Applications of JIMWLK Evolution to Exclusive J/Ψ Production

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Outline

Goal:

Determine cross-section for exclusive J/$\Psi \rightarrow \mu^+\mu^-$ in nuclear collisions

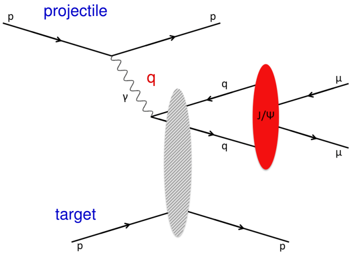
Theory:

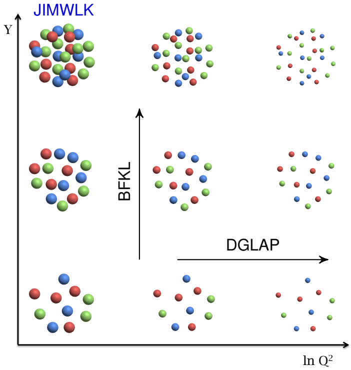
- Colour Glass Condensate (CGC)
- Appropriate effective theory - JIMWLK evolution
- Truncate infinite hierarchy - Gaussian Truncation (GT)

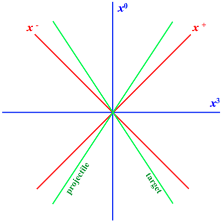
Experiment:

- Vertexing method to identify exclusive events in collision data
- Test method on Starlight Monte Carlo

Saturation Effects



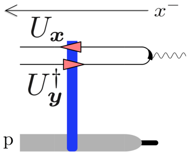




$$Y = \ln\left(\frac{1}{x_{\mathrm{Bj}}}\right) \qquad Q^2 := -q^2$$

Colour Interaction

Colour diagrams



Quark lines $\to$ Wilson lines

$$U_{\boldsymbol{x}} := \mathrm{P}\exp\left\{-ig\int dx^- A^+(0,x^-,\boldsymbol{x})\right\}.$$

Generalise to generic U-correlator $\langle\ldots\rangle_Y$

$\sim$ cross-section

JIMWLK Equation

(Jalilian-Marian - Iancu - McLerran - Weigert - Leonidov - Kovner)

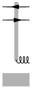
Nonlinear, functional renormalization group equation

$$\frac{d}{dY}\Big\langle \cdots \Big\rangle_Y = -H_{\text{JIMWLK}} \Big\langle \cdots \Big\rangle_Y$$

$$H_{\text{JIMWLK}} := -\frac{\alpha_s}{2\pi^2} \int d^2 z\, \mathcal{K}_{\boldsymbol{xzy}}\, i\nabla^a_{\boldsymbol{x}} \Big[(1 - U^\dagger_{\boldsymbol{x}} U_{\boldsymbol{z}})(1 - U^\dagger_{\boldsymbol{z}} U_{\boldsymbol{y}})\Big]^{ab} i\nabla^b_{\boldsymbol{y}}$$

$$\mathcal{K}_{\boldsymbol{xzy}} = \frac{(\boldsymbol{x}-\boldsymbol{z})\cdot(\boldsymbol{z}-\boldsymbol{y})}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}$$

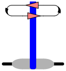
$$i\nabla^a_{\boldsymbol{x}} := -[U_{\boldsymbol{x}} t^a]^{ij} \frac{\delta}{\delta U^{ij}_{\boldsymbol{x}}} \qquad \longrightarrow$$



Evolution

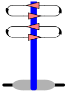
E.g. Typical correlator

$$\left\langle \cdots \right\rangle_Y = \left\langle \frac{\mathrm{tr}(U_{\boldsymbol{x}} U^\dagger_{\boldsymbol{y}})}{N_c} \right\rangle_Y = \frac{1}{\circ}$$



For J/Ψ we need correlators like

$$\left\langle \cdots \right\rangle_Y = \left\langle \frac{\mathrm{tr}(U_{\boldsymbol{x}} U^\dagger_{\boldsymbol{y}})\mathrm{tr}(U_{\boldsymbol{x}'} U^\dagger_{\boldsymbol{y}'})}{N_c^2} \right\rangle_Y = \frac{1}{\circ^2}$$



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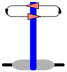

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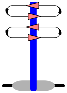
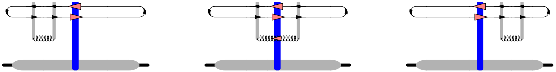
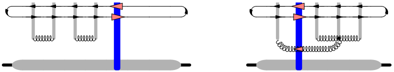


LO $\longrightarrow$



NLO $\longrightarrow$



etc.

