# Elliptic Flow from Non-equilibrium Initial Condition with a Saturation Scale

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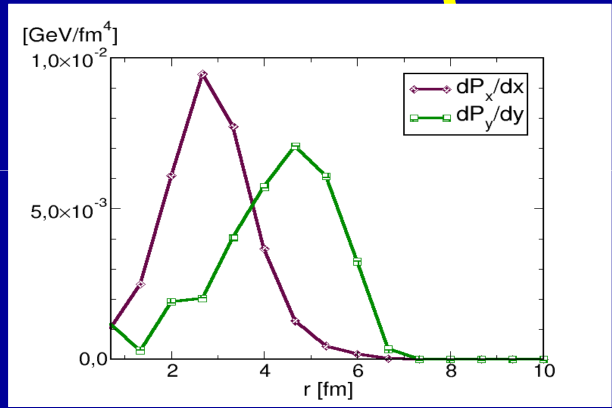
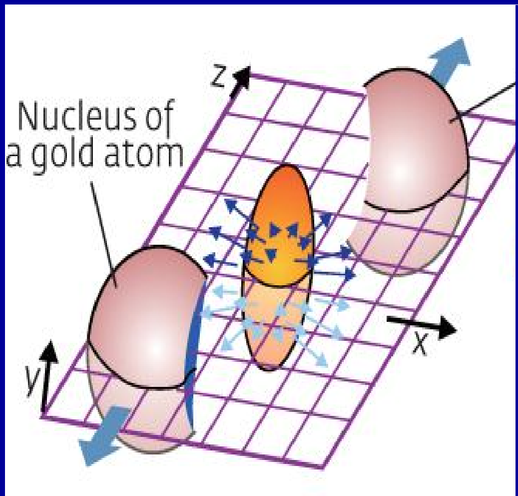
# Outline

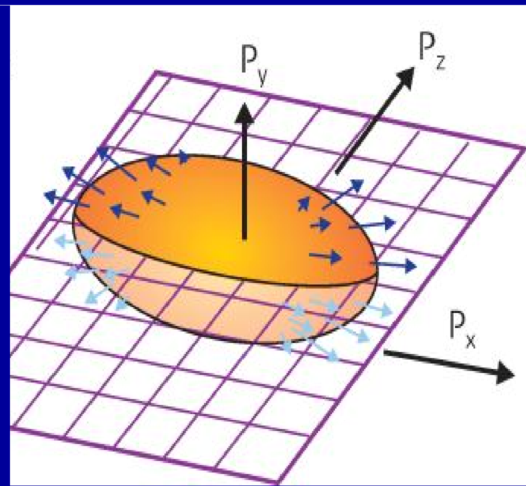
- Elliptic flow and $\eta/s$ in the QGP
- Initial Condition (fKLN)
- Transport Kinetic Theory at Fixed $\eta/s$
- Results
- Conclusions and future developments

# Elliptic flow and η/s in the QGP




**Different gradient pressure**




$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

$$\frac{d^3N}{dyp_T dp_T d\phi} = \frac{1}{2\pi}\frac{d^2N}{dyp_T dp_T}[1 + 2v_2(y, p_T)\cos 2\phi]$$

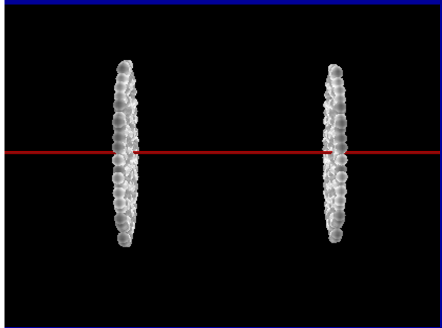
$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

**$v_2$ is sensitive both to the initial condition in the overlap zone and to η/s of the evolving QGP**

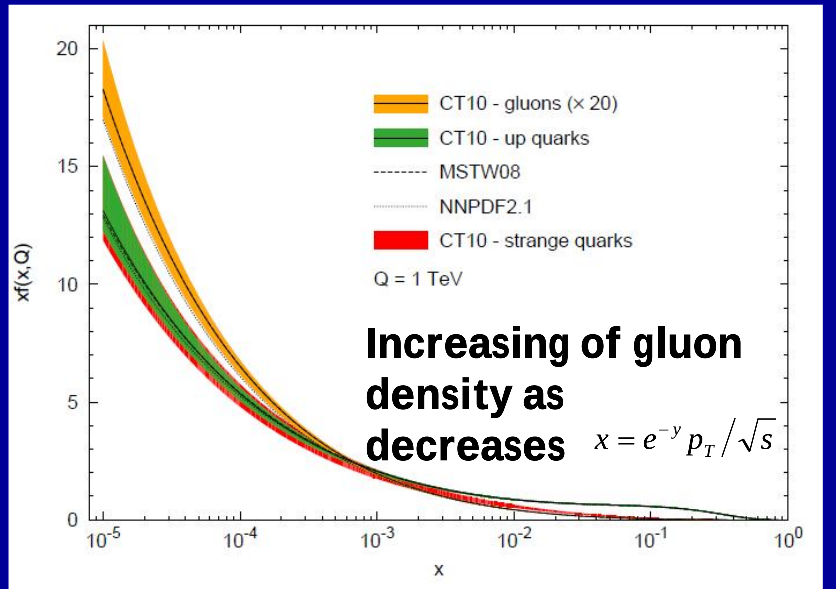
# ➢Initial Conditions :Glauber model

**The initial profile of the fireball is given by the geometrical superposition of the profiles of the two colliding nuclei**

# ➢Initial Conditions: Glasma



**The two nuclei can be described as two tiny disks of Color Glass Condensate (CGC)**




[Brandt and Klasen,arXiv:1305.5677]

**Saturation scale**

$$Q^2_{sat}(s) \propto \alpha_s(Q^2)\frac{xg(x,Q^2)}{\pi R^2} \propto A^{1/3}$$

**At RHIC $Q_s^2 \sim$ 1-2 GeV$^2$**
**At LHC $Q_s^2 \sim$ 1-4 GeV$^2$ ?**

**The production of particle in HIC is controlled by the $Q_s$**

***Reviews***

**McLerran, 2011**
**Iancu, 2009**
**McLerran, 2009**
**Lappi, 2010**
**Gelis, 2010**
**Fukushima, 2011**

# ➢Initial Conditions: Glasma->fKLN

**fKLN coordinate space distribution**

$$\frac{dN_g}{dyd^2\mathbf{x}_\perp} = \int d^2\mathbf{p}_\mathbf{T}\, p_A(\mathbf{x}_\perp) p_B(\mathbf{x}_\perp) \Psi(\mathbf{p}_\mathbf{T}, \mathbf{x}_\perp, y)$$

