Next-to-leading order thermal photon production in a weakly-coupled plasma

Jacopo Ghiglieri, McGill University

based on

JG Hong Kurkela Lu Moore Teaney JHEP0513 (2013)

Hard Probes 2013, Stellenbosch, November 6 2013

Photons from heavy ion collisions

- The hard partonic processes in the heavy ion collision produce quarks, gluons and *primary photons*
- At a later stage, quarks and gluons form a plasma.
  - A jet traveling through the QGP can radiate *jet-thermal photons*
  - Scatterings of thermal partons can produce *thermal photons*
- Later on, partons hadronize. Interactions between charged hadrons produce *hadron gas thermal photons*
- Hadrons may decay into *decay photons*

In this talk

- In this talk: the *thermal, real* photon rate at NLO in an infinite, equilibrated medium (and a sneak peek at jets)
- In this conference:
    - Off-equilibrium, viscous media: C. Shen, U. Heinz and G. Vujanovic
    - Lattice calculations for dileptons: H. T. Ding
    - Large invariant mass dileptons: M. Laine
- This symbol:  $\Rightarrow$ technical, look for details in the paper if you are interested (or just come ask me)

Motivation

- Improve the phenomenological analyses, if not by giving reliable theory error bands
- On the theory side, understand perturbation theory and its convergence better
  - For thermodynamical quantities (p, s, ...) either strict expansion in g, QCD (T) + EQCD (gT) + MQCD (g^2T) (Arnold-Zhai, Braaten Nieto, etc) or non-perturbative solution of EQCD (Kajantie Laine etc)
  - For dynamical quantities? Poor convergence in heavy-quark diffusion coefficient. Need to understand $O(g)$ Caron-Huot Moore **PRL100, JHEP0802** (2008)

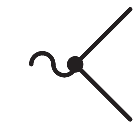
Overview

The basics

- Wightman current-current correlator

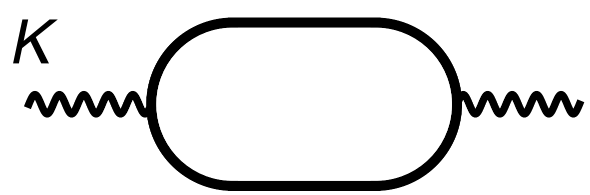
$$\frac{d\Gamma}{d^3k} = \frac{e^2}{(2\pi)^3 2k^0} \int d^4Y e^{-iK\cdot Y} \langle J^\mu(Y) J_\mu(0) \rangle$$

$$J^\mu = \sum_{q=uds} e_q \bar{q} \gamma^\mu q :$$



- Real, hard photon: $k^0 = k \gtrsim T$

- At one loop ($\alpha_{\mathrm{EM}}\, g^0$):



Kinematically forbidden. Need to kick one of the quarks off-shell

- Leading order is $\alpha_{\mathrm{EM}}\, g^2$

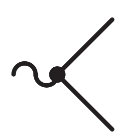
- Strength of the kick (virtuality) determines the momentum region of the calculation

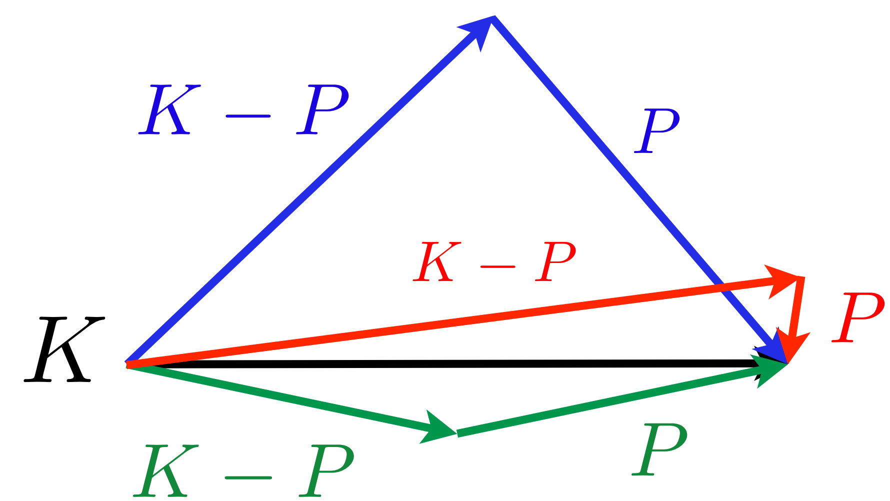
Kinematical regions

- Define a light-cone $K = (k, 0, 0)$

  $P = (p^+, p^-, p_\perp) \qquad p^+ = (p^0 + p^z)/2 \quad p^- = p^0 - p^z$

- Momentum conservation at the current insertion gives three regions

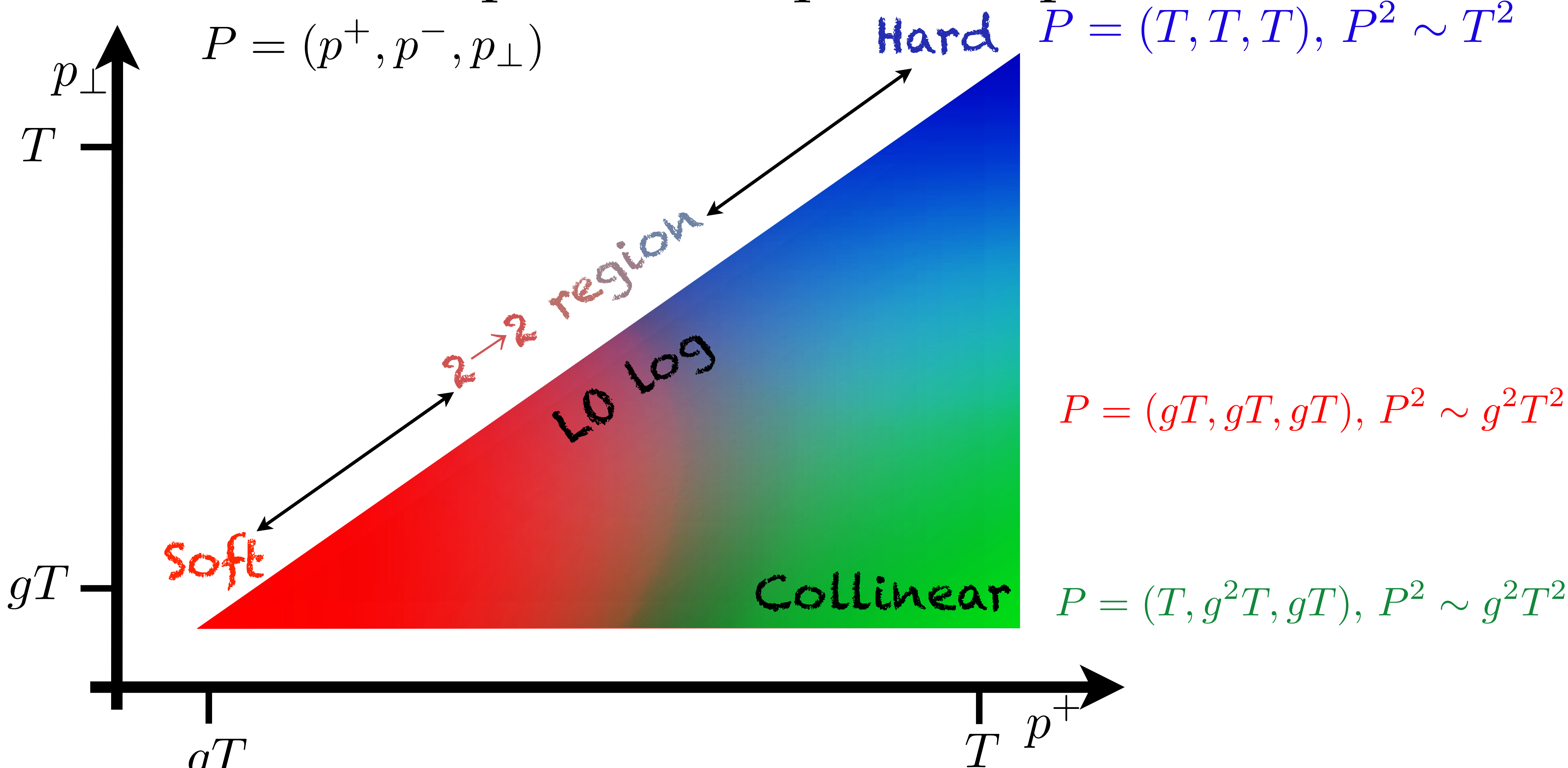
  $J^\mu = \sum_{q=uds} e_q \bar{q} \gamma^\mu q :$ 



  - Hard off-shell
  - Soft, smaller phase space but enhancement
  - Collinear, both nearly on shell and enhanced

Kinematical regions

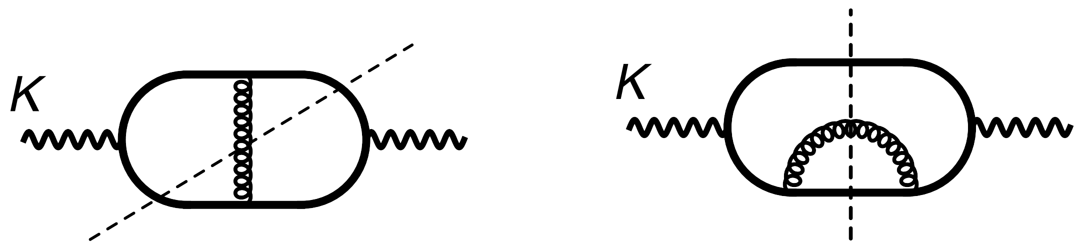
- In the $(p^+, p_\perp)$ plane (P = quark loop momentum)



$p^+ = (p^0 + p^z)/2 \quad p^- = p^0 - p^z$

The 2↔2 region

- Two loop diagrams ($\alpha_{\mathrm{EM}}\, g^2$)



where the cuts correspond to the so-called 2↔2 processes (with their crossings and interferences):

- IR divergence (Compton) when t goes to zero

Introducing the soft scale

- The IR divergence is cured by a proper resummation in the soft sector through the **Hard Thermal Loop** effective theory Braaten Pisarski **NPB337** (1990)

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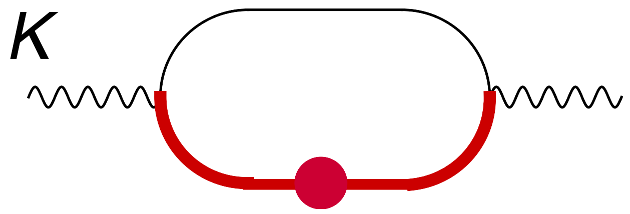

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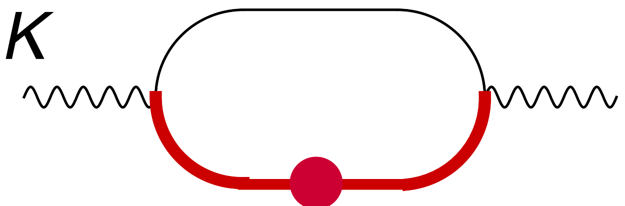


