The tsallis Distribution at the LHC

J. Cleymans
University of Cape Town, South Africa

Discovery Physics at the LHC
3 - 7 December 2012
Kruger Gate, South Africa

Transverse Momentum Distribution

STAR collaboration, B.I. Abelev at al.
arXiv:nucl-ex/0607033; Phys. Rev. C75, 064901 (2007)
PHENIX collaboration, A. Adare et al.
Phys. Rev. **C83**, 064903 (2011)
ALICE collaboration, K. Aamodt et al.
arXiv:1101.4110 [hep-ex]
CMS collaboration, V. Khachatryan et al.
arXiv: 1102.4282 [hep-ex]
ATLAS collaboration, G. Aad et al.
New J. Phys. **13** (2011) 053033.

Transverse Momentum Distribution

STAR, PHENIX, ALICE, CMS, ATLAS use:

$$\frac{d^2N}{dp_t dy} = p_t \times \frac{dN}{dy} \frac{(n-1)(n-2)}{nC(nC + m_0(n-2))} \left(1 + \frac{m_t - m_0}{nC}\right)^{-n}$$

What is the connection with the Tsallis distribution?
Also, the physical significance of the parameters n and C has never been discussed by STAR, PHENIX, ALICE, ATLAS, CMS.

Tsallis Distribution

Possible generalization of Boltzmann-Gibbs statistics

Constantino Tsallis
Rio de janeiro, CBPF
J. Stat. Phys. 52 (1988) 479-487

Citations: 1 389
However:
Citations in HEP: 403

ISSN 0029-3865

MINISTÉRIO DA CIÊNCIA E TECNOLOGIA

CNPq

CBPF

CENTRO BRASILEIRO DE PESQUISAS FÍSICAS

Notas de Física

CBPF-NF-062/87

POSSIBLE GENERALIZATION OF BOLTZMANN-GIBBS STATISTICS

by

Constantino TSALLIS

RIO DE JANEIRO
1987

Multifractal concepts and structures are quickly acquiring importance in many active areas (e.g., non-linear dynamical systems, growth models, commensurate/incommensurate structures). This is due to their utility as well as to their elegance. Within this framework, the quantity which is normally scaled is p_i^q, where p_i is the probability associated to an event and q any real number [1]. We shall use this quantity to generalize the standard expression of the entropy S in information theory, namely $S = -k \sum_{i=1}^{W} p_i \ln p_i$, where $W \in \mathbb{N}$ is the total number of possible (microscopic) configurations and $\{p_i\}$ the associated probabilities. We *postulate* for the entropy

$$S_q \equiv k \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1} \qquad (q \in \mathbb{R}) \qquad (1)$$

where k is a conventional positive constant and $\sum_{i=1}^{W} p_i = 1$. We immediately verify that

$$S_1 \equiv \lim_{q\to 1} S_q = k \lim_{q\to 1} \frac{1 - \sum_{i=1}^{W} p_i e^{(q-1)\ln p_i}}{q-1} = -k \sum_{i=1}^{W} p_i \ln p_i \qquad (1')$$

where we have used the replica-trick type of expansion. We illustrate definition (1) in Fig. 1. S_q may be rewritten as follows:

$$S_q = \frac{k}{q-1} \sum_{i=1}^{W} p_i \left(1 - p_i^{q-1}\right) \qquad (2)$$

For high energy physics a consistent form of Tsallis statistics for the particle number, energy density and pressure is given by

$$N = gV \int \frac{d^3p}{(2\pi)^3} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}, \tag{1}$$

$$\epsilon = g \int \frac{d^3p}{(2\pi)^3} E \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}, \tag{2}$$

$$P = g \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}. \tag{3}$$

where T and μ are the temperature and the chemical potential, V is the volume and g is the degeneracy factor. The Tsallis distribution introduces a new parameter q which for transverse momentum spectra is always close to 1, The expressions for the energy density and the pressure are thermodynamically consistent, e.g. it can be easily shown that relations of the type

$$N = V \left. \frac{\partial P}{\partial \mu} \right|_{T}, \qquad (4)$$

are satisfied.