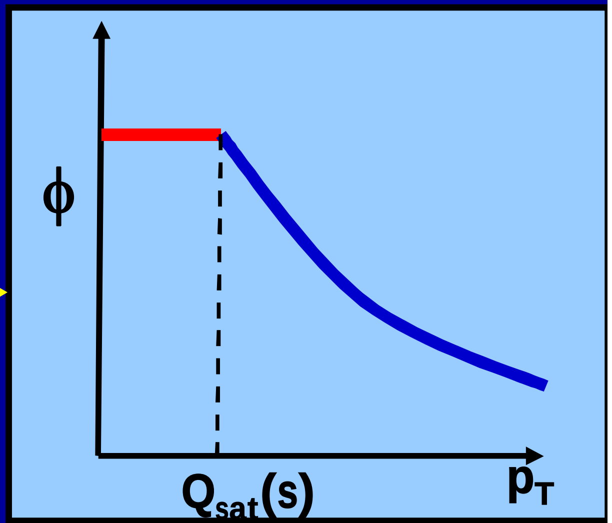
**fKLN momentum space distribution**

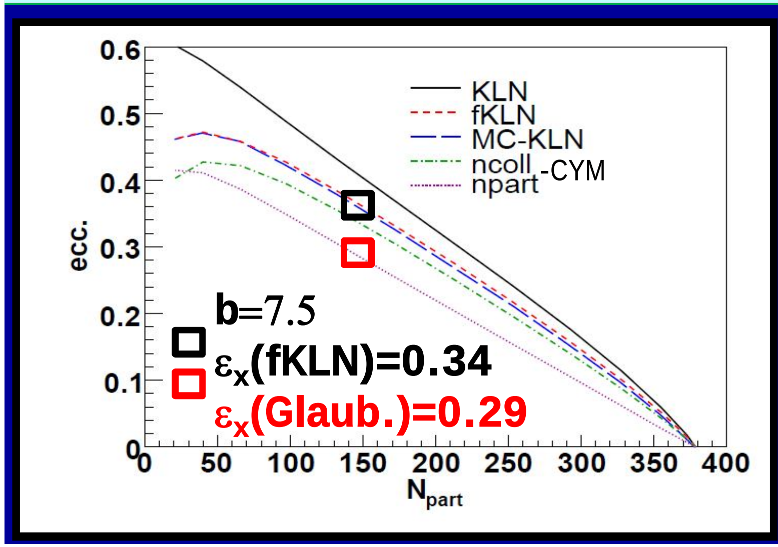
$$\Psi(\mathbf{p}_\mathbf{T}, \mathbf{x}_\perp, y) \propto \frac{1}{p_T^2} \int^{p_T} d^2\mathbf{k}_\mathbf{T}\, \alpha_s(Q^2) \times$$
$$\phi_A(x_1, k_T^2; \mathbf{x}_\mathbf{T})\, \phi_B(x_2, (\mathbf{p}_\mathbf{T} - \mathbf{k}_\mathbf{T})^2; \mathbf{x}_\mathbf{T})$$

**Unintegrated distribution function**

$$\phi_A(x_1, k_T^2; \boldsymbol{x}_\perp) = \frac{\kappa Q_s^2}{\alpha_s(Q_s^2)} \left[ \frac{\theta(Q_s - k_T)}{Q_s^2 + \Lambda^2} + \frac{\theta(k_T - Q_s)}{k_T^2 + \Lambda^2} \right]$$

Kharzeev *et al.*, Phys. Lett. B561, 93 (2003)
Nardi *et al.*, Phys. Lett. B507, 121 (2001)
Drescher and Nara, PRC75, 034905 (2007)
Hirano and Nara, PRC79, 064904 (2009)
Albacete and Dumitru, arXiv:1011.5161[hep-ph]



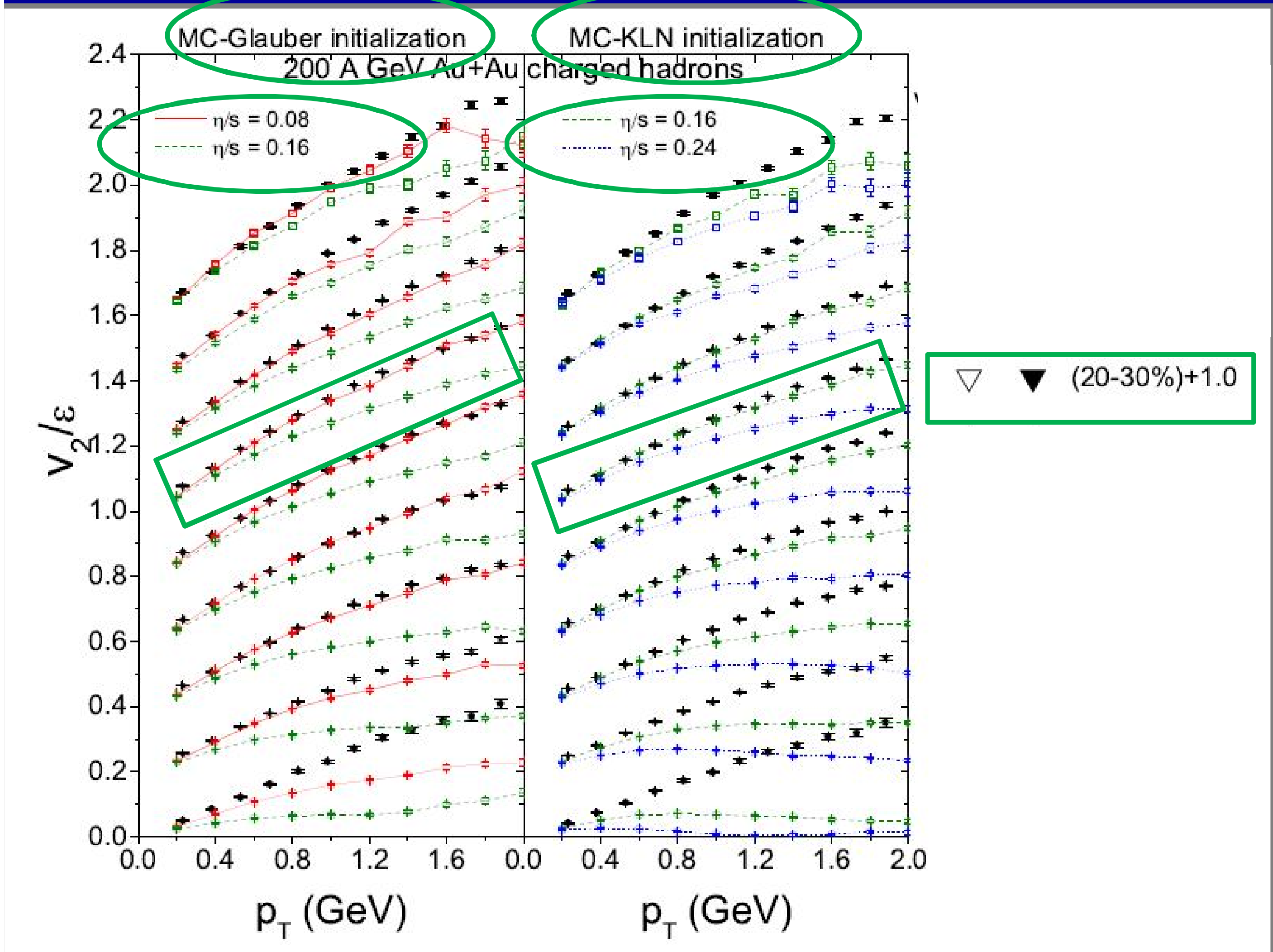

Saturation effects built in the $\phi$

$$Q_{s,A}^2(x, \boldsymbol{x}_\perp) \propto Q_s^2 T_A(\boldsymbol{x}_\perp) x^{-\lambda}$$

***Universal saturation scale*, in agreement with: Lappi and Venugopalan, PRC 74 054905 (2006)**

# $V_2$ from KLN in Hydro

[Heinz *et al.*, PRC 83, 054910 (2011)]




**Glauber η/s≅1/4π**

**fKLN η/s≅2/4π**

**Same efferct also for the $v_2$ of photons ( Shen et al., arXiv:1308.2111)**

***Larger $\varepsilon_x$ - > higher $\eta/s$ to get the same $v_2(p_T)$***

# Transport theory

$$p^{\mu}\partial_{\mu} f(x,p) + M(X)\partial_{\mu} M(X)\partial_{p}^{\mu} f(X,p) = C[f]$$

Free streaming — Mean Field — Collisions

f(x,p) is a one body distribution function

**We map with C[f] the local phase space evolution of a fluid with a fixed η/s**

$$\mathcal{C}_{22} = \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f'_1 f'_2 |\mathcal{M}_{1'2'\to 12}|^2 (2\pi)^4 \delta^{(4)}(p'_1+p'_2-p_1-p_2)$$
$$-\frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f_1 f_2 |\mathcal{M}_{12\to 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1+p_2-p'_1-p'_2)$$