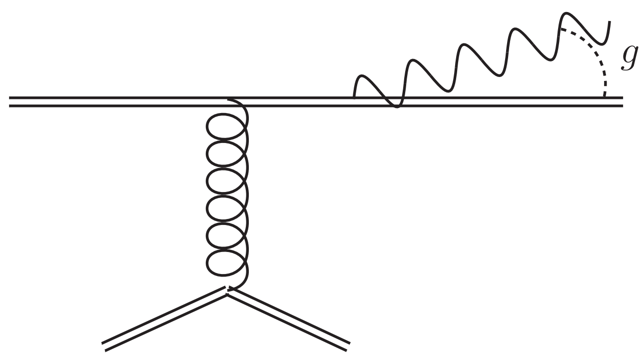
- In the end one obtains the result

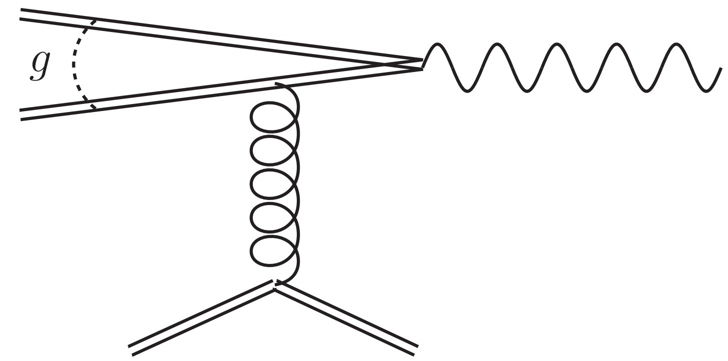
$$\left.\frac{d\Gamma_\gamma}{d^3k}\right|_{2\leftrightarrow 2} \propto e^2 g^2 \left[\log\frac{T}{m_\infty} + C_{2\leftrightarrow 2}\left(\frac{k}{T}\right)\right]$$

The dependence on the cutoff cancels out

Kapusta Lichard Siebert **PRD44** (1991) Baier Nakkagawa Niegawa Redlich **ZPC53** (1992)

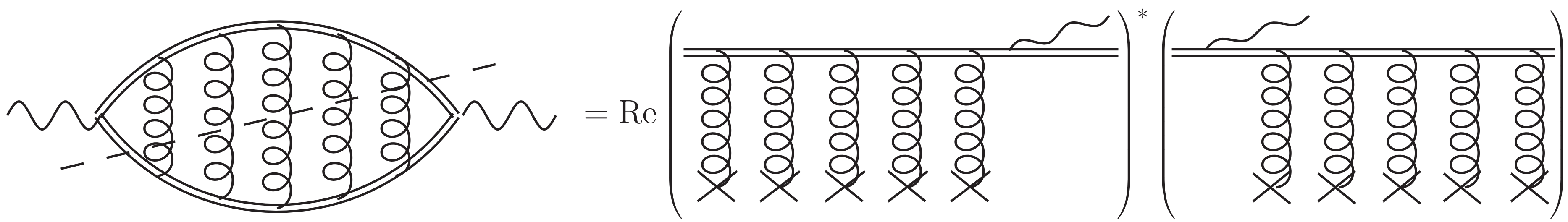
The collinear region





- These diagrams contribute to LO if small (g) angle radiation/annihilation Aurenche Gelis Kobes Petitgirard Zaraket 1998-2000
- Photon formation times is then of the same order of the soft scattering rate ⇒ interference: *LPM effect*
- Requires resummation of infinite number of ladder diagrams

$$\left.\frac{d\Gamma_\gamma}{d^3k}\right|_{\mathrm{coll}} =$$



AMY (Arnold Moore Yaffe) **JHEP** 0111, 0112, 0226 (2001-02)

LPM resummation

- Quark statistical functions × DGLAP splitting × transverse evolution

$$\frac{d\Gamma}{d^3k} = \frac{\alpha}{\pi^2 k}\int \frac{dp^+}{2\pi} n_{\rm F}(k+p^+)[1-n_{\rm F}(p^+)] \frac{(p^+)^2+(p^++k)^2}{2(p^+(p^++k))^2} \lim_{\mathbf{x}_\perp\to 0} 2\mathrm{Re}\boldsymbol{\nabla}_{\mathbf{x}_\perp}\mathbf{f}(x_\perp)$$

$$\begin{array}{ccccc} x^+ & \gg & x_\perp & \gg & x^- \\ 1/g^2T & \gg & 1/gT & \gg & 1/T \end{array}$$

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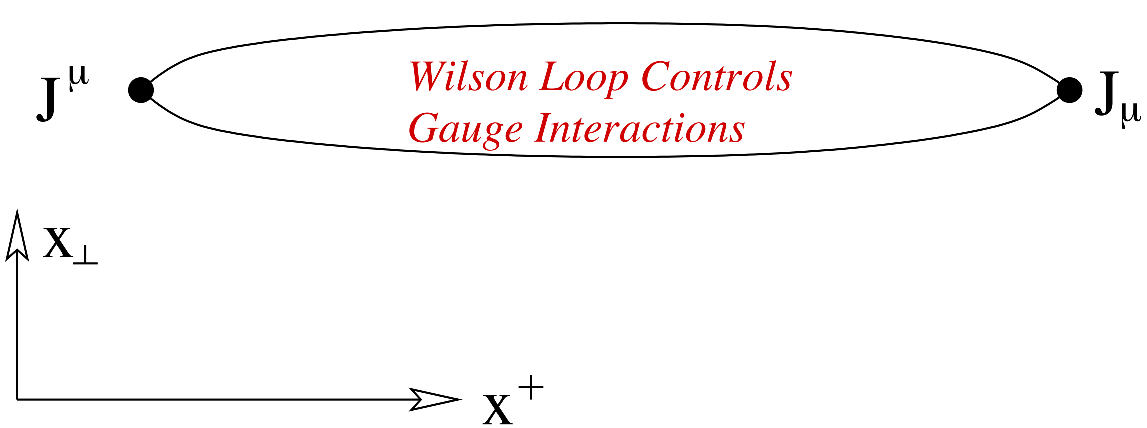
- Transverse diffusion and Wilson-loop correlators evolve the transverse density **f** *along the spacetime light-cone*

$$-2i\nabla\delta^2(\mathbf{x}_\perp) = \left[\frac{ik}{2p^+(k+p^+)}\left(m_\infty^2 - \nabla^2_{\mathbf{x}_\perp}\right) + \mathcal{C}(x_\perp)\right]\mathbf{f}(\mathbf{x}_\perp)$$

$$\begin{array}{ccccc} x^+ & \gg & x_\perp & \gg & x^- \\ 1/g^2T & \gg & 1/gT & \gg & 1/T \end{array}$$

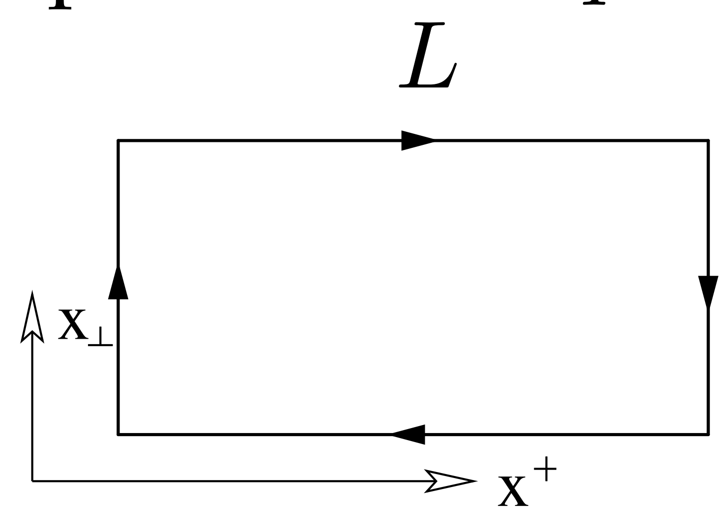


Zakharov 1996-98 AMY 2001-02

LPM resummation: two inputs

- Asymptotic mass $m_\infty^2 = 2g^2 C_R \left(\int \frac{d^3p}{(2\pi)^3} \frac{n_{\rm B}(p)}{p} + \int \frac{d^3p}{(2\pi)^3} \frac{n_{\rm F}(p)}{p} \right)$
- Light-cone Wilson loop, related to $\hat{q}$

$$\hat{q} \equiv \int_0^{q_{\rm max}} \frac{d^2 q_\perp}{(2\pi)^2} q_\perp^2 C(q_\perp)$$



$$\propto e^{\mathcal{C}(x_\perp)L}$$

BDMPS-Z, Wiedemann, Casalderrey-Solana Salgado, D'Eramo Liu Rajagopal, Benzke Brambilla Escobedo Vairo

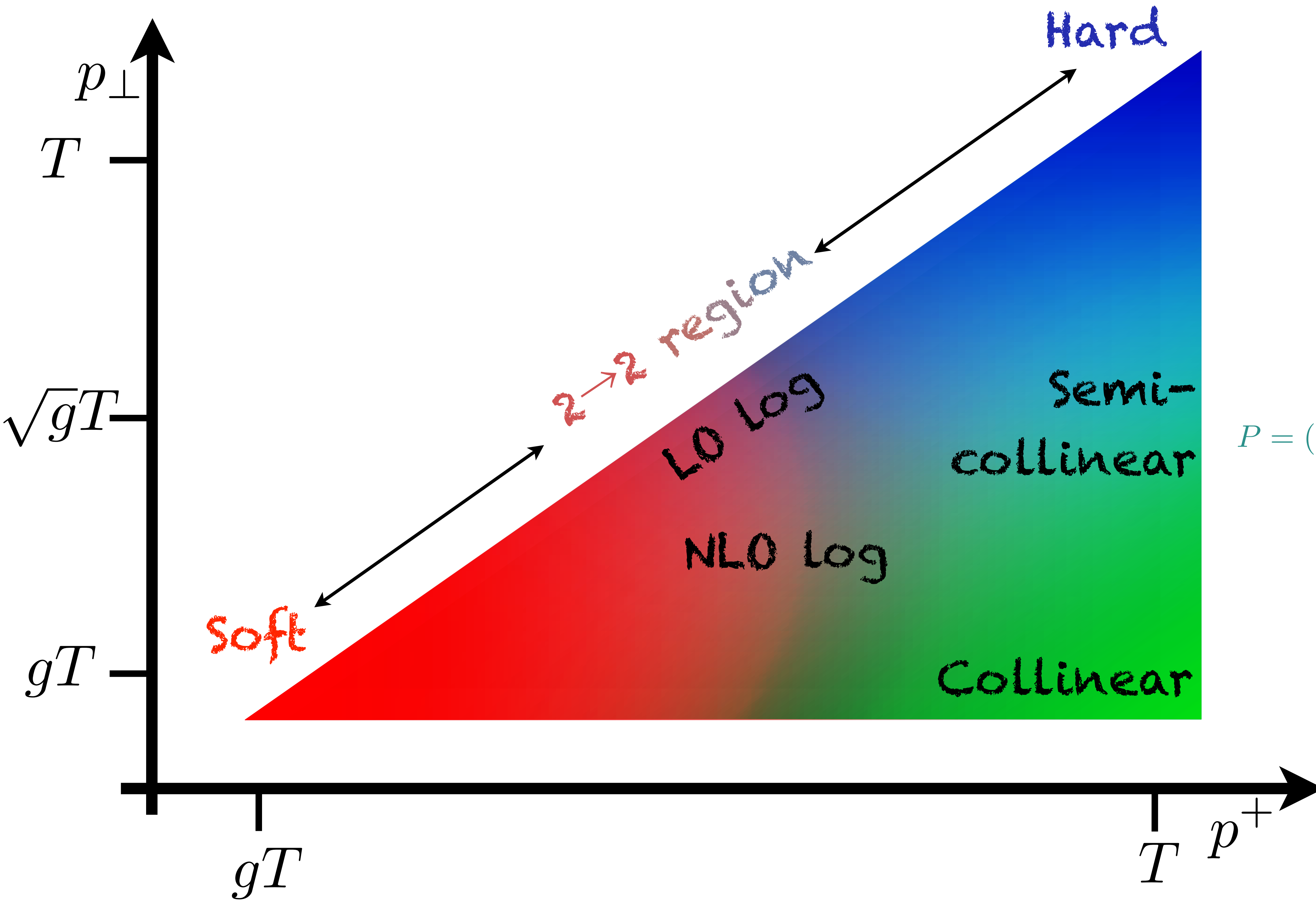
- Soft contribution becomes Euclidean! Caron-Huot **PRD79** (2008), can be "easily" computed in perturbation theory Possible lattice measurements Laine Rothkopf **JHEP1307** (2013) Panero Rummukainen Schäfer **1307.5850** talk by Panero