In the Tsallis distribution the total number of particles is given by:

$$N = gV \int \frac{d^3p}{(2\pi)^3} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{q/(1-q)} ,$$

The corresponding momentum distribution is given by

$$E\frac{dN}{d^3p} = gVE\frac{1}{(2\pi)^3} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{q/(1-q)} ,$$

which, in terms of the rapidity and transverse mass variables, becomes (for $\mu = 0$)

$$\left.\frac{d^2N}{dp_T\, dy}\right|_{y=0} = gV\frac{p_T m_T}{(2\pi)^2} \left[1 + (q-1)\frac{m_T}{T}\right]^{q/(1-q)} ,$$

J.C. and D. Worku, J. Phys. G **G39** (2012) 025006; arXiv:1203.4343[hep-ph].

Rewrite the Tsallis distribution using

$$[1+(q-1)x]^{1/(1-q)} = \exp\left(\frac{1}{1-q}\ln\left[1+(q-1)x\right]\right),$$

and consider the limit $q \to 1$

$$\begin{aligned}\lim_{q\to 1} \quad & [1+(q-1)x]^{1/(1-q)} \\ &= \exp\frac{1}{(1-q)}(q-1)x \\ &= \exp\left(-x\right), \end{aligned} \tag{5}$$

The Tsallis distribution reduces to the Boltzmann distribution in the limit where $q \to 1$

$$\lim_{q\to 1}\frac{d^2N}{dp_T\,dy} = gV\frac{p_T m_T\cosh y}{(2\pi)^2}\exp\left(-\frac{m_T\cosh y-\mu}{T}\right). \tag{6}$$

In all cases q is close to one, typically between 1.05 and 1.2.

Comparison of Tsallis with STAR, ALICE, CMS distributions

$$\frac{\mathrm{d}^2 N}{\mathrm{d}p_\mathrm{t}\mathrm{d}y} = gV\frac{p_T m_T}{(2\pi)^2}\left[1+(q-1)\frac{m_T}{T}\right]^{q/(1-q)}, \quad (7)$$

$$\frac{\mathrm{d}^2 N}{\mathrm{d}p_\mathrm{t}\mathrm{d}y} = p_\mathrm{t}\times\frac{\mathrm{d}N}{\mathrm{d}y}\frac{(n-1)(n-2)}{nC(nC+m_0(n-2))}\left[1+\frac{m_\mathrm{t}-m_0}{nC}\right]^{-n} \quad (8)$$

$$n \to \frac{q}{q-1}$$

$$nC \to \frac{T}{q-1}\frac{m_T-m_0}{m_T}$$

After this substitution one obtains

$$\frac{d^2N}{dp_T\,dy} = p_T\frac{\mathrm{d}N}{\mathrm{d}y}\frac{(n-1)(n-2)}{nC(nC+m_0(n-2))}\left[\frac{T}{T+m_0(q-1)}\right]^{-q/(q-1)}\left[1+(q-1)\frac{m_T}{T}\right]^{-q/(q-1)}. \qquad (9)$$

to be compared with

$$\frac{\mathrm{d}^2N}{\mathrm{d}p_\mathrm{t}\mathrm{d}y} = gV\frac{p_T m_T}{(2\pi)^2}\left[1+(q-1)\frac{m_T}{T}\right]^{-q/(q-1)}, \qquad (10)$$

Only a factor of m_T differs! However, m_0 shouldn't appear as it destroys m_T scaling. The inclusions of the factor m_T leads to a more consistent interpretation of the variables q and T. In particular, no clear pattern emerges for the values of n and C while an interesting regularity is obtained for q and T.

