COMMENTS ON GLUON PRODUCTION AND LARGE N_C COUNTING.

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LEGEND: ALL AMPLITUDES FACTORISE AT LARGE N_C.

NOT TRUE.

FACTORIZABILITY IS NOT AN INHERENT PROPERTY OF THE LARGE N LIMIT.

METHOD 1. **PROOF BY APPEALING TO AUTHORITY.**

MUELLER-HATA (2007) DIPOLE DENSITY CORRELATION IN THE BFKL EVOLVED WAVE FUNCTION OF A SINGLE DIPOLE (PARENT DIPOLE LARGER THAN DAUGHTERS)

$$\langle n(x_1,\bar{x}_1)n(x_2,\bar{x}_2)\rangle - \langle n(x_1,\bar{x}_1)\rangle\langle n(x_2,\bar{x}_2)\rangle \sim \langle n(x_1,\bar{x}_1)\rangle\langle n(x_2,\bar{x}_2)\rangle \left(\frac{b}{x}\right)^{-\lambda}$$

x - DISTANCE BETWEEN THE DIPOLES, b - PARENT DIPOLE SIZE

PROPORTIONAL TO SCATTERING AMPLITUDE OF TWO DIPOLES ON ONE EVOLVED DIPOLE: SO THE AMPLITUDES DO NOT FACTORIZE.

METHOD 2. **PROOF BY TRIVIAL EXAMPLE.**

TARGET T: WITH PROBABLILTY p - NOTHING, WITH PROBABILITY (1-p) - SINGLE DIPOLE D : $|\psi >= \sqrt{p}|0 > +\sqrt{1-p}|D >$.

PROJECTILE P: TWO IDENTICAL DIPOLES, SO THAT EACH ONE SCATTERS ON D WITH PROBABILITY 1.

SCATTERING PROBABILITY OF P ON T: $(1-p) \neq (1-p)^2$

NOT EQUAL TO THE PRODUCT OF PROBABILITIES: ($(1-p)$ FOR EACH DIPOLE IN THE PROJECTILE)

THE TWO DIPOLES ALWAYS SCATTER ON THE SAME "FIELD", AND THUS THEIR SCATTERING PROBABILITIES ARE STRONGLY CORRELATED.

BACK TO TWO GLUON PRODUCTION

(Altinoluk, Kovner, Levin, M.L. - to appear very soon)

"INDEPENDENT PRODUCTION" CONTRIBUTION VALID IN DENSE-DILUTE AND DILUTE-DENSE LIMITS (WITH SOME WISHFUL THINKING EXTENDIBLE TO DENSE-DENSE)

$$\frac{dN}{d^2p d^2k d\eta d\xi} = \left(\frac{1}{32\,\pi^3 \alpha_s N_C}\right)^2 \frac{1}{k^2}\frac{1}{p^2}\int_{x,y,u,v} \cos k(x-y)\cos p(u-v)\left[N_P + N_B\right]$$

where

$$N_P = \frac{1}{4}\frac{\partial}{\partial(ij\bar{i}\bar{j})}[P^T_A(x,y)P^T_A(u,v)]\Delta^{ijkl}\Delta^{\bar{i}\bar{j}\bar{k}\bar{l}}\frac{\partial}{\partial(kl\bar{k}\bar{l})}[P^P_A(x,y)P^P_A(u,v)]$$

$$N_B \approx -\frac{8}{N_C^2}\frac{\partial}{\partial(ij\bar{i}\bar{j})}[B^T_{yuvx}]\Delta^{ijkl}\Delta^{\bar{i}\bar{j}\bar{k}\bar{l}}\frac{\partial}{\partial(kl\bar{k}\bar{l})}[B^P_{xvuy}]$$

with

$$\frac{\partial}{\partial(ijkl)} \equiv \frac{\partial}{\partial x_i}\frac{\partial}{\partial y_j}\frac{\partial}{\partial u_{\bar{i}}}\frac{\partial}{\partial v_{\bar{j}}} \qquad \Delta^{ijkl} \equiv \delta^{ij}\delta^{kl} + \delta^{ik}\delta^{jl} - \delta^{il}\delta^{jk}$$

"ADJOINT POMERON":

$$P_A(z\bar{z}) \equiv \langle 1 - \frac{1}{N_C^2} tr S_A^\dagger(z) S_A(\bar{z}) \rangle = 2P(z\bar{z}) - P^2(z\bar{z})$$

"B"-REGGEON -ESSENTIALLY THE QUADRUPOLE (UP TO SIMPLE SUBTRACTIONS)

$$B(xyuv) = \langle 1 - \frac{1}{N_C} tr[S_x^\dagger S_y S_u^\dagger S_v] - P_{xy} - \ldots \rangle$$

POMERON $\equiv$ DIPOLE SCATTERING AMPLITUDE

$$P(xy) \equiv \langle 1 - \frac{1}{N_C} tr S^\dagger(x) S(y) \rangle$$

COMMENT NUMBER ONE.

WITH A NAKED EYE IT LOOKS THE B-REGGEON CONTRIBUTION IS SUBLEADING AT LARGE N_C (BUT WAIT FOR COMMENT NUMBER THREE).