**Collision integral is solved with a local stochastic sampling**

[ Z. Xhu, C. Greiner, PRC71(04)]
[G. Ferini et al Phys.Lett.B670:325-329,2009]

$$P_{22} = \frac{\Delta N_{coll}^{2\to 2}}{\Delta N_1 \Delta N_2} = v_{rel}\sigma_{22}\frac{\Delta t}{\Delta^3 x}$$

- **CGC $p_T$ non-equilibrium distribution (beyond the difference in $\varepsilon_x$)**
- **valid also at intermediate and high $p_T$**

# Simulate a fixed shear visosity

**Usually a key input ingredient of a transport approach is the knowledge of the cross section $\sigma$ but here we reverse it and start from $\eta/s$ with aim of creating a more direct link to viscous hydro.**

$$P_{22} = \frac{\Delta N_{\rm coll}^{2\to 2}}{\Delta N_1 \Delta N_2} = v_{\rm rel}\sigma_{22}\frac{\Delta t}{\Delta^3 x}$$

**The total Cross section is computed in each configuration space cell according to Chapman-Enskog approximation**

$$\frac{\eta}{s} = \frac{1}{15}\langle p \rangle \cdot \tau_\eta = \frac{1}{15}\frac{\langle p \rangle}{g(\frac{m_D}{T})\sigma_{TOT}\rho}$$

$$\sigma_{tot}(n(\vec{r}),T) = \frac{1}{15}\frac{\langle p_\alpha \rangle}{g(a)n_\alpha}\frac{1}{\eta/s}$$

**Space-Time dependent cross section evaluated locally**

**This approach is valid for a generic differential cross section**

$$\frac{d\sigma}{d\Omega} \propto \frac{\alpha_s^2}{\left[q^2(\theta)+m_D^2\right]^2}$$

**$\alpha$=cell index in the r-space**

$$g(a) = \frac{1}{50}\int dy y^6 \left[(y^2+\frac{1}{3})K_3(2y) - yK_2(2y)\right] h\left(\frac{a^2}{y^2}\right)$$

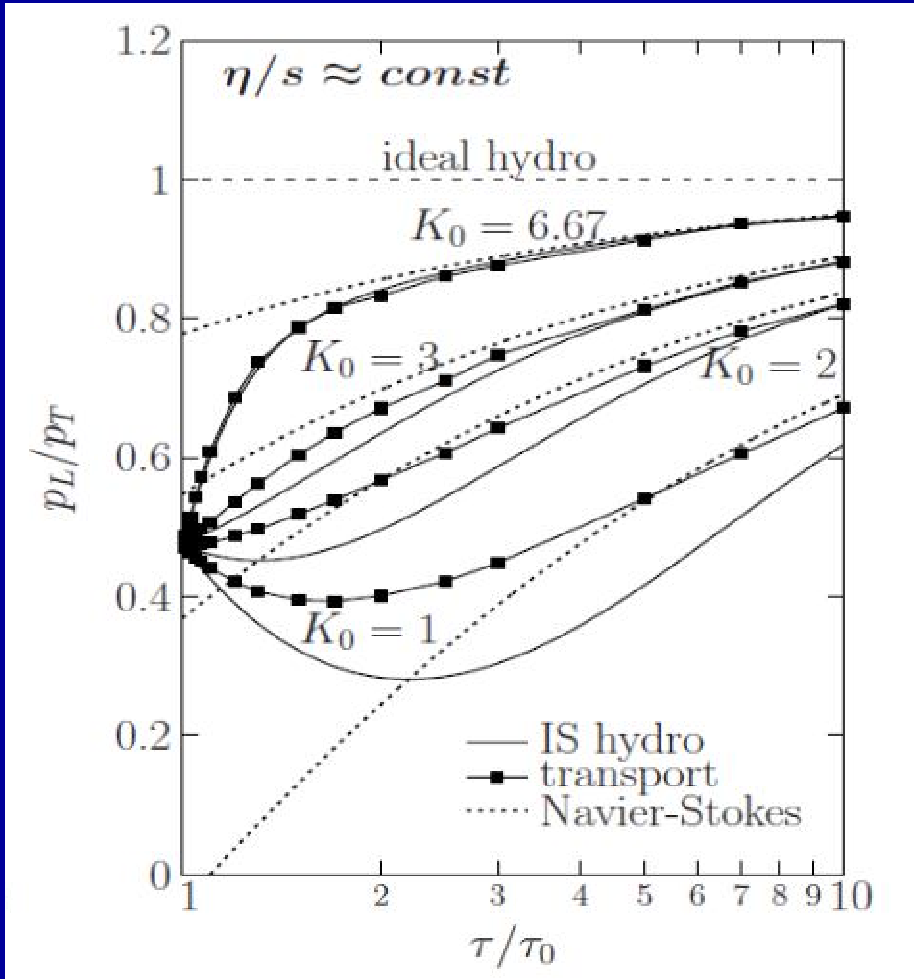
**$g(a=m_D/2T)$ correct function that fixes the relaxation time for the shear motion**


**Plumari *et al.*, Phys. Rev. C86 (2012)**
**Greco *et al.*, Phys. Lett. B670 (2009)**
**Plumari *et al.*, J.Phys.Conf.Ser. 420 (2013)**

