Bragg peak modeling and matter-wave interferometry in gaseous track detectors

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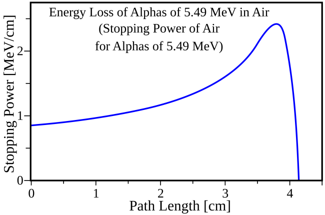
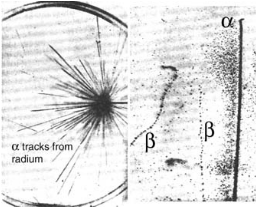
The Bragg peak: from applications. . . to detectors. . . to quantum foundations

- Bragg-peak phenomenon: ionizing particles in matter ⇒ energy-dependent energy loss (stopping power) ⇒ finite range
- Important for p, C. . . radiotherapy
- Important for low-energy gaseous time projection chambers (TPCs)
- Mott problem: α decay in a cloud chamber [Mott PRSA 1929]
    - spherical wave function
    - but linear tracks detected
    - decoherence: where?
    - measurement (w.f. reduction): deterministic?

      [JMS *et al.* EPJwoc 2013]



[Paul, Wikipedia, 2022]



[Wilson PRS 1912]

Stopping power definition

- Stopping "power" (rather stopping force!) of a medium on a beam

$$S_{\text{medium}}(E_{\text{lab}}) = -\left\langle \frac{dE_{\text{lab}}}{dx} \right\rangle, \quad E_{\text{lab}} = \frac{m_1 v_1^2}{2}$$

  - beam = particles of mass m_1, charge Z_1, speed v_1, lab. energy E_{lab}
  - medium: (mono)atomic density n, atomic number Z_2
  - dominated by electronic stopping power in Bragg peak

$$S_{\text{medium},e}(E_{\text{lab}}) = nZ_2 \times S_e(E_{\text{lab}})$$

$\Rightarrow$ target = electrons of mass m_2, charge $z_2 = -1$

- Mean energy transfer to individual electrons = stopping cross section

$$S(E) = \int_{4\pi} \frac{(\hbar q)^2}{2m_2} \frac{d\sigma}{d\Omega} d\Omega, \qquad E = \frac{\mu v^2}{2} = \frac{m_2}{M} E_{\text{lab}} = O(E_{\text{lab}}/2000)$$

  - $E = \hbar^2 k^2 / 2\mu$ = center-of-mass energy, $\boldsymbol{k}, \boldsymbol{k}'$ = initial/final momentum
  - $\mu = m_1 m_2 / M$ = reduced mass, $M = m_1 + m_2$ = total mass
  - $d\sigma/d\Omega$ = elastic-scattering differential cross section
  - momentum transfer $\boldsymbol{q} = \boldsymbol{k} - \boldsymbol{k}'$, for central potential $q = 2k \sin \frac{\theta}{2}$

Born approximation on electron-ion Yukawa potential

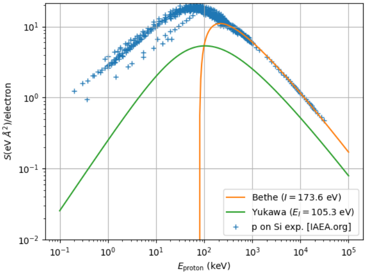
- Born approximation

$$S_{\mathrm{B}}(E) = \int_0^{2k} \frac{(\hbar q)^2}{2m_2} \frac{2\pi}{k^2} |F_{\mathrm{B}}(q)|^2 q dq$$

$$F_{\mathrm{B}}(q) = -\frac{2\mu}{\hbar^2 q} \int_0^\infty \sin qr V(r) r dr$$

- Yukawa potential: electrons in target atoms are screened

$$\Rightarrow \quad V(r) = -Z_1 \alpha \hbar c \, \frac{e^{-r/a}}{r}$$



Example: p^+ on Si

- Exact analytical expression for peak [Yang Chen *et al.* JCE 2002]

$$S_{\mathrm{B}}(E) = (Z_1 \alpha \hbar c)^2 \frac{\pi}{E} \left[\ln\left(1 + \frac{4E}{E_I}\right) - \frac{4E/E_I}{1+4E/E_I} \right], \quad E_I \equiv \frac{\hbar^2}{2\mu a^2}$$

  - factor 2 difference with Bethe formula [Bethe AP 1930]

$$S(E) \underset{E\to\infty}{\sim} (Z_1 \alpha \hbar c)^2 \, \frac{2\pi}{E} \ln \frac{4E}{I}, \qquad I = \text{mean excitation energy}$$