JIMWLK equation $\Longleftrightarrow$ Balitsky Hierarchy

...truncate!

Gaussian Truncation

Approximation:

$$\Big\langle \dots \Big\rangle_Y = \exp\left\{-\frac{1}{2}\int^Y dY' \int d^2u\, d^2v\, G_{Y',\boldsymbol{uv}}\; i\nabla^a_{\boldsymbol{u}}\, i\nabla^a_{\boldsymbol{v}}\right\}\dots$$

or conveniently,

$$\frac{d}{dY}\Big\langle \dots \Big\rangle_Y = -\frac{1}{2}\Big\langle \int d^2u\, d^2v\, G_{Y,\boldsymbol{uv}}\; i\nabla^a_{\boldsymbol{u}}\, i\nabla^a_{\boldsymbol{v}}\dots\Big\rangle_Y$$

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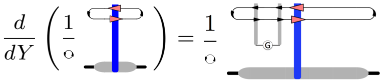
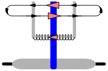
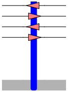
E.g. 

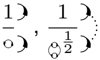
Gaussian Truncation

Use GT to find expressions for $n+1$-order terms in evolution equation at n-order.

- Group theory constraints automatically satisfied exactly (check coincidence limits).
- Unlike BK equation, no large N_c limit necessary.
- Black disc limit vanishes, i.e. $\mathrm{tr}(U_{\boldsymbol{x}} U_{\boldsymbol{y}}^\dagger) \to 0$ when $|\boldsymbol{x} - \boldsymbol{y}| \to \infty$

Parametrisation Equations

At LO $\longrightarrow$  $\longrightarrow$ from  in coincidence limit.

Two dipoles $\Longrightarrow$ multiplet basis 

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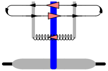
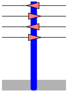
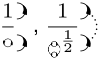
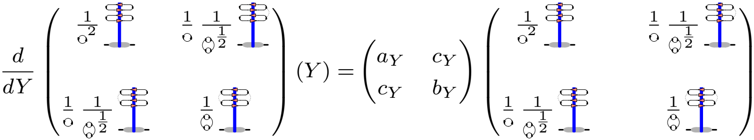
At LO $\longrightarrow$  $\longrightarrow$ from  in coincidence limit.

Two dipoles $\Longrightarrow$ multiplet basis 



Marquet & Weigert, arXiv:1003.0813 (2010)

Beyond GT

Previously,

$$\frac{d}{dY}\Big\langle \cdots \Big\rangle_Y = -\frac{1}{2}\Big\langle \int d^2u\, d^2v\, G_{Y,\boldsymbol{uv}}\; i\nabla^a_{\boldsymbol{u}}\; i\nabla^a_{\boldsymbol{v}} \cdots \Big\rangle_Y$$

Now include on the RHS:

$$\int d^2u\, d^2v\, d^2w \left(f^{abc} G^f_{\boldsymbol{uvw}} + d^{abc} G^d_{\boldsymbol{uvw}} \right) i\nabla^a_{\boldsymbol{u}}\; i\nabla^b_{\boldsymbol{v}}\; i\nabla^c_{\boldsymbol{w}}$$

where $G^f_{\boldsymbol{uvw}}$ antisymmetric and $G^d_{\boldsymbol{uvw}}$ symmetric.

(There is a piece from the antisymmetric term that has a double derivative:

$$\frac{iN_c}{4} \int d^2u\, d^2v \Big(-G^f_{\boldsymbol{uvu}} - G^f_{\boldsymbol{vuv}} + G^f_{\boldsymbol{uvv}} + G^f_{\boldsymbol{vuu}} + G^f_{\boldsymbol{uuv}} + G^f_{\boldsymbol{vvu}} \Big) \{ i\nabla^a_{\boldsymbol{u}}, i\nabla^a_{\boldsymbol{v}} \} \Big)$$