# Transport vs Viscous Hydrodynamics in 1+1D

**Comparison for the relaxation of pressure anisotropy $P_L/P_T$**

**Huovinen and Molnar, PRC79(2009)**




**Knudsen number$^{-1}$**

$$K = \frac{L}{\lambda} \rightarrow \frac{\tau}{\lambda}$$

**Large K small $\eta/s$**

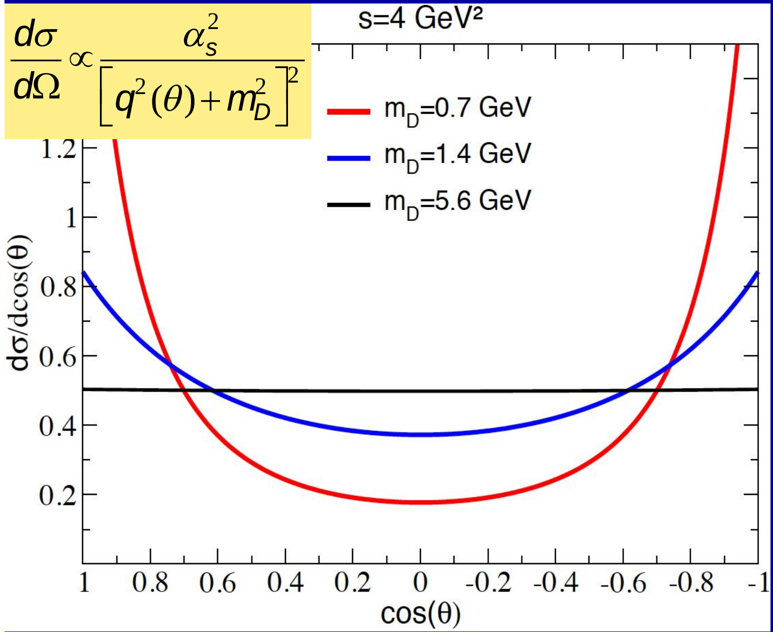
$$K \propto \frac{1}{\eta / s}$$

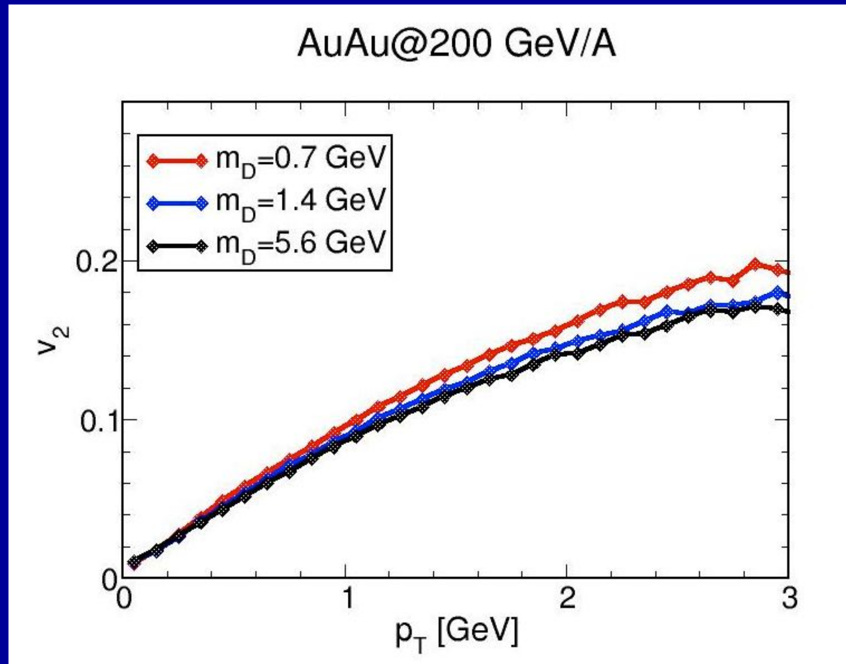
**In the limit of small $\eta/s$ (<0.16) transport reproduce viscous hydro at least for the evolution of $P_L/P_T$**

# Are micro-details important?

**Increasing $m_D$ makes the $\sigma$ isotropic**

$$\frac{d\sigma}{d\Omega} \propto \frac{\alpha_s^2}{\left[q^2(\theta)+m_D^2\right]^2}$$




**We keep the same $\eta/s$**

$$\frac{\eta}{s} = \frac{1}{15}\langle p\rangle \cdot \tau_\eta = \frac{1}{15}\frac{\langle p\rangle}{g(\frac{m_D}{T})\sigma_{TOT}\rho}$$

$$\frac{\sigma_{TOT}(m_{1D})}{\sigma_{TOT}(m_{2D})} = \frac{g(m_{2D})}{g(m_{1D})}$$

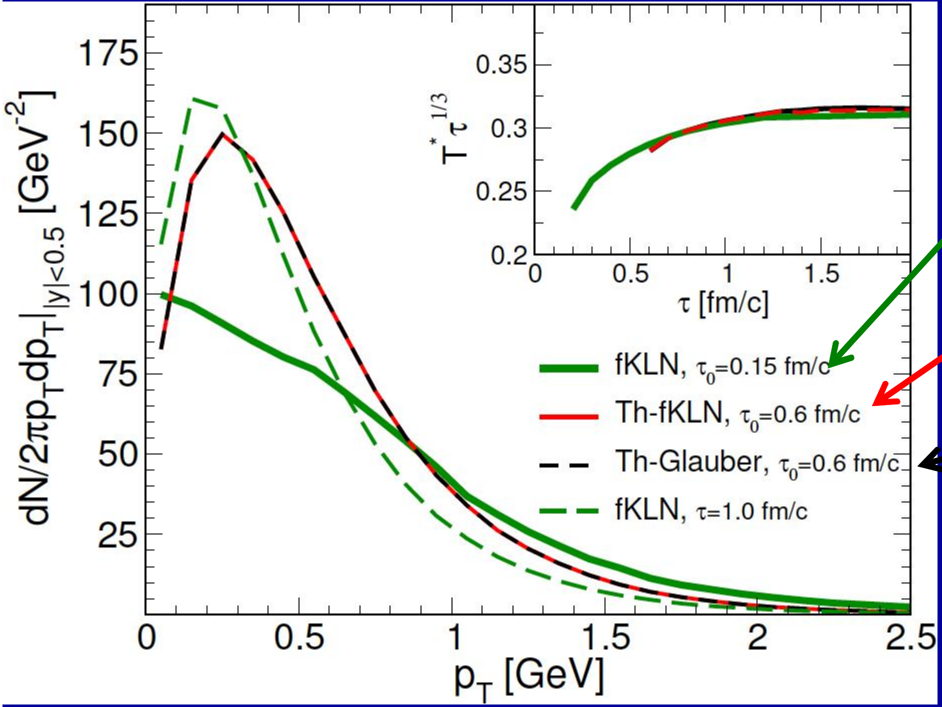
**for $m_D$=1.4 GeV -> 25% smaller $\sigma_{tot}$**

**for $m_D$=5.6 GeV -> 40% smaller $\sigma_{tot}$**

- **Strong change in the angular dependence of $\sigma$ result in a very little change of the elliptic flow at low $p_T$**
- **$\eta/s$ is really the physical parameter determining $v_2$ at least up to 1.5-2 GeV**
- **microscopic details become relevant at higher $p_T$**

# Implementing fKLN $p_T$ distribution

*AuAu@200 GeV - 20-30%*




Using kinetic theory at finite $\eta/s$ we can implement full KLN (x & p space) - $\varepsilon_x$=0.34, <Qs> =1 GeV

KLN only in x space ( like in Hydro) $\varepsilon_x$=0.34, Qs=0

Glauber in x and thermal in p $\varepsilon_x$=0.289 , Qs=0

F. S. *et al.*, 1303.3178 [nucl-th] published on PLB

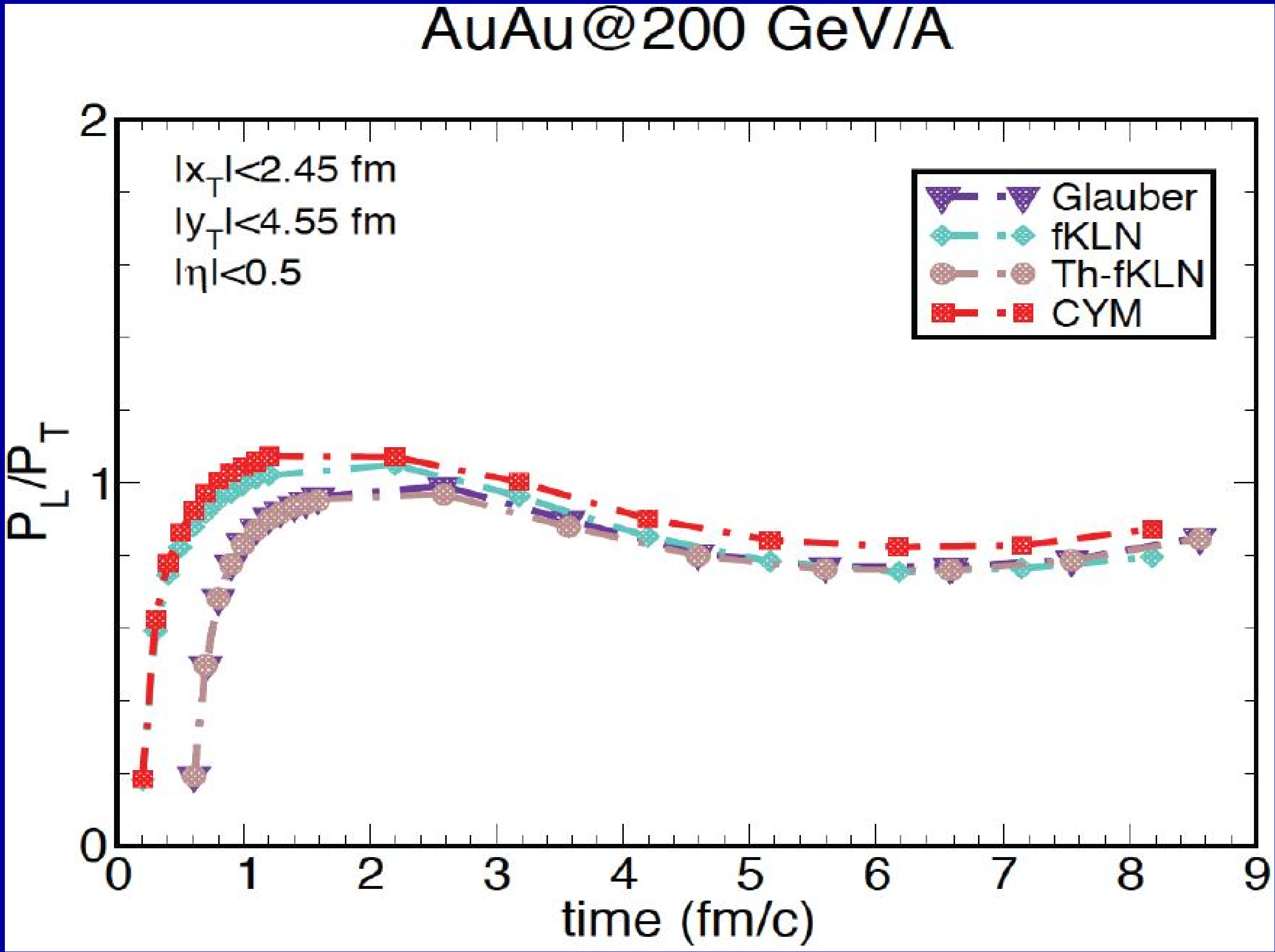
Thermalization in less than 1 fm/c, in agreement with Greiner *et al*., NPA806, 287 (2008).
Not so surprising: $\eta/s$ is small -> large effective scattering rate -> fast thermalization.