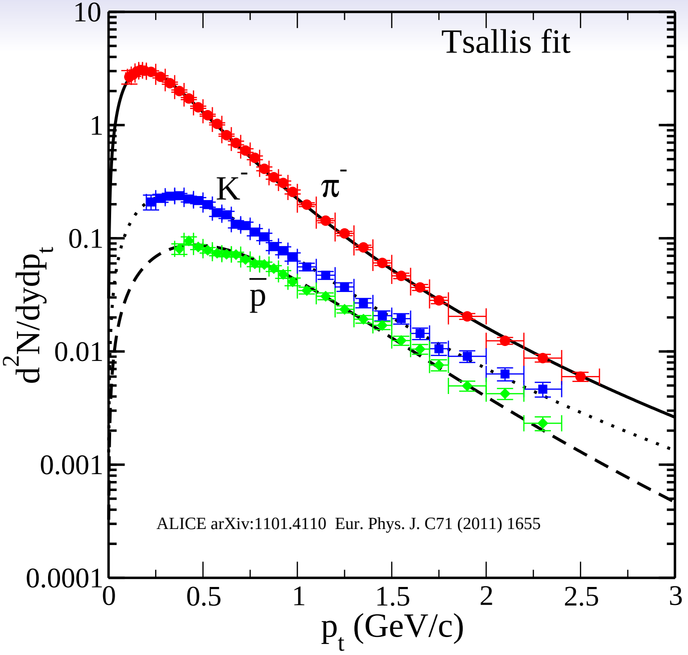
Interpretation of Tsallis Parameter q

G. Wilk and Z. Wlodarczyk, Phys. Rev. Lett. 84 (2000) 2770

$$\left\langle \frac{1}{T_B} \right\rangle = \frac{1}{T}$$

and also

$$\frac{\left\langle \left(\frac{1}{T_B}\right)^2 \right\rangle - \left\langle \frac{1}{T_B} \right\rangle^2}{\left\langle \frac{1}{T_B} \right\rangle^2} = q - 1$$

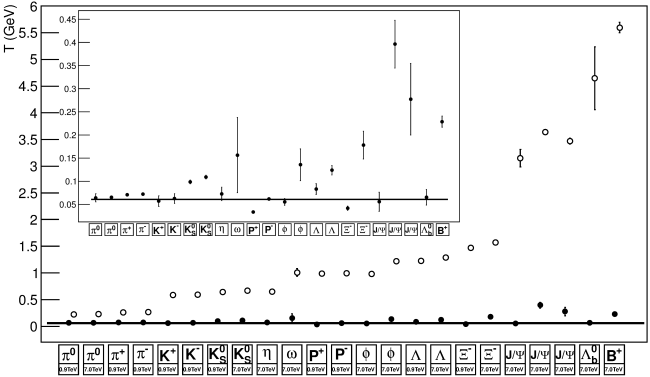


Tsallis fit
K^-
π^-
$\bar{p}$
$d^2N/dydp_t$
p_t (GeV/c)
ALICE arXiv:1101.4110 Eur. Phys. J. C71 (2011) 1655
10
1
0.1
0.01
0.001
0.0001
0
0.5
1
1.5
2
2.5
3

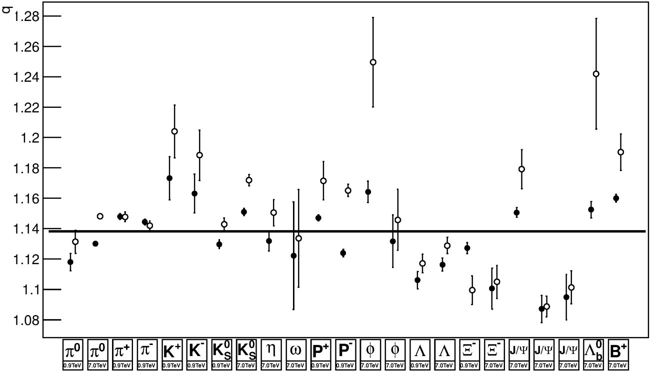
$p-p$ 900 GeV		
Particle	q	T
π^+	1.154 $\pm$0.036	0.0682 $\pm$0.0026
π^-	1.146 $\pm$0.036	0.0704 $\pm$ 0.0027
K^+	1.158 $\pm$0.142	0.0690 $\pm$0.0223
K^-	1.157 $\pm$0.139	0.0681 $\pm$ 0.0217
K_S^0	1.134 $\pm$0.079	0.0923 $\pm$0.0139
p	1.107 $\pm$0.147	0.0730 $\pm$ 0.0425
$\bar{p}$	1.106 $\pm$0.158	0.0764 $\pm$0.0464
Λ	1.114 $\pm$0.047	0.0698 $\pm$ 0.0148
Ξ^-	1.110 $\pm$0.218	0.0440 $\pm$ 0.0752

Table: Fitted values of the T and q parameters measured in $p-p$ collisions by the ALICE and CMS collaborations using the Tsallis form for the momentum distribution.

