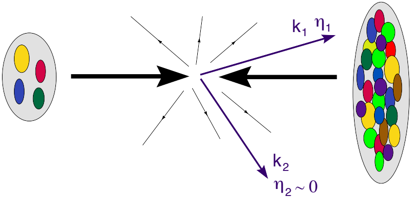
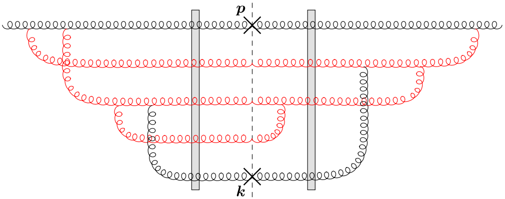
JIMWLK evolution for multi-particle production in Langevin form

Edmond Iancu

IPhT Saclay & CNRS

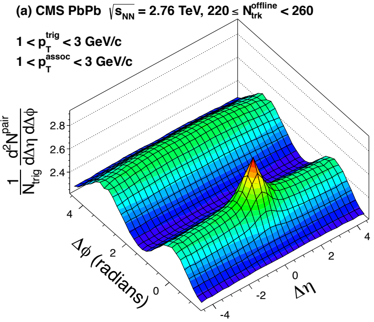
collab. with D.N. Triantafyllopoulos, arXiv: 1307.1559 (JHEP)





Motivation: The ridge in pp and pA collisions

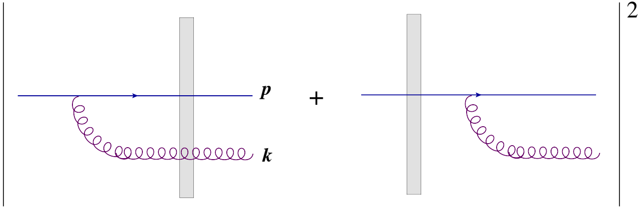
- Di–hadron correlations long–ranged in $\Delta\eta$ & narrow in $\Delta\phi$
- Well known and supposedly understood in AA, but ...
  also seen in p+p and p+A events with high multiplicity



- Final–state interactions might play a role, but to properly answer this one needs to first understand the correlations from the initial state

Quark–gluon production at forward rapidities (1)

- A quark from the proton emits a gluon while scattering off the nucleus



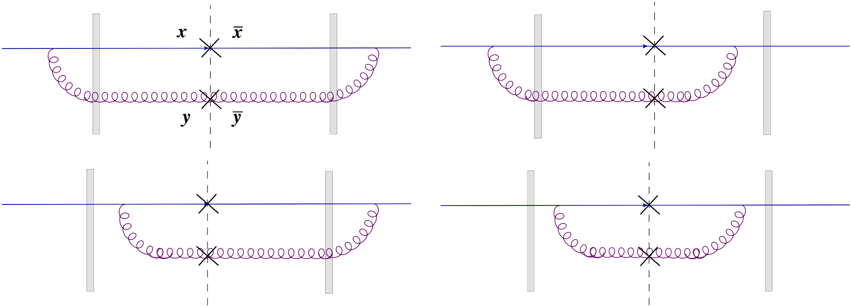
- The quark and the gluon have similar rapidities: $\alpha_s \Delta Y \ll 1$
- The prototype for the CGC calculations of di-hadron production and azimuthal correlations in p+Pb, or d+Au

  (Albacete and Marquet, '10; Dominguez, Marquet, Xiao, Yuan, '11; Stasto, Xiao, Yuan, '11; Lappi and Mäntysaari, '12, E.I. and Laidet, '13)

- The 'tree–level' for the high–energy evolution I shall later discuss

Quark–gluon production at forward rapidities (2)

- Cross–section: direct amplitude × complex conjugate amplitude



▷ emissions in the DA (at $\boldsymbol{x}$, $\boldsymbol{y}$), absorptions in the CCA (at $\bar{\boldsymbol{x}}$, $\bar{\boldsymbol{y}}$)

▷ Fourier transforms: $\boldsymbol{y}-\bar{\boldsymbol{y}} \to \boldsymbol{k}$ & $\boldsymbol{x}-\bar{\boldsymbol{x}} \to \boldsymbol{p}$

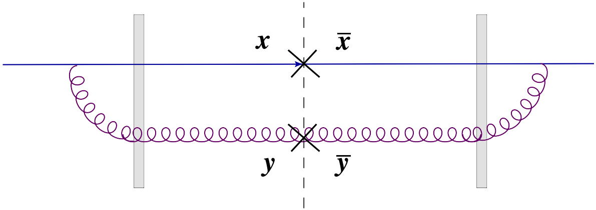
- Each parton that crosses the shockwave acquires a Wilson line

$$U^{\dagger}(\boldsymbol{x}) = \mathrm{T}\exp\left\{ \mathrm{i}g \int \mathrm{d}x^{+} A_a^{-}(x^{+}, \boldsymbol{x})\, T^{a} \right\}$$

- $A_a^{-} \propto \delta(x^{+})$: the (random) color field in the target

Quark–gluon production at forward rapidities (3)

