Investigating jet quenching on the lattice

Marco Panero[a], Kari Rummukainen[b] and Andreas Schäfer[c]

[a] Instituto de Física Teórica UAM/CSIC, Universidad Autónoma de Madrid

[b] Department of Physics and Helsinki Institute of Physics, University of Helsinki

[c] Institute for Theoretical Physics, University of Regensburg

Hard Probes 2013 — Stellenbosch, 4 November 2013

[Based on arXiv:1307.5850]

Outline

Overview of the theoretical approach

Jet quenching Bjorken, 1982 provides important information on heavy-ion physics

Theoretical description involves both perturbative and non-perturbative aspects Casalderrey-Solana and Salgado, 2007

QCD factorization theorems:

$$\sigma_{(M+N\to\text{hadron})} = f_M(x_1, Q^2) \otimes f_N(x_2, Q^2) \otimes \sigma(x_1, x_2, Q^2) \otimes D_{\text{parton}\to\text{hadron}}(z, Q^2)$$

$f_A(x, Q^2)$: parton distribution functions
$\sigma(x, y, Q^2)$: short-distance cross-section
$D_{\text{parton}\to\text{hadron}}(z, Q^2)$: fragmentation function

Overview of the theoretical approach

Jet quenching Bjorken, 1982 provides important information on heavy-ion physics

Theoretical description involves both perturbative and non-perturbative aspects
Casalderrey-Solana and Salgado, 2007

QCD factorization theorems:

$$\sigma_{(M+N\to\text{hadron})} = f_M(x_1, Q^2) \otimes f_N(x_2, Q^2) \otimes \sigma(x_1, x_2, Q^2) \otimes D_{\text{parton}\to\text{hadron}}(z, Q^2)$$

$f_A(x, Q^2)$: parton distribution functions
$\sigma(x, y, Q^2)$: short-distance cross-section
$D_{\text{parton}\to\text{hadron}}(z, Q^2)$: fragmentation function

Here: Focus on propagation of the hard parton in QGP medium

Hard parton propagation in QGP

Multiple soft-scattering description, in the *eikonal approximation* Baier et al., 1997

Leading effect: *transverse momentum broadening*, described by the jet quenching parameter:

$$\hat{q} = \frac{\langle p_\perp^2 \rangle}{L}$$

Can be evaluated in terms of a *collision kernel* $C(p_\perp)$ (differential parton-plasma constituents collision rate)

$$\hat{q} = \int \frac{\mathrm{d}^2 p_\perp}{(2\pi)^2} p_\perp^2 C(p_\perp)$$

$C(p_\perp)$ can be related to a two-point correlator of *light-cone Wilson lines*

Computing the jet quenching parameter

Theoretical tools

- Perturbation theory (PT) expansions
  - ✓ Based on first principles
  - ✓ Well established technology
  - ✓ Problems with infrared divergences are well understood
  - ✗ May not be reliable at RHIC or LHC temperatures
- Holographic computations
  - ✓ Ideally suited for strong coupling
  - ✗ Not directly applied to real-world QCD
- Lattice simulations
  - ✓ Based on first principles
  - ✓ Well established (computer) technology
  - ✓ Do not rely on weak- or strong-coupling assumptions
  - ✗ Euclidean setup, so *generally* unsuitable for real-time phenomena
- Models . . .

Outline

Key idea

Energy scale hierarchy in high-temperature, perturbative QCD:

$$g^2 T/\pi \text{ (ultrasoft) } \ll gT \text{ (soft) } \ll \pi T \text{ (hard)}$$

IR divergences accounted for by 3D effective theories Braaten and Nieto, 1995 Kajantie et al., 1995:

- electrostatic QCD (3D Yang-Mills + adjoint scalar field) for soft scale
- magnetostatic QCD (3D pure Yang-Mills) for ultrasoft scale

Large NLO corrections hindering PT due to *soft*, essentially *classical* fields

Observation: Soft contributions to physics of light-cone partons *insensitive* to parton velocity $\longrightarrow$ Can turn the problem Euclidean! Caron-Huot, 2008

Proof

Spatially separated ($|t| < |z|$) light-like Wilson lines Ghiglieri *et al.*, 2013

$$G^<(t, x_\perp, z) = \int \mathrm{d}\omega \mathrm{d}^2 p_\perp \mathrm{d}p^z \tilde{G}^<(\omega, p_\perp, p^z) e^{-i(\omega t - x_\perp \cdot p_\perp - z p^z)}$$

$$= \int \mathrm{d}\omega \mathrm{d}^2 p_\perp \mathrm{d}p^z \left[\frac{1}{2} + n_\mathrm{B}(\omega)\right] \left[\tilde{G}_\mathrm{R}(\omega, p_\perp, p^z) - \tilde{G}_\mathrm{A}(\omega, p_\perp, p^z)\right] e^{-i(\omega t - x_\perp \cdot p_\perp - z p^z)}$$

