Renormalisation running of masses and mixings in Universal Extra-Dimensional Models

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Motivation

In particle physics a clear feature of the fermion mass spectrum is

$$m_u \ll m_c \ll m_t \ , \ m_d \ll m_s \ll m_b \ , \ m_e \ll m_\mu \ll m_\tau \ ,$$

where a completely satisfactory theory of fermion masses and the related problem of mixing angles is certainly lacking at present.

Among the models which can be used to explain this hierarchy (as well as EWSB, proton stability, gauge hierarchies, dark matter etc.), are those with extra spatial dimensions.

With the LHC now up and running, exploration of the realm of new physics that may operate at the TeV scale has begun

In order to explore the physics at a high energy scale we use RGEs as a probe to study the momentum dependence of the Yukawa couplings, gauge couplings, and the CKM matrix elements.

Note that instead of assuming the RGE goes from the m_Z scale up to the GUT scale by using the $SU_C(3) \times SU_L(2) \times U_Y(1)$ symmetry, we know that models with extra dimensions may bring down the unification scale to a much lower energy.

The UED model

For this talk I shall work with the minimal UED model, i.e. the extra dimension is compactified on a circle of radius R with a Z_2 orbifolding, which identifies the fifth coordinate $y \to -y$.

Note that other UED models exist, where in A. I. Abdalgabar's poster, a 5D MSSM is considered.

In this case, the 5-dimensional KK expansions of the matter fields, the Higgs field and gauge fields are:

$$H(x,y) \quad = \quad \frac{1}{\sqrt{\pi R}} \left\{ H(x) + \sqrt{2} \sum_{n=1}^{\infty} H_n(x) \cos\left(\frac{ny}{R}\right) \right\} \ ,$$

$$u(x,y) = \frac{1}{\sqrt{\pi R}} \left\{ u_R(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[u_R^n(x) \cos\left(\frac{ny}{R}\right) + u_L^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\} ,$$

$$Q(x,y) = \frac{1}{\sqrt{\pi R}} \left\{ Q_L(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[Q_L^n(x) \cos\left(\frac{ny}{R}\right) + Q_R^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\} ,$$

$$d(x,y) = \frac{1}{\sqrt{\pi R}} \left\{ d_R(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[d_R^n(x) \cos\left(\frac{ny}{R}\right) + d_L^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\} ,$$

$$L(x,y) = \frac{1}{\sqrt{\pi R}} \left\{ L_L(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[L_L^n(x) \cos\left(\frac{ny}{R}\right) + L_R^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\} ,$$

$$e(x,y) = \frac{1}{\sqrt{\pi R}} \left\{ e_R(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[e_R^n(x) \cos\left(\frac{ny}{R}\right) + e_L^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\} ,$$

where the corresponding coupling constants among the KK modes are simply equal to the SM couplings up to a normalisation factors,

e.g. $Y_U = \dfrac{Y_U^5}{\sqrt{\pi R}}$.

The zero modes in the above equations are identified with the 4-dimensional SM fields, whilst the complex scalar field H is a Z_2 even field, and there is a left-handed and a right-handed KK mode for each SM chiral fermion.

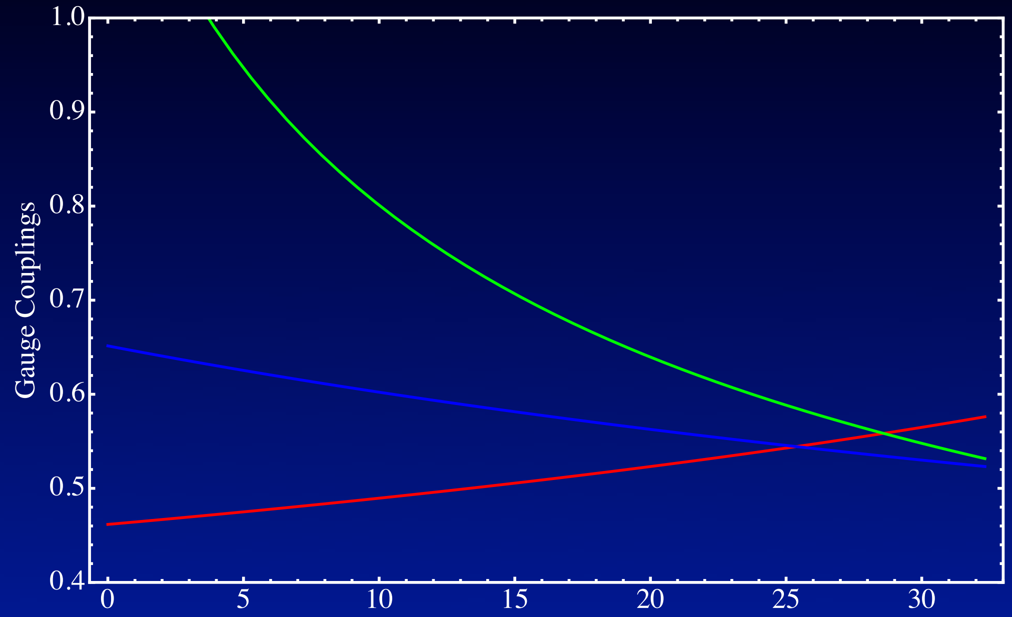
Note that in models with UED momentum conservation in the extra dimensions, we are led to the conservation of KK number at each vertex in the interactions of the 4-dimensional effective theory, where

$$\mathcal{L}_{leptons} = \int_0^{\pi R} dy \left\{ i\bar{L}(x,y)\Gamma^M \mathcal{D}_M L(x,y) + i\bar{e}(x,y)\Gamma^M \mathcal{D}_M e(x,y) \right\} \ ,$$

where $\Gamma^M = (\gamma^\mu, i\gamma^5)$, $M = 0,1,2,3,5$ and similarly for the quark sector etc.