- Most complicated piece: initial–state emissions in both DA & CCA



$$\alpha_s C_F \frac{(\boldsymbol{y}-\boldsymbol{x})^i}{(\boldsymbol{y}-\boldsymbol{x})^2} \frac{(\bar{\boldsymbol{y}}-\bar{\boldsymbol{x}})^i}{(\bar{\boldsymbol{y}}-\bar{\boldsymbol{x}})^2} \times \frac{2}{N_c^2-1} \Big\langle \big(U_{\bar{\boldsymbol{y}}} U_{\boldsymbol{y}}^\dagger\big)^{ab} \mathrm{Tr}\big[V_{\boldsymbol{x}}^\dagger t^b t^a V_{\bar{\boldsymbol{x}}}\big] \Big\rangle_Y$$

  ▷ emission amplitude for a soft gluon at $\boldsymbol{y}$ by a source at $\boldsymbol{x}$ $(i = 1, 2)$

  ▷ Wilson line correlator averaged over the target field

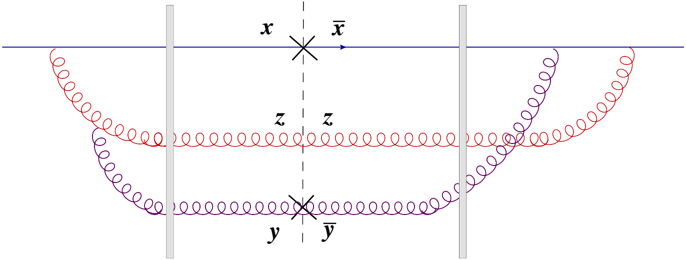
- The most complicated step: the target average at rapidity Y (CGC)

  ▷ Balitsky–JIMWLK evolution for the Wilson line correlators

  ▷ at large N_c : S–matrices for color dipoles and quadrupoles

Forward–central production (1)

- Quark–gluon rapidity difference $\alpha_s \Delta Y \gtrsim 1 \Longrightarrow$ high energy evolution

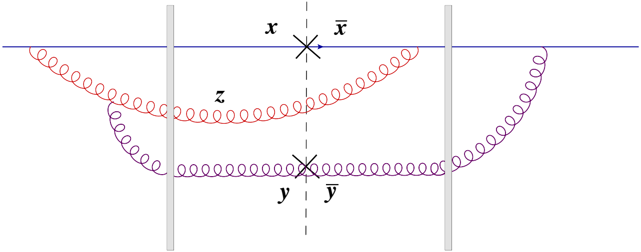


- The evolution gluon at z is not measured $\Longrightarrow$ its interactions with the target cancel between DA and CCA (by unitarity) : $U_z U_z^\dagger = 1$
- 'Initial state evolution' (emission prior to collision) in both DA & CCA

  ▷ no conceptual difficulties by itself

Forward–central production (2)

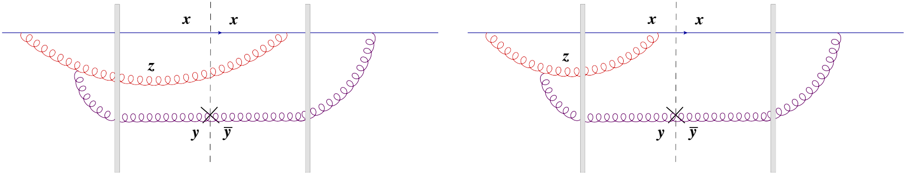
- 'Final state evolution' : emission of a 'red' gluon after the collision
  - ▷ the interaction of the 'red' gluon counts for the final result: $U_{\boldsymbol{z}}^{\dagger}$



- BFKL evolution in a strong background field
  - ▷ both the measured partons & the evolution ones 'know' about the target
- No factorization of the evolution between 'projectile' and 'target'

Forward–central production (2)

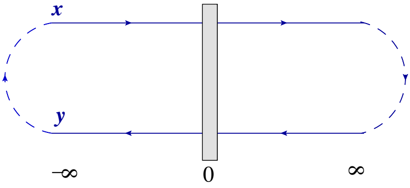
- 'Final state evolution' : emission of a 'red' gluon after the collision
  - ▷ the interaction of the 'red' gluon counts for the final result: $U_{\boldsymbol{z}}^{\dagger}$



- BFKL evolution in a strong background field
  - ▷ both the measured partons & the evolution ones 'know' about the target
- No factorization of the evolution between 'projectile' and 'target'
- k_T–factorization recovered if the quark is not measured $(\boldsymbol{x} = \bar{\boldsymbol{x}})$
  - ▷ the effects of 'final state evolution' cancel out between DA and CCA
  - ▷ BFKL evolution of the gluon distribution in the quark projectile

