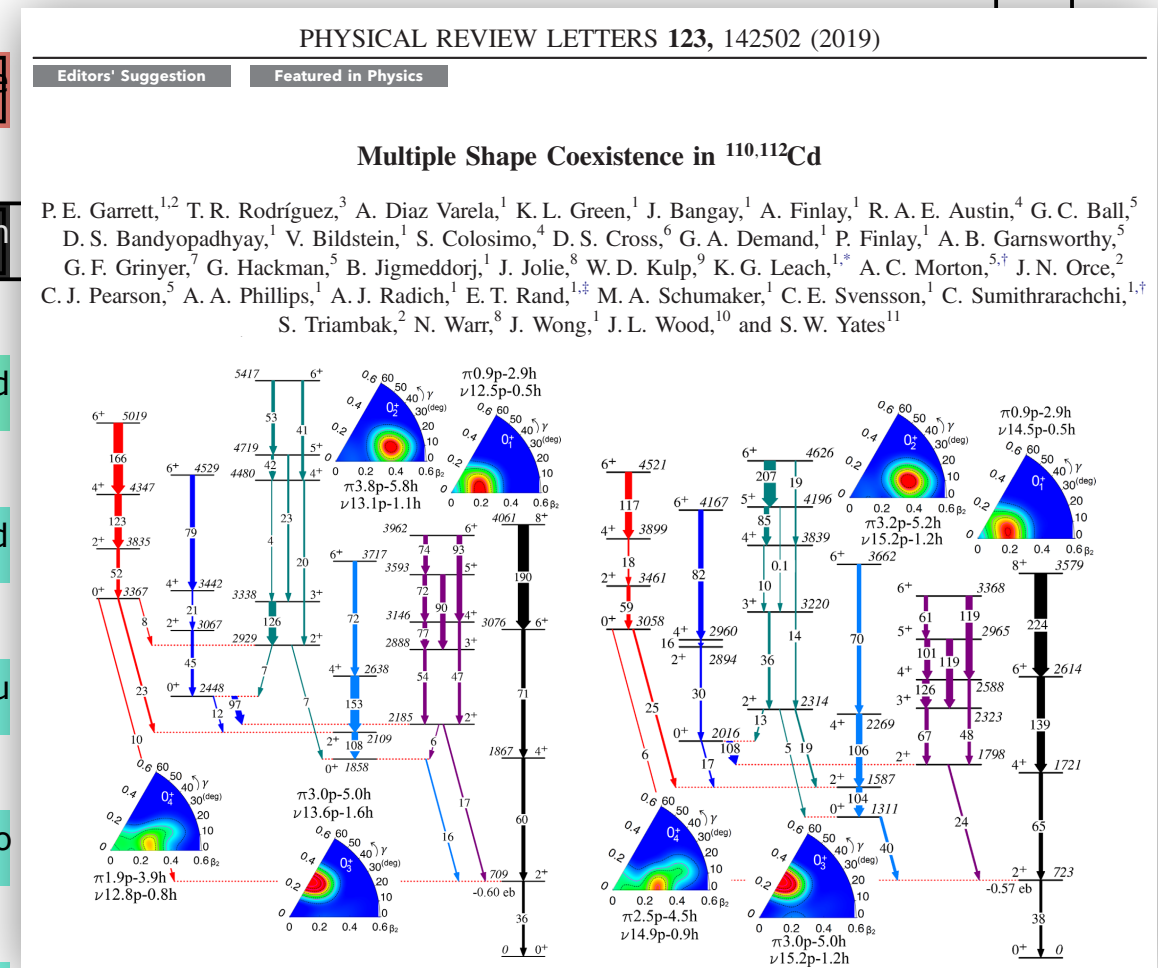
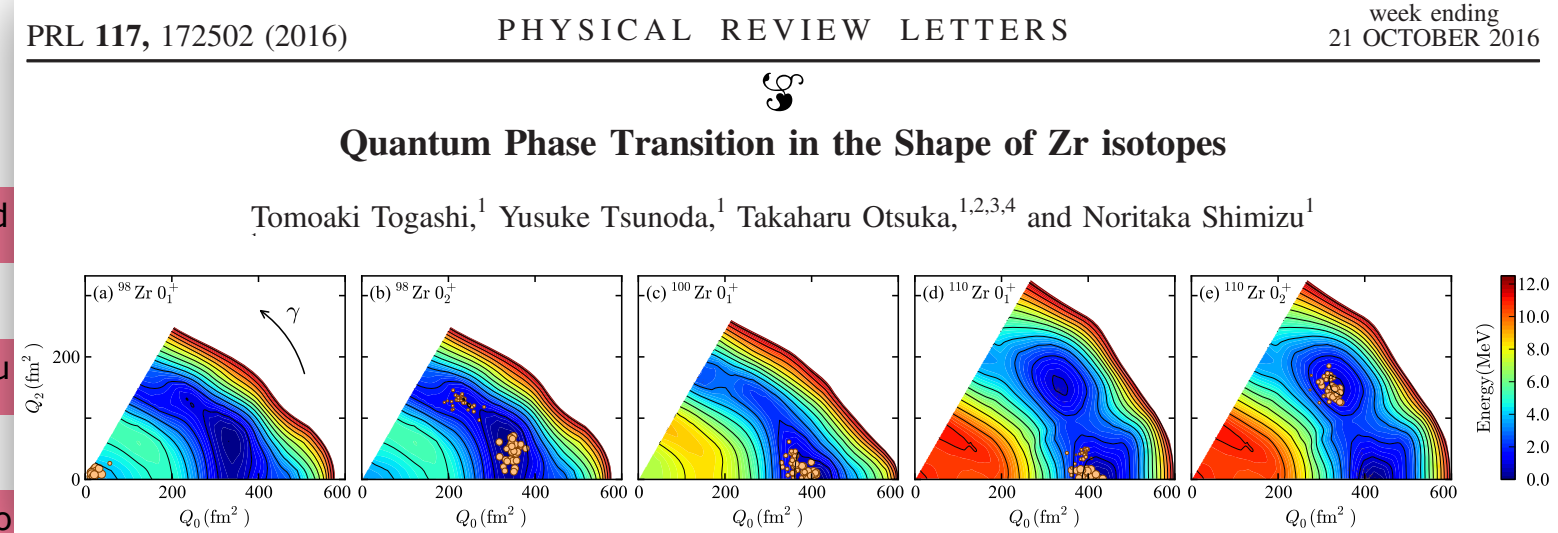
### Do we understand excited $0^+$ states in nuclei?

John L Wood<sup>1</sup>

School of Physics, Georgia Institute of Technology, Atlanta, Georgia, 30332-0430, USA



Shape Coexistence  
(and maybe multiple SC?) seems  
to be going from a rare to an  
ordinary phenomenon





# Zirconium Isotopes

Low-Energy  
Coulomb  
Excitation

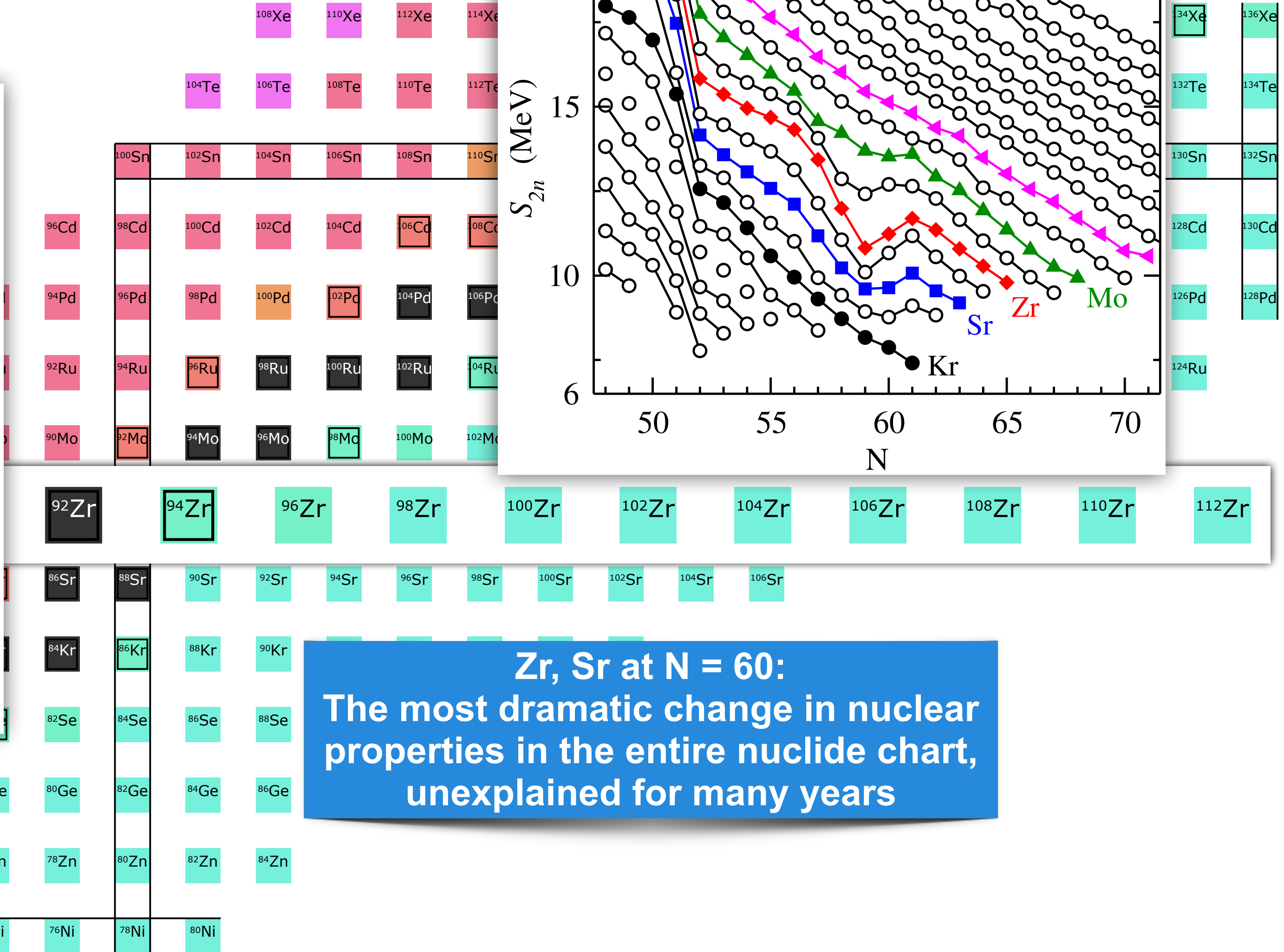
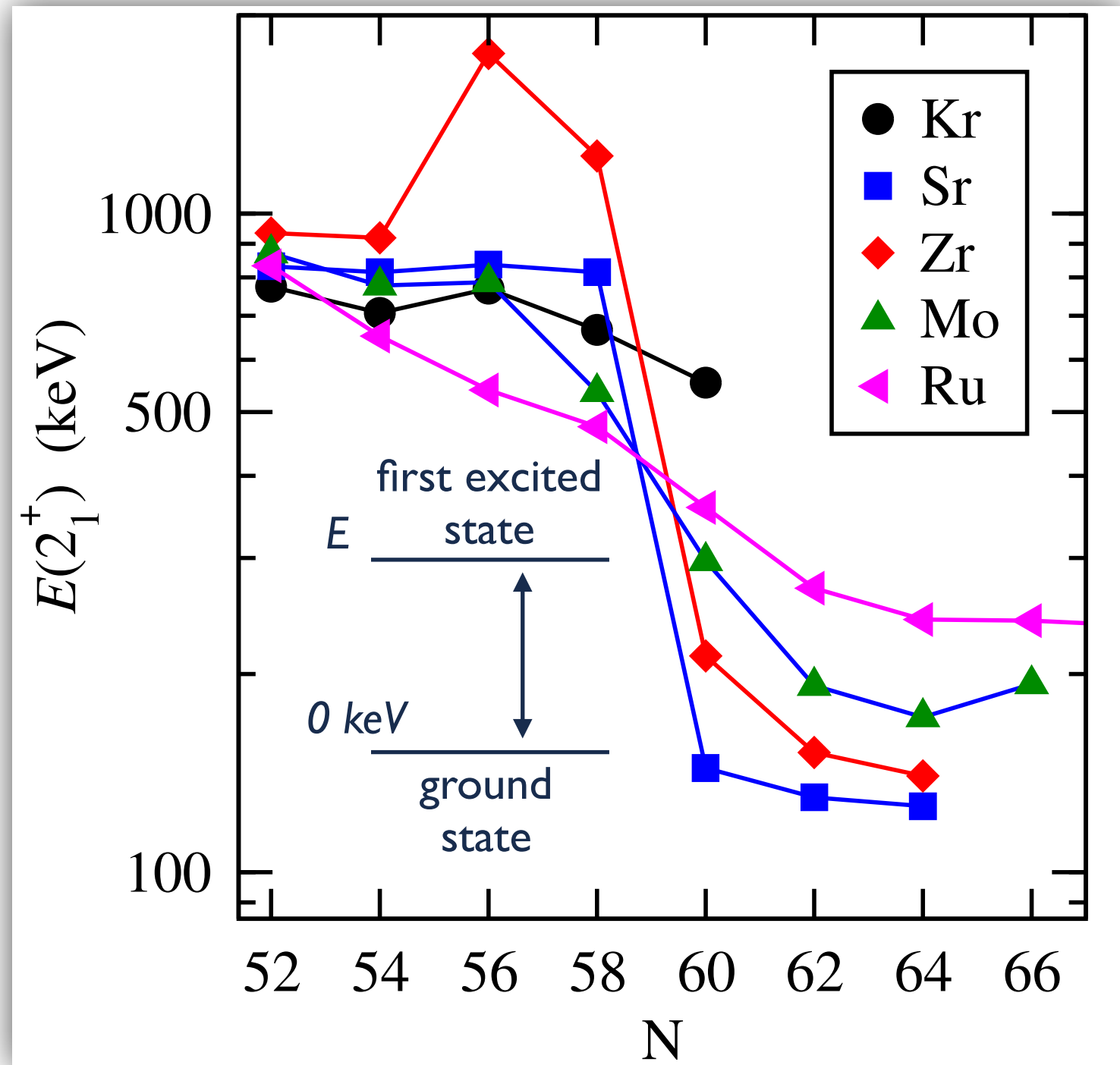
SPIDER with  
GALILEO and  
AGATA