$$\sigma_{tot} = \frac{\langle p \rangle}{\rho\, g(a)} \frac{1}{\eta/s}$$

# Longitudinal and transverse pressure

$$P_T = \frac{1}{V}\int_\Omega d^2\mathbf{x}_\perp d\eta \frac{T_{xx}+T_{yy}}{2}$$

$$P_L = \frac{1}{V}\int_\Omega d^2\mathbf{x}_\perp d\eta T_{zz}$$



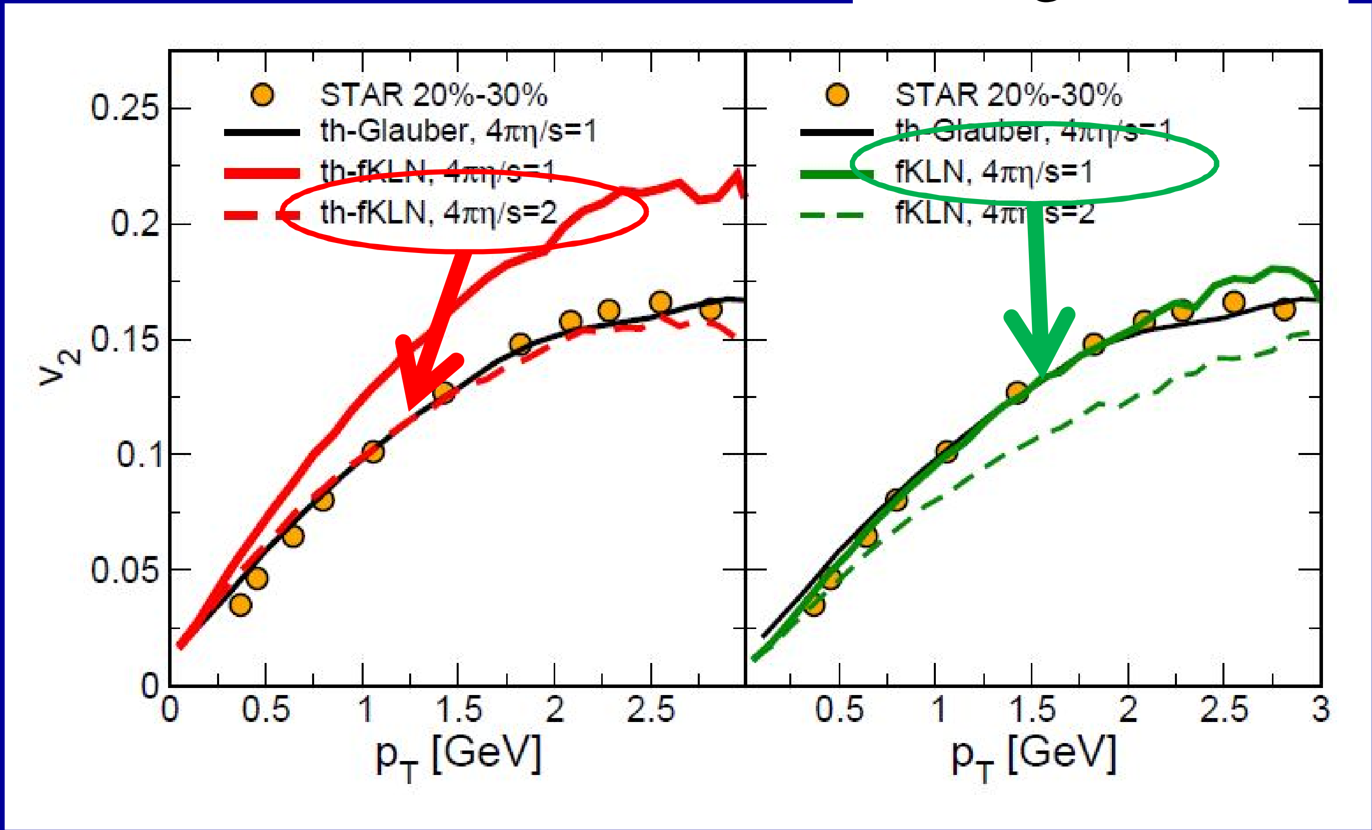

$P_L/P_T$ shows also a fast equilibration

# Elliptic flow at RHIC from: fKLN Glasma

F. S. *et al.*, 1303.3178 [nucl-th] published on PLB

In agreement with:
[Heinz *et al.*, PRC 83, 054910 (2011)]