After integrating out the compactified dimension, the 4-dimensional effective Lagrangian has interactions involving the zero mode and the KK modes. However, these KK modes cannot affect EW processes at tree level, and only contribute to higher order EW processes.

The gauge coupling constants

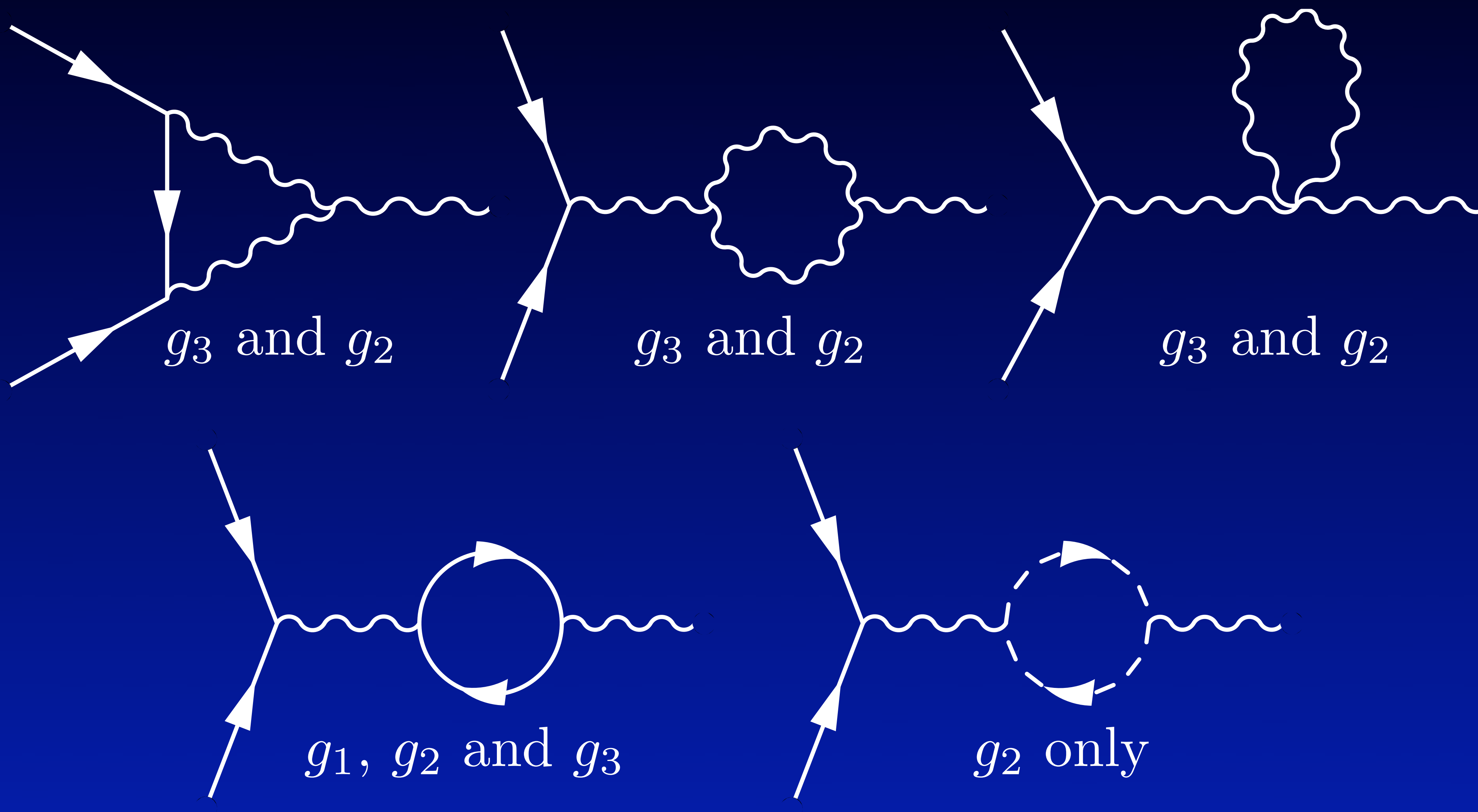


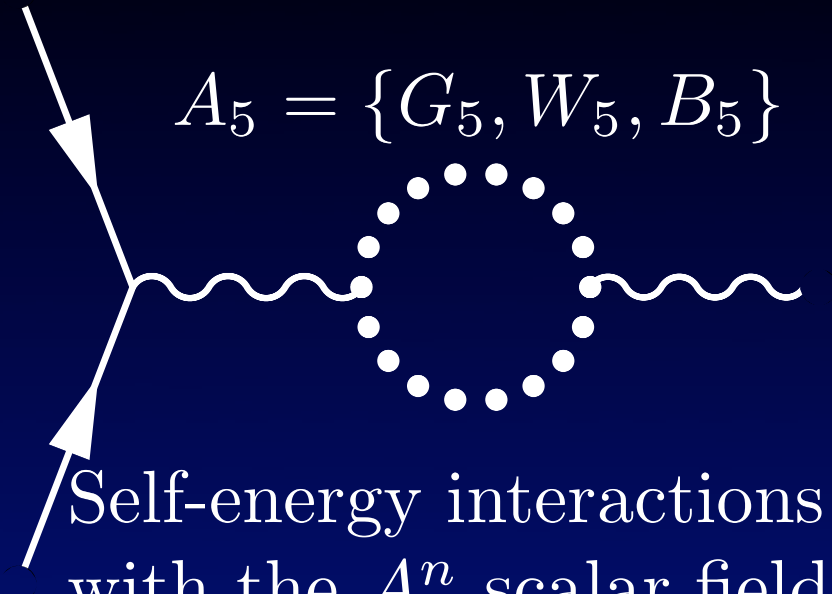
In the SM, the one-loop corrections to the gauge couplings are given by

$$16\pi^2 \frac{dg_i}{dt} = b_i^{SM} g_i^3 \ ,$$

where $b_i^{SM} = \left(\frac{41}{10}\,,\ -\frac{19}{6}\,,\ -7\right)$, $t = \ln(\mu/m_Z)$, and for the MSSM we replace $b_i^{SM} \to b_i^{MSSM} = \left(\frac{33}{5}\,,\ 1\,,\ -3\right)$.

That is, we have diagrams such as:





Self-energy interactions with the A_5^n scalar fields

For the UED model, there will be at each excited KK level the one-loop corrections to the gauge couplings arising from the diagrams exactly mirroring those of the SM ground states

