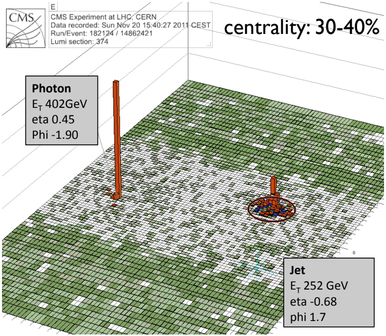
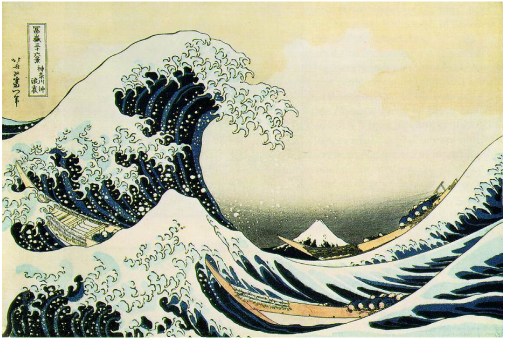
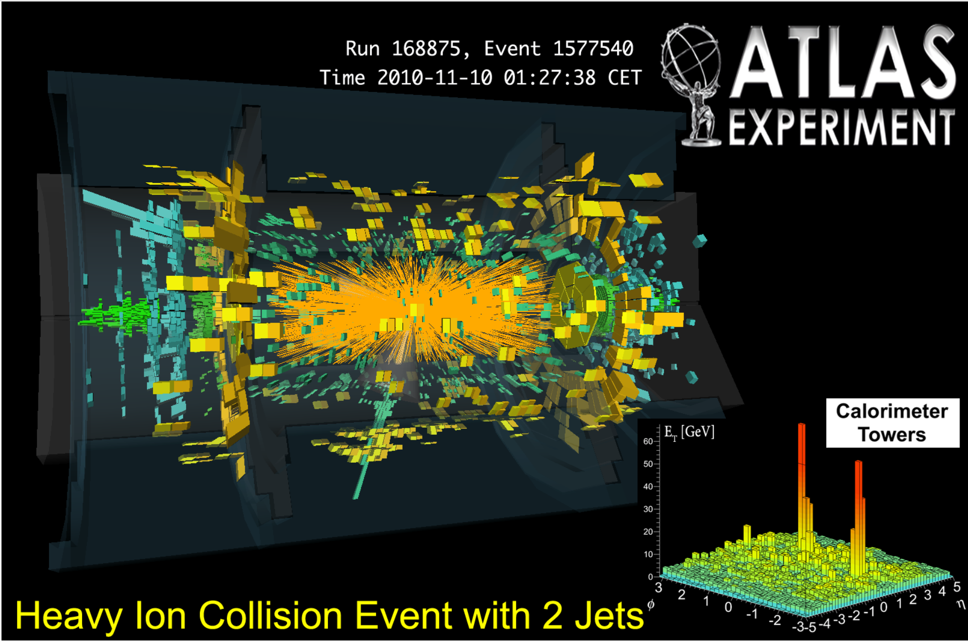
From Jet Quenching to Wave Turbulence

Edmond Iancu
IPhT Saclay & CNRS

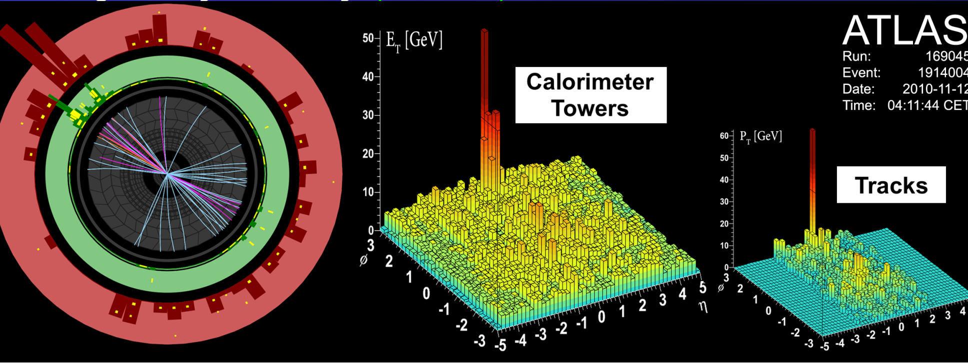




Jet production at the LHC

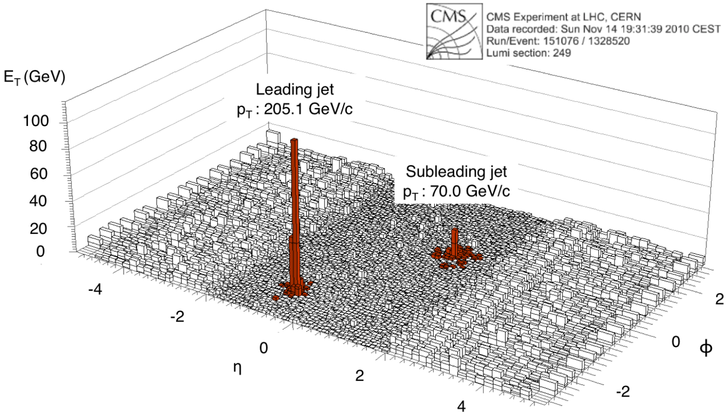


Di–jet asymmetry *(ATLAS)*



- Central Pb+Pb: 'mono–jet' events
- The secondary jet cannot be distinguished from the background: $E_{T1} \geq 100$ GeV, $E_{T2} > 25$ GeV
- Additional energy imbalance as compared to p+p : 20 to 30 GeV
- Remarkably large if compared to the typical scale in the medium: $T \sim 1$ GeV