Going to NLO

Sources of NLO corrections

- As usual in thermal field theory, the soft scale gT introduces NLO $O(g)$ corrections
- The soft region and the collinear region both receive $O(g)$ corrections
- There is a new semi-collinear region
- The NLO calculation is still not sensitive to the magnetic scale g^2T.

NLO regions



$P = (T, gT, \sqrt{g}T),\ P^2 \sim gT^2$

Euclideanization of light-cone soft physics

For $v=x_z/t=\infty$ correlators (such as propagators) are the equal time Euclidean correlators.

$$G^{>}(t=0,\mathbf{x}) = \sum\!\!\!\!\!\!\!\int_p G_E(\omega_n, p)e^{i\mathbf{p}\cdot\mathbf{x}}$$

- Causality: retarded functions analytic for positive imaginary parts of all *timelike* and *lightlike* variables: the above result can be extended to the lightcone

$$G^{>}(t=x_z,\mathbf{x}_\perp) = \sum\!\!\!\!\!\!\!\int_p G_E(\omega_n, p_\perp, p_z + i\omega_n)e^{i(\mathbf{p}_\perp\cdot\mathbf{x}_\perp + p_z x_z)}$$

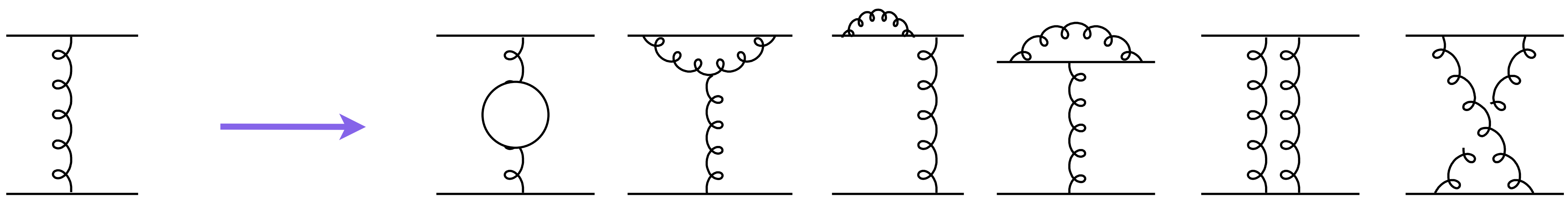
- The sums are dominated by the zero mode for soft physics=>EQCD!
- Equivalent to sum rules   Caron-Huot **PRD79** (2009)

The collinear sector

- Four sources of *O(g)* corrections
- m_∞^2 at NLO, Caron-Huot **PRD79** (2009) 125002

$$\delta m_\infty^2 = 2g^2 C_R T \int \frac{d^3q}{(2\pi)^3} \left(\frac{1}{q^2+m_D^2} - \frac{1}{q^2} \right) = -g^2 C_R \frac{T m_D}{2\pi}$$

- $\mathcal{C}(x_\perp)$ at NLO ⇒ one-loop rungs Caron-Huot **PRD79** (2009) 065039

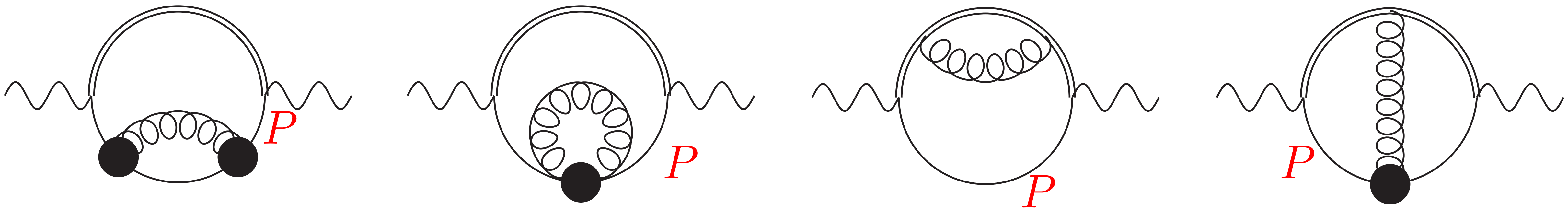


$p^+ \sim gT$ or $p^+ + k \sim gT$. Mistreated soft limit

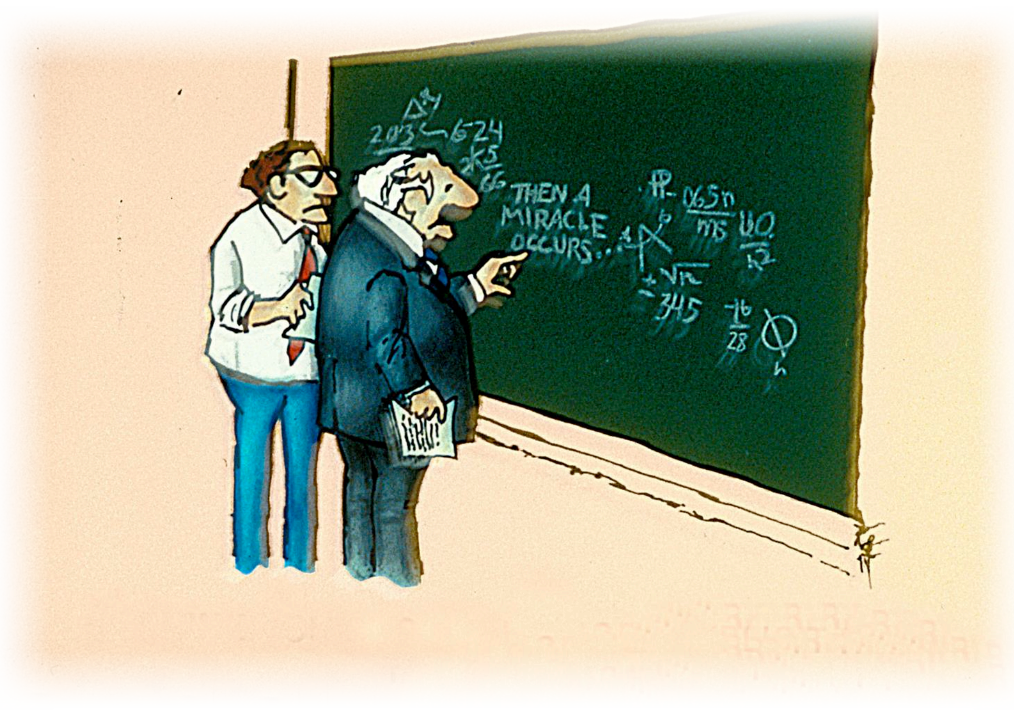
$p_\perp \sim \sqrt{g}T, p^- \sim gT$. Mistreated semi-collinear limit

Identify and subtract the limiting behaviors thereof

The NLO soft region



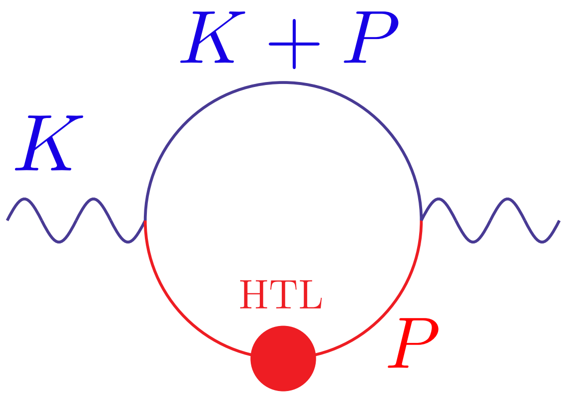
- 4 diagrams with HTL vertices and propagators on the soft line
- Could brute-force them numerically. Or think again about analyticity, light-cones



The soft region: sum rules

- We have found the fermionic analogue of the Aurenche Gelis Zaraket **JHEP0205** (2002) sum rule

- The leading-order soft contribution (P soft)



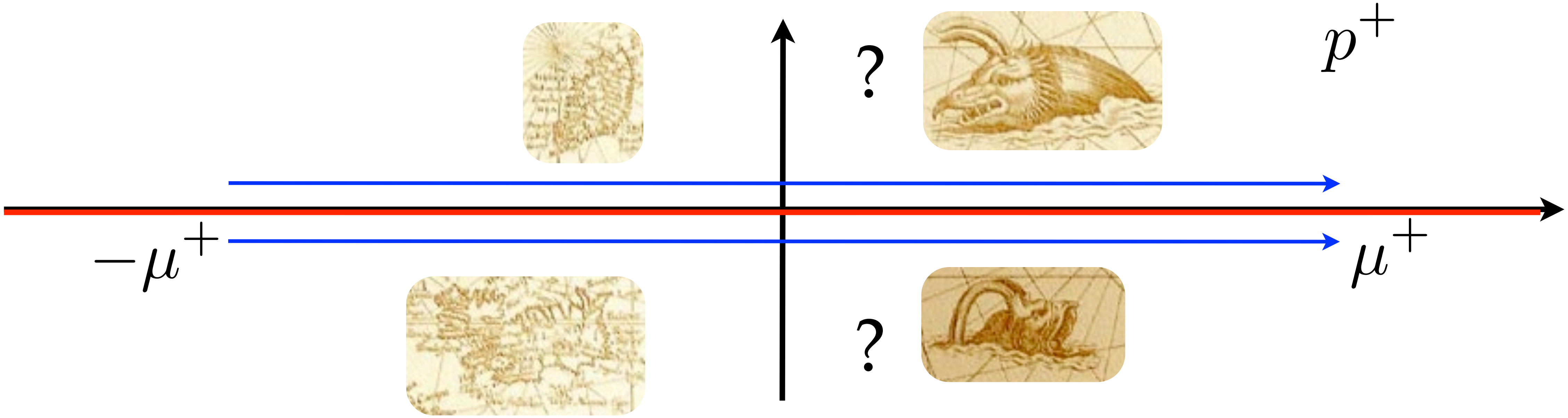
$$(2\pi)^3 \frac{d\Gamma_\gamma}{d^3k}_{\text{soft}} \propto \int \frac{dp^+ \, d^2p_\perp}{(2\pi)^3} \mathrm{Tr}\left[\gamma^-(S_R(P) - S_A(P))\right]_{p^-=0}$$

where $$S(P) = \frac{1}{2}\left[(\gamma^0 - \vec{\gamma}\cdot\hat{p})S^+(P) + (\gamma^0 + \vec{\gamma}\cdot\hat{p})S^-(P)\right]$$

$$S_R^\pm(P) = \left.\frac{i}{p^0 \mp \left[p + \frac{\omega_0^2}{p}\left(1 - \frac{p^0 \mp p}{2p}\ln\left(\frac{p^0+p}{p^0-p}\right)\right)\right]}\right|_{p^0=p^0+i\epsilon}$$