Excited  $0^+$  States  
in Even-Even Mid-  
Mass Nuclei

The Zr Isotopic  
Chain and QPTs

Coulex  
Experiment  
on  $^{94}\text{Zr}$

Model-  
Independent  
Determination of  
Shapes in  $^{94}\text{Zr}$



# Quantum Phase Transition

Low-Energy  
Coulomb  
Excitation

SPIDER with  
GALILEO and  
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Mass Nuclei

The Zr Isotopic  
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Model-  
Independent  
Determination of  
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- ▶ **Monte Carlo Shell Model (MCSM)**  $\Rightarrow$  Excellent reproduction of experimental data, also around  $N = 60$

PRL 117, 172502 (2016)

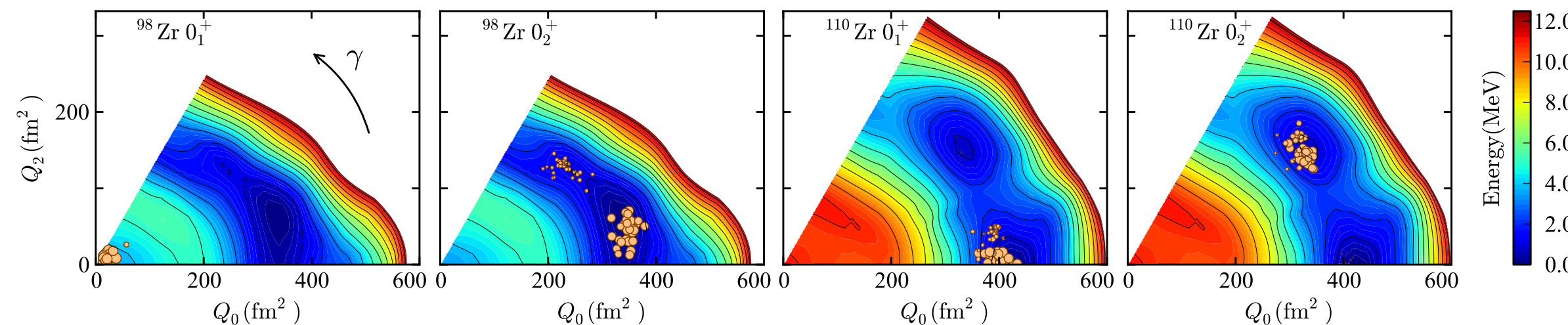
PHYSICAL REVIEW LETTERS

week ending  
21 OCTOBER 2016

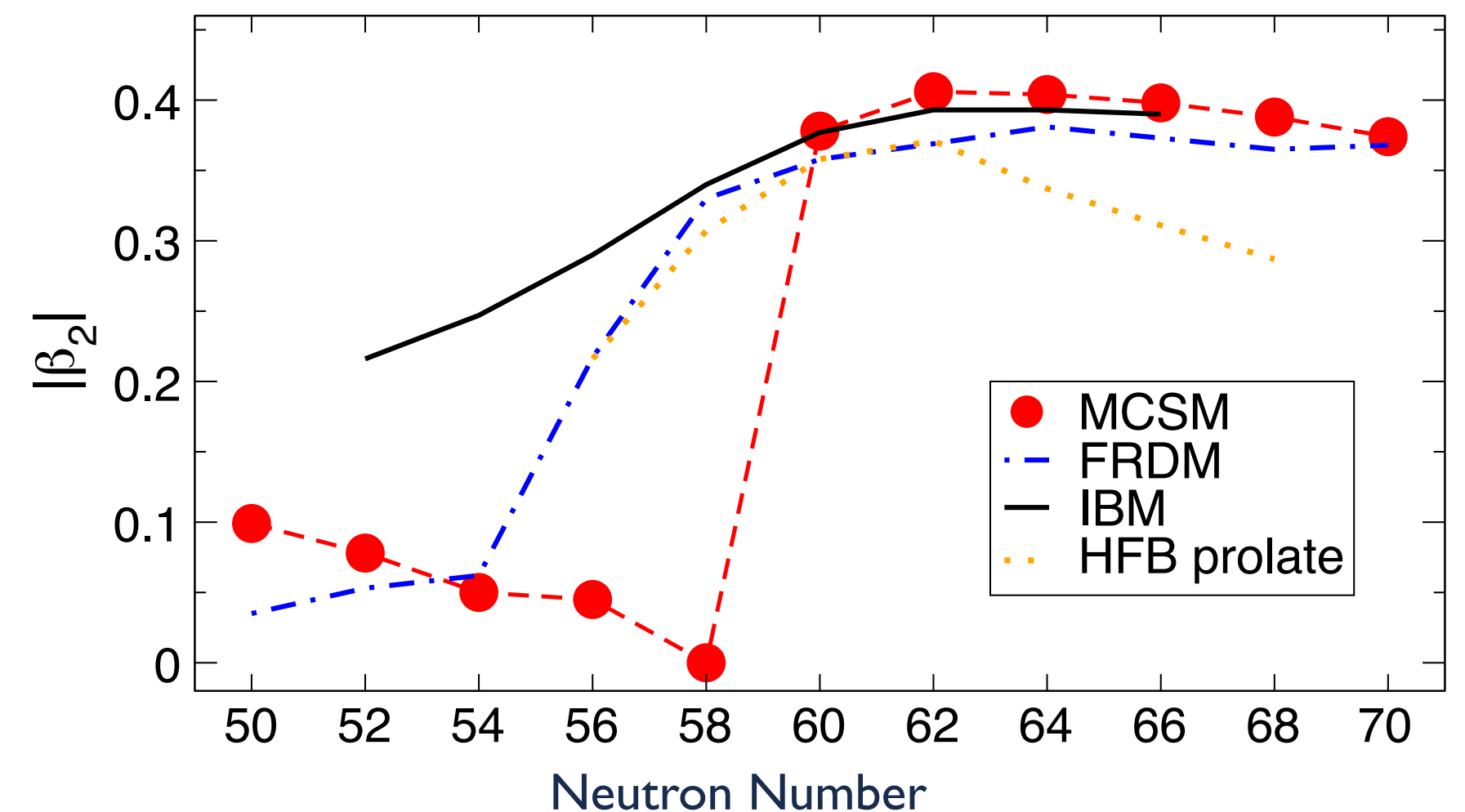
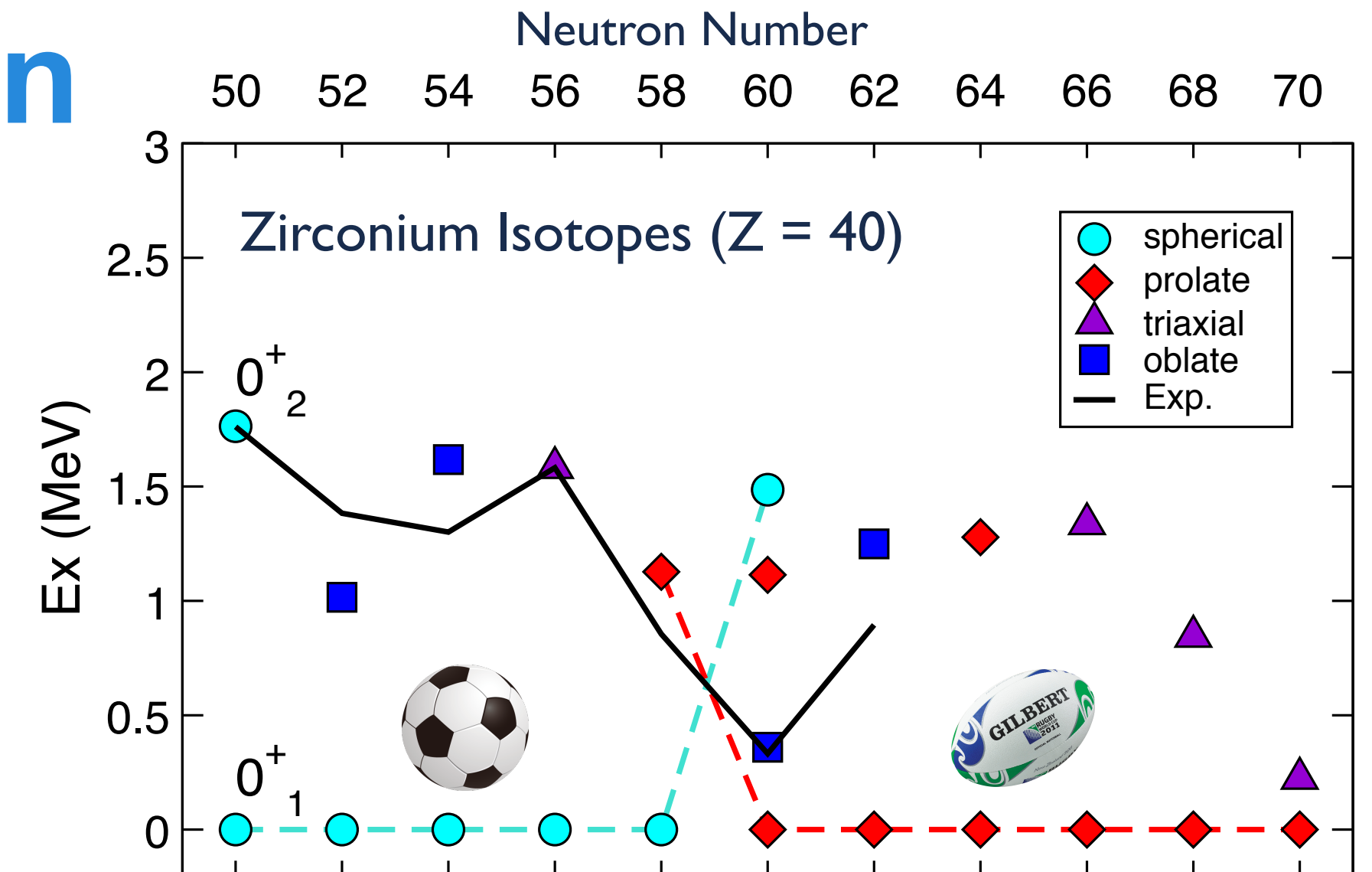
## Quantum Phase Transition in the Shape of Zr isotopes

Tomoaki Togashi,<sup>1</sup> Yusuke Tsunoda,<sup>1</sup> Takaharu Otsuka,<sup>1,2,3,4</sup> and Noritaka Shimizu<sup>1</sup>

- ▶ Use of **T-Plots** for a direct calculation of the nuclear shape:



- ▶ **Quantum Phase Transition (QPT):**
  - ▶ **Control Parameter**  $\Rightarrow$  Neutron number
  - ▶ **“Macroscopic” Quantity**  $\Rightarrow$  Shape





# Intertwined Quantum Phase Transitions

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Coulomb  
Excitation

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Coulex  
Experiment  
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Model-  
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- Interacting Boson Model with Configuration Mixing (IBM-CM)  $\Rightarrow$  Excellent reproduction of experimental data, also around  $N = 60$

PHYSICAL REVIEW C **105**, 014305 (2022)