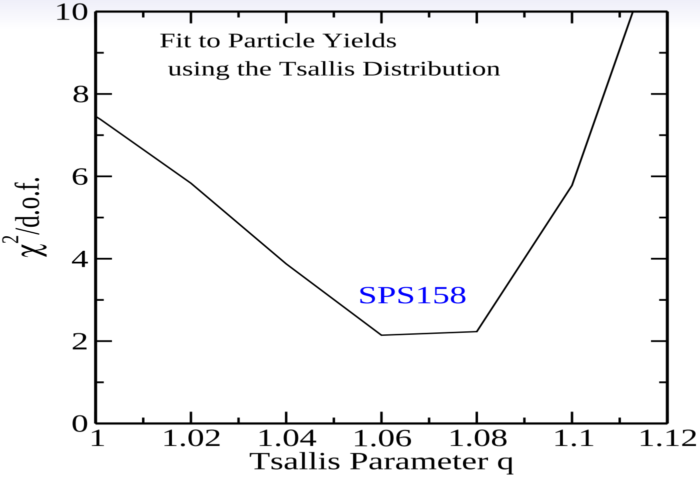
$p-p$ 900 GeV		
Particle	T Tsallis vs C ALICE (MeV)	q
π^+	70 (126)	1.147
K^+	70 (160)	1.156
p	73 (196)	1.110



L. Marques, E. Andrade-II and A. Deppman e-Print: arXiv:1210.1725 [hep-ph]
J.C. and D. Worku e-Print: arXiv:1110.5526 [hep-ph]



L. Marques, E. Andrade-II and A. Deppman e-Print: arXiv:1210.1725 [hep-ph]



J. C., G. Hamar, P. Levai, S. Wheaton
Journal of Physics **G 36** (2009) 064018.

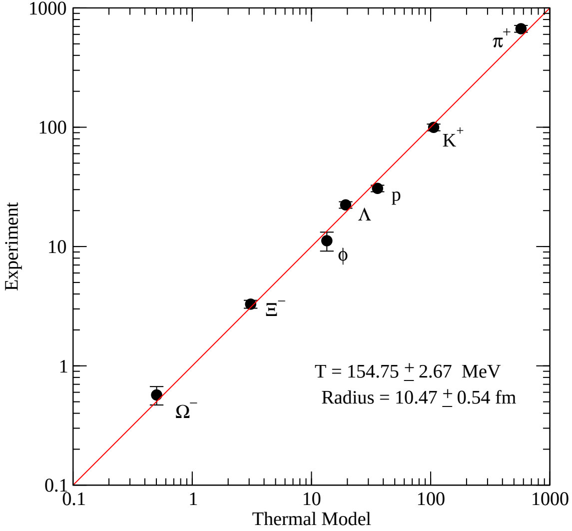
Conclusion:

Use

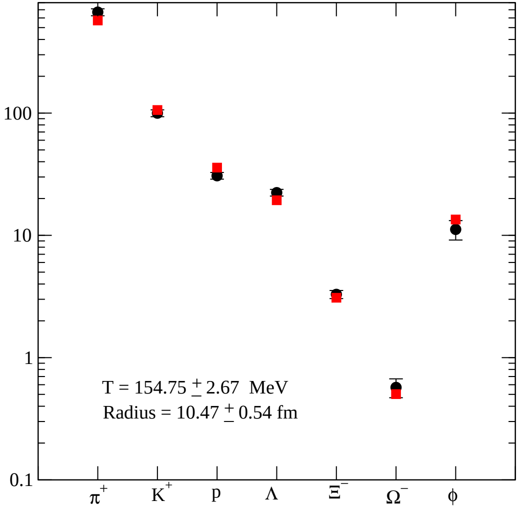
$$\frac{\mathrm{d}^2 N}{\mathrm{d}p_\mathrm{t}\mathrm{d}y} = gV\frac{p_T m_T}{(2\pi)^2}\left[1+(q-1)\frac{m_T}{T}\right]^{q/(1-q)}, \tag{11}$$

instead of

$$\frac{\mathrm{d}^2 N}{\mathrm{d}p_\mathrm{t}\mathrm{d}y} = p_\mathrm{t} \times \frac{\mathrm{d}N}{\mathrm{d}y}\frac{(n-1)(n-2)}{nC(nC+m_0(n-2))}\left[1+\frac{m_\mathrm{t}-m_0}{nC}\right]^{-n} \tag{12}$$



1000
100
10
1
0.1
0.1
1
10
100
1000
Experiment
Thermal Model
π+
K+
p
Λ
ϕ
Ξ−
Ω−
T = 154.75 ± 2.67 MeV
Radius = 10.47 ± 0.54 fm


100
10
1
0.1
T = 154.75 ± 2.67 MeV
Radius = 10.47 ± 0.54 fm
π+
K+
p
Λ
Ξ−
Ω−
ϕ