Plus new contributions to the self-energy of the gauge boson from the fifth component of the 5D gauge field A_M $(M = 0, 1, 2, 3, 5)$ at each KK excited level

$$A_5(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} A_5^n(x) \sin\left(\frac{ny}{R}\right)$$

Note that for the closed fermion one-loop diagrams, one needs to count the contributions from both the left-handed and right-handed KK modes of each chiral fermion to the self-energy of the gauge field

Between the scale R^{-1} where the first KK states are excited and the cutoff scale Λ, there are finite quantum corrections to the Yukawa and gauge couplings from the ΛR number of KK states.

Up to the scale R^{-1}, the first step KK excitation occurs, the RG evolution is logarithmic, controlled by the SM beta functions.

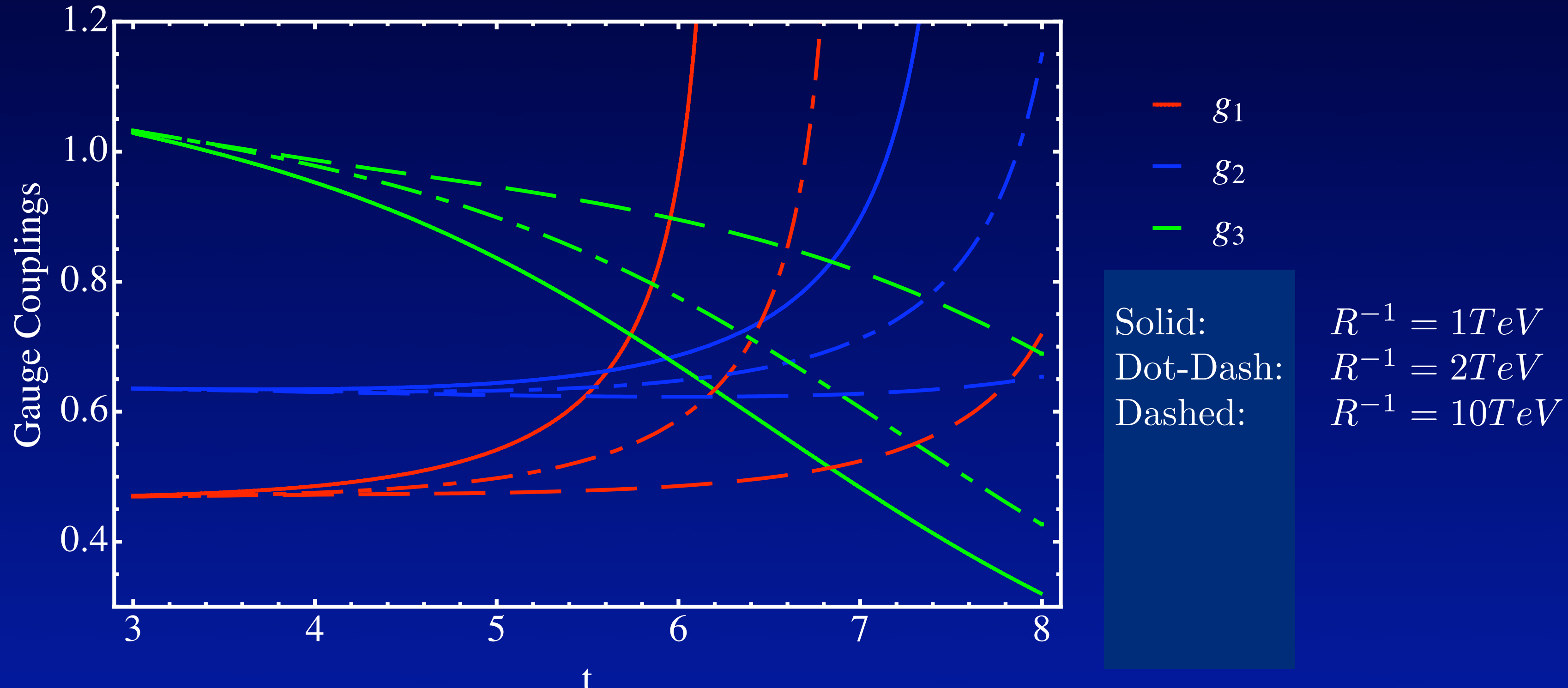
With increasing energy, that is, as each KK threshold is crossed, new excitations come into play and govern new sets of beta functions until the next threshold is reached, leading to