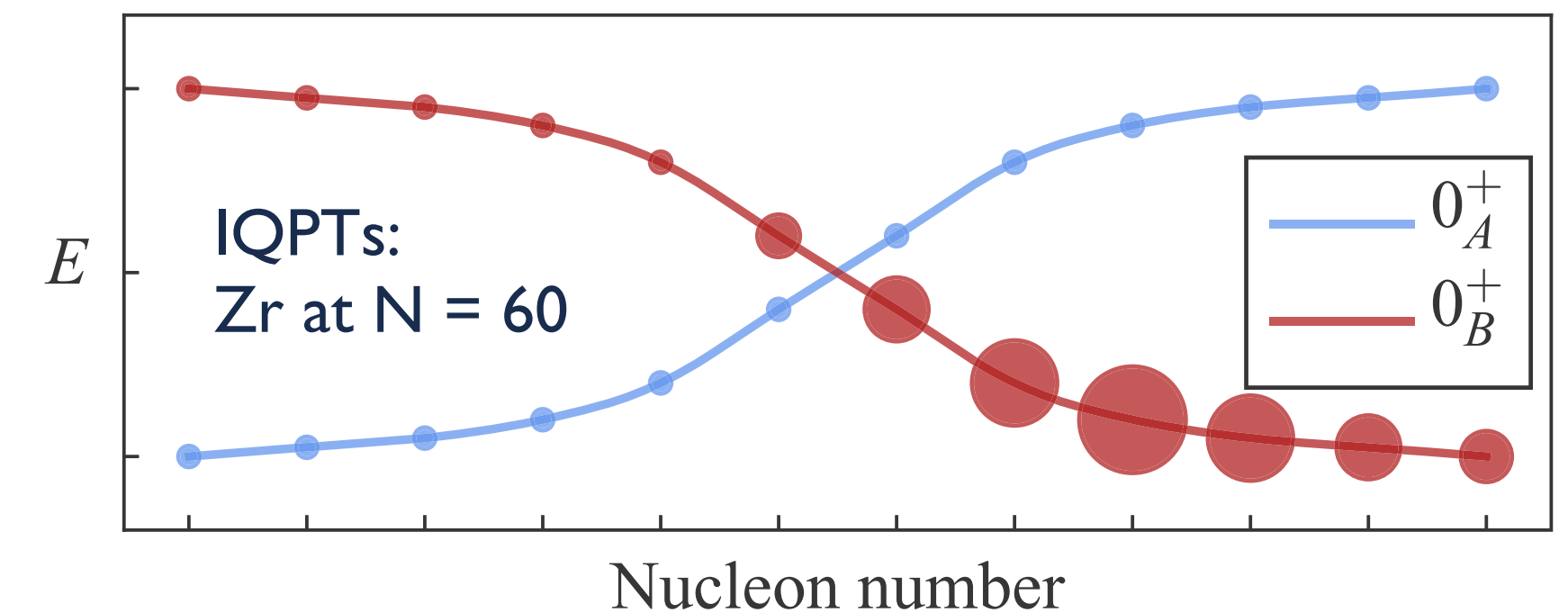
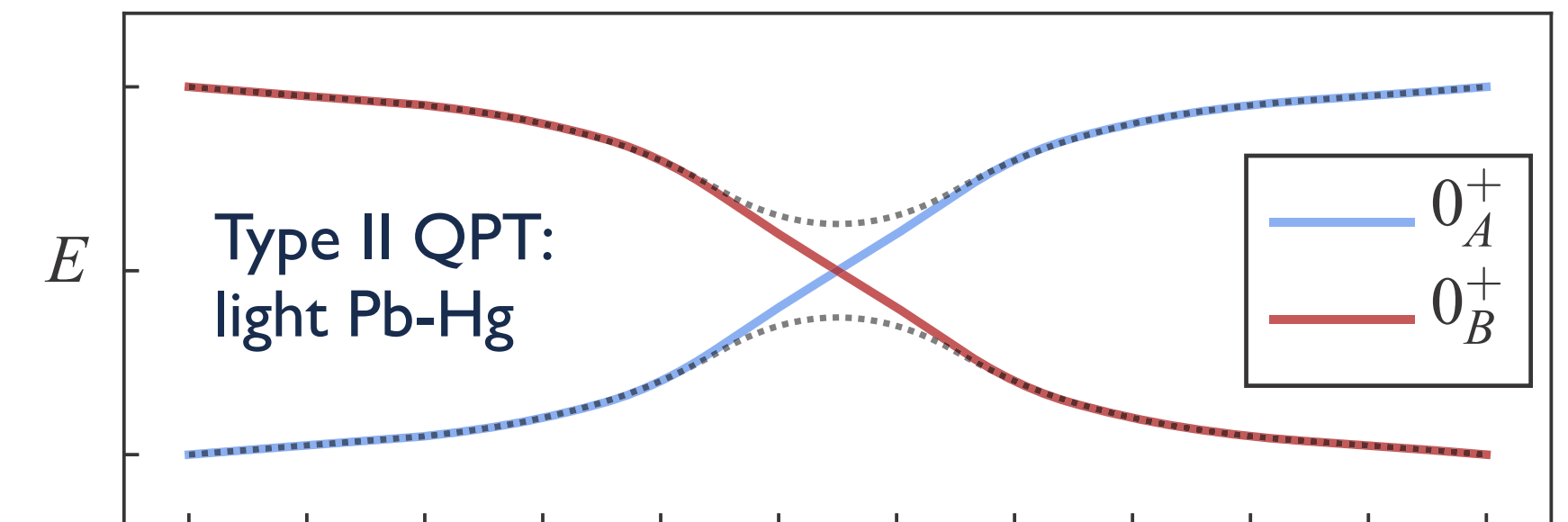
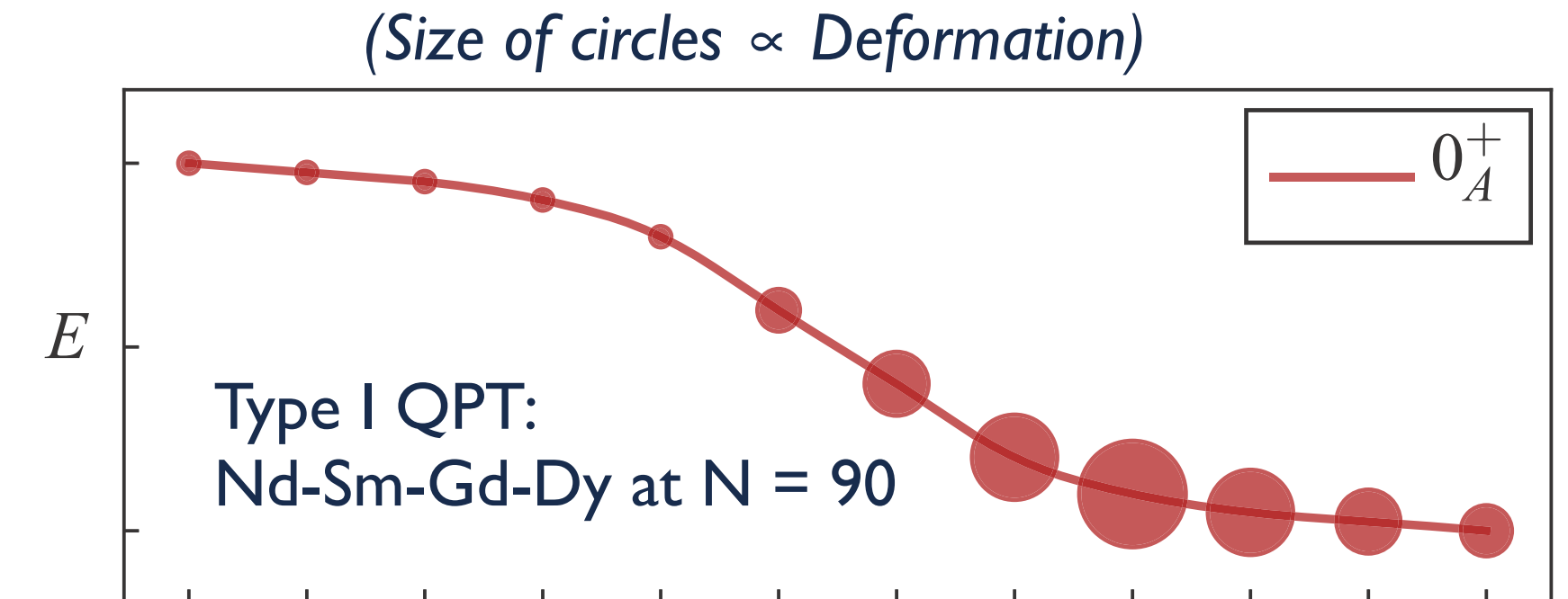
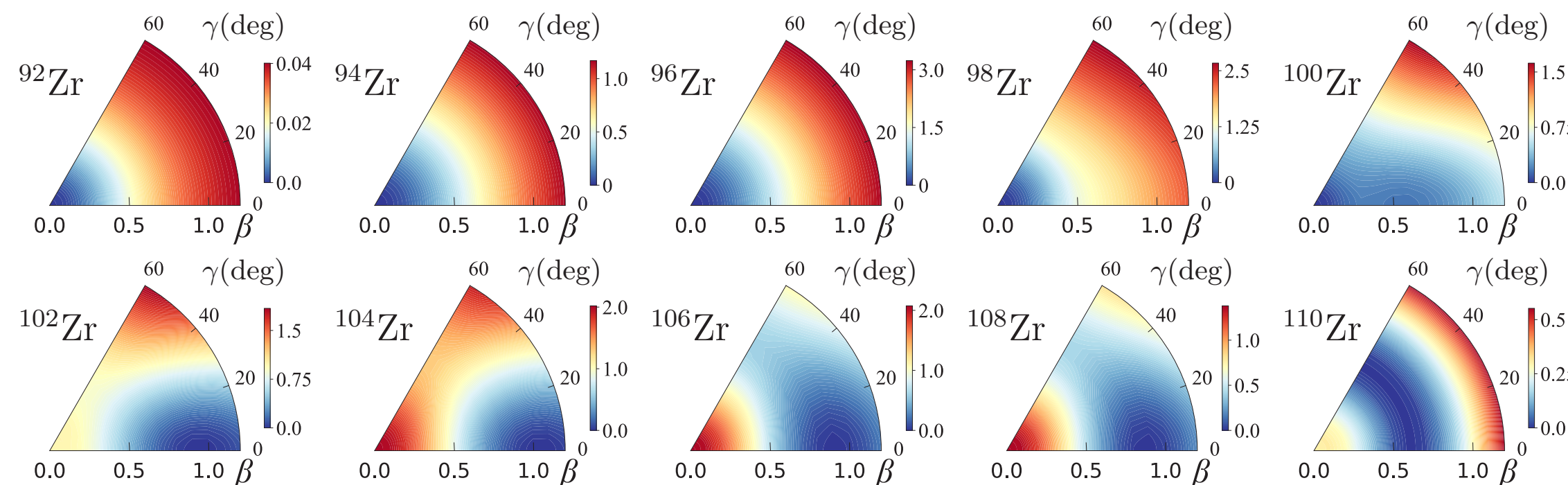
Zr isotopes as a region of intertwined quantum phase transitions

N. Gavrielov<sup>1,2,\*</sup>, A. Leviatan<sup>1,†</sup> and F. Iachello<sup>2,‡</sup>

<sup>1</sup>Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel

<sup>2</sup>Center for Theoretical Physics, Sloane Physics Laboratory, Yale University, New Haven, Connecticut 06520-8120, USA

- Two Intertwined Quantum Phase Transitions (IQPTs) exist in the Zr isotopic chain, appearing in the first two  $0^+$  states:
  - As in MCSM, the mixing between the coexisting  $0^+$  states is weak
  - At variance with MCSM, the ground state for  $N < 60$  ( $0_2^+$  state for  $N > 60$ ) is always U(5)-like (i.e., weakly deformed)