***AuAu@200 GeV***

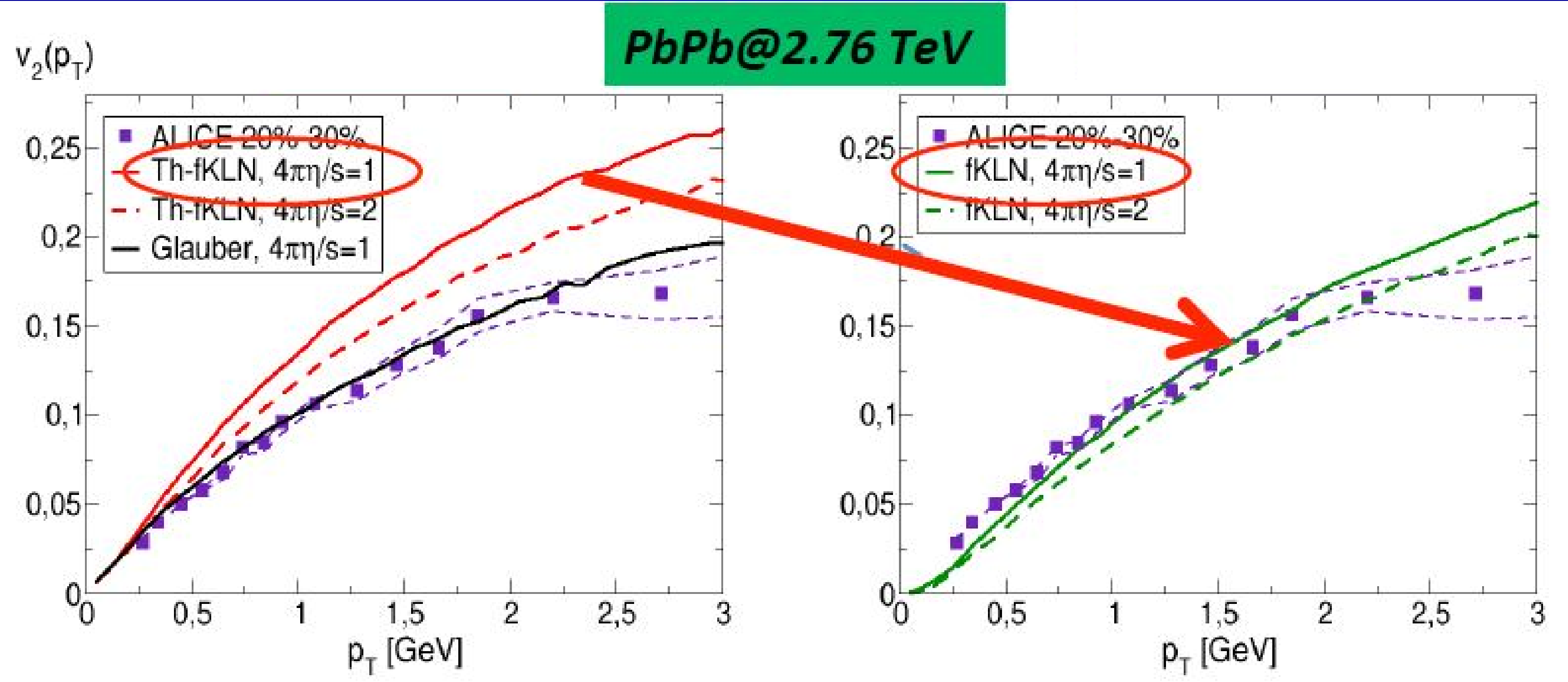



**When implementing KLN and Glauber like Hydro we get the same results**

**When implementing the full fKLN we get close to the data with 4πη/s=1: larger $\varepsilon_x$ compensated by the nonequilibrium distribution**

# Elliptic flow at LHC from: fKLN Glasma

**Prelimiary results**




- $4\pi\eta/s=2$ not sufficent to get close the data for Th-fKLN
- $4\pi\eta/s=1$ it is enough if one implements both x & p

# Conclusions

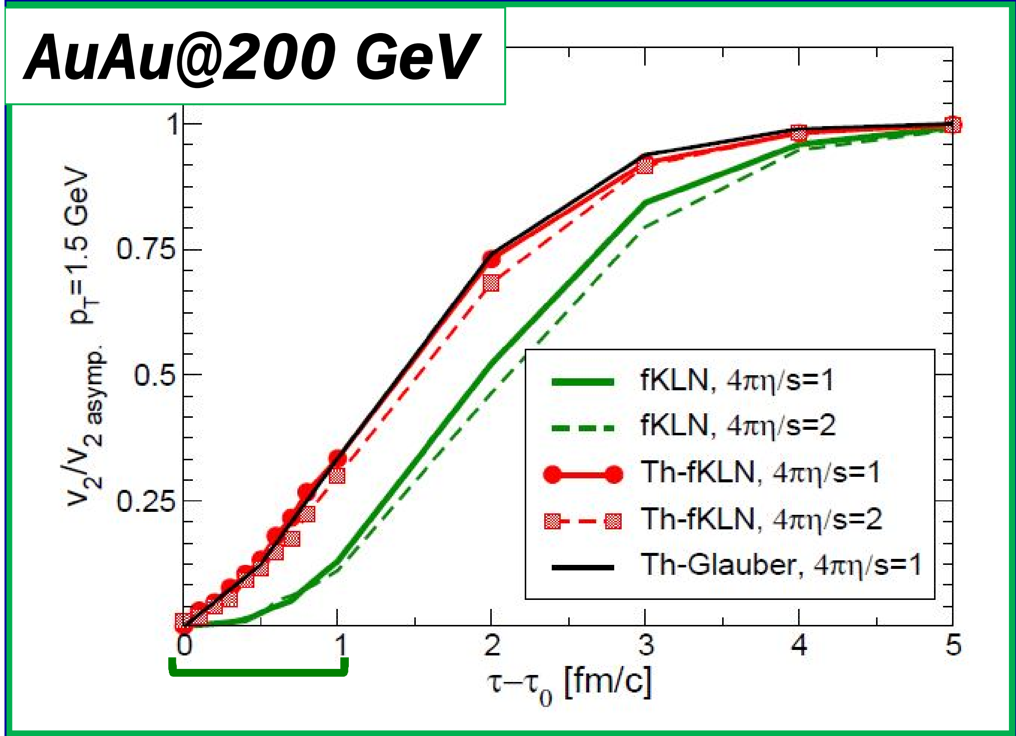
- ✓ We have used Kinetic Theory to compute the elliptic flow of a fireball produced in heavy ion collisions, both at RHIC and LHC energies
- ✓ v2 depends not only on the initial $\varepsilon_x$ and on $\eta$/s but it also depends on the initial distributions in p
- ✓ Initial non-equilibrium distribution with a saturation scale damps the $v_2(p_T)$ compensating the larger $\varepsilon_x$

# Outlook

- ✓ Using also the Classic Yang-Mills initial condition
- ✓ Including the initial state fluctuations in order to study also the impact on the $v_3$

# What is going on?

## $V_2$ normalized time evolution




Put it very simplistic:

$$f(p_T,\phi) = e^{-\frac{p_T - \delta p \cos(2\phi)}{T^*}}$$

$$v_2(p_T,\phi) \cong \frac{\delta p}{4T^*}$$

CGC at can be faken by very large T (un linking dN/dy from it)