Di–jet asymmetry *(CMS)*

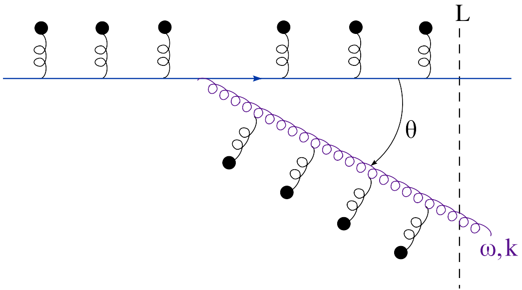


- Central Pb+Pb: the secondary jet is barely visible
- Detailed studies show that the 'missing energy' is carried by many soft ($p_\perp < 4$ GeV) hadrons propagating at large angles
- Can we understand that from first principles ?

pQCD : the BDMPSZ mechanism

Baier, Dokshitzer, Mueller, Peigné, and Schiff; Zakharov (96–97)

- Additional gluon radiation triggered by interactions in the medium



- Several related approaches from first–principles (pQCD) ...

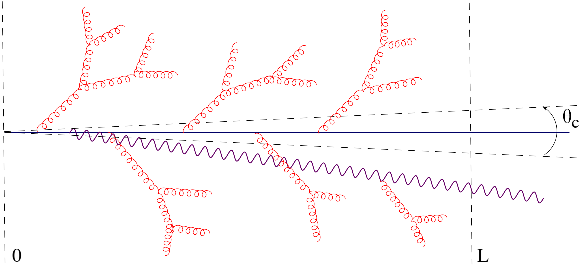
  Wiedemann (2000); Guylassy, Levai, Vitev (2000); Guo and Wang (2000); Arnold, Moore, Yaffe (2002); Armesto, Salgado, and Wiedemann (2003) ...

- ... which have mostly focused on a single gluon emission

  ▷ the total energy loss by the leading particle

Medium–induced jet evolution

- The LHC data call for a global understanding of the jet evolution



- Heuristic treatment of multiple emissions : independent branchings

  Baier, Dokshitzer, Mueller, Schiff (2001); Jeon, Moore (2003) ...

- Implicitly assumed in the Monte–Carlo event generators
  - *Q–PYTHIA : Armesto, Cunqueiro, and Salgado (09) [BDMPSZ]*
  - *MARTINI : Schenke, Gale, Jeon, Young (since 09) [BDMPSZ]*
  - *JEWEL : Zapp, Stachel, Wiedemann, Krauss ... (since 08)*
  - *YaJEM : Renk (since 2009)*

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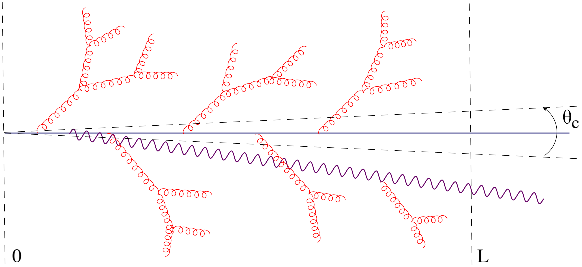

- Recent extension of the theory to multiple medium–induced emissions
  Mehtar-Tani, Salgado, Tywoniuk (10–12); Casalderrey-Solana, E. I. (11); Blaizot, Dominguez, E.I., Mehtar-Tani (2012–13)
- Intense ongoing activity with many contributions to this meeting
  Wang, Salgado, Apolinario, Mehtar-Tani, Tywoniuk, Rodriguez-Calvo, ...

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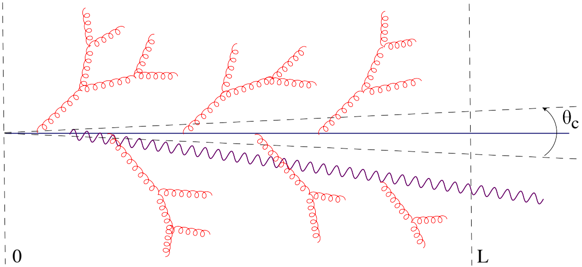

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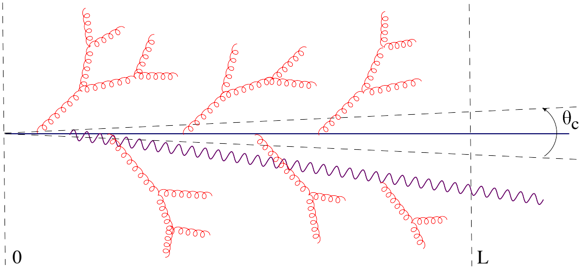
  ▷ *Thanks to Carlos for an enlightening introduction !*