Shift $p'^z = p^z - \omega t/z$, integrate over frequencies by analytical continuation into upper (lower) half-plane for retarded (advanced) contribution $\longrightarrow$ sum over Matsubara frequencies

$$G^<(t, x_\perp, z) = T \sum_{n \in \mathbb{Z}} \int \mathrm{d}^2 p_\perp \mathrm{d}p'^z \tilde{G}_\mathrm{E}(2\pi n T, p_\perp, p'^z + 2\pi i n T t/z) e^{i\left(x_\perp \cdot p_\perp + z p'^z\right)}$$

- $n \neq 0$ contributions: exponentially suppressed at large separations
- Soft contribution: from $n = 0$ mode. Time-independent: evaluate in EQCD

Outline

Electrostatic QCD on the lattice

Super-renormalizable EQCD Lagrangian

$$\mathcal{L} = \frac{1}{4} F_{ij}^a F_{ij}^a + \mathrm{Tr}\left((D_i A_0)^2\right) + m_{\mathrm{E}}^2 \mathrm{Tr}\left(A_0^2\right) + \lambda_3 \left(\mathrm{Tr}\left(A_0^2\right)\right)^2$$

Parameters chosen (by matching) to reproduce soft physics of high-T QCD

- 3D gauge coupling: $g_{\mathrm{E}}^2 = g^2 T + \dots$
- Debye mass parameter: $m_{\mathrm{E}}^2 = \left(1 + \frac{n_f}{6}\right) g^2 T + \dots$
- 3D quartic coupling: $\lambda_3 = \frac{9 - n_f}{24\pi^2} g^4 T + \dots$

Standard Wilson lattice regularization Hietanen *et al.*, 2008

Our setup: QCD with $n_f = 2$ light flavors, two temperature ensembles:

- $T \simeq 398$ MeV
- $T \simeq 2$ GeV

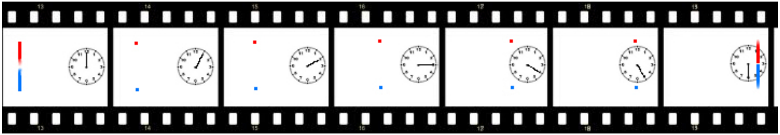
Closely related studies in MQCD Laine, 2012 Benzke *et al.*, 2012

Operator implementation

Effective theory: purely spatial

but

Operator describes *real time* evolution



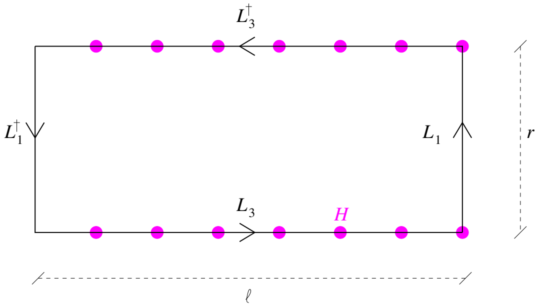
Operator implementation

Light-cone Wilson line correlator

$$\langle W(\ell, r)\rangle = \left\langle \mathrm{Tr}\left(L_3 L_1 L_3^\dagger L_1^\dagger\right)\right\rangle \sim \exp\left[-\ell V(r)\right]$$