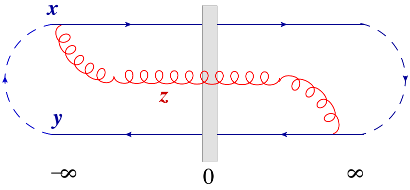
The Balitsky–JIMWLK evolution

- BFKL evolution in a strong background field: scattering amplitudes
- Example : the dipole S–matrix $\langle S_{\boldsymbol{xy}} \rangle_Y$



$$\langle S_{\boldsymbol{xy}} \rangle_Y = \int [DU] \underbrace{W_Y[U]}_{\text{CGC}} \underbrace{\frac{1}{N_c} \mathrm{tr}\big[V_{\boldsymbol{y}} V_{\boldsymbol{x}}^\dagger\big]}_{\text{operator}}$$

- BFKL evolution in a strong background field: scattering amplitudes
- Example : the dipole S–matrix $\langle S_{\boldsymbol{xy}} \rangle_Y$



▷ both initial–state and final–state emissions

▷ the evolution ('red') gluons can interact as well

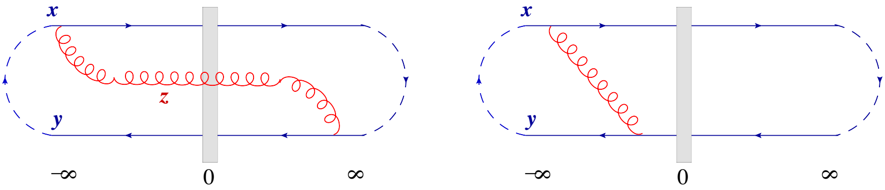
- This evolution is described by the JIMWLK Hamiltonian

$$\frac{\partial}{\partial Y} S_{\boldsymbol{xy}} = H_{\text{JIMWLK}} S_{\boldsymbol{xy}}$$

▷ the change in the scattering operator (projectile) for a fixed target field

The Balitsky–JIMWLK evolution

- BFKL evolution in a strong background field: scattering amplitudes
- Example : the dipole S–matrix $\langle S_{\boldsymbol{xy}} \rangle_Y$



▷ both initial–state and final–state emissions

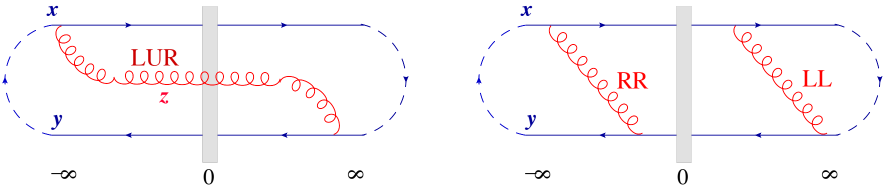
▷ the evolution ('red') gluons can interact as well

- This evolution is described by the JIMWLK Hamiltonian

$$\frac{\partial}{\partial Y} \langle S_{\boldsymbol{xy}} \rangle_Y = \langle H_{\text{JIMWLK}} S_{\boldsymbol{xy}} \rangle_Y$$

- After averaging over the target $\Longrightarrow$ Balitsky–JIMWLK equations

The JIMWLK Hamiltonian



$$H_{\text{JIMWLK}} = \frac{1}{8\pi^3}\int_{\boldsymbol{uvz}} \frac{(\boldsymbol{z}-\boldsymbol{u})^i}{(\boldsymbol{z}-\boldsymbol{u})^2}\,\frac{(\boldsymbol{z}-\boldsymbol{v})^i}{(\boldsymbol{z}-\boldsymbol{v})^2}\left[L^a_{\boldsymbol{u}} - U^{\dagger ab}_{\boldsymbol{z}} R^b_{\boldsymbol{u}}\right]\left[L^a_{\boldsymbol{v}} - U^{\dagger ac}_{\boldsymbol{z}} R^c_{\boldsymbol{v}}\right]$$

- 'Right'/'Left' Lie derivatives: gluon emissions before/after scattering

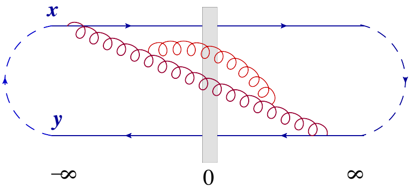
$$R^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{x}} = \mathrm{i}g\delta_{\boldsymbol{ux}} U^\dagger_{\boldsymbol{x}} T^a, \qquad L^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{x}} = \mathrm{i}g\delta_{\boldsymbol{ux}} T^a U^\dagger_{\boldsymbol{x}}$$

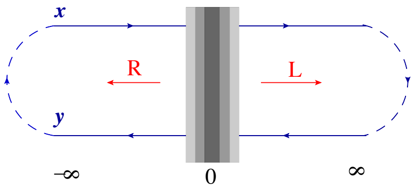
  ▷ the color charge density operators ($L^a_{\boldsymbol{u}} = U^{\dagger ab}_{\boldsymbol{u}} R^b_{\boldsymbol{u}}$)

- When acting on the dipole & for large N_c ⇒ Balitsky-Kovchegov eqn

  ▷ finite N_c : infinite hierarchy of coupled equations

  ▷ even at large N_c, the quadrupole equation is extremely complicated





- The ‘red’ gluons can also be viewed as a result of target evolution

$$\frac{\partial W_Y[U]}{\partial Y} = H_{\rm JIMWLK} W_Y[U] \qquad \text{(original JIMWLK)}$$