Medium–induced jet evolution

- The LHC data call for a global understanding of the jet evolution



- New, qualitative, phenomenon (no analog in pQCD) which naturally explains the energy transport via soft quanta at large angles:

  ▷ turbulent flow of the energy throughout the cascade

Medium–induced jet evolution

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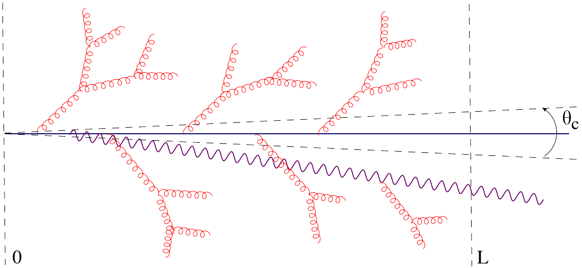

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- This talk: from single to multiple emissions ... pedagogically

  See the closely related talks by Y. Mehtar-Tani and K. Tywoniuk for more details and applications to phenomenology

Medium–induced emissions à la BDMPSZ

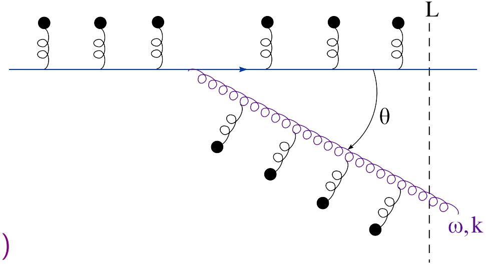
- Gluon emission is linked to transverse momentum broadening
  - transverse kicks provide acceleration and thus allow for radiation
  - they increase the emission angle θ
  - they occur randomly $\Longrightarrow$ Brownian motion in $k_\perp$

$$\langle k_\perp^2 \rangle \simeq \hat{q}\, \Delta t$$

$$\hat{q} \simeq \frac{m_D^2}{\lambda} = \frac{(\text{Debye mass})^2}{\text{mean free path}}$$

'jet quenching parameter'

(quasi–local transport coefficient)



- Gluon emissions can occur anywhere inside the medium (with size L)
  - ... but they are not instantaneous: formation time $\tau_f \simeq 1/\Delta E$

Formation time & emission angle

- By the uncertainty principle, $\tau_f \simeq \omega/k_\perp^2$
- During formation, the gluon acquires a $k_\perp$ momentum $k_\perp^2 \sim \hat{q}\tau_f$

$$\tau_f(\omega) \simeq \sqrt{\frac{\omega}{\hat{q}}} \qquad \& \qquad \theta_f(\omega) \simeq \frac{k_\perp}{\omega} \sim \left(\frac{\hat{q}}{\omega^3}\right)^{1/4}$$

- Maximal ω for this mechanism : $\tau_f \simeq L \Rightarrow \omega_c = \hat{q}L^2$
- Minimal emission angle: $\theta_c \equiv \theta(\omega_c) \sim 1/\sqrt{\hat{q}L^3}$
- Soft gluons have short formation times & large emission angles

$$\omega \ll \omega_c \quad \Longrightarrow \qquad \tau_f \ll L \qquad \& \qquad \theta_f \gg \theta_c$$

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$$\omega \ll \omega_c \quad \Longrightarrow \qquad \tau_f \ll L \quad \& \quad \theta_f \gg \theta_c$$

- Some typical value (consistent with the phenomenology) :

$$\hat{q} \simeq (1 \div 2)\ \text{GeV}^2/\text{fm}, \ \ L \simeq 5\ \text{fm}, \ \ \omega_c \simeq 40\ \text{GeV}, \ \ \theta_c \simeq 0.1$$

Emission probability

- Spectrum : Bremsstrahlung $\times$ average number of emissions

$$\omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq \alpha_s \frac{L}{\tau_f(\omega)} \simeq \alpha_s \sqrt{\frac{\omega_c}{\omega}} \qquad (\omega_c = \hat{q}L^2)$$

- LPM effect : the emission rate decreases with increasing ω

  (from Landau, Pomeranchuk, Migdal, within QED)

  - coherence: many collisions contribute to a single, hard, emission
  - formation time $\tau_f(\omega) \gg$ mean free path λ

- Energy loss by the leading particle :

$$\Delta E = \int^{\omega_c} \mathrm{d}\omega\, \omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \sim \alpha_s \omega_c \sim \alpha_s \hat{q} L^2$$

  - integral dominated by its upper limit $\omega = \omega_c$
  - energy loss scales like L^2

Emission probability

- Spectrum : Bremsstrahlung $\times$ average number of emissions

$$\omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq \alpha_s \frac{L}{\tau_f(\omega)} \simeq \alpha_s \sqrt{\frac{\omega_c}{\omega}} \qquad (\omega_c = \hat{q}L^2)$$

- One is naturally led to distinguish between 2 types of emissions
- Relatively hard emissions with $\omega \sim \omega_c$:
  - large formation times: $\tau_f \sim L$
  - rare events : probability of $\mathcal{O}(\alpha_s)$
  - control the energy loss by the leading particle
  - small emission angle $\theta_c \Rightarrow$ the energy remains inside the jet
- Arguably, not so important for the di–jet asymmetry