$$16\pi^2 \frac{dg_i}{dt} = \left[b_i^{SM} + (S(t) - 1)\tilde{b}_i \right] g_i^3 \ ,$$

where $S(t) = e^t m_Z R$, and $\tilde{b}_i = \left(\frac{81}{10}, \frac{7}{6}, -\frac{5}{2} \right)$, corresponding to each of the gauge couplings.

Note again, that in the 5D MSSM case $b_i^{SM} \to b_i^{MSSM}$ and $\tilde{b}_i = \left(\frac{6}{5}, \ -2\, , \ -6 \right) + 4\eta$ where η represents the number of generations of fermions which propagate in the bulk

We can see that the dependence of the gauge couplings on the energy scale drastically changes the normal one-loop running of the gauge couplings, and lowers the unification scale considerably.



The extra dimensions naturally lead to the appearance of GUTs at scales substantially below the usual GUT scale.

The SM Yukawa RGEs

Starting with SM, which is based on the group structure $SU_C(3) \times SU_L(2) \times U_Y(1)$ with one Higgs doublet, the mass matrices arise from the Yukawa sector of the theory as given by

$$\mathcal{L}_Y = Y_U \epsilon H \bar{u} Q + Y_D H^* \bar{d} Q + Y_E H^* \bar{e} L \ ,$$

where ϵ is the 2×2 antisymmetric tensor with $\epsilon_{12} = -\epsilon_{21} = -1$.

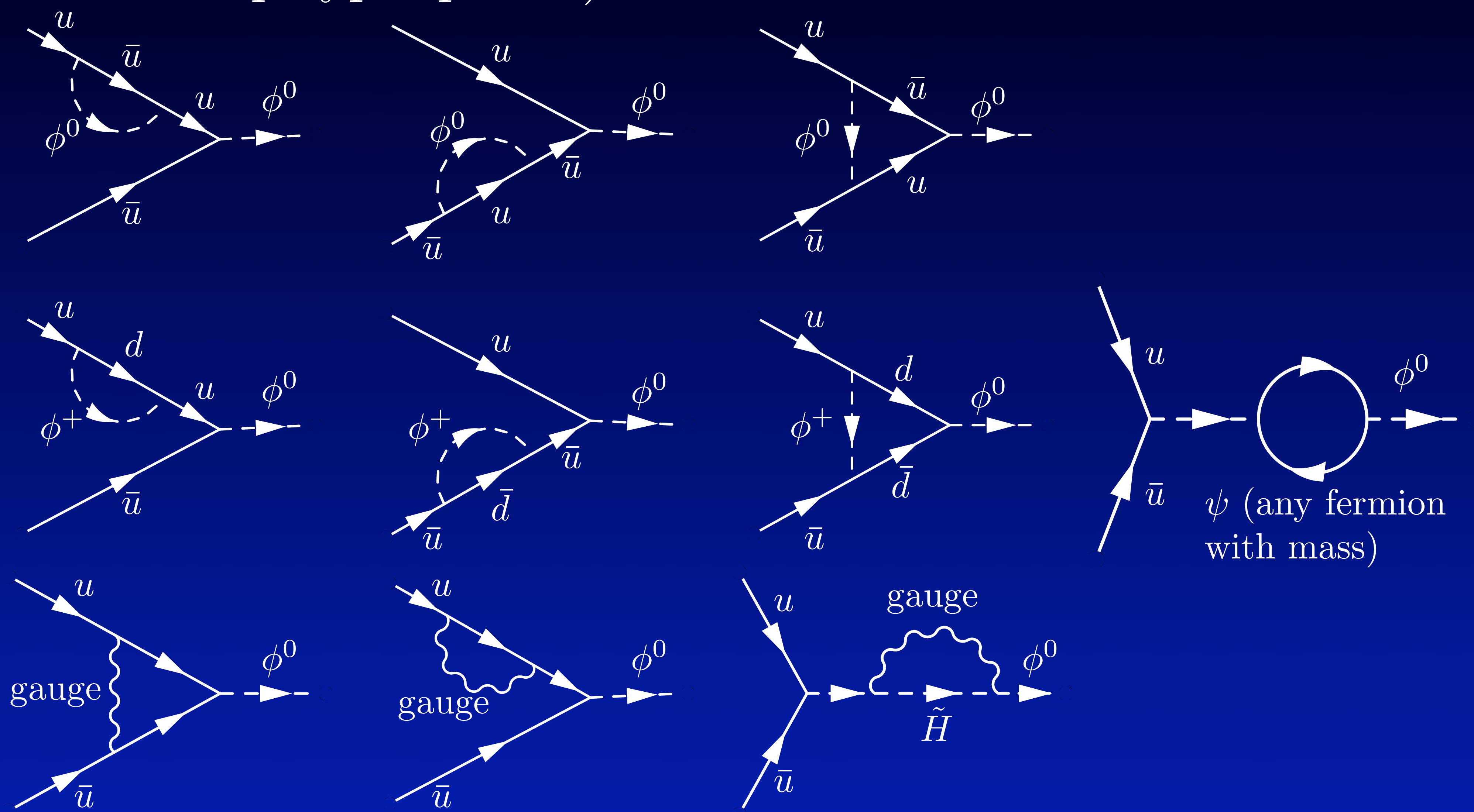
It is well known that the evolution of the generic Yukawa coupling, which describes the fermion-boson interactions, is given by the beta function. As a result the Yukawa coupling renormalisation depends on the corresponding beta functions, including contributions from the anomalous dimensions of the field operators.