- R/L derivatives add new layers of target field at larger values of $|x^+|$
- The new fields are random (quantum flucts) ⇒ stochastic process

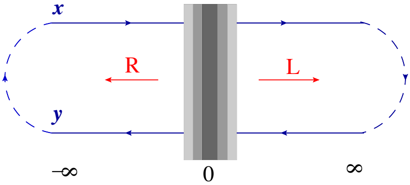
  ▷ Langevin equation *(Blaizot, E.I., Weigert, 2003)*

- Well suited for numerics *(Weigert & Rummukainen; Lappi; Schenke et al)*

  ▷ see the next talk by Tuomas Lappi !

JIMWLK evolution in Langevin form (2)

- A random walk in the space of Wilson lines
  - ▷ discretize the rapidity interval between projectile and target $Y = N\epsilon$



$$\langle S_{\boldsymbol{xy}}\rangle_Y = \frac{1}{N_c}\left\langle \mathrm{tr}\left[U_{N,\boldsymbol{y}}U^\dagger_{N,\boldsymbol{x}}\right]\right\rangle_\nu$$

$$U^\dagger_{n,\boldsymbol{x}} = \mathrm{e}^{\mathrm{i}\varepsilon g\alpha^n_{L,\boldsymbol{x}}}\, U^\dagger_{n-1,\boldsymbol{x}}\, \mathrm{e}^{-\mathrm{i}\varepsilon g\alpha^n_{R,\boldsymbol{x}}}$$

$$\alpha^n_{L,\boldsymbol{x}} = \int_{\boldsymbol{z}} \frac{x^i - z^i}{(\boldsymbol{x}-\boldsymbol{z})^2}\,\nu^{ia}_{n,\boldsymbol{z}}T^a\,, \qquad \alpha^n_{R,\boldsymbol{x}} = \int_{\boldsymbol{z}} \frac{x^i - z^i}{(\boldsymbol{x}-\boldsymbol{z})^2}\,U^{ab}_{n-1,\boldsymbol{z}}\,\nu^{ib}_{n,\boldsymbol{z}}T^a$$

- 'White' noise $\nu^{ia}_{n,\boldsymbol{z}}$: color charge of the evolution gluon

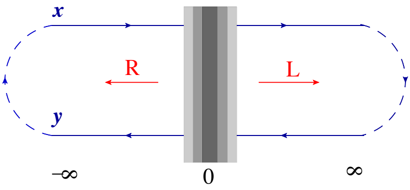
$$\left\langle \nu^{ia}_{m,\boldsymbol{x}}\,\nu^{jb}_{n,\boldsymbol{y}}\right\rangle = \frac{1}{\varepsilon}\,\delta_{mn}\delta^{ij}\delta^{ab}\delta_{\boldsymbol{xy}}$$

- Multiplicative noise $U^{ab}_{n-1,\boldsymbol{z}}\nu^{ib}_{n,\boldsymbol{z}} \Longrightarrow$ BFKL cascade

JIMWLK evolution in Langevin form (2)

- A random walk in the space of Wilson lines

  ▷ discretize the rapidity interval between projectile and target $Y = N\epsilon$



$$\langle S_{\boldsymbol{xy}} \rangle_Y = \frac{1}{N_c} \left\langle \mathrm{tr} \left[U_{N,\boldsymbol{y}} U^\dagger_{N,\boldsymbol{x}} \right] \right\rangle_\nu$$

$$U^\dagger_{n,\boldsymbol{x}} = \mathrm{e}^{\mathrm{i}\varepsilon g \alpha^n_{L,\boldsymbol{x}}} \, U^\dagger_{n-1,\boldsymbol{x}} \, \mathrm{e}^{-\mathrm{i}\varepsilon g \alpha^n_{R,\boldsymbol{x}}}$$

$$\alpha^n_{L,\boldsymbol{x}} = \int_{\boldsymbol{z}} \frac{x^i - z^i}{(\boldsymbol{x}-\boldsymbol{z})^2} \, \nu^{ia}_{n,\boldsymbol{z}} T^a \,, \qquad \alpha^n_{R,\boldsymbol{x}} = \int_{\boldsymbol{z}} \frac{x^i - z^i}{(\boldsymbol{x}-\boldsymbol{z})^2} \, U^{ab}_{n-1,\boldsymbol{z}} \, \nu^{ib}_{n,\boldsymbol{z}} T^a$$