# Experimental Knowledge on $0^+$ States in Zr

Low-Energy  
Coulomb  
Excitation

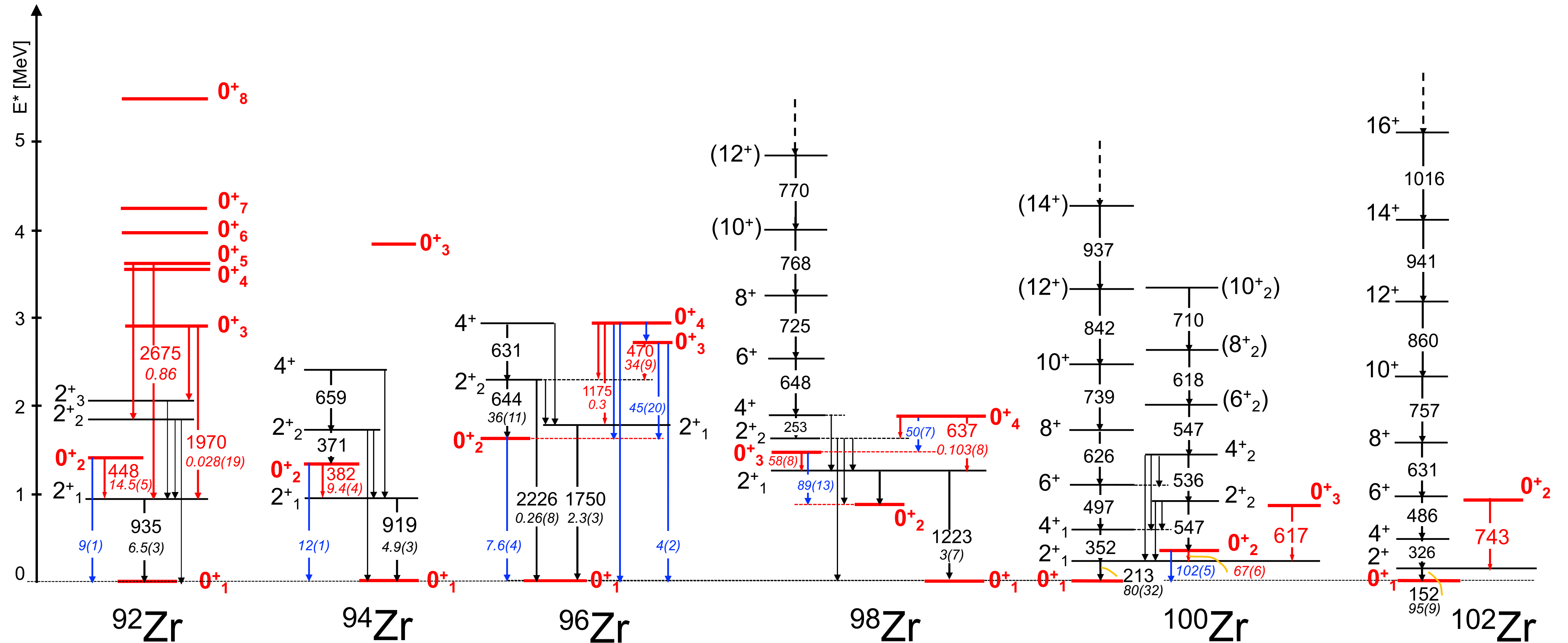
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S. Leoni et al. Progress in Particle and Nuclear Physics 139 (2024) 104119

# Experimental Knowledge on $0^+$ States in Zr

Low-Energy  
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Excitation

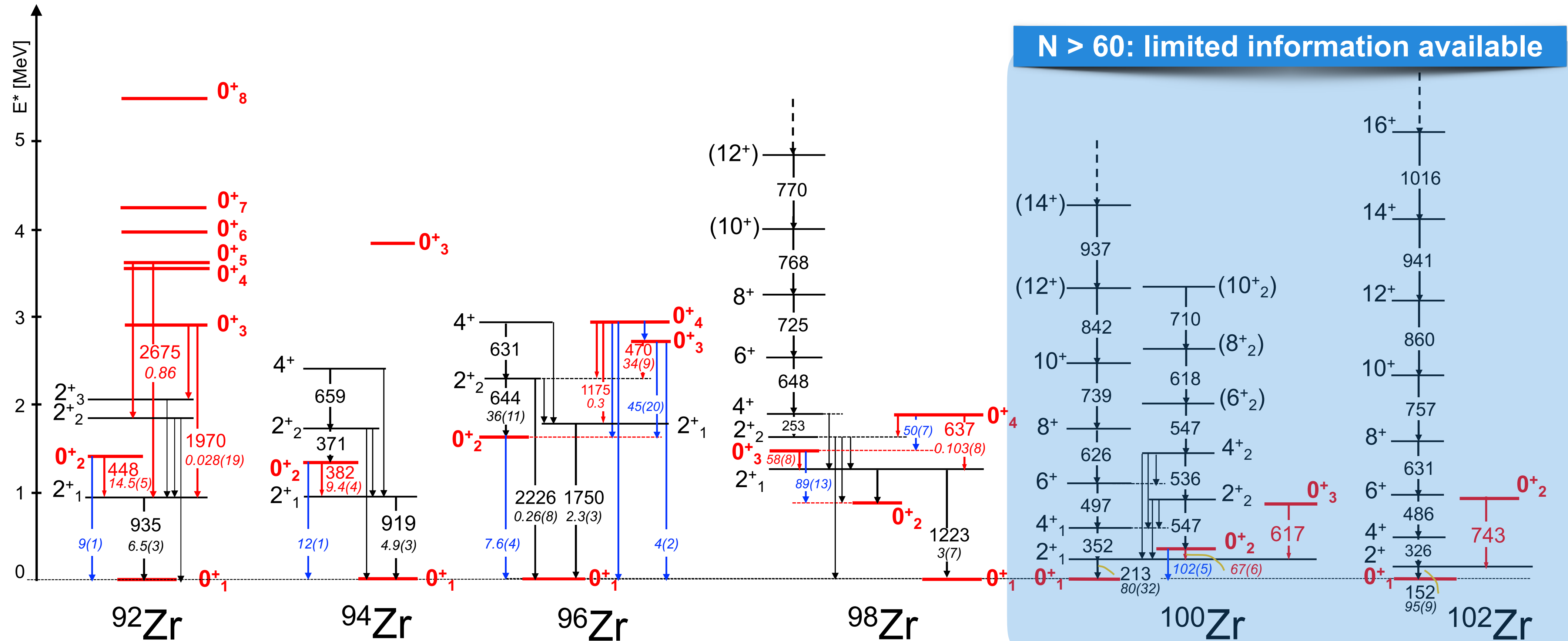
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S. Leoni et al. Progress in Particle and Nuclear Physics 139 (2024) 104119



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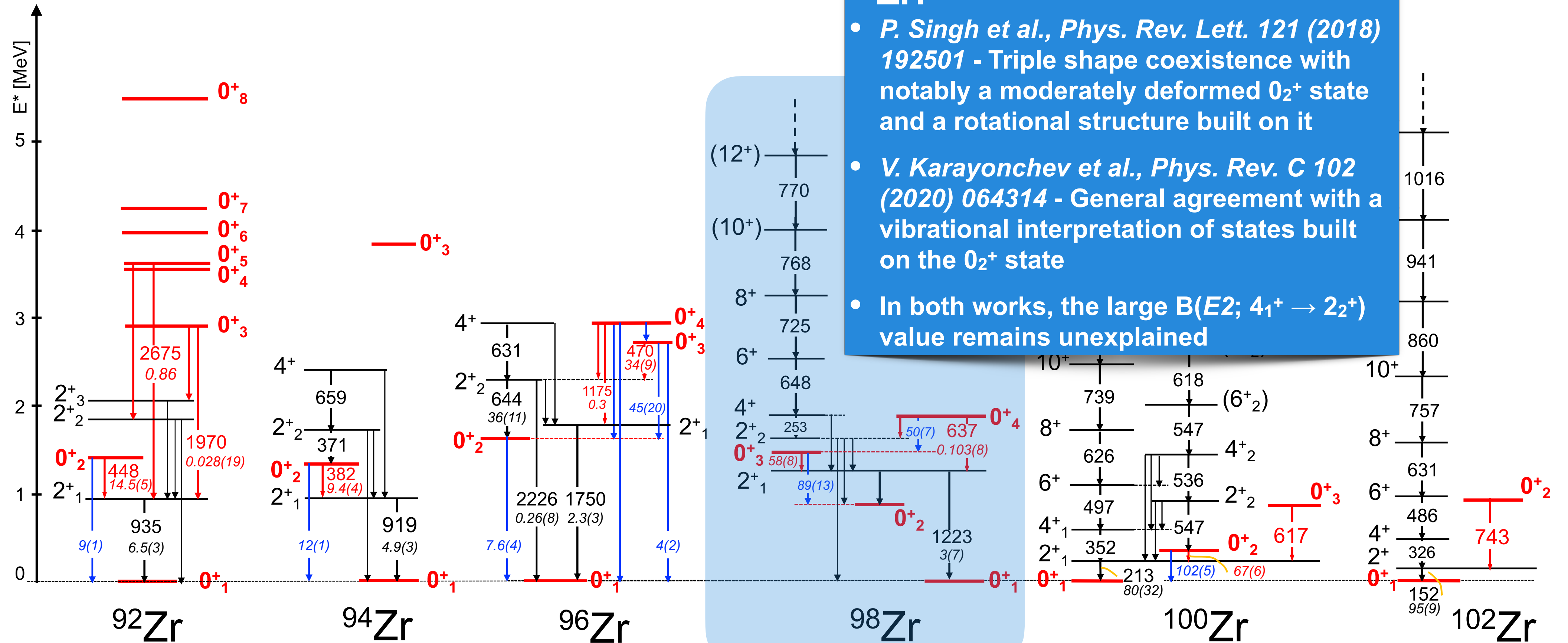
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S. Leoni et al. *Progress in Particle and Nuclear Physics* 139 (2024) 104119



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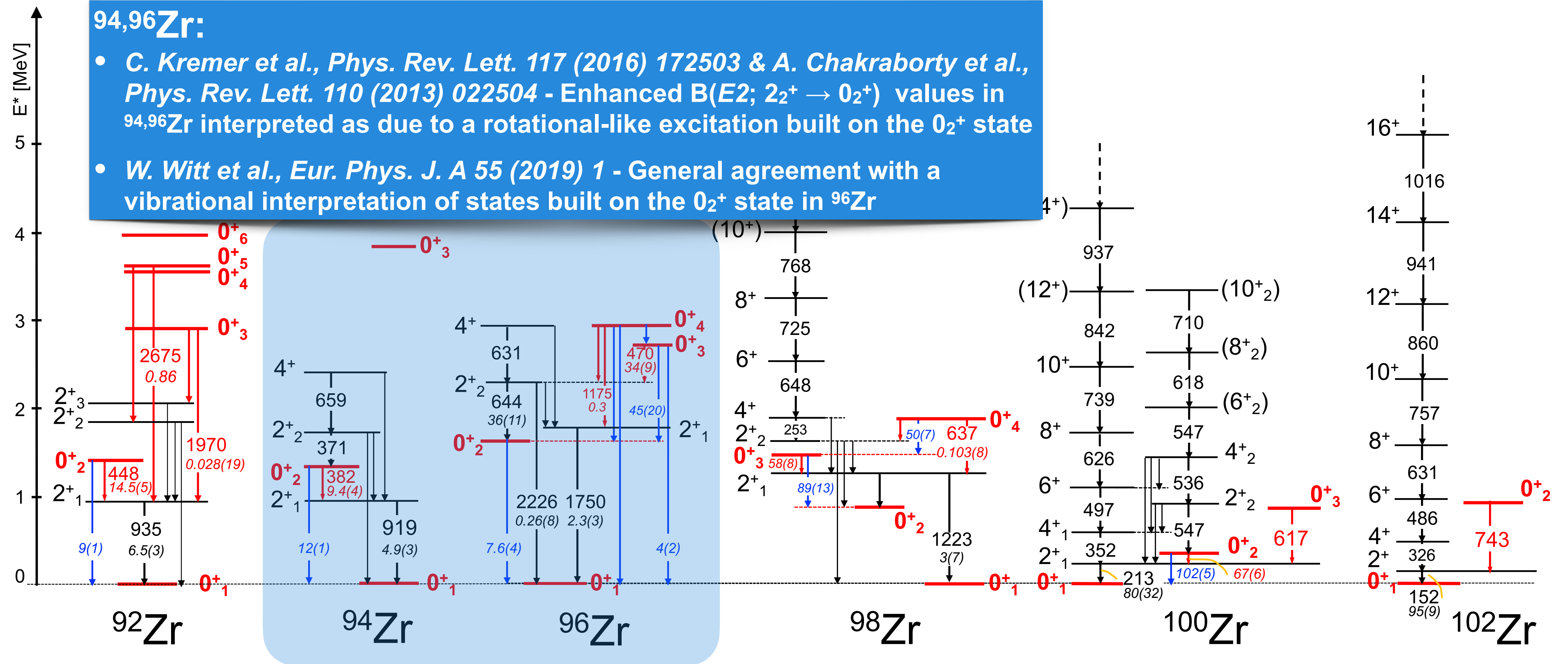
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S. Leoni et al. Progress in Particle and Nuclear Physics 139 (2024) 104119