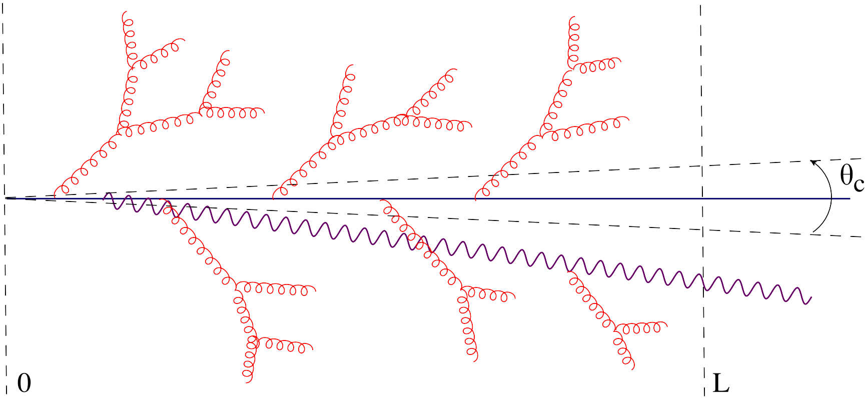
Emission probability

- Spectrum : Bremsstrahlung $\times$ average number of emissions

$$\omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq \alpha_s \frac{L}{\tau_f(\omega)} \simeq \alpha_s \sqrt{\frac{\omega_c}{\omega}} \qquad (\omega_c = \hat{q}L^2)$$

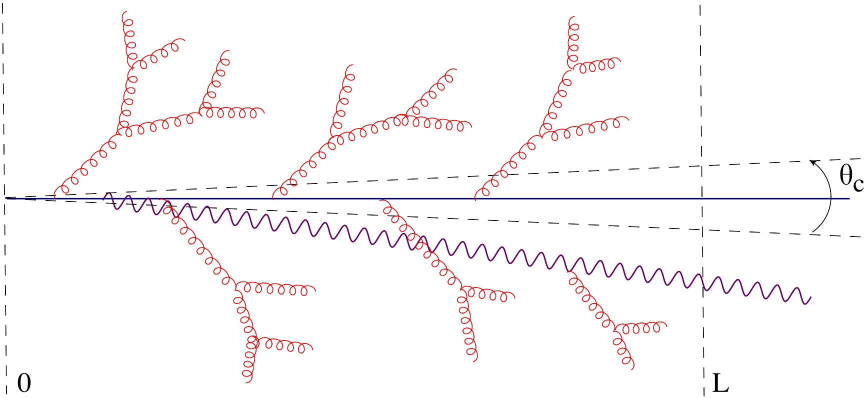
- Relatively soft emissions with $\omega \ll \omega_c$:
  - small formation times : $\tau_f \ll L$
  - quasi–deterministic : probability of $\mathcal{O}(1)$ for $\omega \lesssim \alpha_s^2 \omega_c$
  - a relatively smaller contribution to the energy loss : $\Delta E_{\rm soft} \sim \alpha_s^2 \omega_c$
  - ... but this can be lost at very large angles
- Potentially relevant for the di–jet asymmetry ☺
- When probability of $\mathcal{O}(1) \Longrightarrow$ multiple branchings become important

A typical gluon cascade



- A 'rain' of soft gluons plus (sometimes) a harder one ($\omega \sim \hat{q}L^2$)
- From now on, we shall focus on the 'rain' !
- To that aim, we need to understand multiple branchings

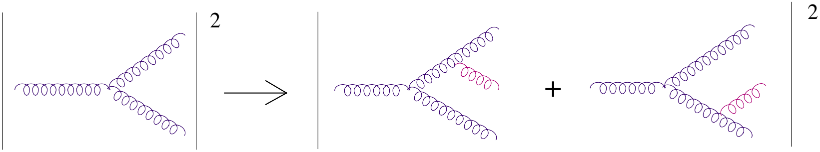
A typical gluon cascade



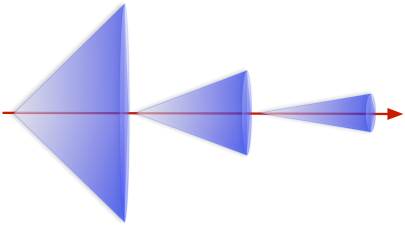
- A 'rain' of soft gluons plus (sometimes) a harder one ($\omega \sim \hat{q}L^2$)
- From now on, we shall focus on the 'rain' !
- Potential difficulty : interference effects between different sources

Angular ordering in the vacuum

- Quantum interference: one sums the amplitudes



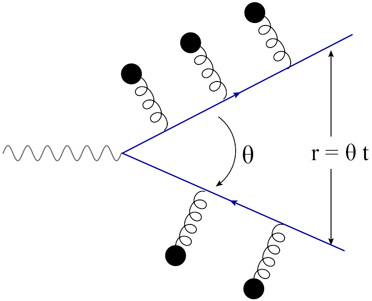
- Vacuum : daughter gluons keep color coherence till the next emission