Fermionic sum rules

$$(2\pi)^3 \frac{d\Gamma_\gamma}{d^3k}_{\,\mathrm{soft}} \propto \int \frac{dp^+ \, d^2p_\perp}{(2\pi)^3} \mathrm{Tr}\left[\gamma^-(S_R(P) - S_A(P))\right]_{p^-=0}$$



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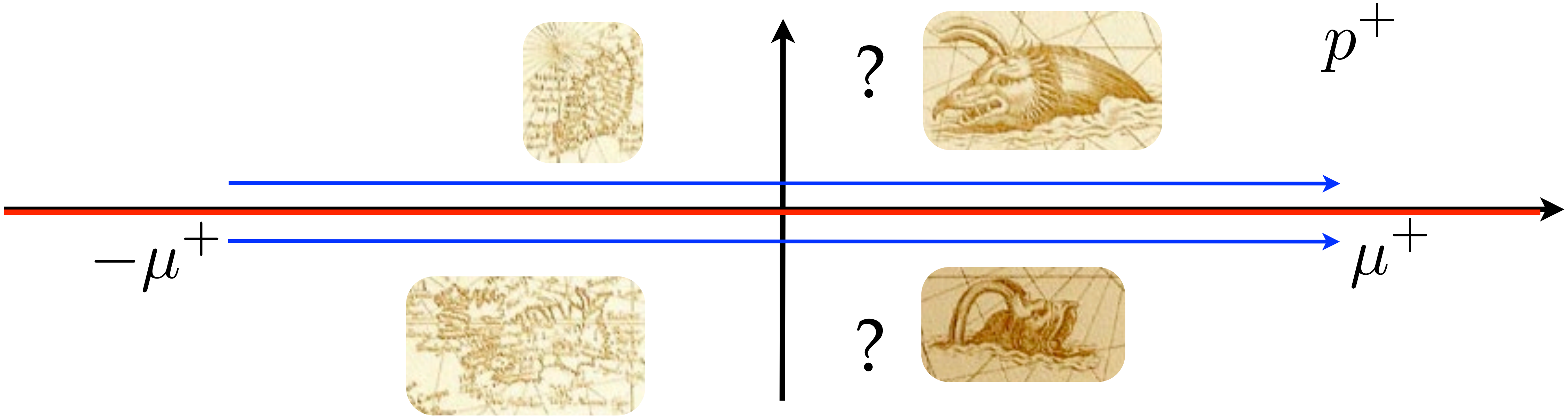

- A retarded propagator is an analytic function of Q in the upper half-plane not just in the frequency, but in any time-like or light-like variable

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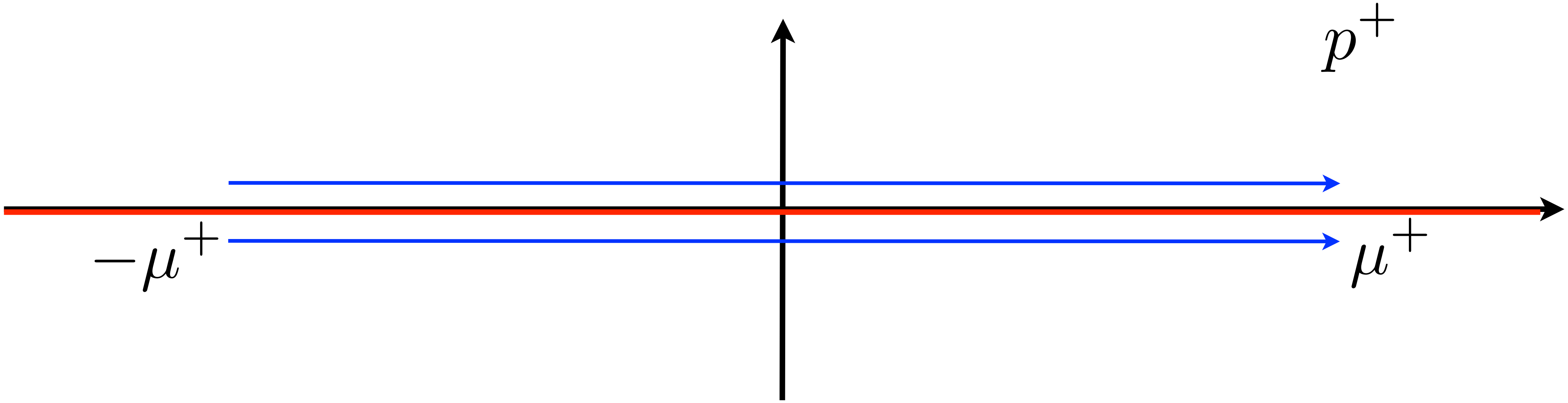

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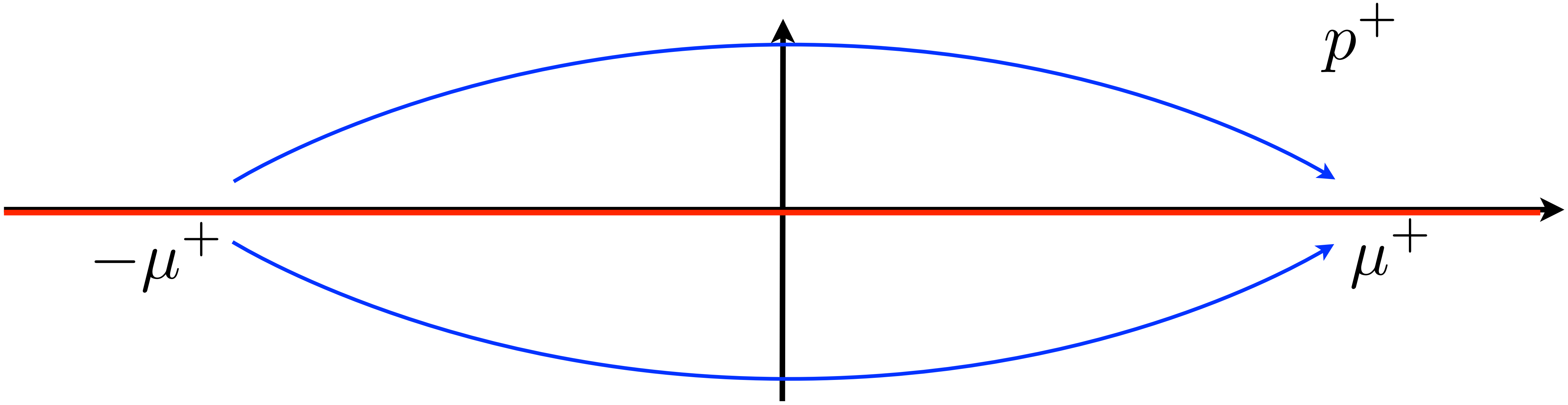

- A retarded propagator is an analytic function of Q in the upper half-plane not just in the frequency, but in any time-like or light-like variable
- Deform the contour away from the real axis

Fermionic sum rules

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- Along the arcs at large complex p^+ the integrand has a very simple behavior

$$\mathrm{Tr}\left[\gamma^-(S_R(P) - S_A(P)\right]_{p^-=0} = \frac{i}{p^+}\frac{m_\infty^2}{p_\perp^2 + m_\infty^2} + \mathcal{O}\left(\frac{1}{(p^+)^2}\right)$$

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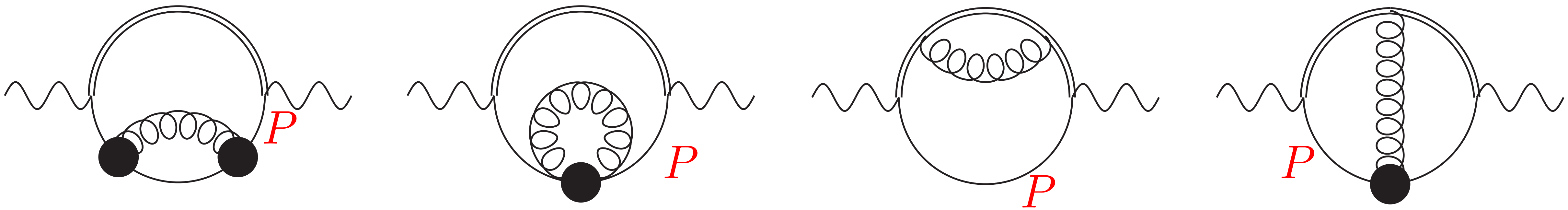
- The $p_\perp$ integral is UV-log divergent, giving the LO UV-divergence that cancels the IR divergence at the hard scale, now analytically

Independently obtained by Besak Bödeker **JCAP1203** (2012)

The NLO soft region



- At NLO one can use the KMS relations and the *ra* basis to write the diagrams in terms of fully retarded and fully advanced functions of P. The hard only depend on p^-.

The contour deformations are then again possible and the diagrams can be expanded for large complex p^+. On general grounds we expect

$$(2\pi)^3 \frac{d\delta\Gamma_\gamma}{d^3k}\bigg|_{\mathrm{soft}} \propto \int \frac{dp^+ d^2p_\perp}{(2\pi)^3}\left[C_0\left(\frac{1}{p^+}\right)^0 + C_1\left(\frac{1}{p^+}\right)^1 + \dots\right]$$

The soft region

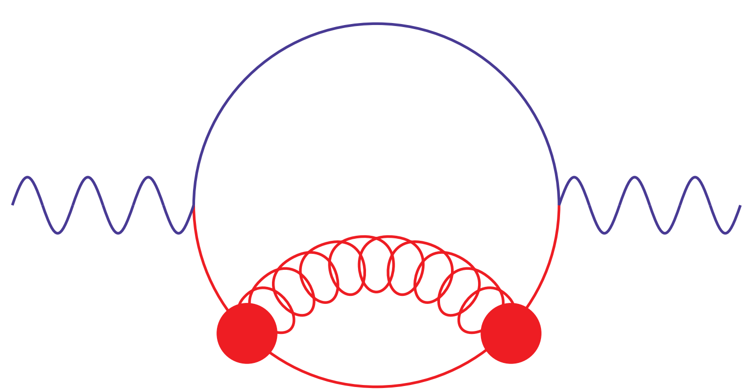
The $(1/p^+)^0$ term has to be *exactly* the subtraction term we have mentioned before in the collinear region, to cancel the cutoff dependence. Confirmed by explicit calculation

- At order $1/p^+$ we had the LO result. We can expect

$$\frac{m_\infty^2}{p_\perp^2+m_\infty^2} \to \frac{m_\infty^2+\delta m_\infty^2}{p_\perp^2+m_\infty^2+\delta m_\infty^2} = \left(\frac{m_\infty^2}{p_\perp^2+m_\infty^2} + \frac{\delta m_\infty^2\, p_\perp^2}{(p_\perp^2+m_\infty^2)^2} + \mathcal{O}(g^2)\right)$$

The explicit calculation finds just this contribution.