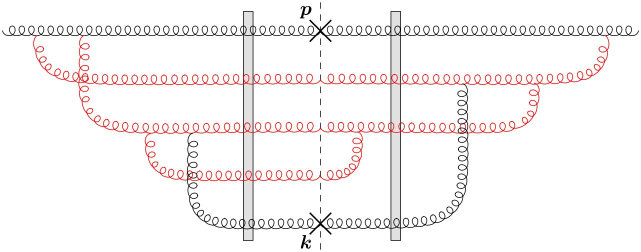
- 'White' noise $\nu^{ia}_{n,\boldsymbol{z}}$: color charge of the evolution gluon

$$\left\langle \nu^{ia}_{m,\boldsymbol{x}} \, \nu^{jb}_{n,\boldsymbol{y}} \right\rangle = \frac{1}{\varepsilon} \, \delta_{mn} \delta^{ij} \delta^{ab} \delta_{\boldsymbol{xy}}$$

- Initial condition $U^\dagger_{0,\boldsymbol{x}}$ randomly selected according to the MV model

Generalization to particle production in p+A

- 2 gluon production with large rapidity separation $\Delta Y \gtrsim 1/\alpha_s$
- How to systematically generate all such graphs ?



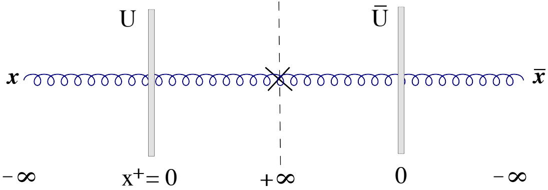
▷ build the wavefunction of the 'fast' gluon via evolution over ΔY and in the presence of the target background field (Wilson line U)

▷ emit the 'slow' gluon from any of the gluons in the wavefunction

▷ average over the target field with the weight function $W_{Y_A}[U]$, where $Y_A = Y - \Delta Y$

Wavefunction squared

- Generating functional for soft gluon emissions (resolved or not)
- Gluon emission $\Longleftrightarrow$ (R/L) Lie derivative w.r.t. U
- One needs to distinguish between the DA and the CCA : U, $\bar{U}$



Bare gluon

$$S_{12}(\boldsymbol{x}\bar{\boldsymbol{x}}) = \frac{1}{N_c^2 - 1}\,\mathrm{Tr}\big[\bar{U}_{\bar{\boldsymbol{x}}} U^\dagger_{\boldsymbol{x}}\big]$$

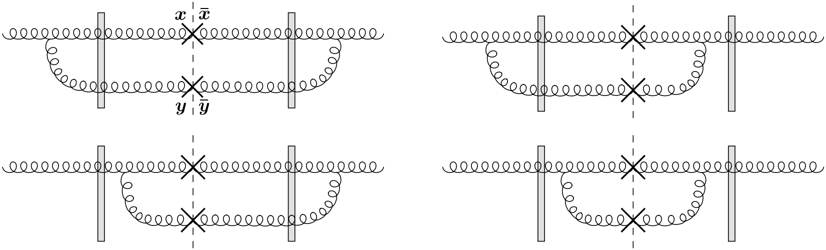
- A physical gluon $\longleftrightarrow$ a mathematical dipole
- A physical dipole $\longleftrightarrow$ a mathematical quadrupole, etc
- U, $\bar{U}$: arguments of the generating functional

  $\triangleright$ one sets $U = \bar{U} =$ physical Wilson line after computing observables

Two gluon production: similar rapidites

$$\frac{\mathrm{d}\sigma_{2g}}{\mathrm{d}Y_p \mathrm{d}^2\boldsymbol{p}\,\mathrm{d}Y_k \mathrm{d}^2\boldsymbol{k}} = \frac{1}{(2\pi)^4}\int_{\boldsymbol{x}\bar{\boldsymbol{x}}} \mathrm{e}^{-\mathrm{i}\boldsymbol{p}\cdot(\boldsymbol{x}-\bar{\boldsymbol{x}})} \left\langle H_{\mathrm{prod}}(\boldsymbol{k}) S_{12}(\boldsymbol{x}\bar{\boldsymbol{x}})\big|_{\bar{U}=U}\right\rangle_Y$$

- H_{prod} generates gluons which 'cross the cut' (measured in final state)

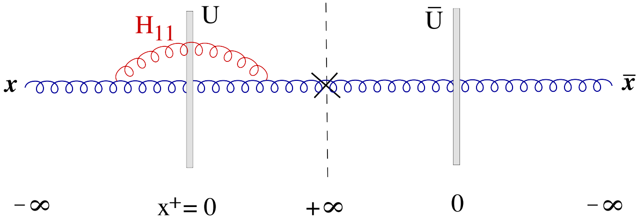


$$H_{\mathrm{prod}}(\boldsymbol{k}) = \int_{\boldsymbol{uvy}\bar{\boldsymbol{y}}} \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{y}-\bar{\boldsymbol{y}})} \frac{(\boldsymbol{y}-\boldsymbol{u})^i}{(\boldsymbol{y}-\boldsymbol{u})^2} \frac{(\bar{\boldsymbol{y}}-\boldsymbol{v})^i}{(\bar{\boldsymbol{y}}-\boldsymbol{v})^2} \left[L^a_{\boldsymbol{u}} - U^{\dagger ab}_{\boldsymbol{y}} R^b_{\boldsymbol{u}}\right] \left[\bar{L}^a_{\boldsymbol{v}} - \bar{U}^{\dagger ac}_{\bar{\boldsymbol{y}}} \bar{R}^c_{\boldsymbol{v}}\right]$$