IT IS QUITE SIMILAR TO THE EXPRESSION USED BY DUSLING-VENUGOPALAN BUT NOT QUITE THE SAME - THEY USED P INSTEAD OF P_A

$$N_P = \frac{1}{4}\frac{\partial}{\partial(ij\bar{i}\bar{j})}[P^T(x,y)P^T(u,v)]\Delta^{ijkl}\Delta^{\bar{i}\bar{j}\bar{k}\bar{l}}\frac{\partial}{\partial(kl\bar{k}\bar{l})}[P^P(x,y)P^P(u,v)]$$

AT SMALL MOMENTA (BELOW Q_S) THE TWO ARE THE SAME.

BUT THE CORRELATIONS WERE CALCULATED FOR MOMENTA ABOVE Q_S - AND THERE IS $O(1)$ DIFFERENCE IN THAT REGIME.

SO WOULD BE INTERESTING TO SEE HOW MUCH DIFFERENCE IT MAKES FOR THE NUMERICAL RESULTS.

COMMENT NUMBER TWO.

$B(xyuv)$ IN PRINCIPLE IS A NONTRIVIAL NONFACTORIZABLE FUNCTION OF ALL FOUR VARIABLES, AND THUS IS A VERY GOOD CANDIDATE TO PRODUCE ANGULAR CORRELATIONS.

ITS CONTRIBUTION IS OF THE SAME ORDER AS THAT TAKEN INTO ACCOUNT IN THE CALCULATION OF DUSLING-VENUGOPALAN, AND THUS ALSO HAS TO BE RETAINED.

COMMENT NUMBER THREE.

THE B-REGGEON CONTRIBUTION IS LEADING IN N_C IN THE DENSE-DILUTE LIMIT.

HOW IS THAT?

N_C COUNTING IS VERY DIFFERENT IN DENSE AND DILUTE LIMITS.

TAKE POMERON:

$$P(xy) \equiv \langle 1 - \frac{1}{N_C} trS^{\dagger}(x)S(y)\rangle$$

IN THE DILUTE LIMIT $P \sim \alpha_s^2$.

BUT REMEMBER $\alpha_s N_C = \lambda \rightarrow_{N_C \rightarrow \infty} constant$

SO IN DILUTE LIMIT $P = O(1/N_C^2)$!

SIMILARLY B-REGGEON IN DIUTE LIMIT $B = O(1/N_C^2)$

BUT IN THE GLUON DOUBLE INCLUSIVE CROSS SECTION WE HAVE TO COMPARE B VERSUS P^2.

$$N_P = \frac{1}{4}\frac{\partial}{\partial(ij\bar{i}\bar{j})}[P^T_A(x,y)P^T_A(u,v)]\Delta^{ijkl}\Delta^{\bar{i}\bar{j}\bar{k}\bar{l}}\frac{\partial}{\partial(kl\bar{k}\bar{l})}[P^P_A(x,y)P^P_A(u,v)]$$

$$N_B \approx -\frac{8}{N_C^2}\frac{\partial}{\partial(ij\bar{i}\bar{j})}[B^T_{yuvx}]\Delta^{ijkl}\Delta^{\bar{i}\bar{j}\bar{k}\bar{l}}\frac{\partial}{\partial(kl\bar{k}\bar{l})}[B^P_{xvuy}]$$

SO IF ONE OF THE COLLIDING OBJECTS IS DILUTE, THE B TERM RECOVERS THE FACTOR N_C^2 RELATIVE TO THE P^2 TERM AND BECOMES COMPARABLE.

AS DISCUSSED BY PREVIOUS SPEAKER, MOMENTA IMPORTANT FOR CORRELATIONS ARE ABOVE Q_S - POINTS SHOULD BE INSIDE THE CORRELATION LENGTH. SO WHICH N_C COUNTING IS RELEVANT FOR DETERMINING RELATIVE IMPORTANCE OF THE TERMS IS NOT OBVIOUS. IT MAY BE THAT THE B-REGGEON CONTRIBUTION HAS TO BE CONSIDERED AS THE LEADING N_C EFFECT.

HOW DID WE GET INTO SUCH A MESS WITH N_C COUNTING?

THIS IS NOT NEW. AT LARGE N_C FORMALLY THERE IS NEVER A PROBLEM WITH UNITARITY AND THERE IS NEVER SATURATION.

BFKL-LIKE BEHAVIOR OF THE POMERON AMPLITUDE

$$P(\eta) \sim \alpha_s^2 e^{\lambda\eta}$$

THE UNITARITY IS VIOLATED, AND SATURATION IS APPROACHED ONLY AT "INFINITE" RAPIDITIES

$$\eta > \frac{1}{\lambda} \ln(N_C^2/\lambda^2)$$

AT LARGE N_C WE NEED TO EVOLVE VERY FAR IN RAPIDITY IN ORDER TO REACH SATURATION. ONCE WE HAVE APPROACHED SATURATION, THE AMPLITUDES ARE $O(1)$. THE PRICE WE PAY, IS THAT THE COLOR CHARGE DENSITY IS VERY LARGE - $O(N_C^2)$

IN THIS CONTEXT, ESPECIALLY IF WE ARE INTERESTED IN MOMENTA CLOSE TO THE SATURAION BOUNDARY, COUNTING POWERS OF N_C IS A TRICKY BUSINESS.