- Destructive interference effects leading to angular ordering

Color decoherence in the medium

- In medium, color coherence is rapidly lost via rescattering
  Mehtar-Tani, Salgado, Tywoniuk; Casalderrey-Solana, E. I. (10 –11)
- Originally demonstrated for a 'frozen antenna' with fixed opening angle



$$S(t,\theta) \simeq \exp\left\{-\,\hat{q}\theta^2 t^3\right\}$$

$$t_{\rm decoh} \simeq \frac{1}{(\hat{q}\theta^2)^{1/3}}$$

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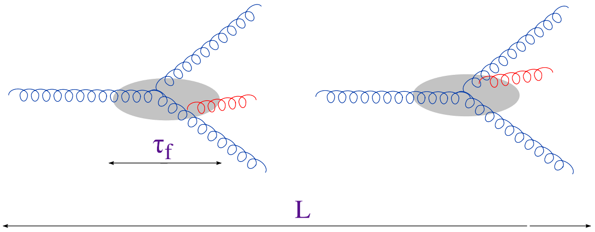
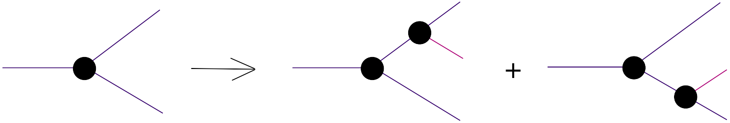
  Mehtar-Tani, Salgado, Tywoniuk; Casalderrey-Solana, E. I. (10 –11)
- Generalization to a dynamical antenna : $g \to gg$ branching
  Blaizot, Dominguez, E.I., Mehtar-Tani (arXiv: 1209.4585)



- The daughter gluons lose color coherence already by the time of formation $\Rightarrow$ interference effects are suppressed by a factor τ_f/L
- Successive emissions of soft gluons ($\omega \ll \omega_c$) are independent

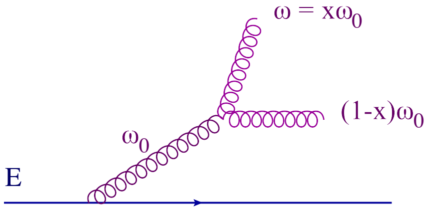
A classical branching process



- Markovian process in $D = 3 + 1$: ω, $k_\perp$, time t (or medium size L)
  - the $g \to gg$ splitting vertex (the 'blob') : the BDMPSZ spectrum
  - the propagator (the 'line') : transverse momentum broadening
- Well suited for Monte Carlo simulations
- The $D = 1 + 1$ version $(\omega$ & $t)$ already studied in the literature
  - *Baier, Mueller, Schiff, Son, 2001 : 'bottom-up thermalization'*
  - *Jeon, Moore, 2003 : jet quenching $\Longrightarrow$ MARTINI*
- Interesting new physics that has been missed by such previous studies
  - *J.-P. Blaizot, E. I., Y. Mehtar-Tani, PRL 111, 052001 (2013)*

Quasi–democratic branchings

- Previous studies focused on the energy loss by the leading particle
- Here: the energy flow towards soft modes ($\omega \ll \omega_c$), or large angles
- The branchings of the soft gluons are quasi–democratic
  - the daughter gluons carry comparable energy fractions: $x \sim 1/2$
- Non–trivial ! Not true for bremsstrahlung in the vacuum !



$$\mathrm{d}\mathcal{N} \sim \alpha_s \frac{\mathrm{d}\omega}{\omega} \sim \alpha_s \frac{\mathrm{d}x}{x}$$

- probability of $\mathcal{O}(1)$ when $\alpha_s \ln(1/x) \sim 1 \Longrightarrow$ favors $x \ll 1$
- argument independent of the parent energy ω_0

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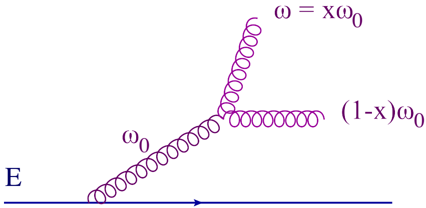
- In–medium radiation : a consequence of the LPM effect



$$\mathrm{d}\mathcal{N} \sim \alpha_s \frac{\mathrm{d}\omega}{\omega} \sqrt{\frac{\omega_c}{\omega}}$$

$$\sim \alpha_s \frac{\mathrm{d}x}{x} \sqrt{\frac{\omega_c}{x\omega_0}}$$

- the rate also depends upon the parent gluon energy ω_0
- probability of $\mathcal{O}(1)$ when $\omega_0 \sim \alpha_s^2 \omega_c$ for any value of x