- Very similar to JIMWLK Hamiltonian (except that $\boldsymbol{y} \neq \bar{\boldsymbol{y}}$)

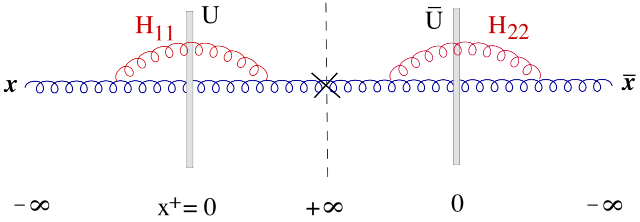
High energy evolution

- One step of quantum evolution : DA $\Rightarrow H_{11} = H_{\text{JIMWLK}}[U]$



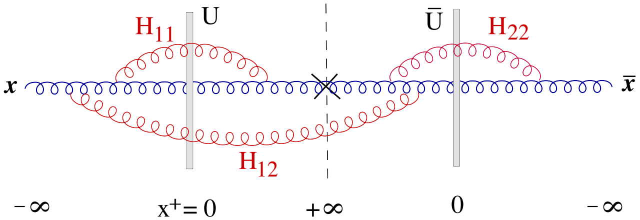
High energy evolution

- One step of quantum evolution : DA, CCA $\Rightarrow H_{22} = H_{\text{JIMWLK}}[\bar{U}]$



High energy evolution

- One step of quantum evolution : DA, CCA, and mixed



$$H_{\rm evol}[U,\bar U] \,=\, H_{11}[U] \,+\, H_{22}[\bar U] \,+\, 2H_{12}[U,\bar U]$$

- Extension of JIMWLK Hamiltonian to the Keldysh time contour

  (Hentschinski, Weigert, Schafer, 05 — study of DIS diffraction)

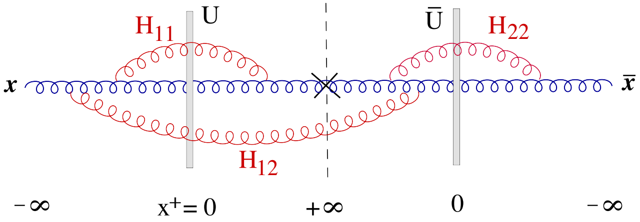
- Generalized B–JIMWLK equations for the generating functionals (or directly for the n–particle cross–sections)

  ▷ large N_c: *Jalilian-Marian, Kovchegov, 04; Kovner, Lublinsky, Weigert, 06*

  ▷ finite N_c: *E. I., Triantafyllopoulos, 13*

High energy evolution

- One step of quantum evolution : DA, CCA, and mixed



$$H_{\rm evol}[U,\bar U] \,=\, H_{11}[U] \,+\, H_{22}[\bar U] \,+\, 2H_{12}[U,\bar U]$$

- Extension of JIMWLK Hamiltonian to the Keldysh time contour

  (Hentschinski, Weigert, Schafer, 05 — study of DIS diffraction)

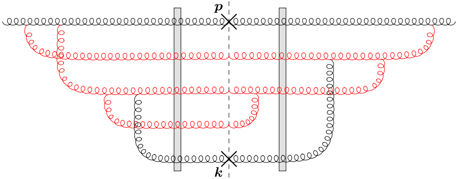
- These equations are extremely complicated, even for large N_c ☹

  ▷ similar degree of complexity as the equation for the quadrupole amplitude

Langevin reformulation *(E. I., Triantafyllopoulos, 13)*

- Evolution of the gluon wavefunction squared over rapidity interval ΔY

  ▷ target evolution from Y_A up to $Y = Y_A + \Delta Y$, starting with $U_A^\dagger \,, \bar{U}_A^\dagger$



$$U^\dagger_{n,\boldsymbol{x}} \;=\; \mathrm{e}^{\mathrm{i}\varepsilon g \alpha^n_{L,\boldsymbol{x}}}\, U^\dagger_{n-1,\boldsymbol{x}}\, \mathrm{e}^{-\mathrm{i}\varepsilon g \alpha^n_{R,\boldsymbol{x}}}$$

$$\bar{U}^\dagger_{n,\boldsymbol{x}} \;=\; \mathrm{e}^{\mathrm{i}\varepsilon g \bar{\alpha}^n_{L,\boldsymbol{x}}}\, \bar{U}^\dagger_{n-1,\boldsymbol{x}}\, \mathrm{e}^{-\mathrm{i}\varepsilon g \bar{\alpha}^n_{R,\boldsymbol{x}}}$$

- Two correlated Langevin processes: DA $(U^\dagger_{n,\boldsymbol{x}})$ and CCA $(\bar{U}^\dagger_{n,\boldsymbol{x}})$

  ▷ different initial conditions: $U_0^\dagger = U_A^\dagger$ and resp. $\bar{U}_0^\dagger = \bar{U}_A^\dagger$