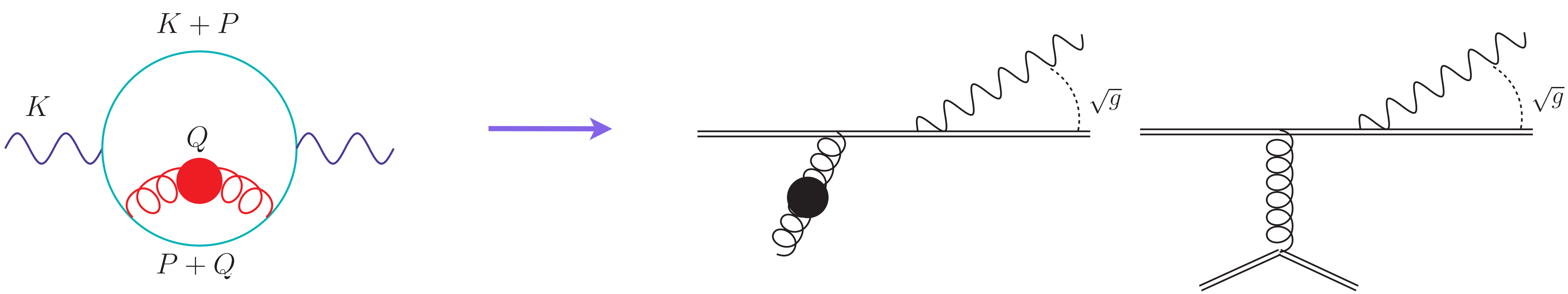
The contribution from HTL vertices goes like $(1/p^+)^2$ or smaller on the arcs.



$$\sim \frac{1}{(p^+)^2}$$

The semi-collinear region

- Seemingly different processes boiling down to wider-angle radiation



P semi-collinear
Q soft

Q soft plasmon, timelike

Q soft cut, spacelike

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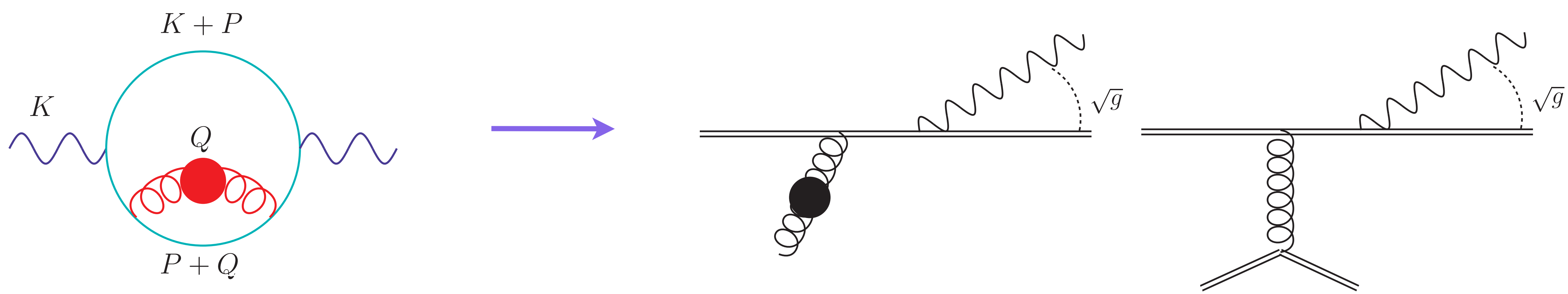

P semi-collinear
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- Evaluation: introduce "*modified* $\hat{q}$" that keep tracks of the changes in the small light-cone component p^- of the quarks

"*standard*" $$\frac{\hat{q}}{g^2C_R} \equiv \frac{1}{g^2C_R}\int \frac{d^2q_\perp}{(2\pi)^2} q_\perp^2 \mathcal{C}(q_\perp) \propto \int d^4Q \langle F^{+\mu}(Q)F^{+}{}_{\mu}(-Q)\rangle_{q^-=0}$$

"*modified*" $$\frac{\hat{q}(\delta E)}{g^2C_R} \propto \int d^4Q \langle F^{+\mu}(Q)F^{+}{}_{\mu}(-Q)\rangle_{q^-=\delta E}$$

- The "*modified* $\hat{q}$" can also be evaluated in EQCD

Results

Summary

- LO rate

$$(2\pi)^3 \frac{d\Gamma}{d^3k}\Big|_{\rm LO} = \mathcal{A}(k) \overbrace{\left[\log \frac{T}{m_\infty} + C_{2\to 2}(k) + C_{\rm coll}(k)\right]}^{C_{\rm LO}(k)}$$

$$\mathcal{A}(k) = \alpha_{\rm EM}\, g^2 \; C_F T^2 \frac{n_{\rm F}(k)}{2k} \sum_f Q_f^2 d_f$$

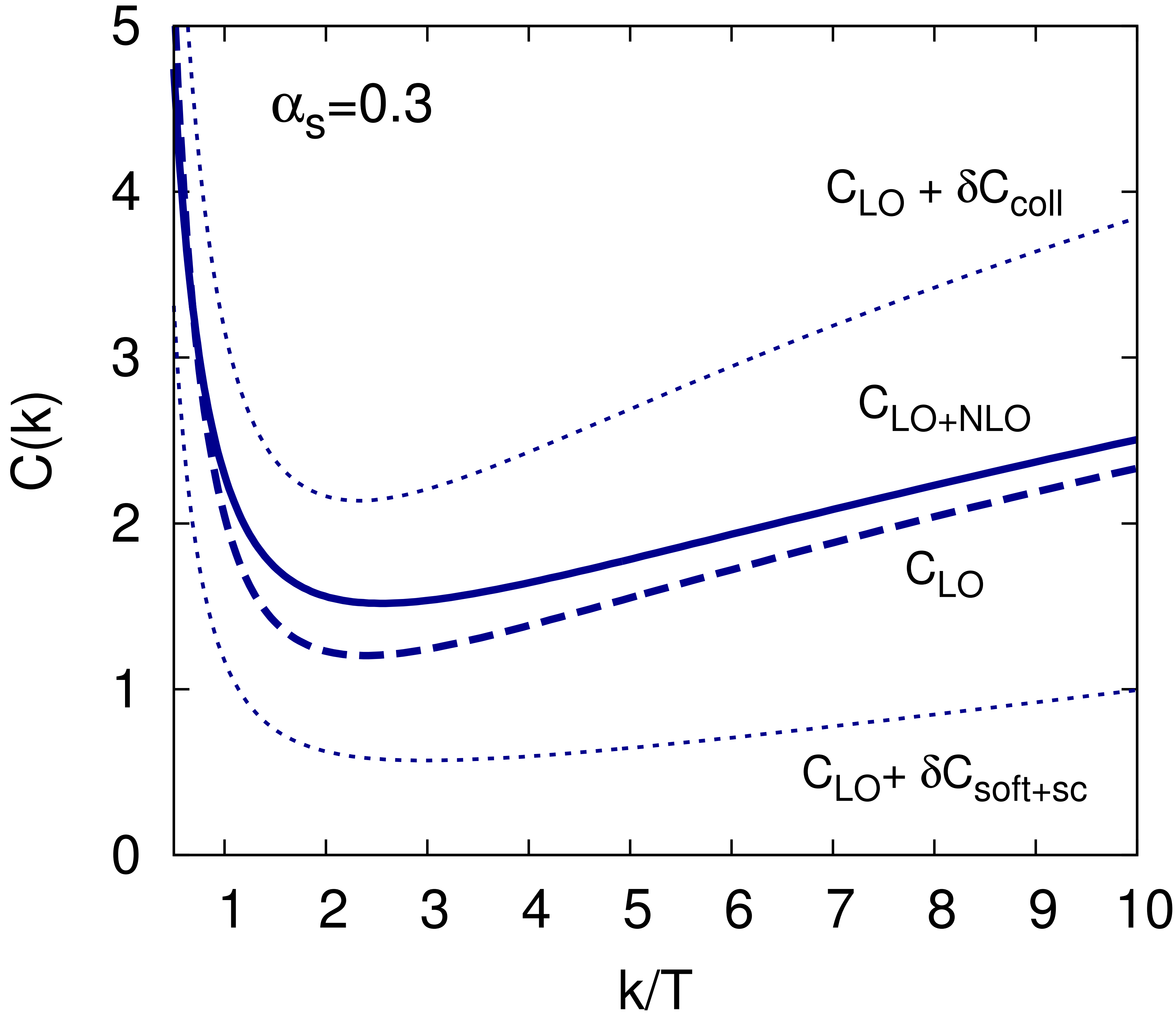
- NLO correction

$$(2\pi)^3 \frac{d\delta\Gamma}{d^3k}\Big|_{\rm NLO} = \mathcal{A}(k) \overbrace{\Bigg[\underbrace{\frac{\delta m_\infty^2}{m_\infty^2} \log \frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m_\infty^2}{m_\infty^2} C_{\rm soft+sc}(k)}_{\delta C_{\rm soft+sc}(k)} + \underbrace{\frac{\delta m_\infty^2}{m_\infty^2} C_{\rm coll}^{\delta m}(k) + \frac{g^2 C_A T}{m_D} C_{\rm coll}^{\delta C}(k)}_{\delta C_{\rm coll}(k)} \Bigg]}^{\delta C_{\rm NLO}(k)}$$

- Fits available in the paper

  JG Hong Kurkela Lu Moore Teaney **JHEP0513 (2013)**

$$(2\pi)^3 \frac{d\delta\Gamma}{d^3k}\Big|_{\rm NLO} = \mathcal{A}(k)\Bigg[\overbrace{\underbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m_\infty^2}{m_\infty^2}C_{\rm soft+sc}(k)}_{\delta C_{\rm soft+sc}(k)} + \underbrace{\frac{\delta m_\infty^2}{m_\infty^2}C^{\delta m}_{\rm coll}(k) + \frac{g^2C_AT}{m_D}C^{\delta\mathcal{C}}_{\rm coll}(k)}_{\delta C_{\rm coll}(k)}}^{\delta C_{\rm NLO}(k)}\Bigg]$$