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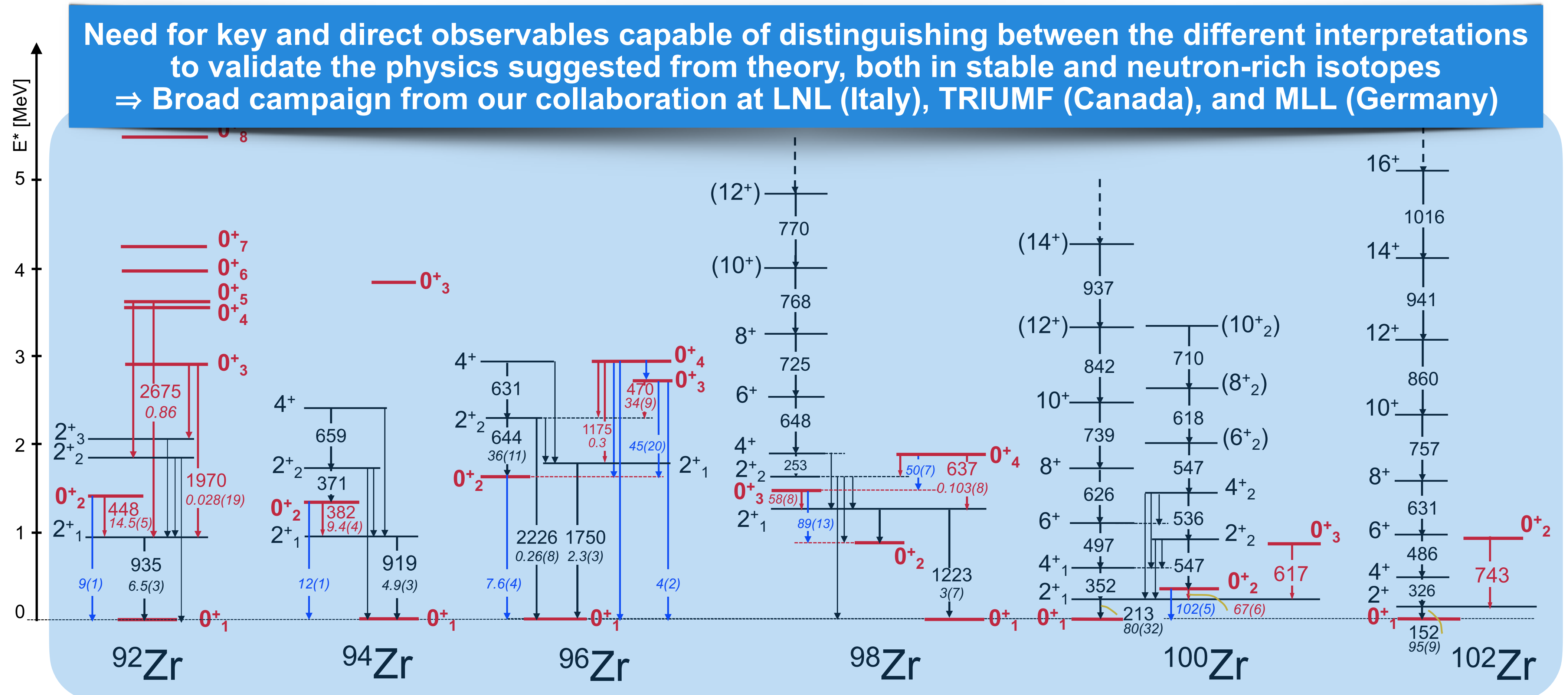
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S. Leoni et al. Progress in Particle and Nuclear Physics 139 (2024) 104119



# Coulomb Excitation of $^{94}\text{Zr}$

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Coulomb  
Excitation

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GALILEO and  
AGATA

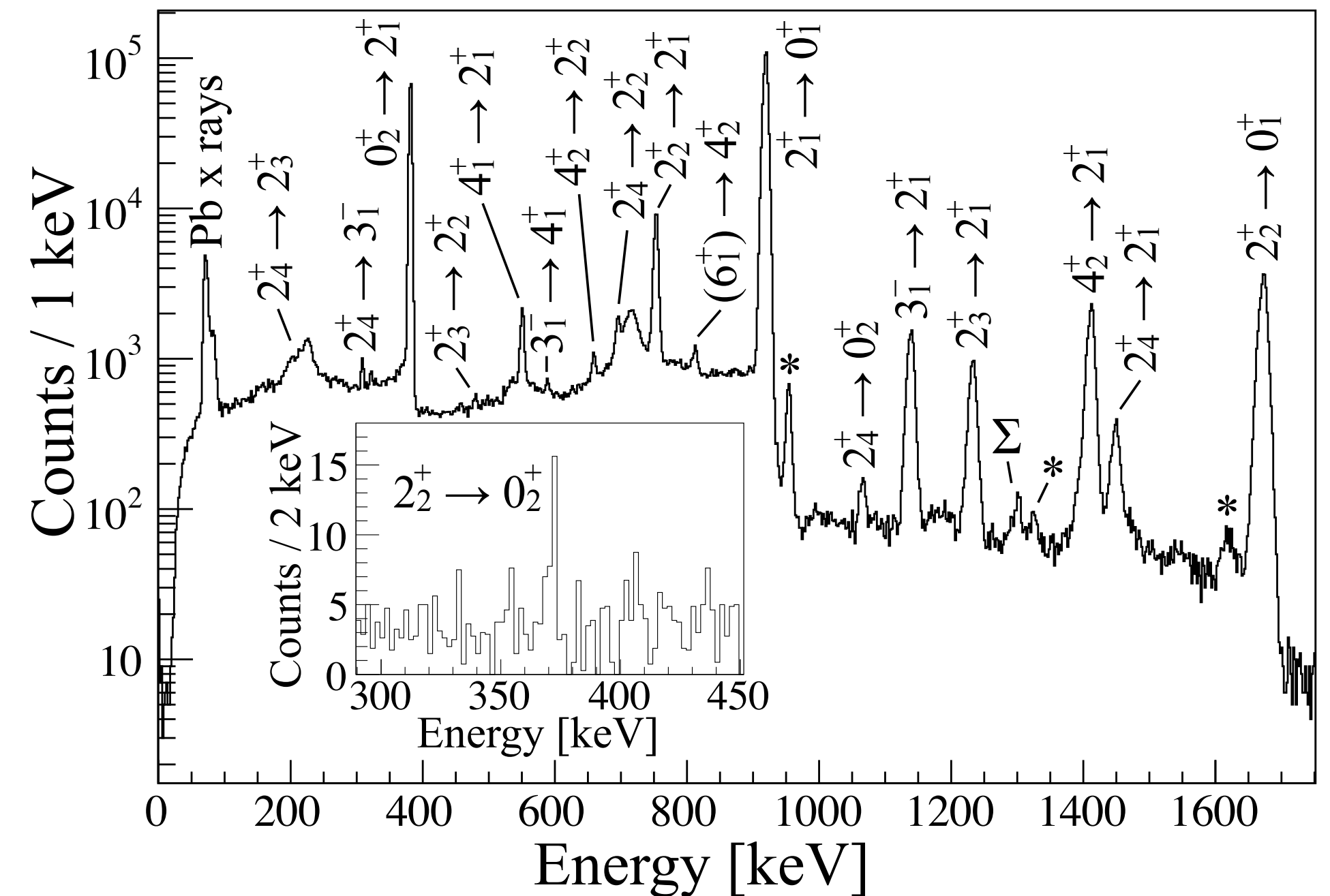
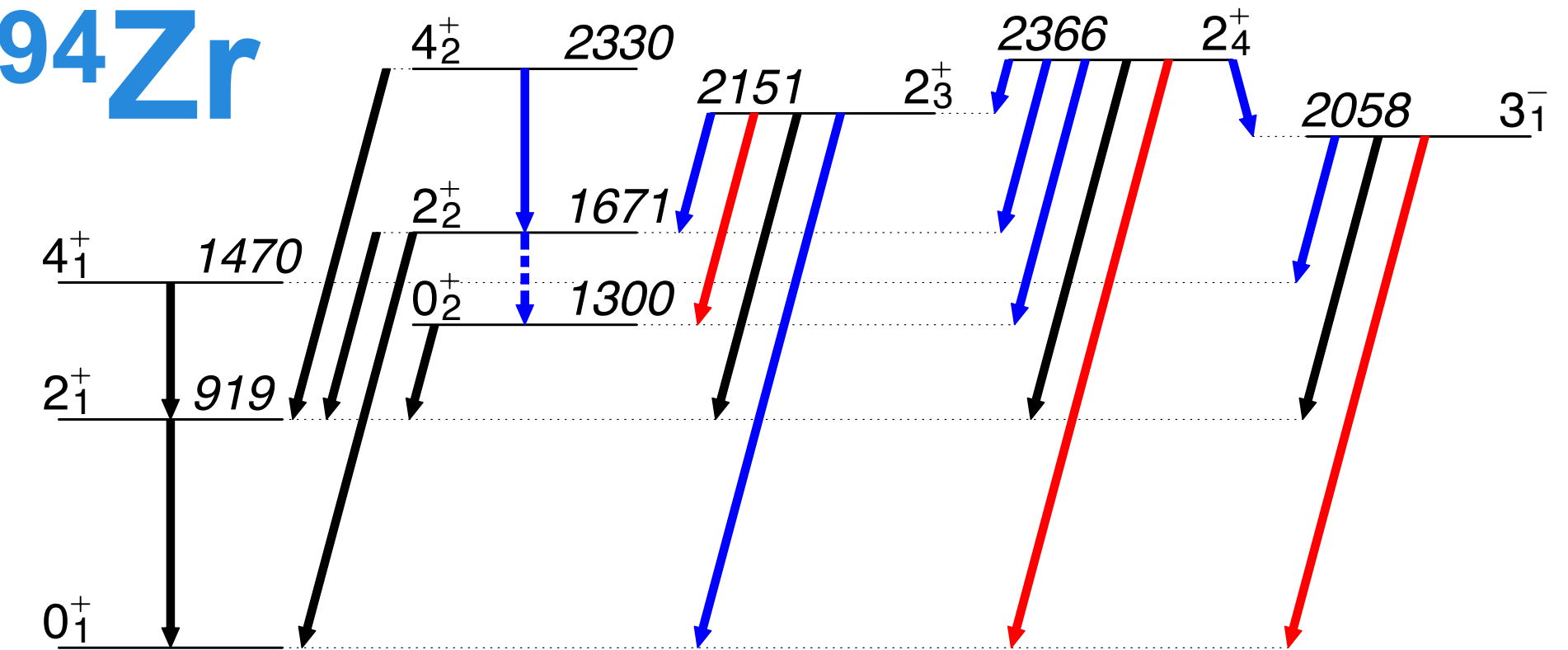
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Model-  
Independent  
Determination of  
Shapes in  $^{94}\text{Zr}$

- ▶ Experiment performed with **GALILEO** and **SPIDER**:
  - ▶  $^{94}\text{Zr}$  **beam** @370 MeV (0.1 pA) on a self-supporting  $^{208}\text{Pb}$  **target** with 1 mg/cm<sup>2</sup> thickness
  - ▶ GALILEO with **25 HPGe** Compton-suppressed detectors
  - ▶ SPIDER at **backward angles** (126° - 162°)
  - ▶ 6 **LaBr<sub>3</sub>:Ce** detectors to increase the efficiency in  $\gamma$ - $\gamma$  coinc.
  - ▶ *Spokespersons: D.T. Doherty (University of Surrey, UK), M. Rocchini, M. Zielinska (CEA Saclay, France)*
  - ▶ *Analysis by: N. Marchini (INFN Firenze), M. Rocchini*
- ▶ 18 gamma-ray transitions observed, **8** states populated
  - ▶ **Known spectroscopic data** added in the analysis:
    - ▶ **13** branching ratios, **10** lifetimes, **7** mixing ratios