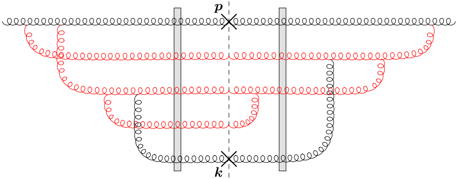
  ▷ ...but the same noise term : $\nu_n = \bar{\nu}_n$

- The evolved generating functional :

$$\big\langle S_{12}(\boldsymbol{x}\bar{\boldsymbol{x}})\big\rangle_{\Delta Y}[U_A, \bar{U}_A] \;=\; \frac{1}{N_c^2-1}\,\big\langle \mathrm{Tr}\big[\bar{U}_{N,\bar{\boldsymbol{x}}} U^\dagger_{N,\boldsymbol{x}}\big]\big\rangle_\nu \quad (\Delta Y = N\epsilon)$$

Langevin reformulation *(E. I., Triantafyllopoulos, 13)*

- Evolution of the gluon wavefunction squared over rapidity interval ΔY
  - ▷ target evolution from Y_A up to $Y = Y_A + \Delta Y$, starting with $U_A^\dagger, \bar{U}_A^\dagger$



$$U^\dagger_{n,\boldsymbol{x}} = \mathrm{e}^{\mathrm{i}\varepsilon g \alpha^n_{L,\boldsymbol{x}}} U^\dagger_{n-1,\boldsymbol{x}} \, \mathrm{e}^{-\mathrm{i}\varepsilon g \alpha^n_{R,\boldsymbol{x}}}$$

$$\bar{U}^\dagger_{n,\boldsymbol{x}} = \mathrm{e}^{\mathrm{i}\varepsilon g \bar{\alpha}^n_{L,\boldsymbol{x}}} \bar{U}^\dagger_{n-1,\boldsymbol{x}} \, \mathrm{e}^{-\mathrm{i}\varepsilon g \bar{\alpha}^n_{R,\boldsymbol{x}}}$$

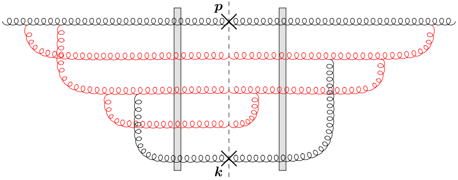
- Two correlated Langevin processes: DA $(U^\dagger_{n,\boldsymbol{x}})$ and CCA $(\bar{U}^\dagger_{n,\boldsymbol{x}})$
  - ▷ different initial conditions: $U_0^\dagger = U_A^\dagger$ and resp. $\bar{U}_0^\dagger = \bar{U}_A^\dagger$
  - ▷ ...but the same noise term : $\nu_n = \bar{\nu}_n$
- The emission of the second gluon (at rapidity Y_A)

$$H_{\rm prod}(\boldsymbol{k})[U_A, \bar{U}_A] \big\langle S_{12}(\boldsymbol{x}\bar{\boldsymbol{x}}) \big\rangle_{\Delta Y} \Big|_{\bar{U}_A = U_A}$$

Langevin reformulation *(E. I., Triantafyllopoulos, 13)*

- Evolution of the gluon wavefunction squared over rapidity interval ΔY

  ▷ target evolution from Y_A up to $Y = Y_A + \Delta Y$, starting with $U_A^\dagger, \bar{U}_A^\dagger$



$$U_{n,\boldsymbol{x}}^\dagger = \mathrm{e}^{\mathrm{i}\varepsilon g \alpha_{L,\boldsymbol{x}}^n} U_{n-1,\boldsymbol{x}}^\dagger \, \mathrm{e}^{-\mathrm{i}\varepsilon g \alpha_{R,\boldsymbol{x}}^n}$$

$$\bar{U}_{n,\boldsymbol{x}}^\dagger = \mathrm{e}^{\mathrm{i}\varepsilon g \bar{\alpha}_{L,\boldsymbol{x}}^n} \bar{U}_{n-1,\boldsymbol{x}}^\dagger \, \mathrm{e}^{-\mathrm{i}\varepsilon g \bar{\alpha}_{R,\boldsymbol{x}}^n}$$

- Two correlated Langevin processes: DA $(U_{n,\boldsymbol{x}}^\dagger)$ and CCA $(\bar{U}_{n,\boldsymbol{x}}^\dagger)$

  ▷ different initial conditions: $U_0^\dagger = U_A^\dagger$ and resp. $\bar{U}_0^\dagger = \bar{U}_A^\dagger$

  ▷ ...but the same noise term : $\nu_n = \bar{\nu}_n$

- Average over the target with the CGC weight function at Y_A

$$\int [DU_A] \, W_{Y_A}[U_A] \, H_{\rm prod}(\boldsymbol{k})[U_A, \bar{U}_A] \langle S_{12}(\boldsymbol{x}\bar{\boldsymbol{x}}) \rangle_{\Delta Y} \Big|_{\bar{U}_A = U_A}$$

Langevin reformulation (2)