*If the momentum shift is the same we can expect smaller $v_2$ in CGC!*

**[M. Ruggieri *et al.*, 1303.3178 [nucl-th]]**

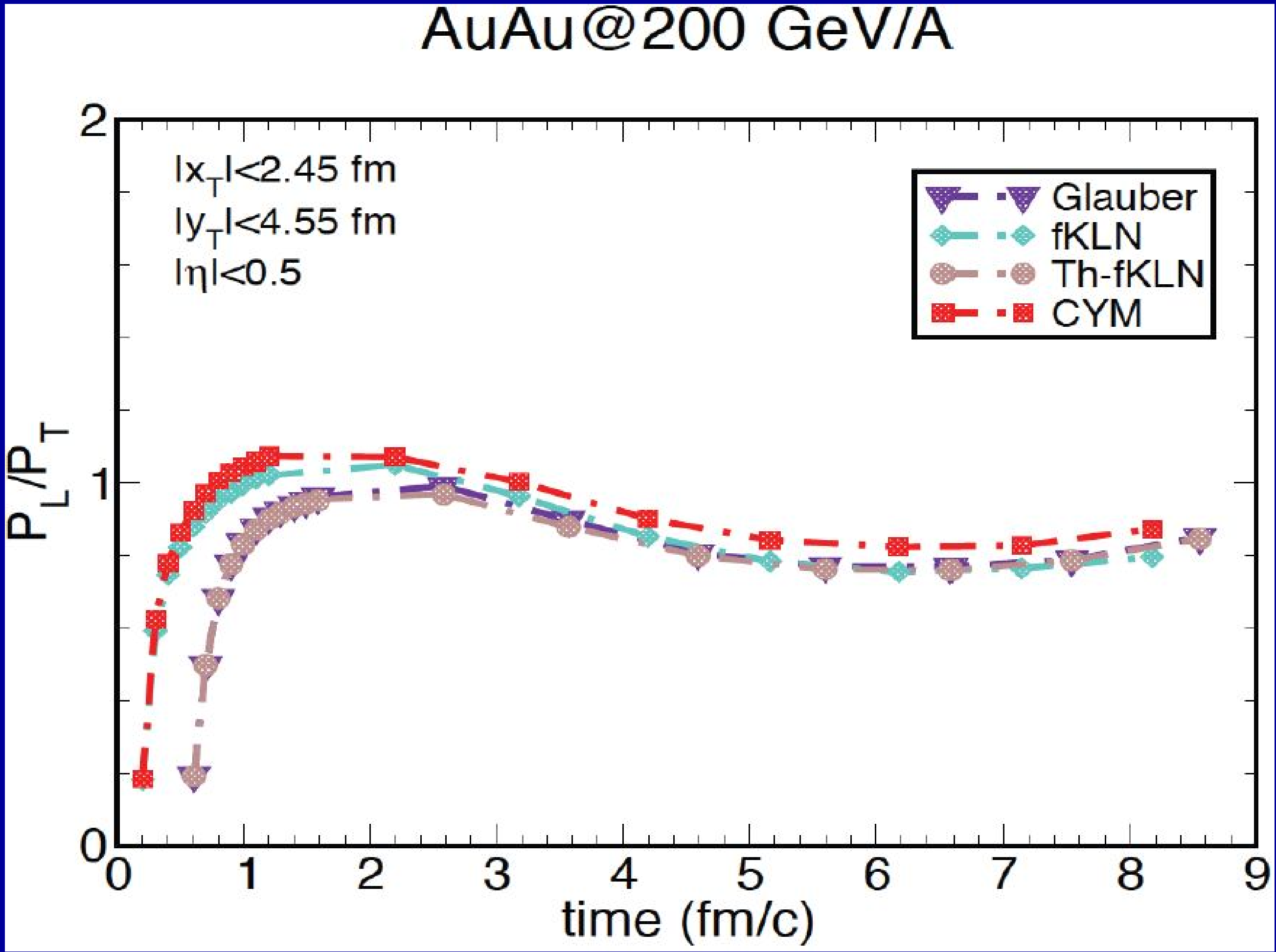
**We clearly see that when non-equilibrium distribution is implemented in the initial stage (≈ 1 fm/c) $v_2$ grows slowly respect to thermal one**

# Longitudinal and transverse pressure

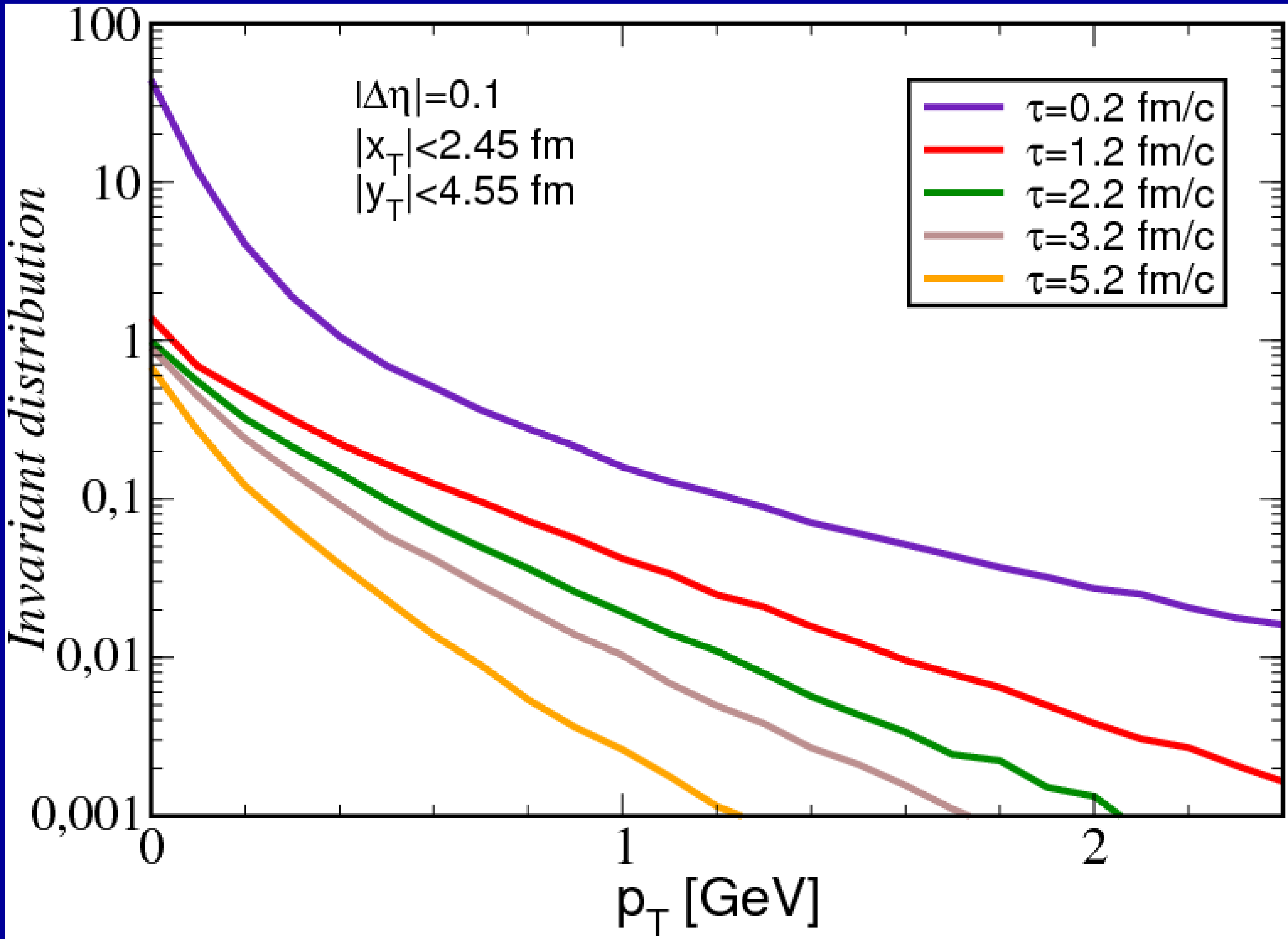
**$t = 1/Q_s \cong 0.1$ fm/c -> $P_L/P_T > 0$**
**Gelis & Epelbaum**

$$P_T = \frac{1}{V}\int_\Omega d^2\mathbf{x}_\perp d\eta \frac{T_{xx}+T_{yy}}{2}$$

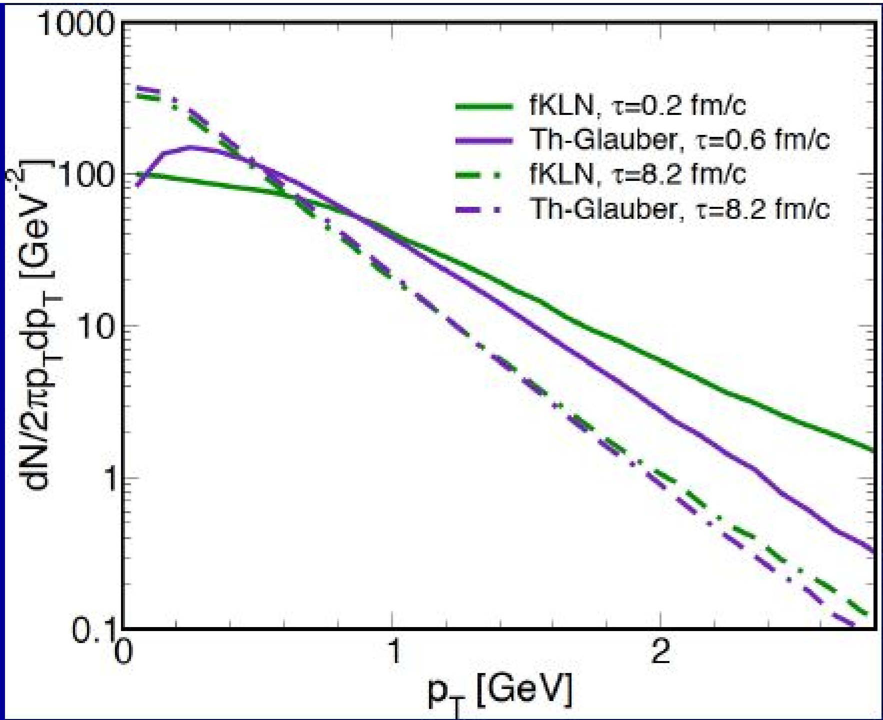
$$P_L = \frac{1}{V}\int_\Omega d^2\mathbf{x}_\perp d\eta T_{ZZ}$$