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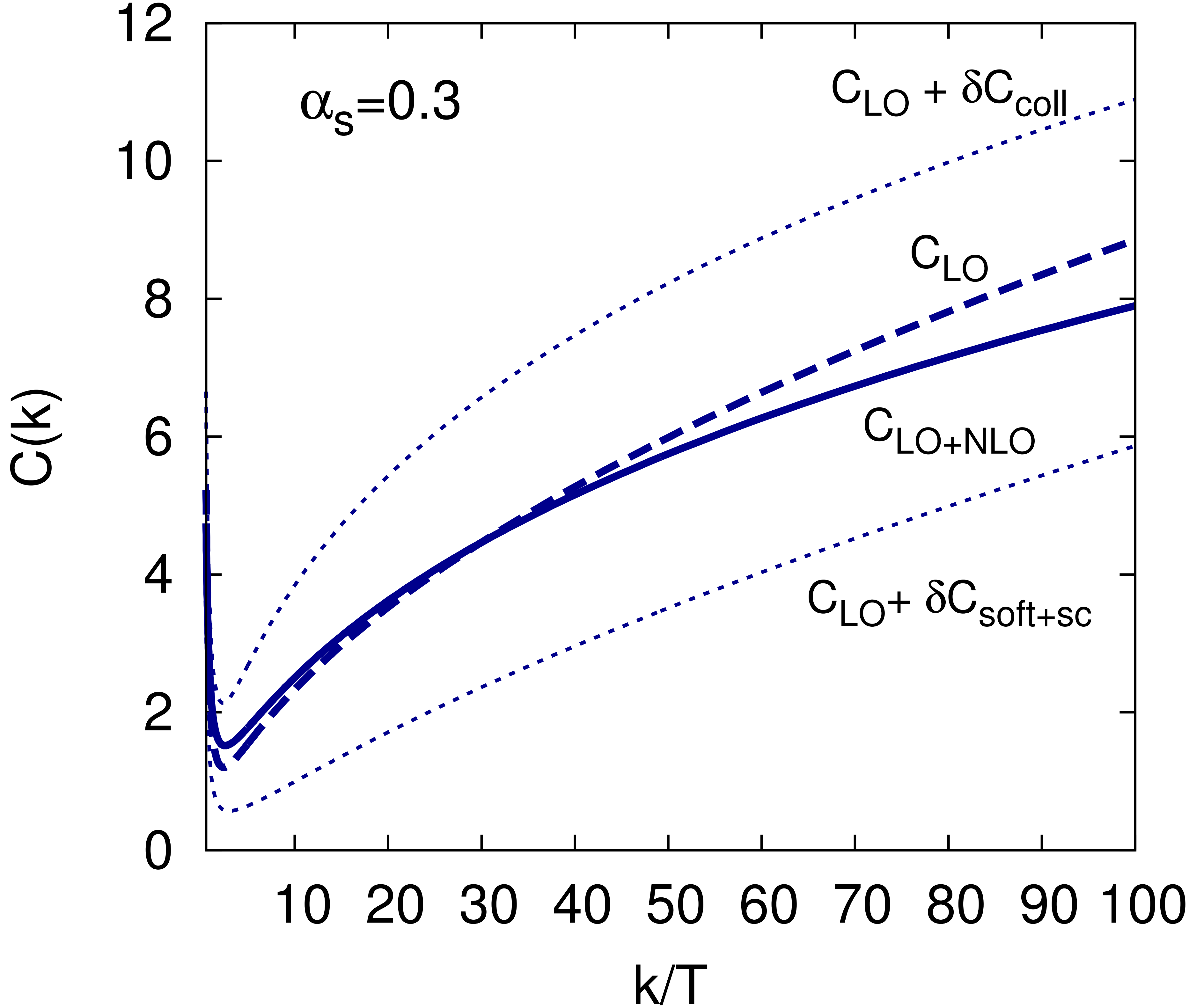

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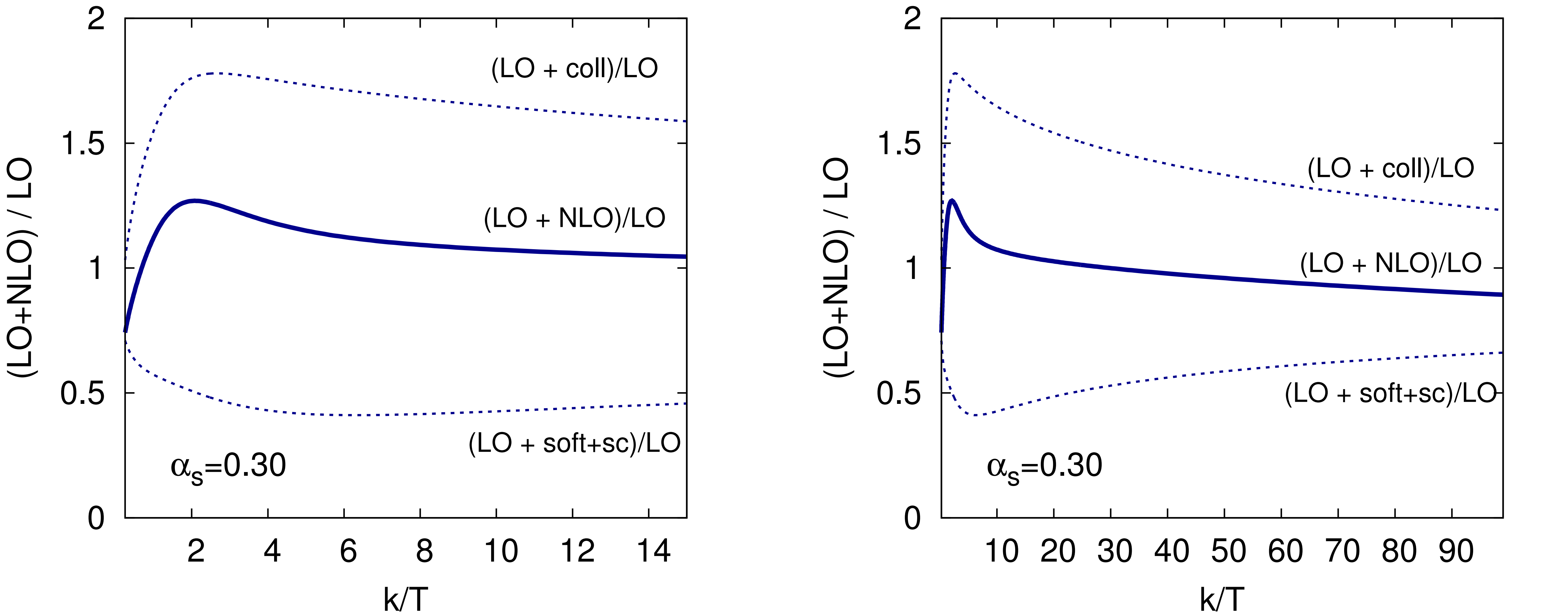
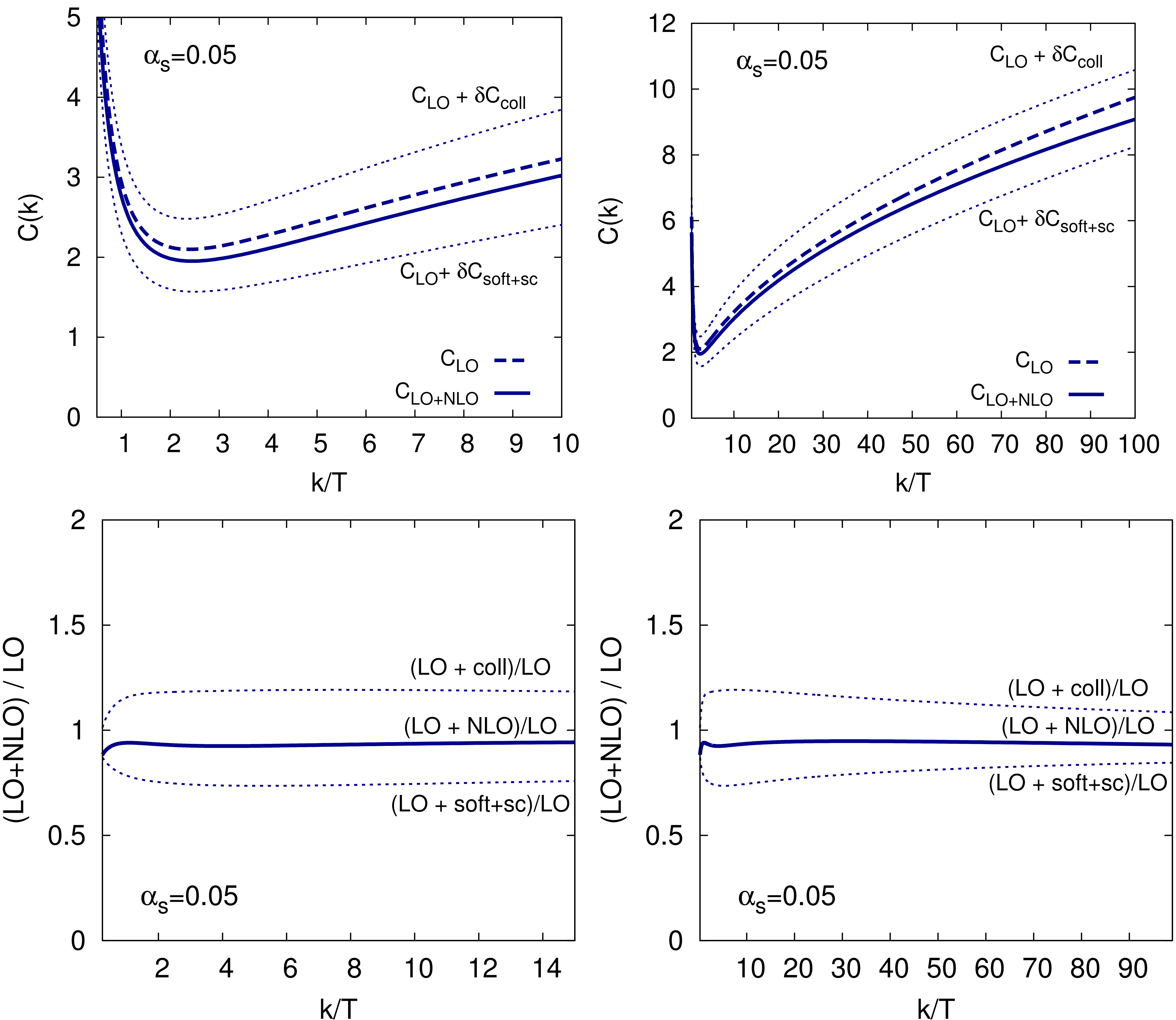




α_s=0.05
C_{LO} + δC_{coll}
C_{LO}+ $\delta C_{soft+sc}$
C_{LO}
C_{LO+NLO}
C(k)
k/T
(LO+NLO) / LO
(LO + coll)/LO
(LO + NLO)/LO
(LO + soft+sc)/LO

A sneak peek at jets/E-loss

A sneak peek at jets/E-loss

PRELIMINARY

McGill-AMY-MARTINI at NLO

- Apply similar technologies to jet evolution and E-loss
- Start from effective Boltzmann-Fokker-Planck approach