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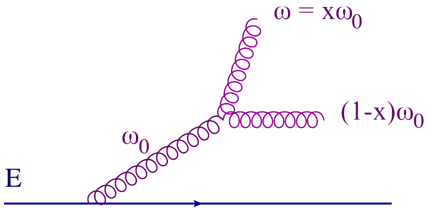
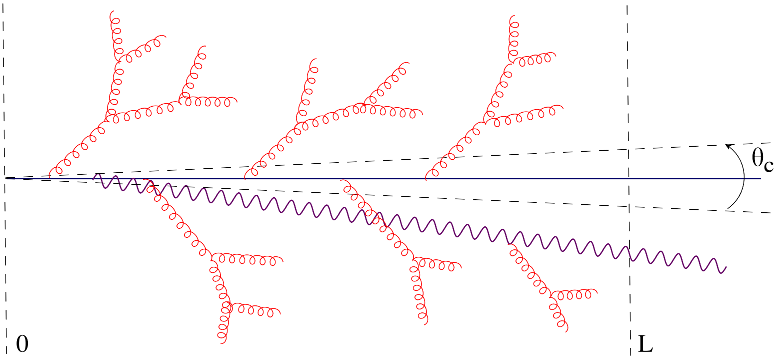


$$\mathrm{d}\mathcal{N} \sim \alpha_s \frac{\mathrm{d}\omega}{\omega}\sqrt{\frac{\omega_c}{\omega}} \sim \alpha_s \frac{\mathrm{d}x}{x}\sqrt{\frac{\omega_c}{x\omega_0}}$$

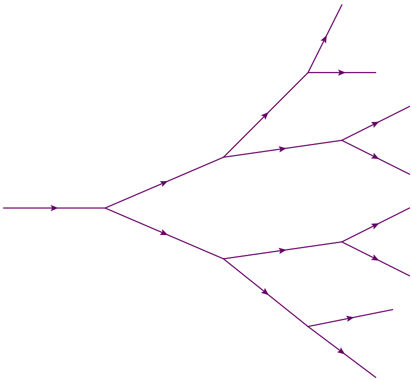
- A similar scenario at strong coupling *(Y. Hatta, E.I., Al Mueller '08)*
- ... but no other example in a weakly coupled gauge theory

Wave turbulence



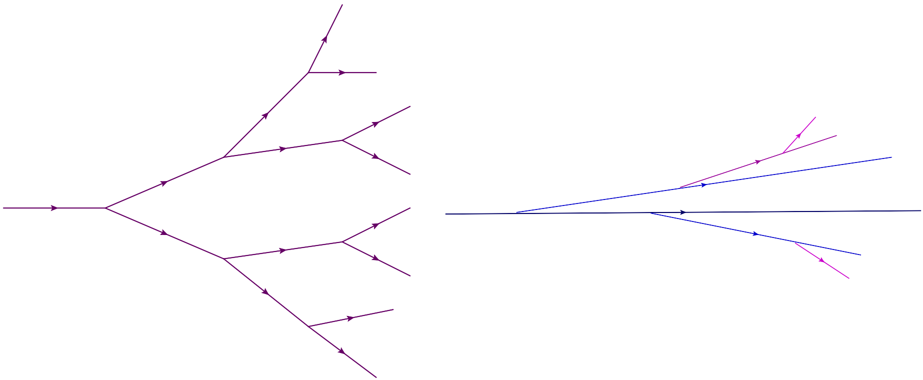
- The leading particle emits mostly soft gluons ($x \ll 1$)
- The subsequent branchings of these soft gluons are quasi–democratic
- The quasi–democratic cascade develops wave turbulence
  - the most efficient mechanism to transport energy between 2 widely separated scales *(Richardson, '21; Kolmogorov, '41; Zakharov, '92 ...)*

Wave turbulence



- The rate for energy transfer from one parton generation to the next one is independent of the generation (i.e. of x)
  - via successive branchings, the energy flows from large x to small x, without accumulating at any intermediate value of x

Wave turbulence

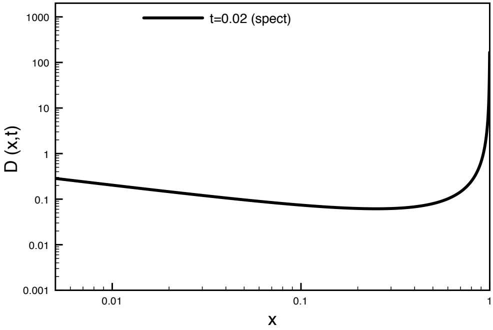


- The rate for energy transfer from one parton generation to the next one is independent of the generation (i.e. of x)
- This is not what happens for a jet in the vacuum (DGLAP equation)
  - splittings are typically asymmetric and the energy remains in the partons with the largest values of x

The gluon spectrum

- Time evolution of $D(x,t) \equiv x\frac{\mathrm{d}N}{\mathrm{d}x}$ where $x = \omega/E$
- Described by a rate equation : $\partial D/\partial t$ = Gain - Loss
- At small times: single branching $\Rightarrow$ BDMPSZ spectrum :