  - wrong behaviour at low energy

Above Bragg peak: energy-dependent screening

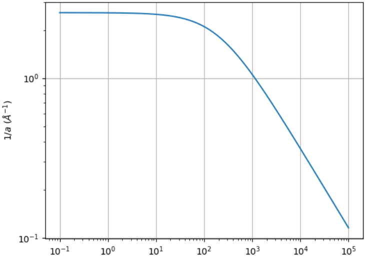
- Phenomenological screening

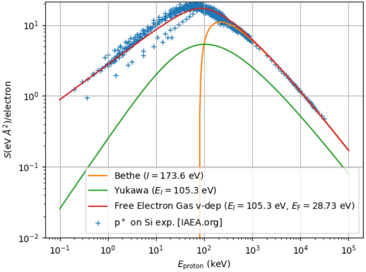
$$a(k, k_I, k_F) = \sqrt{\frac{4k^2}{k_I^4} + \frac{\pi a_0}{4k_F}}$$
$$\equiv \sqrt{\frac{v^2}{\omega^2} + \frac{v_F^2}{3\omega_p^2}}$$

- Inspired by [Lifschitz and Arista, PRA 1998; Sigmund, 2006]
- High-energy: Bohr adiabatic radius

$$a \underset{k\to\infty}{\to} a_B = \frac{v}{\omega}$$

  - related to Bethe $E_I \equiv \frac{\hbar^2 k_I^2}{m_e} = \frac{I}{\sqrt{e}}$
  - tabulated, hard to calculate
  - example: p on Si: $E_I = 105.3$ eV
- Good fit on whole Bragg peak with Fermi energy $E_F \equiv \frac{\hbar^2 k_F^2}{m_e} = 28.7$ eV



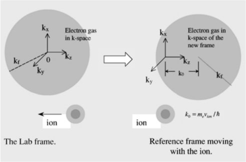


Below Bragg peak: Free Electron Gas [Yang Chen *et al.* JCE 2002]

- Electronic cloud $\approx$ Fermi gas
  $\Rightarrow$ analytical expression for $S_{\mathrm{B}}(E)$
- Fermi speed $\quad v_{\mathrm{F}} = \hbar k_{\mathrm{F}}/m_e$
  related to electron gas density
  $$n_e = k_{\mathrm{F}}^3/3\pi^2$$
  - predictive for solids: $n_e = nZ_2$
  - $\approx$ predictive for gases, using condensed-matter density

gas	H_2	He	N_2	Ar
E_I (eV)	11.6	25.4	49.7	113.4
E_{F}(eV)	4.43	4.09	14.2	19.1

- Low-energy limit
  $$a \underset{k\to 0}{\longrightarrow} \sqrt{\frac{\pi a_0}{4k_{\mathrm{F}}}} \equiv \sqrt{\frac{1}{3}}\frac{v_{\mathrm{F}}}{\omega_{\mathrm{p}}}$$
  - ω_{p} = plasma frequency
  - $S_{\mathrm{B}}(E)$ reduces to Ritchie formula, linear in k [Ritchie PR 1959]



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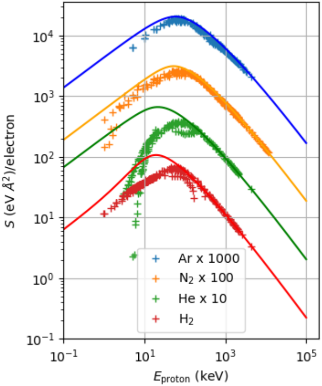

Below Bragg peak: Free Electron Gas [Yang Chen *et al.* JCE 2002]

- Electronic cloud ≈ Fermi gas
  ⇒ analytical expression for $S_\mathrm{B}(E)$
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  related to electron gas density
  $$n_e = k_\mathrm{F}^3/3\pi^2$$
  - predictive for solids: $n_e = nZ_2$
  - ≈ good fit for gases
    ⇒ good phenomenological model

gas	H_2	He	N_2	Ar
E_I (eV)	11.6	25.4	60.7	113.4
E_F(eV)	11.6	25.4	16.7	26

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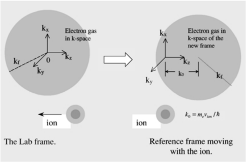

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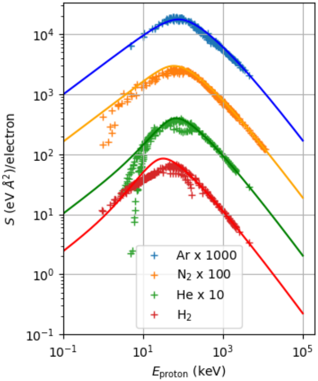
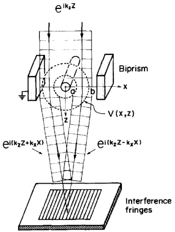
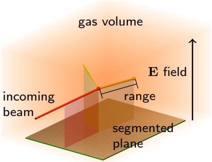


When does decoherence/measurement happen in a TPC?