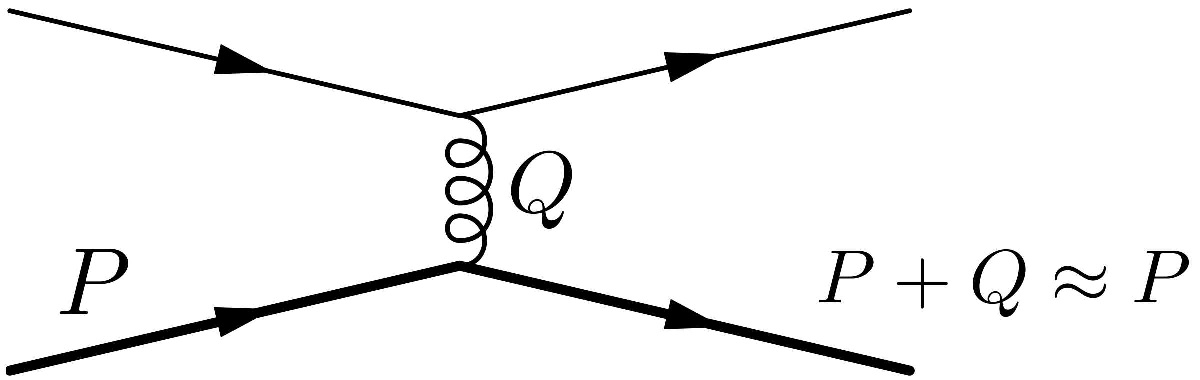
$$\frac{dP(p)}{dt} = \int_{-\infty}^{+\infty} dk \left(P(p+k)\frac{d\Gamma(p+k,k)}{dk} - P(p)\frac{d\Gamma(p,k)}{dk} \right)$$

AMY **JHEP0301** (2003) Jeon Moore **PRC71** (2005)

- 1↔2 and 2↔2 processes in the rates. The former a generalization of the collinear photon emission to gluons. The latter require HTL resummation. In both cases everything but the jet is in equilibrium
- LO rates implemented in MARTINI Schenke Gale Jeon **PRC80** (2009)

NLO @ work

- Again, need to account for NLO corrections in collinear, semi-collinear and soft regions
- The first two are rather straightforward generalizations of the photon case
- The latter requires some work. In the soft limit 2↔2 exchanges reduce to an energy-loss/momentum diffusion picture



The soft limit

- Soft limit of the Fokker-Planck equation

$$\frac{dP(p)}{dt} = \int_{-\mu^+}^{+\mu^+} dq^+ \frac{d\Gamma(p,q^+)}{dq^+} \left(q^+ \frac{dP(p)}{dp^+} + \frac{(q^+)^2}{2} \frac{d^2P(p)}{d(p^+)^2} \right) + \frac{1}{4}\nabla_\perp^2 P(p) \int d^2q_\perp q_\perp^2 \frac{d\Gamma(p,q^+)}{d^2q_\perp}$$

- Energy loss term dE/dt unknown to NLO
- Longitudinal momentum diffusion $\hat{q}_L$ unknown to NLO
- Transverse momentum diffusion $\hat{q}$, known to LO and NLO
- Fluctuation-dissipation $\hat{q}_L = 2T dE/dt$

Longitudinal momentum diffusion

- Field-theoretical lightcone definition

$$\hat{q}_L \equiv \frac{g^2}{d_R}\int_{-\infty}^{+\infty} dx^+ \mathrm{Tr}\left\langle U(-\infty,x^+)F^{+-}(x^+)U(x^+,0)F^{+-}(0)U(0,-\infty)\right\rangle$$

$F^{+-}=E^z$, longitudinal Lorentz force correlator

- At leading order $\hat{q}_L \propto \int \frac{dq^+ d^2q_\perp}{(2\pi)^3}(q^+)^2 G^>_{++}(q^+,q_\perp,0)$

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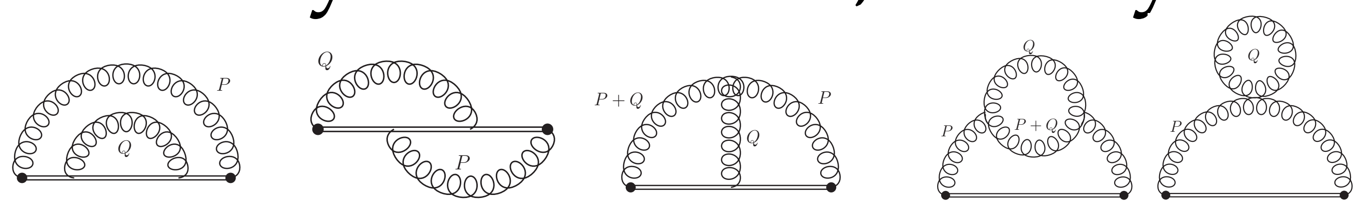
- Not dominated by zero-mode, but by arcs. LO + NLO



$$\hat{q}_L \propto \int \frac{d^2q_\perp}{(2\pi)^2}\frac{T(m_\infty^2+\delta m_\infty^2)}{q_\perp^2+m_\infty^2+\delta m_\infty^2} = T\int \frac{d^2q_\perp}{(2\pi)^2}\left[\frac{m_\infty^2}{q_\perp^2+m_\infty^2}+\frac{\delta m_\infty^2 q_\perp^2}{(q_\perp^2+m_\infty^2)^2}\right]$$

LO analytical result also in Peigné Peshier **PRD77** (2008)

- Implementation of these results in MARTINI is underway (Gervais JG Moore Schenke Teaney)

Conclusions

$$(2\pi)^3 \frac{d\delta\Gamma}{d^3k}\bigg|_{\mathrm{NLO}} = \mathcal{A}(k)\overbrace{\Bigg[\underbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m_\infty^2}{m_\infty^2}C_{\mathrm{soft+sc}}(k)}_{\delta C_{\mathrm{soft+sc}}(k)} + \underbrace{\frac{\delta m_\infty^2}{m_\infty^2}C_{\mathrm{coll}}^{\delta m}(k) + \frac{g^2 C_A T}{m_D}C_{\mathrm{coll}}^{\delta\mathcal{C}}(k)}_{\delta C_{\mathrm{coll}}(k)}\Bigg]}^{\delta C_{\mathrm{NLO}}(k)}$$

- The NLO contribution arises from three kinematical regions that are mutually sensitive to each other
- The result is given by two large and opposite contributions that largely cancel giving a relatively small NLO correction. Is the cancellation accidental?
- In the phenomenologically interesting window up to the NLO correction is 10%-20% for α_s=0.3

Conclusions

- On the lightcone, apparently complicated dynamical quantities factor into simpler light-cone condensates or operators, which are basically of two kinds
    - Energy-dependent: thermal masses
    - Energy-independent: correlators of the 3D theory
- The NLO-dynamical-calculation train has departed. Next stops:
    - Jets
    - Low invariant mass dileptons
    - Transport coefficients

Backup

NLO transport coefficients

- The only transport coefficient known so far at NLO is the *heavy quark momentum diffusion coefficient,* which is defined through the noise-noise correlator in a Langevin formalism. In field theory it can be written as

$$\kappa = \frac{g^2}{3N_c}\int_{-\infty}^{+\infty} dt \mathrm{Tr}\langle U(t,-\infty)^\dagger E_i(t) U(t,0) E_i(0) U(0,-\infty)\rangle$$

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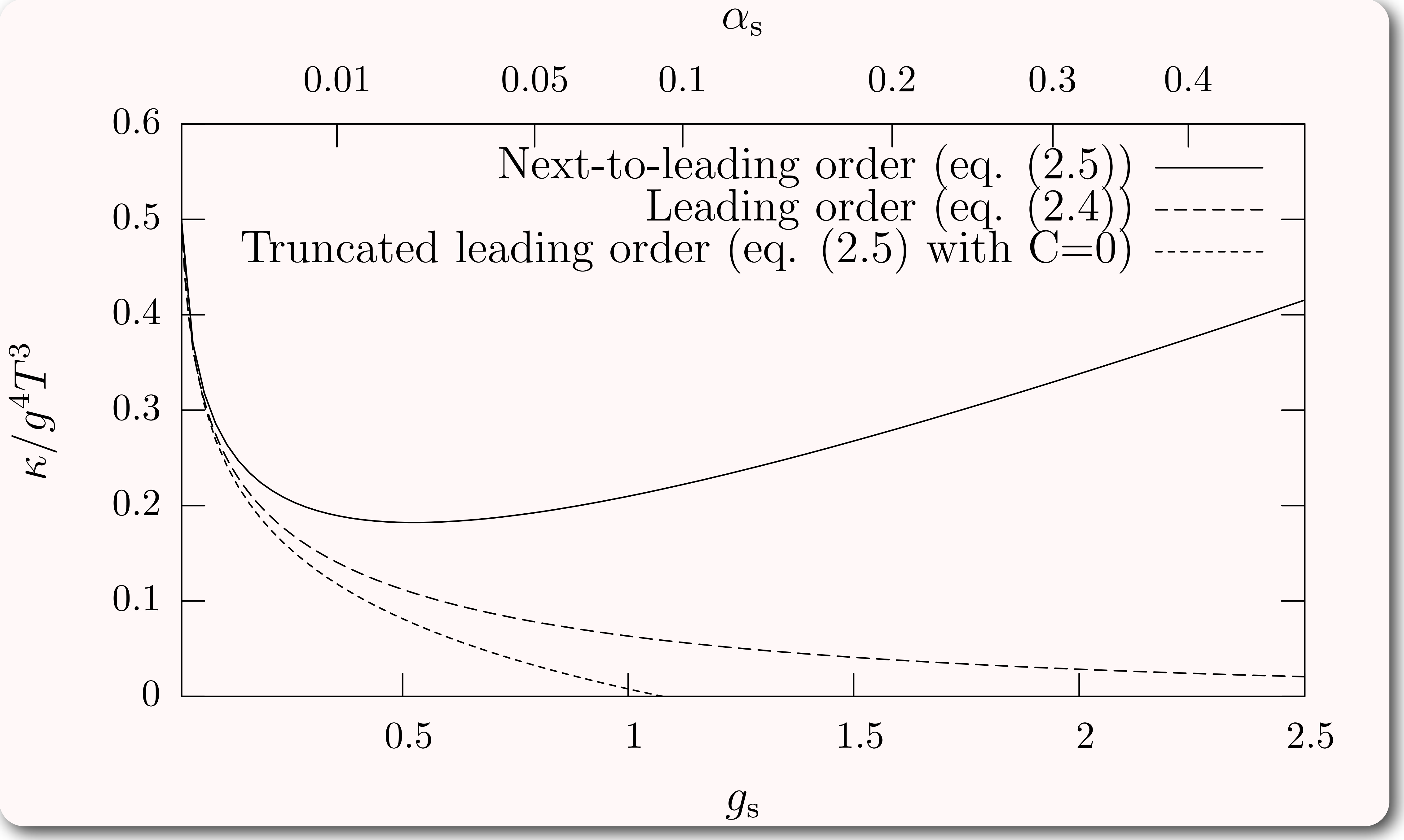
- The NLO computation factors in the coefficient C, which turns out to be sizeable

$$\kappa = \frac{C_H g^4 T^3}{18\pi}\left(\left[N_c + \frac{N_f}{2}\right]\left[\ln\frac{2T}{m_D} + \xi\right] + \frac{N_f \ln 2}{2} + \frac{N_c m_D}{T} C + \mathcal{O}(g^2)\right) \qquad \xi = \frac{1}{2} - \gamma_E + \frac{\zeta'(2)}{\zeta(2)}$$