That is, its evolution is given by:

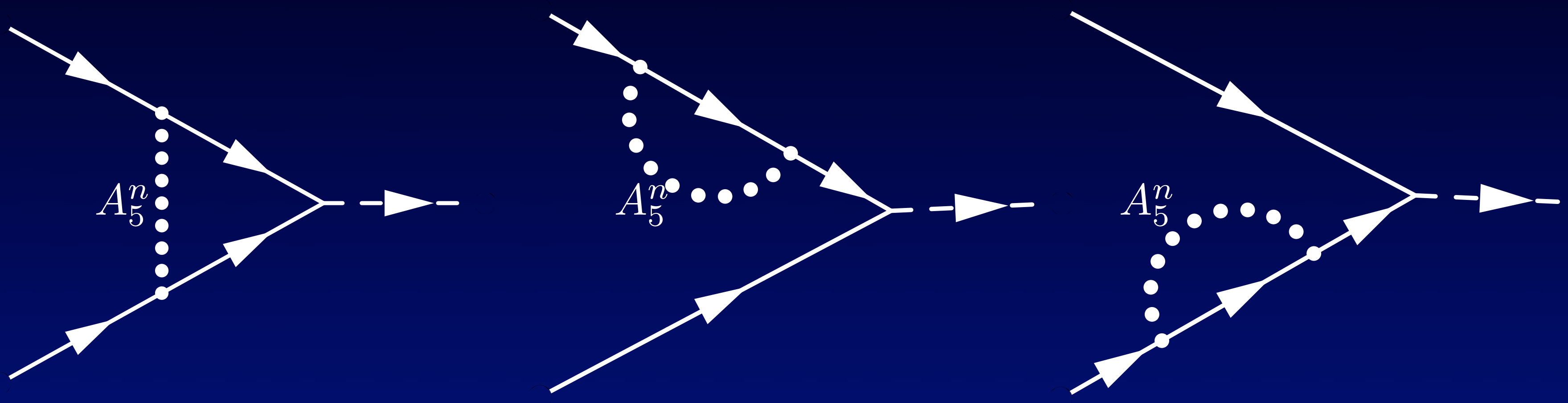
$$\mu\frac{\partial}{\partial\mu}\ln Y_R = \frac{1}{2}\mu\frac{\partial}{\partial\mu}\ln Z_{\psi_L} + \frac{1}{2}\mu\frac{\partial}{\partial\mu}\ln Z_{\psi_R} + \frac{1}{2}\mu\frac{\partial}{\partial\mu}\ln Z_{\phi} - \mu\frac{\partial}{\partial\mu}\ln Z_{coupling} \ ,$$

where Y_R is the renormalised Yukawa coupling constant, and Z_{ψ_L}, Z_{ψ_R} and Z_ϕ are the wave function renormalisation constants related to left-handed, right-handed fermions and Higgs boson respectively, and $Z_{coupling}$ is the vertex renormalisation constant.

So the diagrams we need to consider (in the SM for the up-type quarks) are:



The UED Yukawa RGEs



As discussed earlier, we will have additional diagrams from the KK modes in complete analogy to the SM diagrams (where the fermion loop will have both left and right-handed KK modes) plus the above contributions from the $A_5^n = \{G_5^n, W_5^n, B_5^n\}$ scalar fields.

In which case, if we write:

$$16\pi^2 \frac{dY_i}{dt} = \beta_i^{SM} + \beta_i^{UED} \ ,$$

$$\begin{aligned}
\beta_U^{UED} &= Y_U \left\{ (S(t)-1) \left[-\left(\frac{28}{3} g_3^2 + \frac{15}{8} g_2^2 + \frac{101}{120} g_1^2 \right) + \frac{3}{2} \left(Y_U^\dagger Y_U - Y_D^\dagger Y_D \right) \right] \right. \\
&\quad \left. + 2(S(t)-1) \mathrm{Tr} \left[2Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E \right] \right\} \ , \\
\beta_D^{UED} &= Y_D \left\{ (S(t)-1) \left[-\left(\frac{28}{3} g_3^2 + \frac{15}{8} g_2^2 + \frac{17}{120} g_1^2 \right) + \frac{3}{2} \left(Y_D^\dagger Y_D - Y_U^\dagger Y_U \right) \right] \right. \\
&\quad \left. + 2(S(t)-1) \mathrm{Tr} \left[2Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E \right] \right\} \ , \\
\beta_E^{UED} &= Y_U \left\{ (S(t)-1) \left[-\left(\frac{15}{8} g_2^2 + \frac{99}{40} g_1^2 \right) + \frac{3}{2} Y_E^\dagger Y_E \right] \right. \\
&\quad \left. + 2(S(t)-1) \mathrm{Tr} \left[2Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E \right] \right\} \ .
\end{aligned}$$

The square of the up-type and down-type Yukawa coupling matrices can be diagonalised by using two unitary matrices U and V

$$UY_U^\dagger Y_U U^\dagger = \mathrm{diag}(f_u^2\,,\ f_c^2\,,\ f_t^2)\ ,\ VY_D^\dagger Y_D V^\dagger = \mathrm{diag}(h_d^2\,,\ h_s^2\,,\ h_b^2)\ .$$

The evolution of these eigenvalues, and similarly in the lepton can be found in the literature, and for compactness will not be presented here.