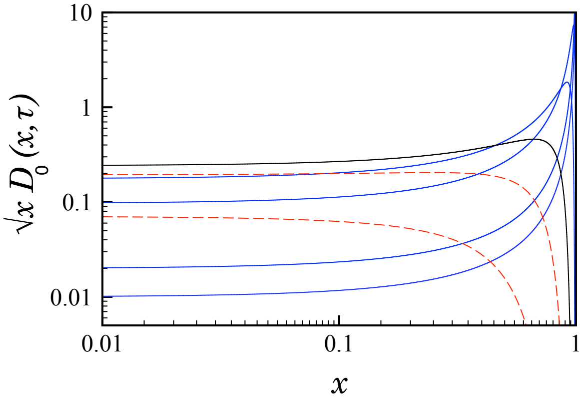
$$D^{(1)}(x,L) \simeq \alpha_s \frac{L}{\tau_f(\omega)} = \frac{t}{\sqrt{x}} \qquad (t = L \text{ in appropriate units})$$



The scaling spectrum

- The spectrum at later times : exact solution to the rate equation

$$D(x \ll 1, t) \simeq \frac{t}{\sqrt{x}}\, \mathrm{e}^{-\pi t^2}$$



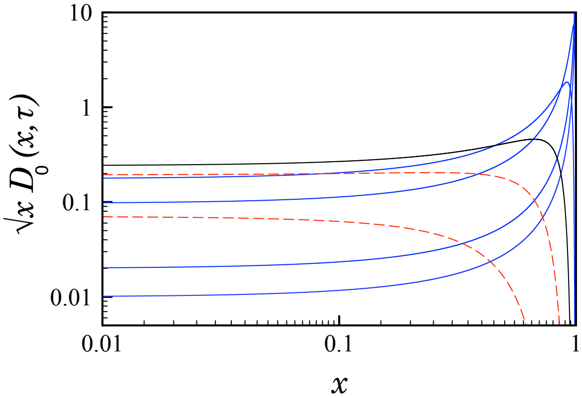
- "BDMPSZ" $\times$ "survival probability for the leading particle"
- The 'scaling' spectrum $\frac{1}{\sqrt{x}}$ is regenerated by the evolution: fixed point
  - 'Gain' = 'Loss' $\Longrightarrow$ the energy flux is independent of x

The scaling spectrum

- The spectrum at later times : exact solution to the rate equation

$$D(x \ll 1, t) \simeq \frac{t}{\sqrt{x}}\, \mathrm{e}^{-\pi t^2}$$

$$\int_0^1 \mathrm{d}x\, D(x,t) = \mathrm{e}^{-\pi t^2}$$



- "BDMPSZ" $\times$ "survival probability for the leading particle"
- The 'scaling' spectrum $\frac{1}{\sqrt{x}}$ is regenerated by the evolution: fixed point
- The energy flows out from the spectrum

Where does the energy go ?

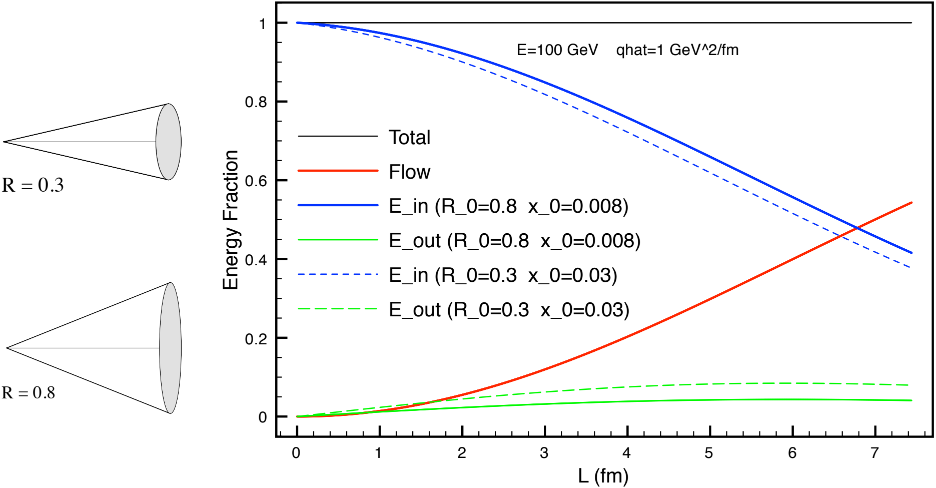
- Via successive branchings, the energy flows down to $x = 0$
  - formally, it accumulates into a 'condensate' at $x = 0$
  - physically, it goes below $x_{\rm th} = T/E \ll 1$, meaning it thermalizes
- The energy fraction carried away by this flow

$$\mathcal{E}_{\rm flow}(t) \equiv 1 - \int_0^1 {\rm d}x\, D(x,t) = 1 - {\rm e}^{-\pi t^2}$$

... ends up at arbitrarily large angles

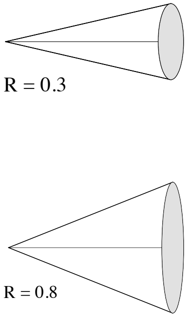
- In practice : $t = \alpha_s \sqrt{2\omega_c/E} \sim 0.3$ for $E = 100$ GeV
  - $1 - {\rm e}^{-\pi t^2} \sim 0.25 \Rightarrow$ about 25% of the energy is lost at large angles
  - ... irrespective of the details of the thermalization mechanism $(x_{\rm th})$
- A property of the gluon cascade, not of the in–medium dissipation