Beyond the GT

Final result for the evolution equation for a quark dipole

$$\frac{d}{dY}\Big\langle\Big(G'_{\boldsymbol{xy}} + 3iG^{\mathcal{O}}_{\boldsymbol{xy}}\Big)(Y)\Big\rangle = \frac{\alpha_s}{\pi^2}\tilde{\mathcal{K}}_{\boldsymbol{xzy}}\Big\langle 1 - e^{\frac{N_c}{2}\left\{\left(G'_{\boldsymbol{xy}} - G'_{\boldsymbol{xz}} - G'_{\boldsymbol{zy}}\right) + 3i\left(G^{\mathcal{O}}_{\boldsymbol{xy}} - G^{\mathcal{O}}_{\boldsymbol{xz}} - G^{\mathcal{O}}_{\boldsymbol{zy}}\right)\right\}(Y)}\Big\rangle_Y$$

where $G^{\mathcal{O}}_{\boldsymbol{xy}} := \frac{C_d}{2}\Big(G^d_{\boldsymbol{yxx}} - G^d_{\boldsymbol{yyx}}\Big)$

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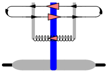
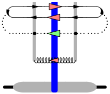
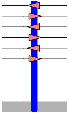
Further investigation

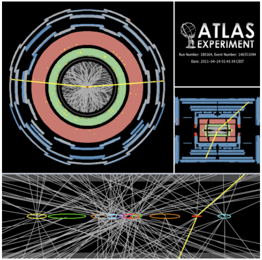
To understand $G^{\mathcal{O}}_{\boldsymbol{xy}}$, need to find evolution equation for 

$\Longrightarrow$ parametrise  $\Longrightarrow$ go to 6 Wilson line correlators 

Finally, result for $\left\langle \dots \right\rangle_Y$ goes into cross-section.

Experiment

Determine cross-section from collision data.



Selection Criteria:

- Each event has 2 muons within detector acceptance
- The muons come from a single vertex
- No other tracks come from the vertex
- The muon pair has a low p_T

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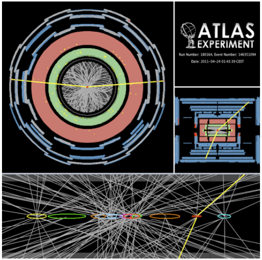

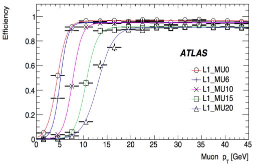

Eur. Phys. J. C72 (2012) 1849

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Challenges:

- Low p_T muon efficiency is poor
- No guarantee the protons did not dissociate
- Pile-up when luminosity is high $\Rightarrow$ go to p-Pb

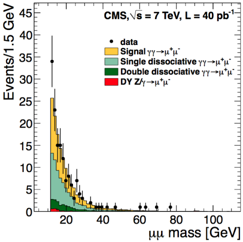
CMS and LHCb Exclusive Studies

CMS [arXiv:1111.5536 (2012)]

Exclusive $\gamma\gamma \to \mu^+\mu^-$

148 events; approx. half are exclusive

$\sigma(pp \to p\mu^+\mu^-p) = 3.38 + 0.58(\text{stat.}) \pm 0.16(\text{syst.}) \pm 0.14(\text{lumi.})$ pb



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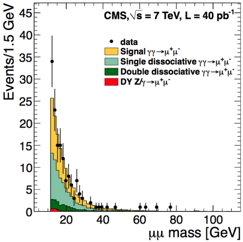
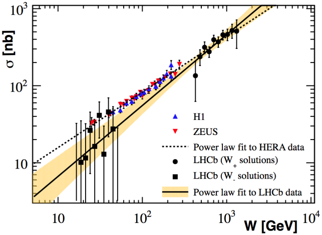
LHCb [arXiv:1301.7084 (2013)]

$\mathcal{L} = 36$ pb^{-1}

$\sigma_{pp\to J/\Psi(\to\mu^+\mu^-)}(2.0 < \eta_{\mu^\pm} < 4.5) = 307 \pm 21(\text{stat.}) \pm 36(\text{syst.})$ pb





Conclusions

- The Gaussian truncation is a useful tool to truncate the infinite hierarchy of equations, the Balitsky hierarchy (equivalent to JIMWLK equation).
- Parametrisation equations help to find simple expressions for correlators.
- Several experimental challenges in detecting exclusive events $\rightarrow$ ongoing work with ATLAS.

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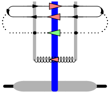
Outlook

- Determine 6-dipole correlator parametrisation equations in order to find evolution equation for  .
- Go beyond three-derivative operators.

End