- Functional initial conditions are not well suited for numerics ☹

  ▷ first build $U_N^\dagger$ as a functional of U_A, then act with R_A ; e.g.

$$R^a_{A,\boldsymbol{u}}\,\bar{R}^b_{A,\boldsymbol{v}}\,\big\langle S_{12}(\boldsymbol{x}\bar{\boldsymbol{x}})\big\rangle_{\Delta Y}\Big|_{\bar{U}_A=U_A}=\frac{1}{N_g}\,\Big\langle \mathrm{Tr}\Big[\big(R^b_{A,\boldsymbol{v}}U_{N,\bar{\boldsymbol{x}}}\big)\big(R^a_{A,\boldsymbol{u}}U^\dagger_{N,\boldsymbol{x}}\big)\Big]\Big\rangle_\nu$$

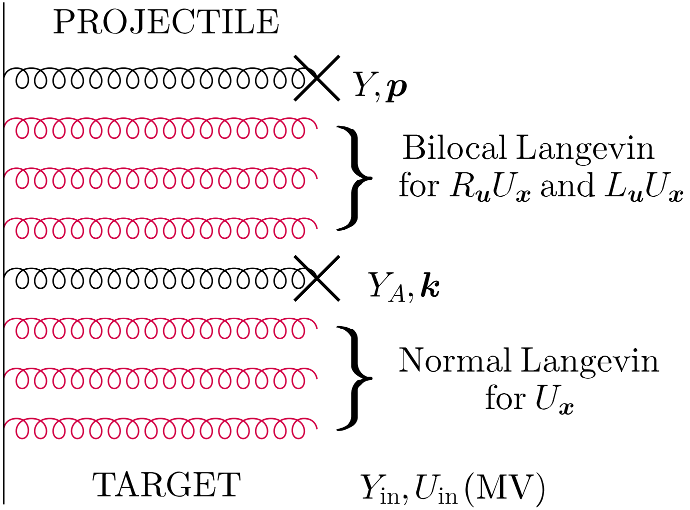
  ▷ the difference between DA and CCA disappears after differentiation

- Alternatively: a recurrence formula for the action of the Lie derivatives

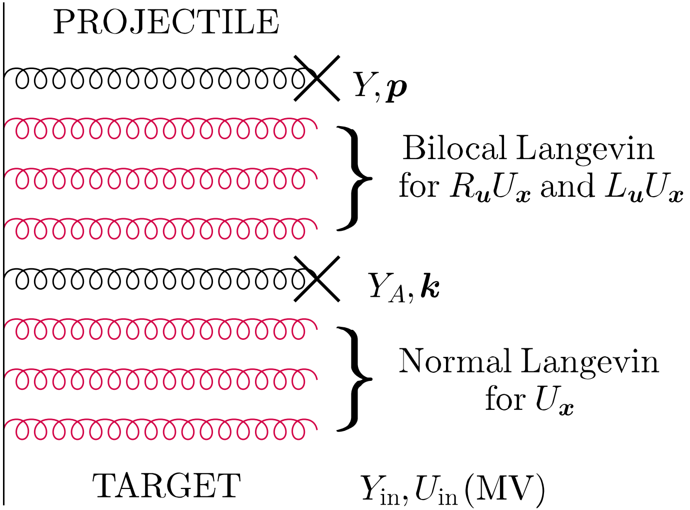
$$\mathcal{R}^a_{n,\boldsymbol{ux}}=\mathrm{e}^{\mathrm{i}\varepsilon g\alpha^n_{R,\boldsymbol{x}}}\,\mathcal{R}^a_{n-1,\boldsymbol{ux}}\,\mathrm{e}^{-\mathrm{i}\varepsilon g\alpha^n_{R,\boldsymbol{x}}}$$
$$-\frac{\mathrm{i}\varepsilon g}{\sqrt{4\pi^3}}\,\mathrm{e}^{\mathrm{i}\varepsilon g\alpha^n_{R,\boldsymbol{x}}}\int_{\boldsymbol{z}}\frac{x^i-z^i}{(\boldsymbol{x}-\boldsymbol{z})^2}\,U^{bc}_{n-1,\boldsymbol{z}}\,\nu^{ic}_{n,\boldsymbol{z}}\big[T^b,\,\mathcal{R}^a_{n-1,\boldsymbol{uz}}\big]$$

- Langevin process for the bi–local quantity $\mathcal{R}^a_{n,\boldsymbol{ux}}\equiv U_{n,\boldsymbol{x}}\,R^a_{A,\boldsymbol{u}}\,U^\dagger_{n,\boldsymbol{x}}$

- No functional initial condition anymore : $\mathcal{R}^a_{0,\boldsymbol{ux}}=\mathrm{i}g\delta_{\boldsymbol{ux}}T^a$



Feasible ? I think so ... but better ask Tuomas !


PROJECTILE
Y, p
Bilocal Langevin
for R_u U_x and L_u U_x
Y_A, k
Normal Langevin
for U_x
TARGET
Y_in, U_in (MV)

THANK YOU !!