N. Marchini, M. Rocchini, M. Zielinska et al. submitted to Phys. Lett. B



## Results

Low-Energy  
Coulomb  
Excitation

SPIDER with  
GALILEO and  
AGATA

Excited  $0^+$  States  
in Even-Even Mid-  
Mass Nuclei

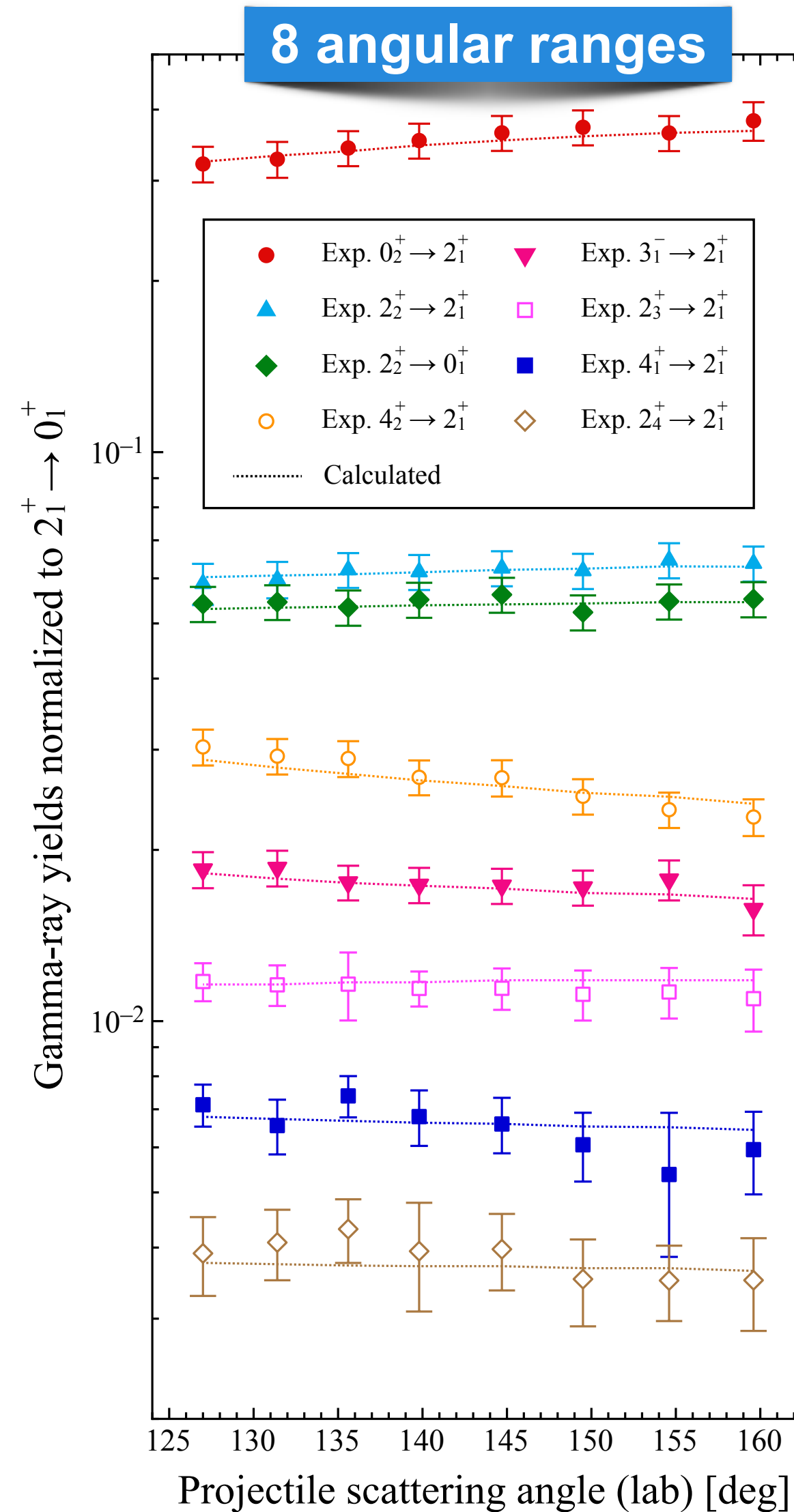
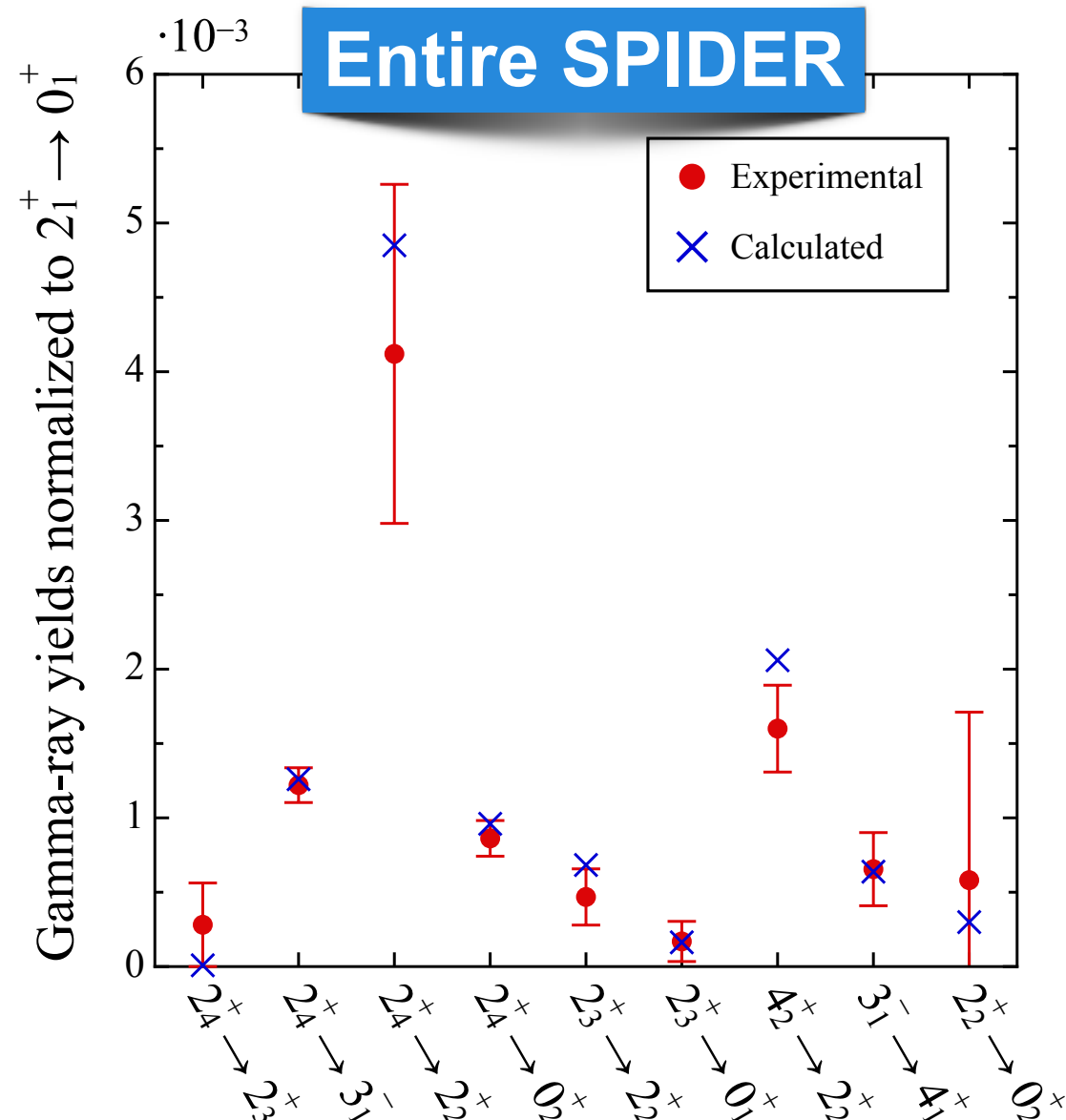
The Zr Isotopic  
Chain and QPTs

Coulex  
Experiment  
on  $^{94}\text{Zr}$

Model-  
Independent  
Determination of  
Shapes in  $^{94}\text{Zr}$

- 8 transitions analysed by dividing the statistics into 8 angular ranges, 9 transitions considering the entire SPIDER

- GOSIA analysis: excellent reproduction of experimental yields



### 17 reduced transition probabilities extracted + new IBM-CM calculations

$J_i \rightarrow J_f$	$\langle J_f    E2    J_i \rangle$ [eb]	$B(E2; J_i \rightarrow J_f)$ [W.u.]	
	Exp.	Exp.	IBM-CM
$2_1^+ \rightarrow 0_1^+$	$+0.250(7)^{ab}$	$4.8(2)^b$	2.7
$0_2^+ \rightarrow 2_1^+$	$+0.155(4)^a$	$9.5(5)$	9.3
$4_1^+ \rightarrow 2_1^+$	$+0.141(4)^a$	$0.87(5)$	— <sup>c</sup>
$2_2^+ \rightarrow 0_2^+$	$+0.484(12)^a$	$18.5(9)$	20.2
$2_2^+ \rightarrow 2_1^+$	$+0.03(3)$	$< 0.3$	1.49
$2_2^+ \rightarrow 0_1^+$	$+0.221(6)$	$3.9(2)$	0.82
$4_2^+ \rightarrow 2_2^+$	$+0.96(3)^a$	$40(3)$	26.6
$4_2^+ \rightarrow 2_1^+$	$+0.607(16)$	$16.1(9)$	2.1
$2_3^+ \rightarrow 2_2^+$	$+0.249^{+0.019}_{-0.040}$	$4.9^{+0.7}_{-1.6}$	17.3
$2_3^+ \rightarrow 0_2^+$	$< 0.13^d$	$< 1.3$	0.07
$2_3^+ \rightarrow 2_1^+$	$+0.290^{+0.014a}_{-0.012}$	$6.6^{+0.6}_{-0.5}$	1.2
$2_3^+ \rightarrow 0_1^+$	$-0.0182^{+0.0016}_{-0.0020}$	$0.026^{+0.005}_{-0.006}$	0.001
$2_4^+ \rightarrow 2_3^+$	$-0.02^{+0.06}_{-0.03}$	$< 0.2$	2.44
$2_4^+ \rightarrow 2_2^+$	$\pm 0.073^{+0.030}_{-0.018}$	$0.4^{+0.3}_{-0.2}$	0.1
$2_4^+ \rightarrow 0_2^+$	$\pm 0.177^{+0.010}_{-0.005}$	$2.47^{+0.30}_{-0.14}$	0.06
$2_4^+ \rightarrow 2_1^+$	$+0.092^{+0.006a}_{-0.004}$	$0.67^{+0.09}_{-0.06}$	0.001
$2_4^+ \rightarrow 0_1^+$	$-0.001^{+0.003}_{-0.006}$	$< 4 \cdot 10^{-3}$	$3 \cdot 10^{-4}$

<sup>a</sup> Sign imposed in the analysis (see text for details).

<sup>b</sup> Determined from literature data.

<sup>c</sup> Outside IBM-CM model space (see Ref. [24] for details).

<sup>d</sup> Positive sign determined.

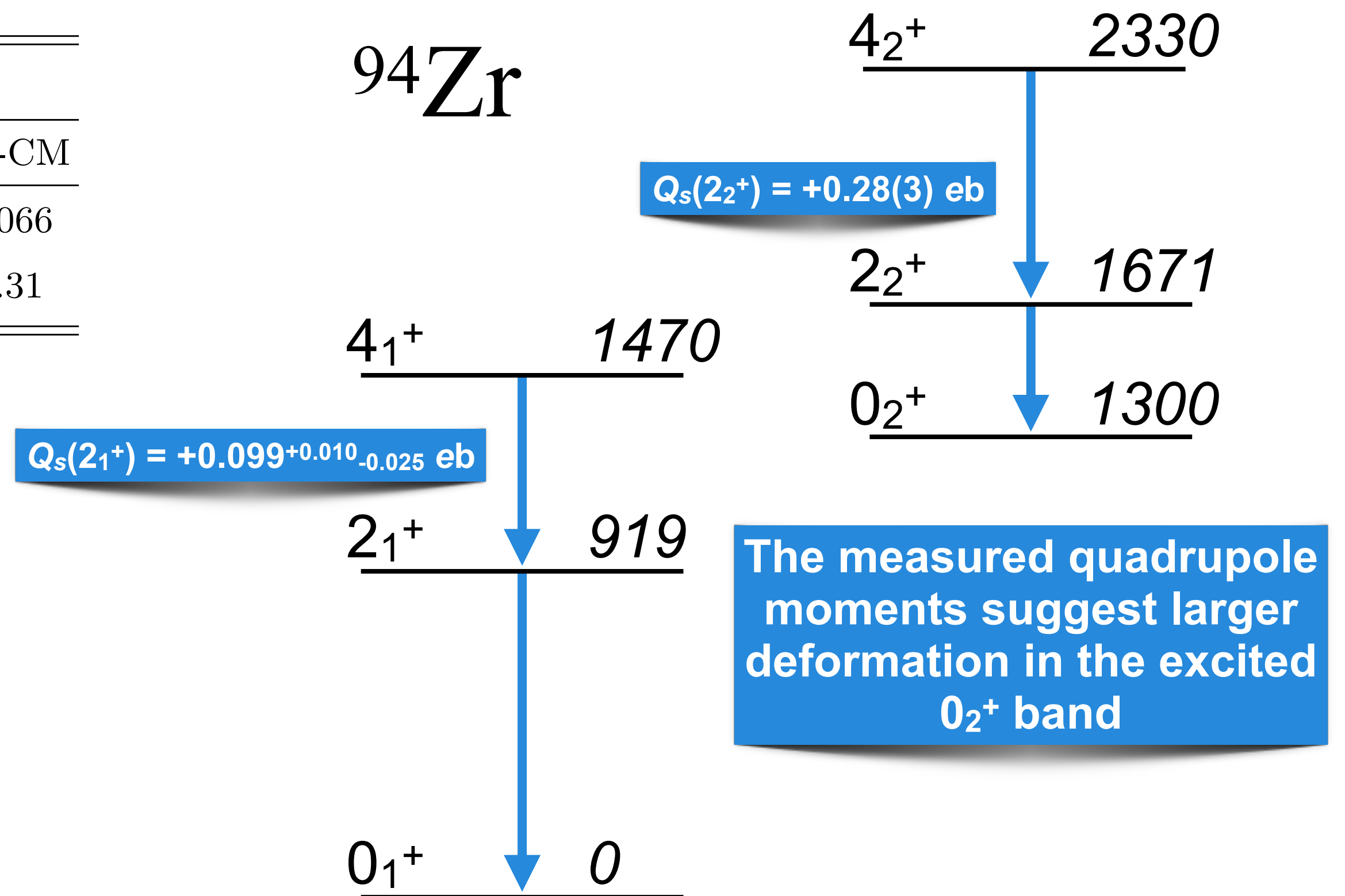
N. Marchini, M. Rocchini, M. Zielinska et al. submitted to Phys. Lett. B