Note that the CKM matrix describing the quark flavour mixing in the charged current is given by

$$V_{CKM} = UV^\dagger\ .$$

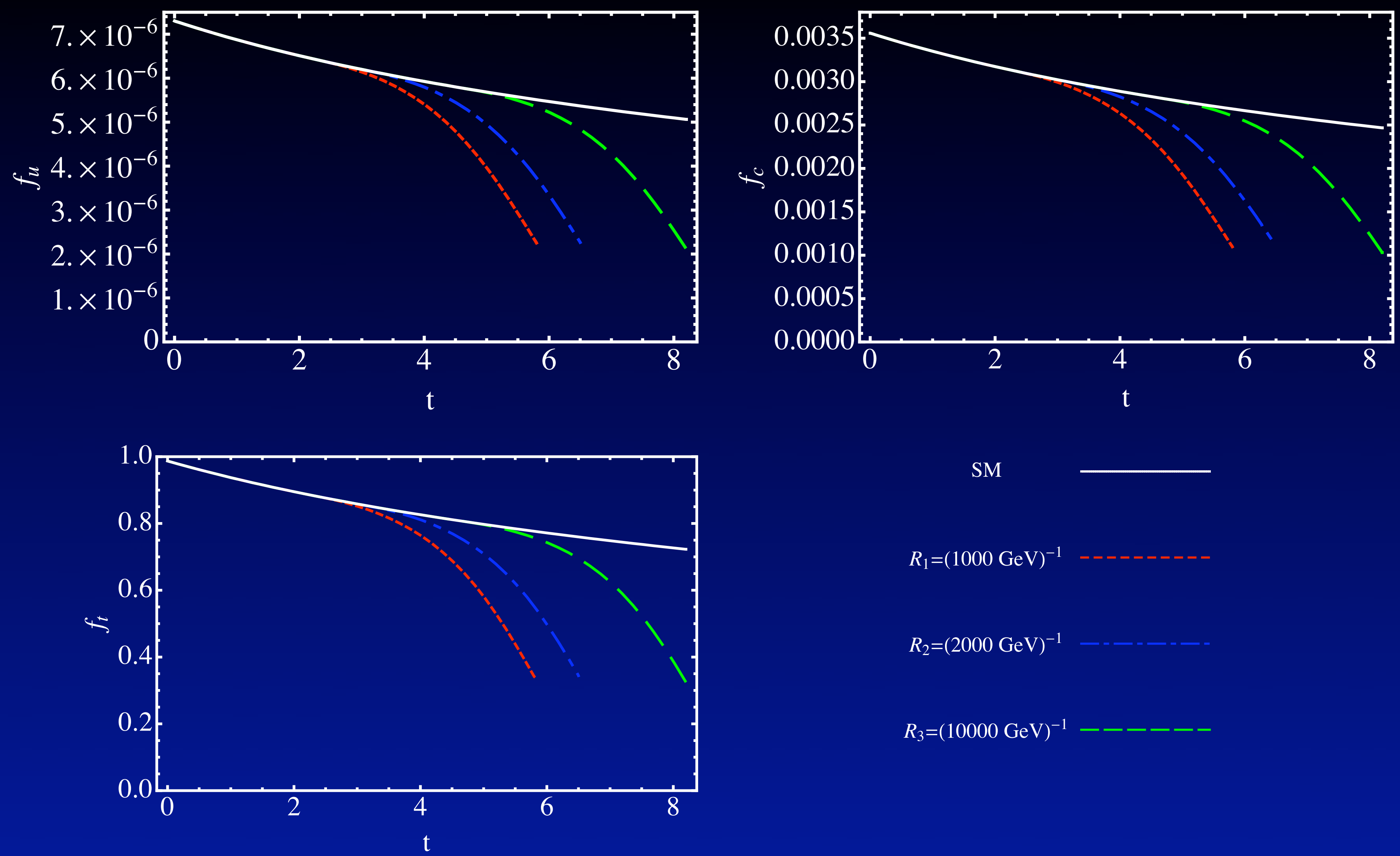
Where we can now also obtain the RGE of the elements of the CKM matrix. Note that we can see from these equations that one needs to know the running of the Yukawas to obtain the evolution of the CKM matrix, where beyond the R^{-1} threshold:

$$16\pi^2 \frac{dV_{ik}}{dt} = -\frac{3}{2} S(t) \left[\sum_{m,j \neq i} \frac{f_i^2 + f_j^2}{f_i^2 - f_j^2} h_m^2 V_{im} V_{jm}^* V_{jk} + \sum_{m,j \neq i} \frac{h_k^2 + h_m^2}{h_k^2 - h_m^2} f_j^2 V_{jm}^* V_{jk} V_{im} \right] .$$