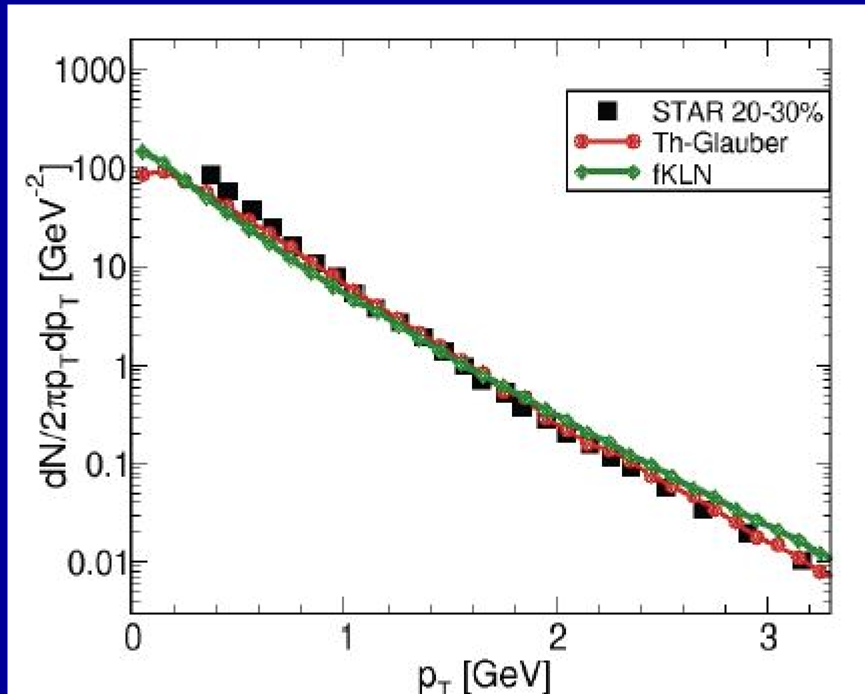

**$P_L/P_T$ shows also a fast equilibration**



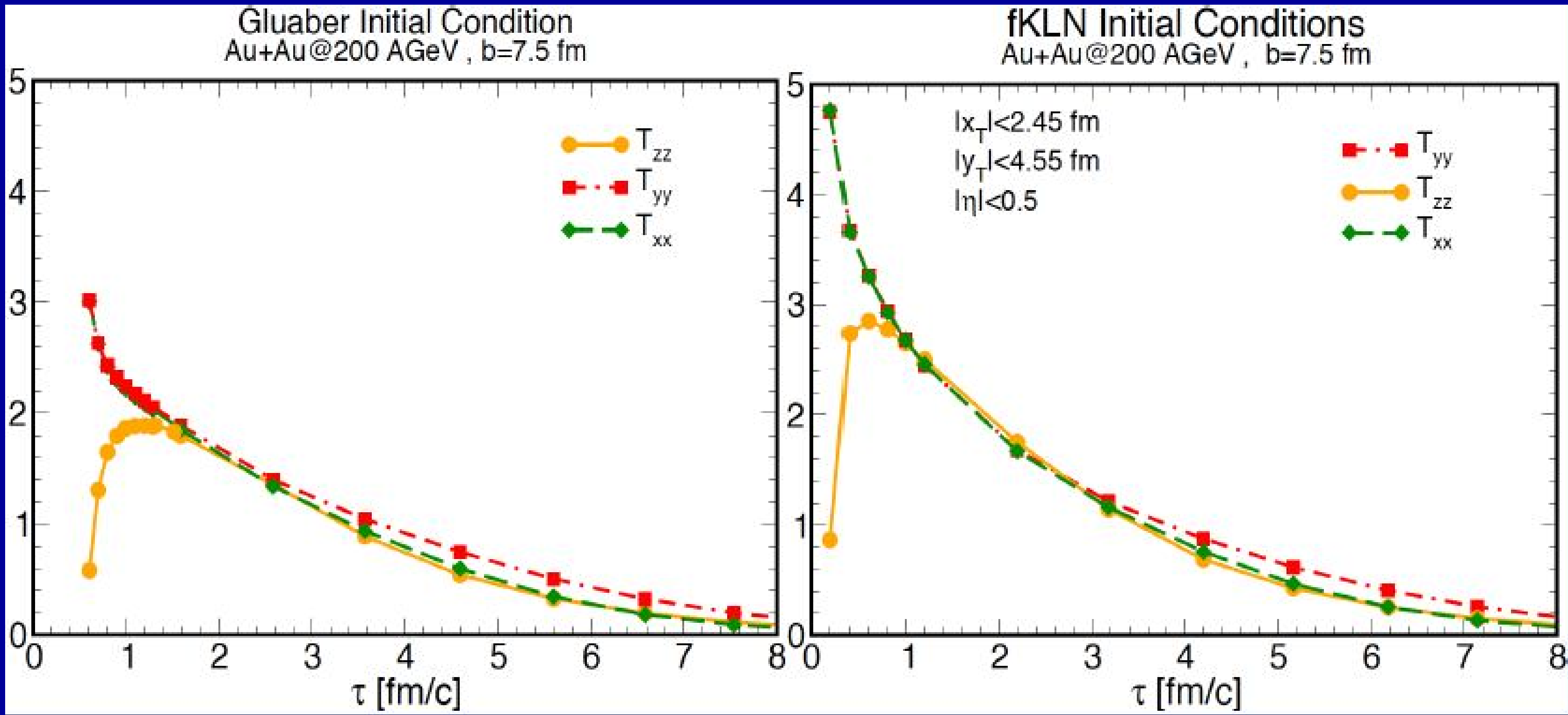
|Δη|=0.1
|x_T|<2.45 fm
|y_T|<4.55 fm
τ=0.2 fm/c
τ=1.2 fm/c
τ=2.2 fm/c
τ=3.2 fm/c
τ=5.2 fm/c
Invariant distribution
100
10
1
0,1
0,01
0,001
0
1
2
p_T [GeV]



fKLN, τ=0.2 fm/c
Th-Glauber, τ=0.6 fm/c
fKLN, τ=8.2 fm/c
Th-Glauber, τ=8.2 fm/c
dN/2πp$_T$dp$_T$ [GeV$^{-2}$]
p$_T$ [GeV]

STAR 20-30%
Th-Glauber
fKLN
dN/2πp$_T$dp$_T$ [GeV$^{-2}$]
p$_T$ [GeV]



Gluaber Initial Condition
Au+Au@200 AGeV , b=7.5 fm
T_zz
T_yy
T_xx
τ [fm/c]
fKLN Initial Conditions
Au+Au@200 AGeV , b=7.5 fm
|x_T|<2.45 fm
|y_T|<4.55 fm
|η|<0.5
T_yy
T_zz
T_xx
τ [fm/c]