# Shapes of the $0_{1,2}^+$ States

- First determination of spectroscopic quadrupole moments for the  $2_{1,2}^+$  states in the Zr isotopic chain:

$J_i$	$\langle J_i    E2    J_i \rangle$ [eb]	$Q_s(J_i)$ [eb]	
	Exp.	Exp.	IBM-CM
$2_1^+$	$+0.131^{+0.013}_{-0.030}$	$+0.099^{+0.010}_{-0.025}$	$+0.066$
$2_2^+$	$+0.37(4)$	$+0.28(3)$	$+0.31$



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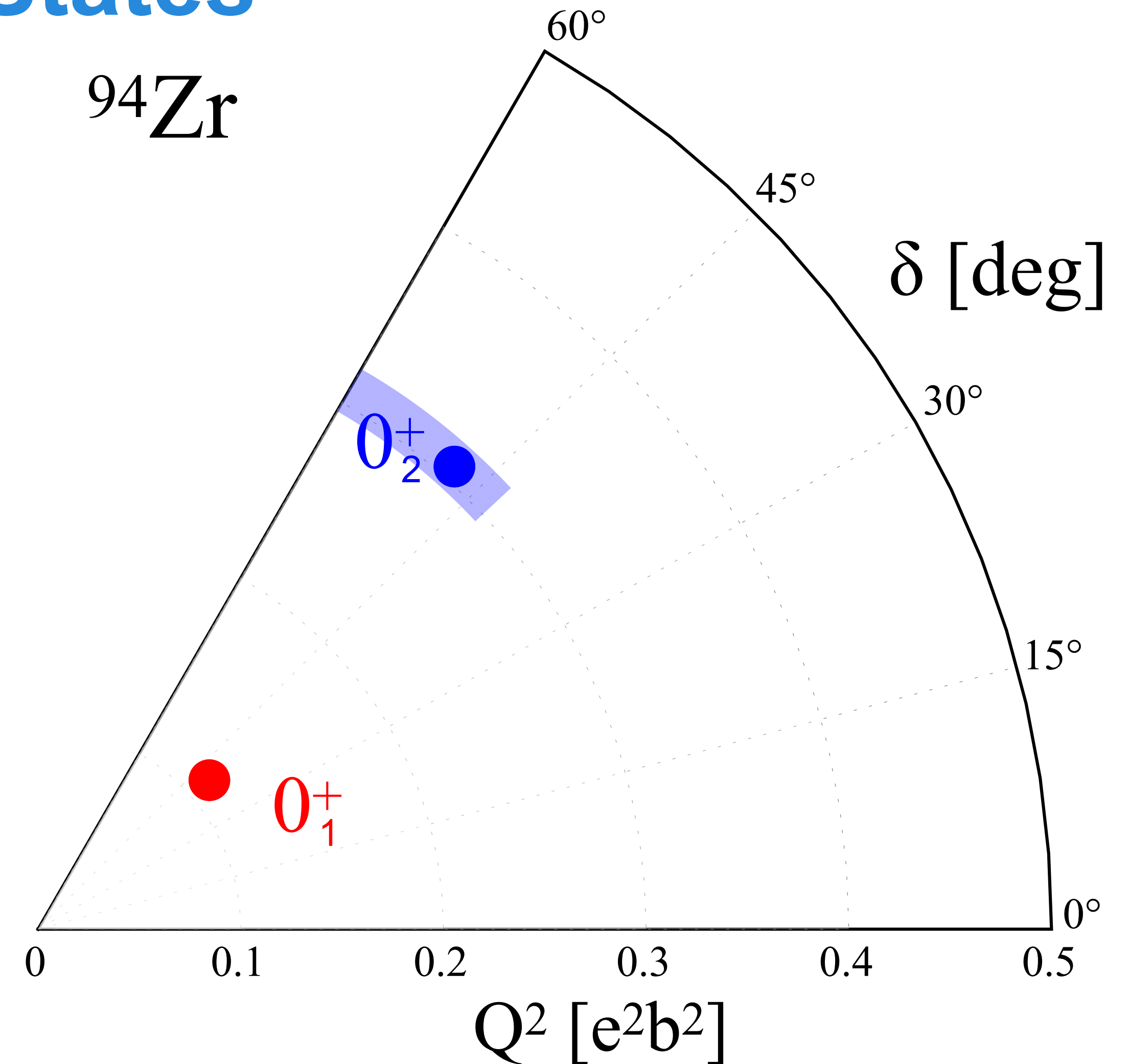
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- First experimental application of **quadrupole sum rules** to the  $0_{1,2}^+$  states in the Zr isotopic chain, and one of the few determinations of  $\sigma(Q^2) = (\langle Q^4 \rangle - (\langle Q^2 \rangle)^2)^{1/2}$  in the entire nuclide chart:

$J_i$	$\langle Q^2 \rangle$ [ $e^2 b^2$ ]		$\sigma(Q^2)$ [ $e^2 b^2$ ]		$\langle \cos(3\delta) \rangle$	
	Exp.	Th.	Exp.	Th.	Exp.	Th.
$0_1^+$	0.112(4)				-0.37(7)	
$0_2^+$	0.305(12)				-0.8(2)	

$^{94}\text{Zr}$



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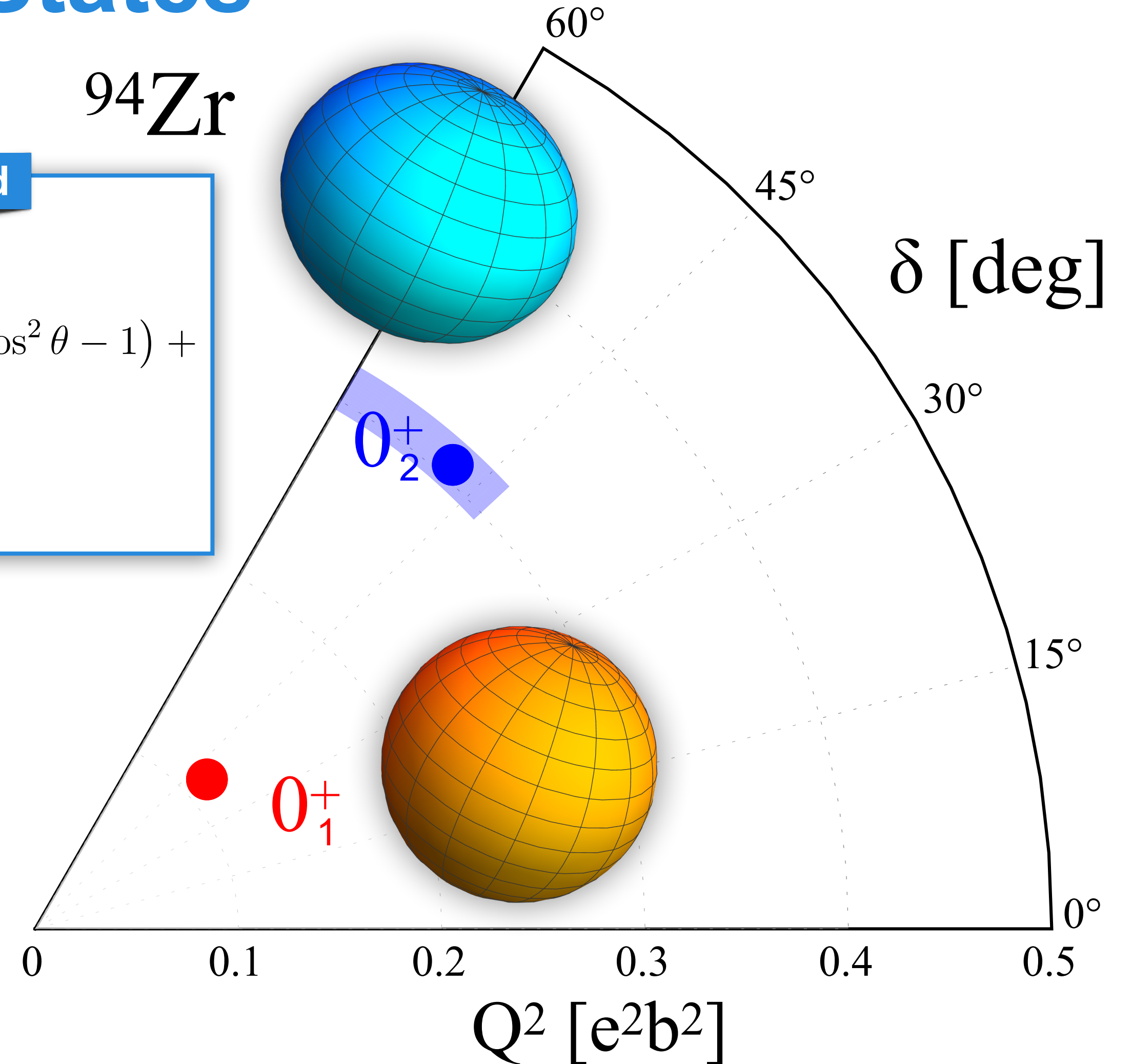
**Generic Ellipsoid**

$$R(\theta, \phi) = R_0 \left[ 1 + \beta \sqrt{\frac{5}{16\pi}} \left( \cos \gamma (3 \cos^2 \theta - 1) + \sqrt{3} \sin \gamma \sin^2 \theta \cos 2\phi \right) \right]$$

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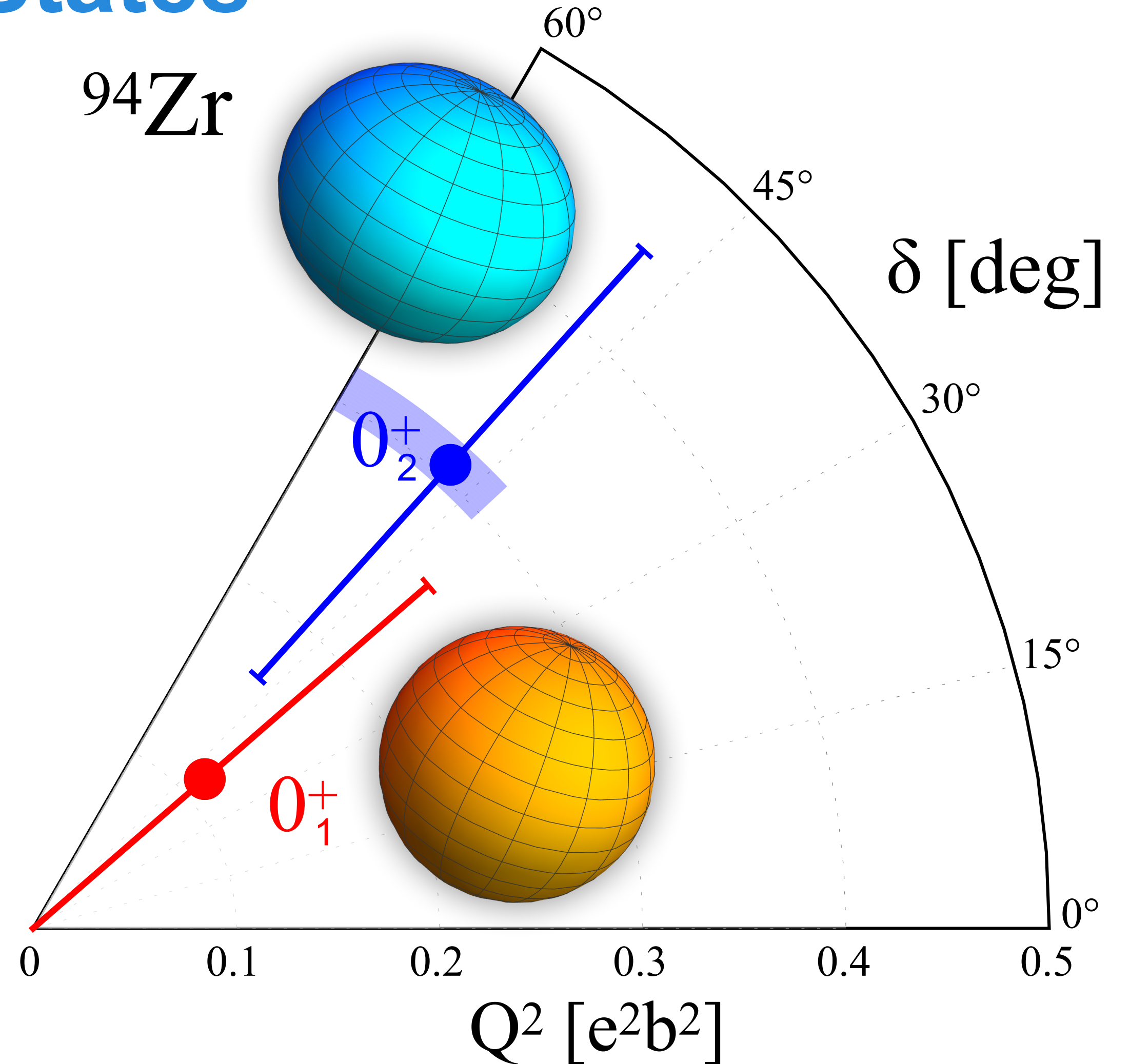
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$0_1^+$	0.112(4)		0.143(4)		-0.37(7)	
$0_2^+$	0.305(12)		0.14(3)		-0.8(2)	