Caron-Huot Moore **PRL100, JHEP0802** (2008)

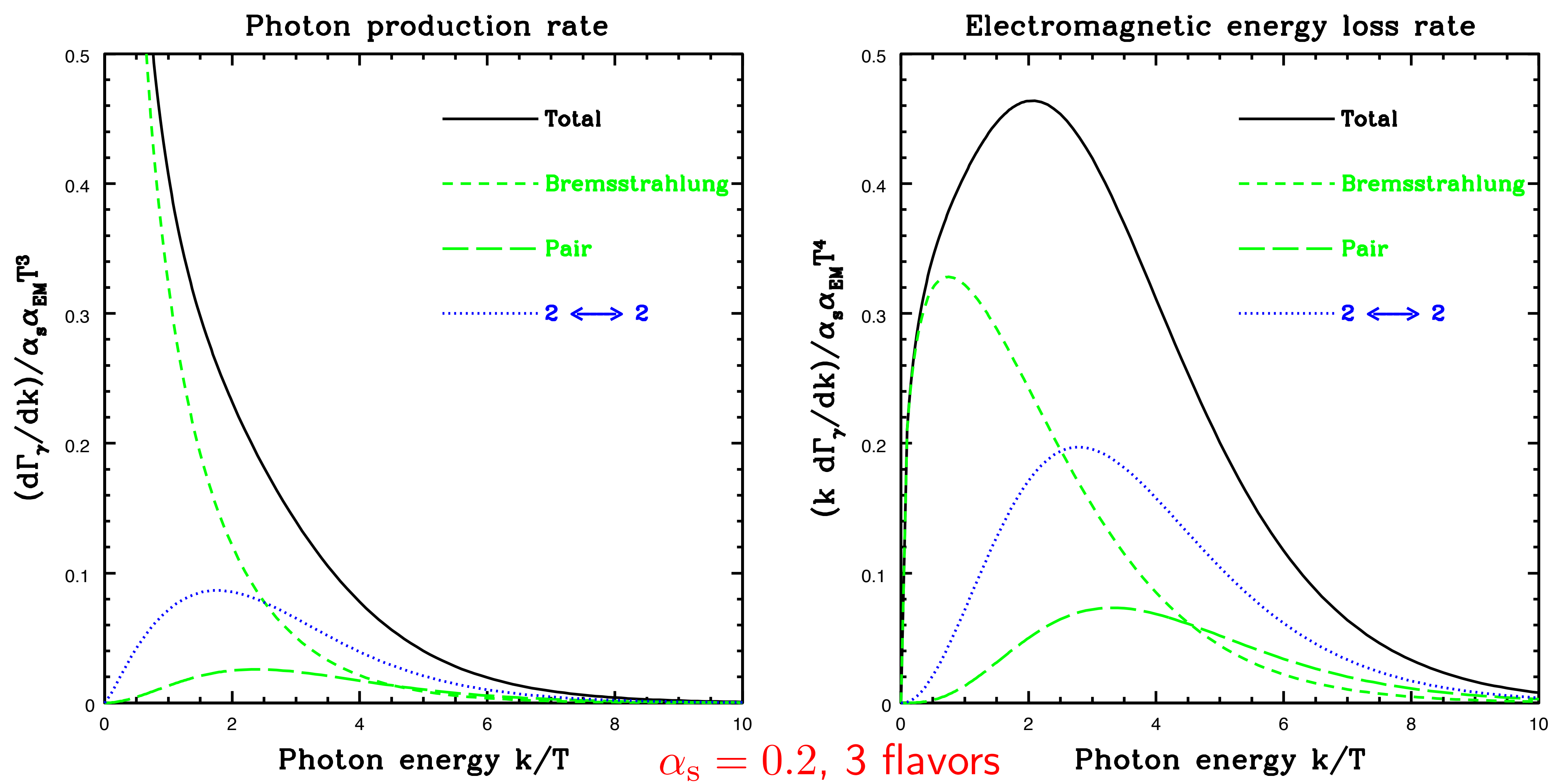
NLO transport coefficients



Caron-Huot Moore **PRL100, JHEP0802** (2008)

Full LO results

- Numerically solving the implicit equation for the collinear region yields the full LO results for the thermal photon production rate



$\alpha_s = 0.2$, 3 flavors

Arnold Moore Yaffe **JHEP0112** (2001)

Euclideanization of light-cone soft physics

- For $t/x_z = 0$: equal time Euclidean correlators.

$$G_{rr}(t=0,\mathbf{x}) = \sum\!\!\!\!\!\!\!\int_p G_E(\omega_n, p) e^{i\mathbf{p}\cdot\mathbf{x}}$$

Caron-Huot **PRD79** (2009)

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- Soft physics dominated by $n=0$ (and t-independent) =>EQCD!

Caron-Huot **PRD79** (2009)

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$$G_{rr}(t,\mathbf{x})_{\rm soft} = T\int d^3p\, e^{i\mathbf{p}\cdot\mathbf{x}}\, G_E(\omega_n=0,\mathbf{p})$$

- Soft physics dominated by $n=0$ (and t-independent) =>EQCD!

Caron-Huot **PRD79** (2009)

The semi-collinear region

- Subtraction term from the collinear region

$$\frac{d\delta\Gamma_\gamma}{d^3k}\bigg|^{\text{coll subtr.}}_{\text{semi-coll}} = 2\frac{\mathcal{A}(k)}{(2\pi)^3}\int dp^+ \left[\frac{(p^+)^2+(p^++k)^2}{(p^+)^2(p^++k)^2}\right]\frac{n_F(k+p^+)[1-n_F(p^+)]}{n_F(k)}$$
$$\times\frac{1}{g^2C_RT^2}\int\frac{d^2p_\perp}{(2\pi)^2}\frac{4(p^+)^2(p^++k)^2}{k^2p_\perp^4}\int\frac{d^2q_\perp}{(2\pi)^2}\,q_\perp^2\,\mathcal{C}(q_\perp)\,.$$

- Proper evaluation: replace

$$\frac{\hat{q}}{g^2C_R}\equiv\frac{1}{g^2C_R}\int\frac{d^2q_\perp}{(2\pi)^2}q_\perp^2\mathcal{C}(q_\perp)\ \propto\int d^4Q\langle F^\mu_+F_{+\mu}(Q)\rangle_{q^-=0}$$

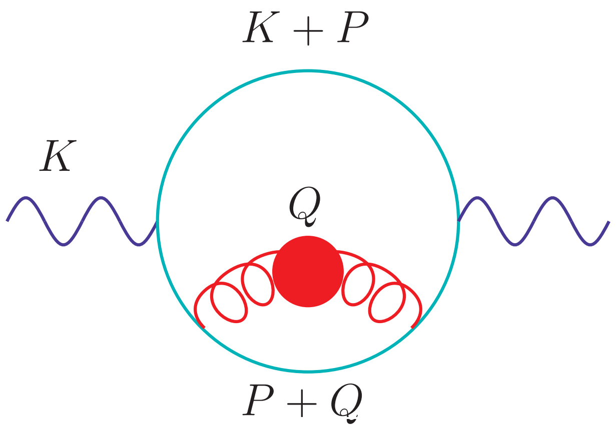
with

$$\frac{\hat{q}(\delta E)}{g^2C_R}\ \propto\int d^4Q\langle F^\mu_+F_{+\mu}(Q)\rangle_{q^-=\delta E}$$

because $\delta E \sim gT$ is no longer negligible


The latter object too can be evaluated in Euclidean spacetime

The semi-collinear region



P semi-collinear
Q soft

- Limits and divergences

  ↑ $p_\perp \to \infty \, (\delta E \to \infty)$ subtract the hard limit

  ↓ $\mathrm{p}_\perp \to 0$ subtract the collinear limit $(p_\perp \gg q_\perp)$

  ↙ $\mathrm{p}_\perp \to 0 \wedge p^+ \to 0$ IR log, combines with UV soft log (NLO log)

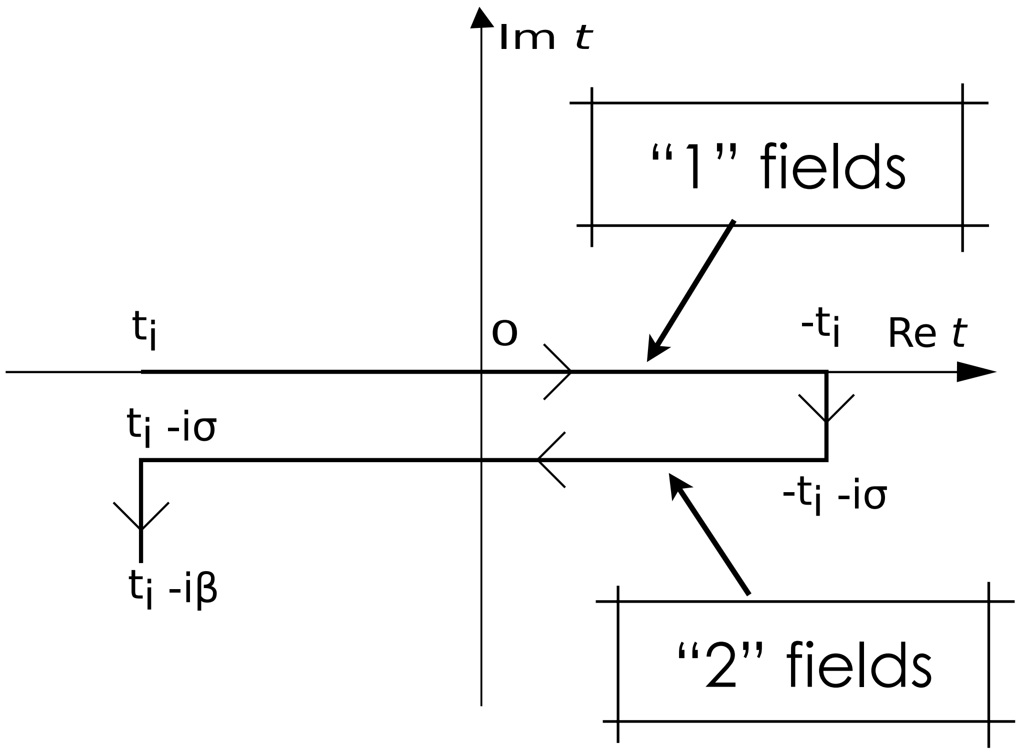
- Aside from the IR-log, the general behaviour of the P integration can only be obtained numerically.

The *ra* formalism

- Alternative to the "12" formulation of the real-time formalism. Define

$$\phi_r = (\phi_1 + \phi_2)/2$$

$$\phi_a = \phi_1 - \phi_2$$



- The propagators become

$$G \equiv \begin{pmatrix} G_{rr} & G_{ra} \\ G_{ar} & G_{aa} \end{pmatrix} = \begin{pmatrix} (G_R - G_A)\left(\frac{1}{2} \pm n(p^0)\right) & G_R \\ G_A & 0 \end{pmatrix}$$

- Graphical notation