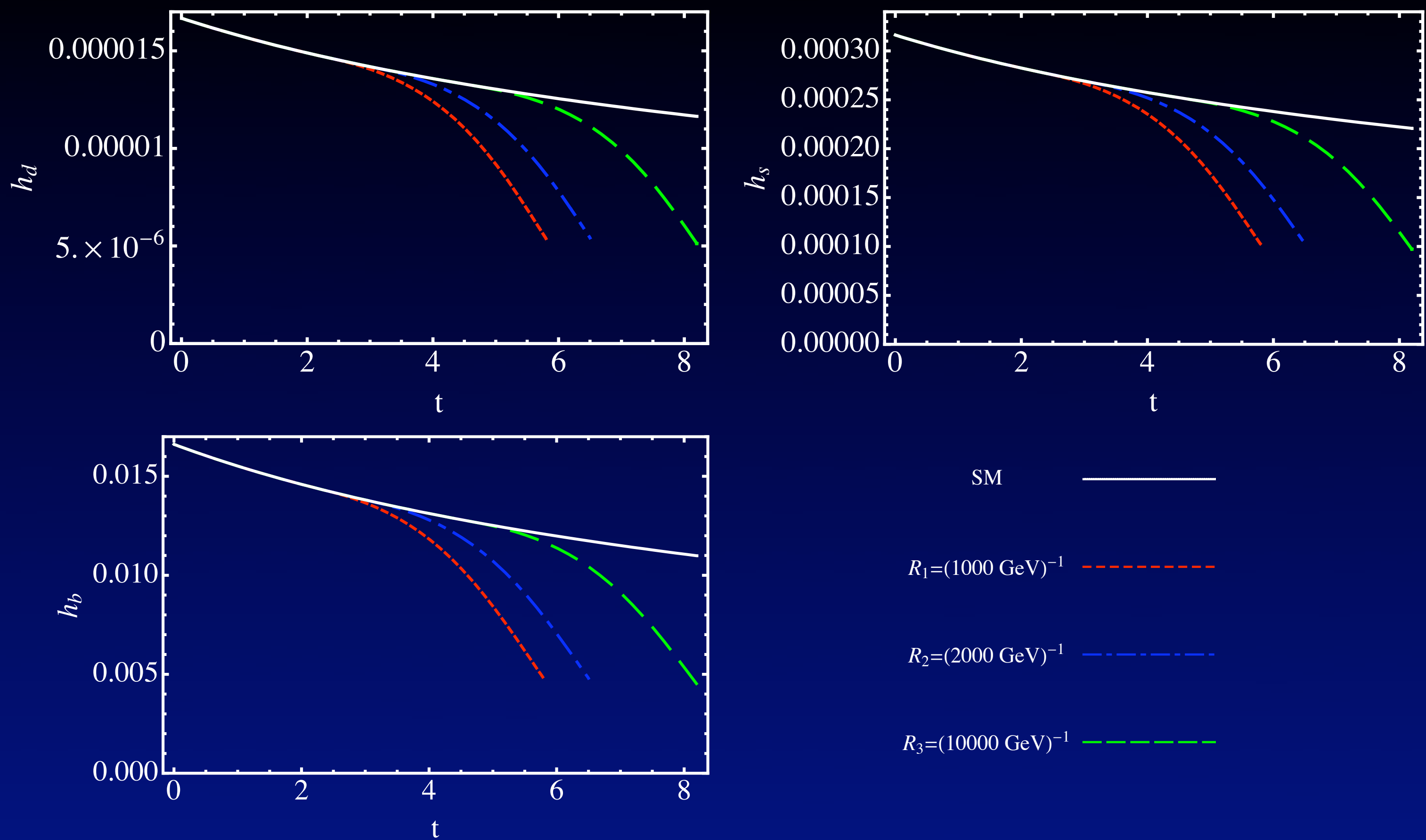
Properties of the RGE evolution

From this full set of one-loop coupled RGE for the Yukawa couplings and the CKM matrix, together with those for the gauge coupling equations, one can obtain the renormalisation group flow of all observables related to up- and down-quark masses and the CKM matrix elements

We assume the fundamental scale is not far from the range of the LHC scale, and set the compactification radii to be $R^{-1} = 1$ TeV, 2 TeV, and 10 TeV. The extra dimensions naturally lead to gauge coupling unification at an intermediate mass scale.



The Yukawa couplings evolve in the usual logarithmic fashion when the energy is below 1 TeV, 2 TeV, and 10 TeV for the three different cases



However, once the first KK threshold is reached, the contributions from the KK states become more and more significant, and the running of the Yukawa couplings, or more precisely, the one-loop KK corrected effective four dimensional Yukawa couplings, begins to deviate from their normal orbits and start to evolve faster and faster