1. Imagine any matter-wave interferometry (Young-type) experiment in an empty time projection chamber and measure interference pattern
2. Increase pressure continuously…
   check pattern
3. Switch on voltage…
   check pattern
4. Switch on electronics readout…
   check pattern
5. Become aware of tracks…
   check pattern

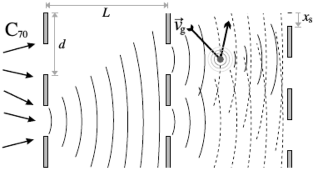


electron biprism, $E_e = 50$ keV
[Tonomura et al. 1989]

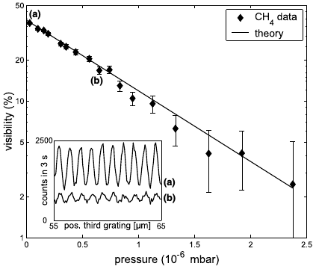


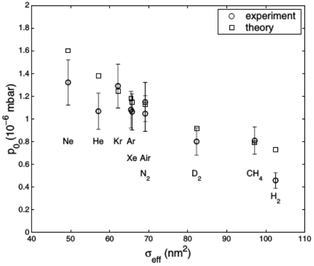
ACTAR TPC principle
[CDR 2012]

A similar experiment for (heavy!) molecules



- Matter-wave interferences for fullerene molecules [Hornberger *et al.* PRL 2003]
- Collisional decoherence due to background gas
  - fringe visibility $V(p) = V_0 e^{-p/p_0}$
  - decoherence pressure $p_0 = k_\mathrm{B}T/2L\sigma_\mathrm{eff}$
  - effective total cross section σ_eff
- Gas dependence well understood
- But no track measurement!





Link between decoherence and stopping power

- Quantum master equation for fast particle (a) in a gas of slow particles (b) ⇒ Focker-Planck eq.

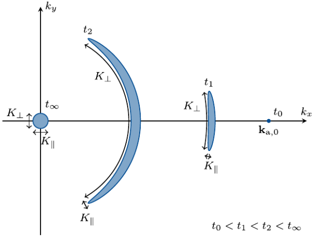
$$\frac{\partial f_a}{\partial t} + \mathbf{v}_a \cdot \nabla_\mathbf{r} f_a = \eta \nabla_{\mathbf{k}_a} \cdot (\mathbf{k}_a f_a) + \gamma \nabla^2_{\perp \mathbf{k}_a} f_a + \xi \nabla^2_{\mathbf{k}_a} f_a$$

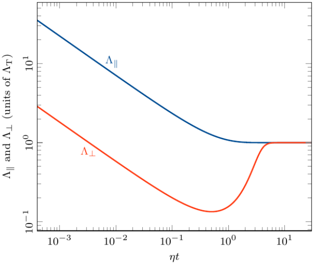
  - η = friction parameter $\propto S$
  - γ = transverse diffusivity $\propto \eta$
  - ξ = momentum diffusivity $\propto \eta$

  ⇒ momentum distribution $\boldsymbol{k}_a$

- Longitudinal and transverse coherence lengths $\Lambda_\parallel$, $\Lambda_\perp$
- Thermal wavelength $\Lambda_\mathrm{T} \approx 10$ pm for 5 MeV α particle ($T = 300$ K) ⇒ to be adapted for electrons

[DG, PRA 2022; DG & JMS, PRA 2023]





Conclusions and perspectives

- Promising (phenomenological) two-parameter formula for stopping power below and above Bragg peak
  - electron-ion energy-dependent Yukawa potential, Born approximated
  - low-energy: reproduces Ritchie formula, free-electron gas model, predictive, based on (condensed-matter) electron density
  - high-energy: reproduces Bethe formula, based on phenomenological mean excitation energy
  - works both for condensed matter and gases
- Call to experimentalists
  - matter-wave interferometry experiment
  - under varying gas pressure $\Rightarrow$ decoherence pressure
  - test of stopping-power/cross section
  - step-by-step check of measurement process in time-projection-chambers
- Longer-term research project (coll. B. Lorent): deterministic quantum statistical model for quantum particle in gaseous environment
  - new approach to decoherence, localisation and measurement
  - also applies to molecular collisions

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