with

$$L_3 = \prod U_3 H \qquad L_1 = \prod U_1 \qquad H = \exp(-a g_E^2 A_0)$$



Outline

$\hat{q}$ estimate

Contribution to $\hat{q}$ related to the curvature of $V(r)$ near the origin

$\hat{q}$ estimate

Contribution to $\hat{q}$ related to the curvature of $V(r)$ near the origin

Data fitted with a procedure similar to Laine, 2012

$$V/g_E^2 = Arg_E^2 + B(rg_E^2)^2 + C(rg_E^2)^2 \ln(rg_E^2) + \dots$$

$\hat{q}$ estimate

Contribution to $\hat{q}$ related to the curvature of $V(r)$ near the origin

Data fitted with a procedure similar to Laine, 2012

$$V/g_E^2 = Arg_E^2 + B(rg_E^2)^2 + C(rg_E^2)^2 \ln(rg_E^2) + \dots$$

Purely NP soft contribution to $\hat{q}$ is quite large

$$\hat{q}_{\rm EQCD}^{\rm NP} \sim 0.5 g_E^6$$

$\hat{q}$ estimate

Contribution to $\hat{q}$ related to the curvature of $V(r)$ near the origin

Data fitted with a procedure similar to Laine, 2012

$$V/g_E^2 = Arg_E^2 + B(rg_E^2)^2 + C(rg_E^2)^2 \ln(rg_E^2) + \ldots$$

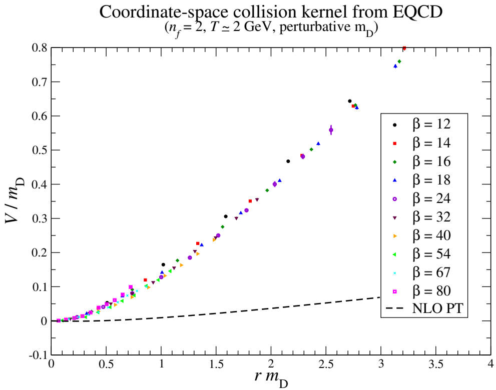
Purely NP soft contribution to $\hat{q}$ is quite large

$$\hat{q}_{\rm EQCD}^{\rm NP} \sim 0.5 g_E^6$$

Approximate estimate $\hat{q} \sim 6$ GeV^2/fm at RHIC temperatures

Lattice versus perturbation theory

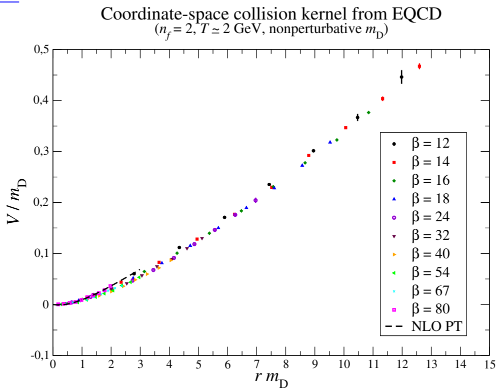
"Naïve" comparison with NLO PT



Lattice versus perturbation theory

Discrepancy reduced if data are plotted in terms of non-perturbative m_D

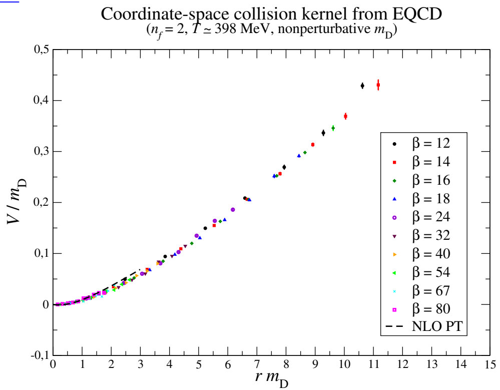
Laine and Philipsen, 2008



Lattice versus perturbation theory

Discrepancy reduced if data are plotted in terms of non-perturbative m_D

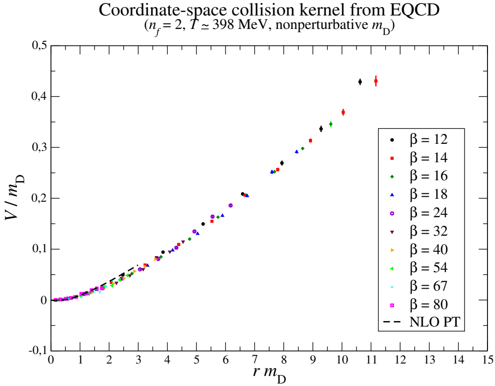
Laine and Philipsen, 2008



Lattice versus perturbation theory

Discrepancy reduced if data are plotted in terms of non-perturbative m_D

Laine and Philipsen, 2008



Using NP value for m_D in

$$\hat{q}^{\rm NLO}_{\rm EQCD} = g^4 T^2 m_D \mathcal{C}_f \mathcal{C}_a \frac{3\pi^2 + 10 - 4\ln 2}{32\pi^2}$$

yields again $\hat{q} \sim 6\ {\rm GeV}^2/{\rm fm}$ at RHIC temperatures

Outline

Conclusions

- Lattice approach *possible* for certain real-time problems—see also Ji, 2013
- Here: focus on soft physics in thermal QCD Laine and Rothkopf, 2013 Cherednikov *et al.*, 2013
- Outlined approach is *systematic*
- Tentative estimate of jet quenching parameter
- Clear indication for large non-perturbative effects
- Results in ballpark of
    - holographic computations Liu, Rajagopal and Wiedemann, 2006 Armesto, Edelstein and Mas, 2006 ✓
    - phenomenological estimates Dainese *et al.*, 2004 Eskola *et al.*, 2004 ✓