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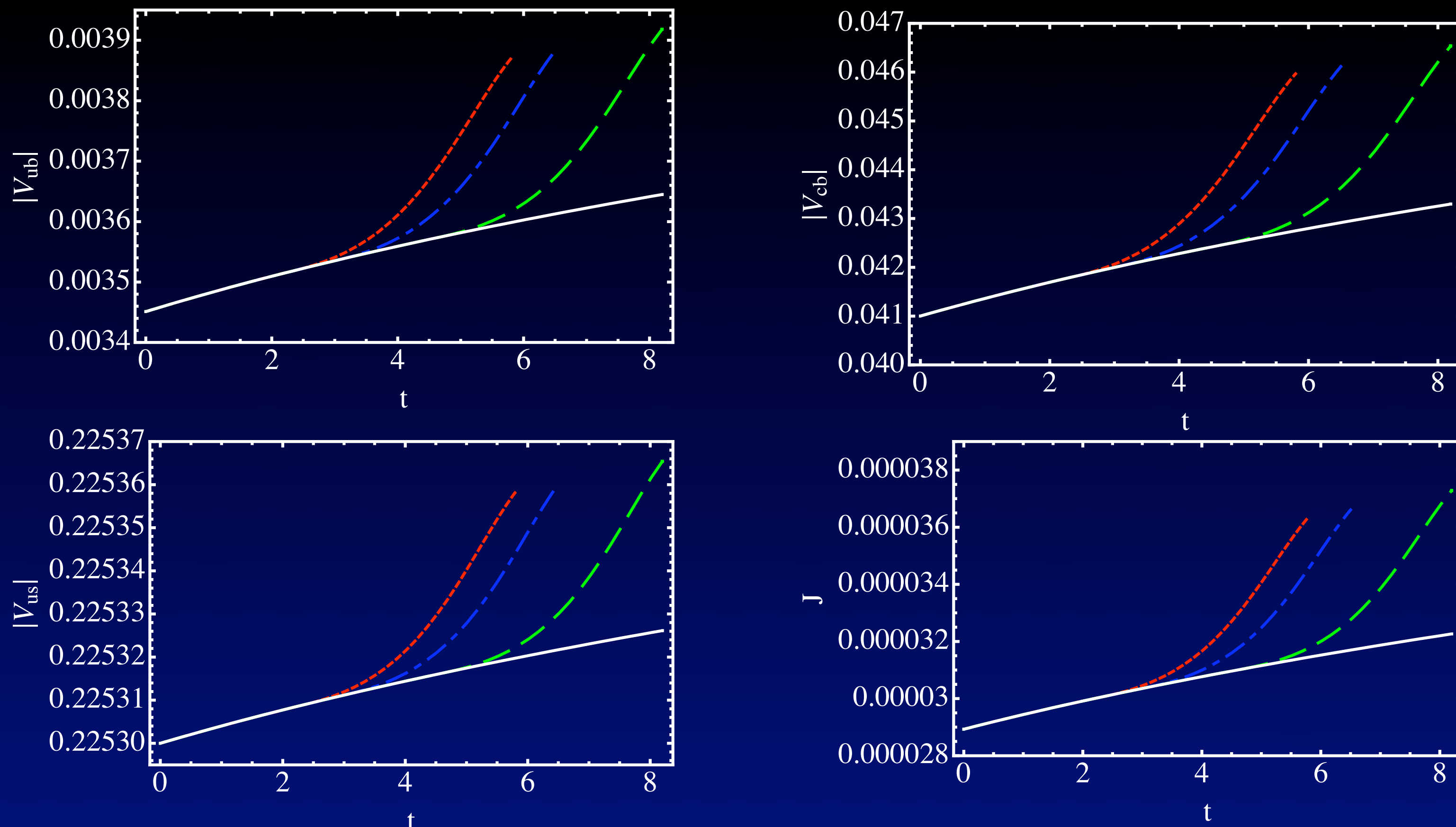
For the compactification radius $R^{-1} = 1$ TeV, the Yukawa couplings evolve faster than the other two, reaching its maximum value at the unification scale around 30 TeV, after that point their evolution will "blow-up" due to the faster running of the gauge couplings and new physics would come into play

We also observe that the Yukawa couplings are quickly evolving to zero, however, a satisfactory unification of these seems to still be lacking in this scenario

The first generation f_u and h_d are driven to the order of 10^{-6}, while the f_t, the heaviest one, is driven to the order of 10^{-1}

We next turn our attention to the quark flavour mixings, where we choose to look at $|V_{ub}|$, $|V_{cb}|$, $|V_{us}|$ and J (the Jarlskog rephasing invariant) as the four independent parameters of V_{CKM}



Take the initial values $|V_{ub}| = 0.00347$, $|V_{cb}| = 0.0410$, $|V_{us}| = 0.2253$ and $J = 2.91 \times 10^{-5}$, we observe from these plots the following;
the CKM matrix elements $V_{ub} \simeq \theta_{13} e^{-i\delta}$, $V_{cb} \simeq \theta_{23}$, can be used to observe the mixing angles θ_{13} and θ_{23} and that they increase with the energy scale; the variation rate becoming faster once the KK threshold is passed

For mixings related to the third family, the UED effects become sizable and the mixing angles θ_{13} and θ_{23} change at a level of 15% between m_Z and the unification scale, in contrast with the SM, in which the angles only rise by around 5% at similar energy scales.

By contrast, the variation of the Cabibbo angle appears to be the least sensitive

However, for the parameter J, the characteristic parameter for the CP non-conservation effects, its variation becomes very significant. The larger the value of the compactification radius R, the faster J evolves to reach its maximum

Conclusions

In this talk we investigated the consequences of the UED model on the gauge and Yukawa couplings, as well as the CKM matrix elements evolution

The energy dependence of the first two generations is very weak, and qualitatively different from mixing behaviours involving the third generation

That is, while the evoluation of the Cabibbo angle is tiny, the elements $|V_{ub}|$ and $|V_{cb}|$ increase sizably, relative deviations can be up to 15% in the whole range.

As for the energy scaling of J, the variation here can be raised to more than 30%