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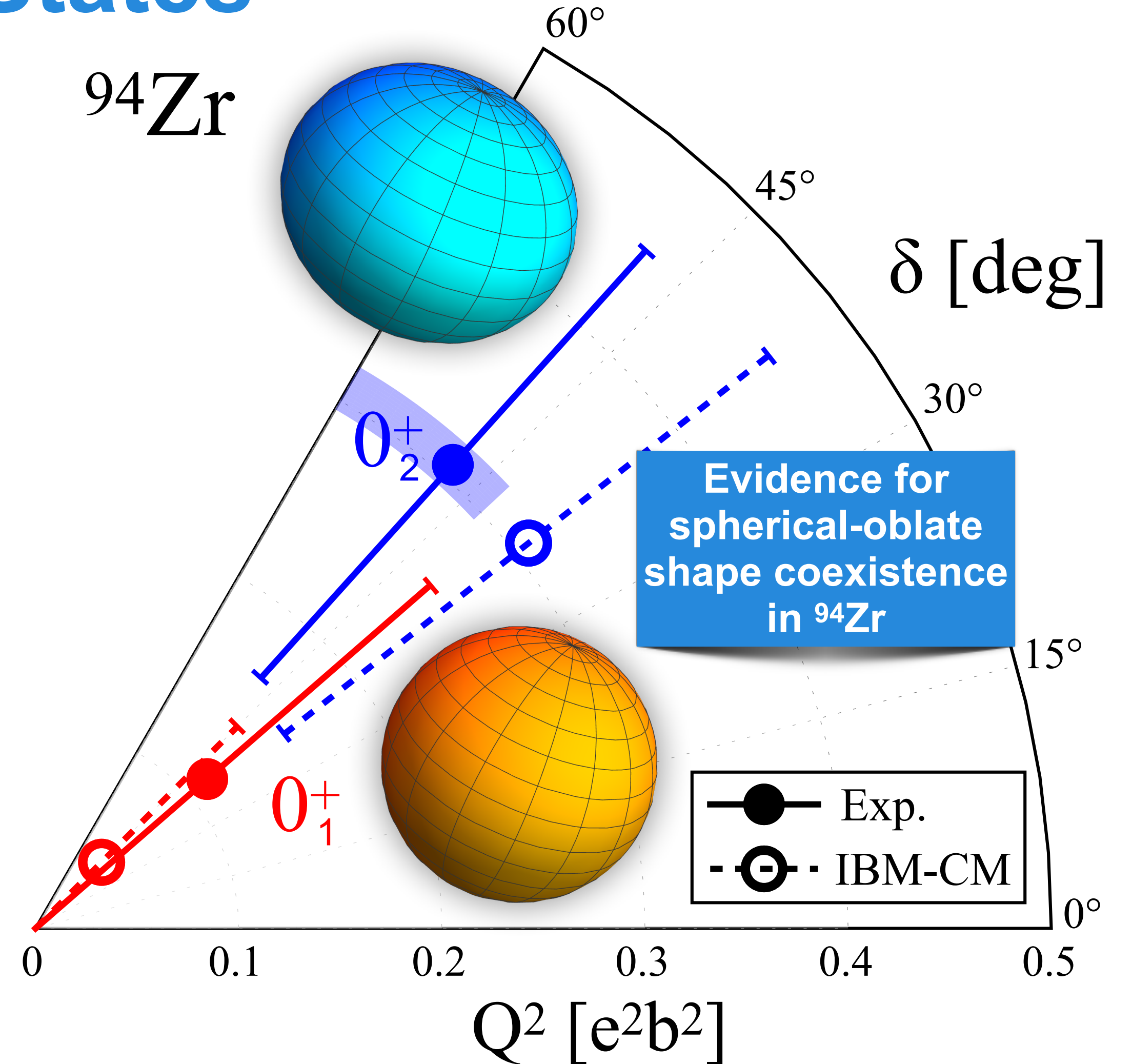
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	Exp.	Th.	Exp.	Th.	Exp.	Th.
$0_1^+$	0.112(4)	0.046	0.143(4)	0.094	-0.37(7)	-0.72
$0_2^+$	0.305(12)	0.308	0.14(3)	0.153	-0.8(2)	-0.4



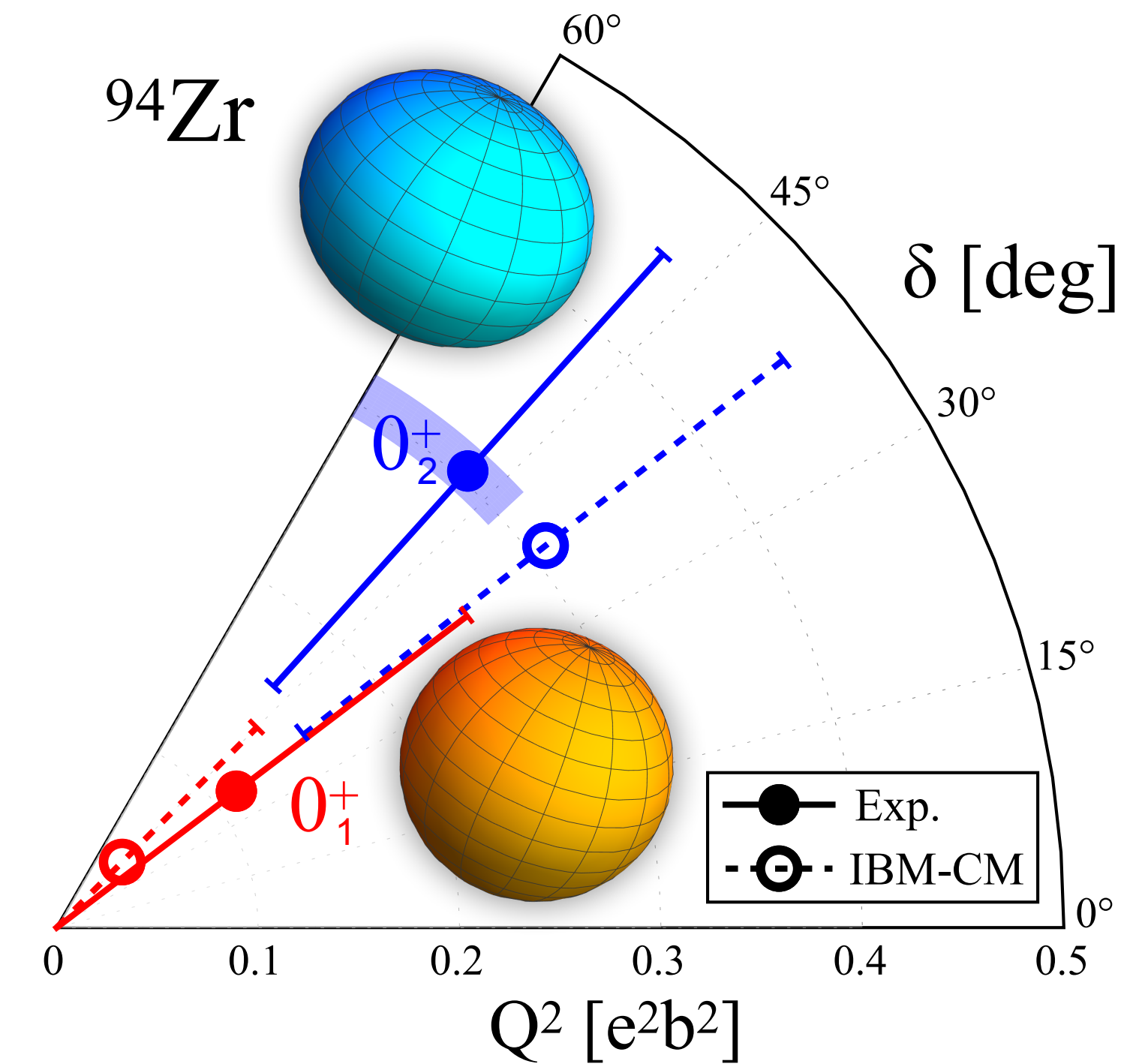
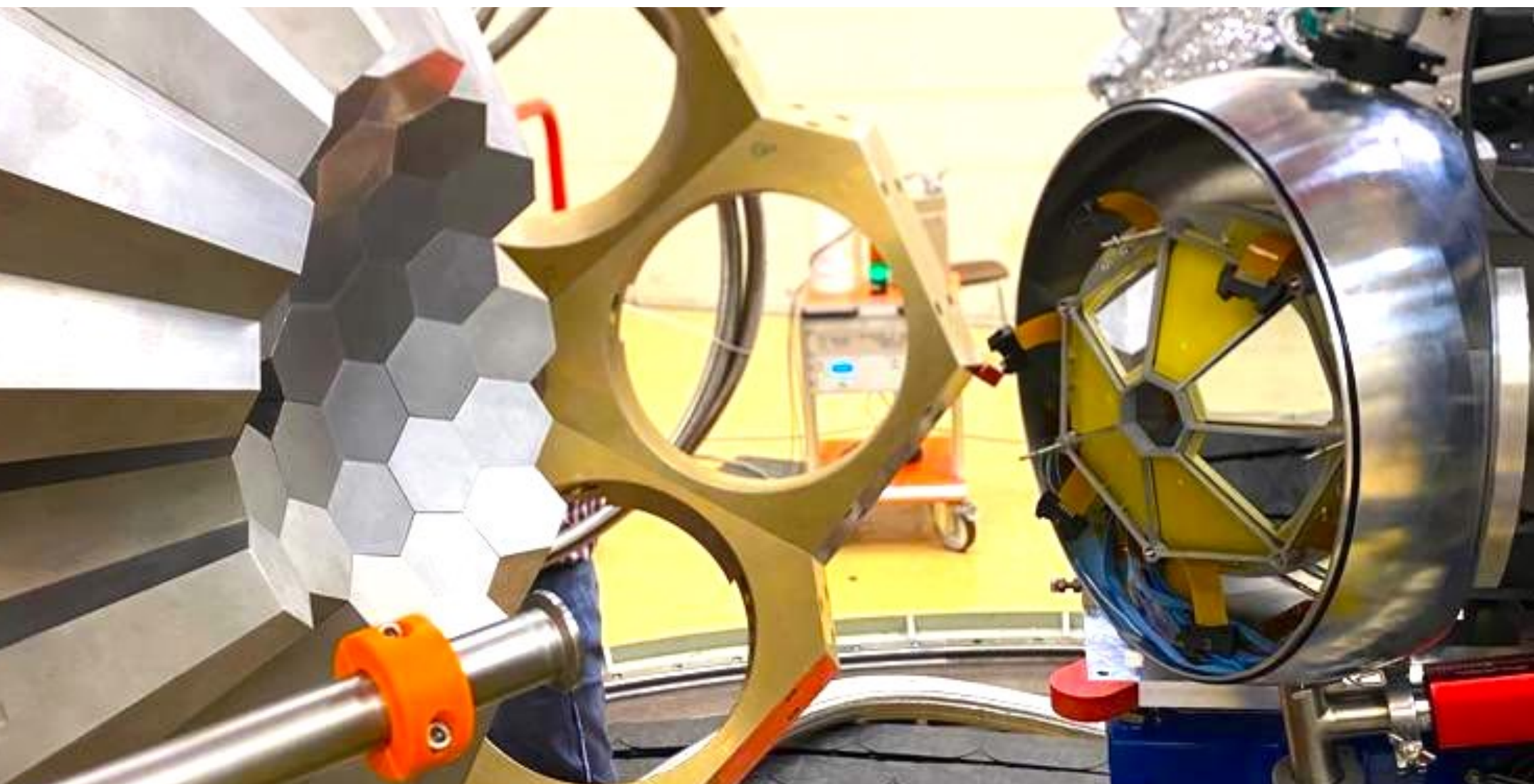
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# Summary

## Low-energy Coulomb excitation at LNL

- ▶ Well-established activity which involves many users and institutions
- ▶ Currently exploiting AGATA and SPIDER with stable beams
- ▶ New possibilities will be offered soon by SPES



## Coulex of $^{94}\text{Zr}$

- ▶ A high-statistics Coulomb-excitation experiment was performed at LNL
- ▶ 17 transitional matrix elements with relative signs and 2 diagonal matrix elements with absolute signs extracted
- ▶ Model-independent determination of spherical-oblate shape coexistence from the obtained quadrupole moments and rotational invariants





Marco Rocchini  
INFN - Istituto Nazionale di Fisica Nucleare  
FIRENZE DIVISION

THANK YOU FOR THE ATTENTION