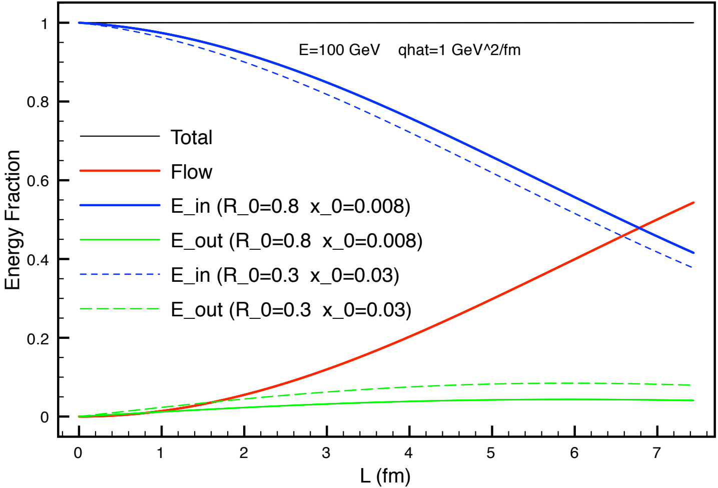
Energy flow at large angles



- The energy inside the jet is only weakly increasing with the jet angular opening R, within a wide range of values for R ☺

Energy flow at large angles

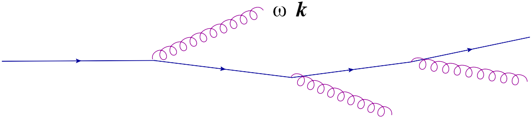




$$E_{\text{flow}} \simeq \pi\, \alpha_s^2\, \hat{q} L^2 \qquad (\sim 20\,\text{GeV for } L = 5 \text{ fm})$$

A large radiative correction to $\hat{q}$

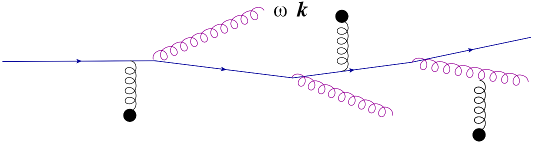
- $\hat{q}$: the result of collisions in the medium ... but not only !
- Gluon emissions contribute to momentum broadening, via their recoil !



$$\langle p_\perp^2 \rangle_{\rm rad} \sim \int_\omega \int_{\boldsymbol{k}} \boldsymbol{k}^2 \frac{{\rm d}N}{{\rm d}\omega {\rm d}^2\boldsymbol{k}}$$

A large radiative correction to $\hat{q}$

- $\hat{q}$: the result of collisions in the medium ... but not only !
- Gluon emissions contribute to momentum broadening, via their recoil !



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- Dominant effect from relatively hard emissions (large $k_\perp$), as triggered by a single scattering (Gunion–Bertsch spectrum)

$$\frac{{\rm d}N}{{\rm d}\omega\, {\rm d}^2\boldsymbol{k}} \simeq \frac{\alpha_s}{\omega} \frac{\hat{q}L}{k_\perp^4} \implies \langle p_\perp^2 \rangle_{\rm rad} \sim L\, \alpha_s \hat{q} \int \frac{{\rm d}\omega}{\omega} \int \frac{{\rm d}k_\perp^2}{k_\perp^4} \equiv L\, \Delta\hat{q}$$

- Formally NLO but enhanced by a double-log *(Liou, Mueller, Wu, 13)*

$$\frac{\Delta\hat{q}}{\hat{q}} \simeq \frac{\alpha_s N_c}{2\pi} \ln^2(LT) \simeq 0.75\ (!) \implies \text{need for resummation}$$

A large radiative correction to $\hat{q}$

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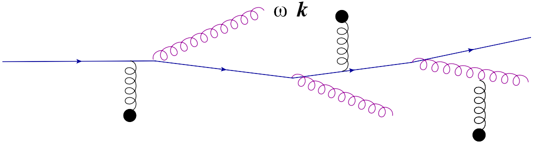

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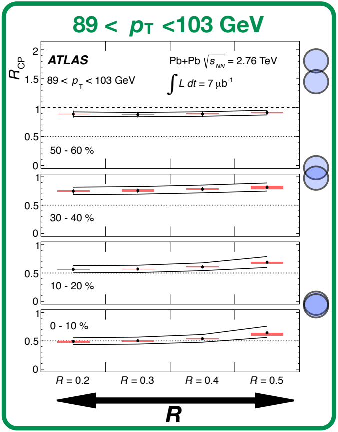
- The beginning of QCD evolution for jet quenching ... to be continued
- Not included in the recent lattice calculation *(cf. talk by Panero)*

Conclusions

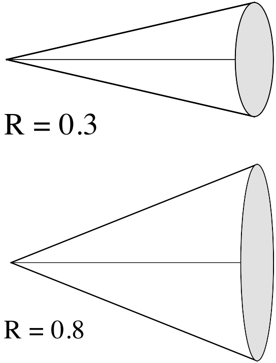
- Remarkable progress in understanding medium–induced jet evolution
  - a new kind of branching process in pQCD
  - hard emissions at small angles (energy loss by leading particle, R_{AA})
  - soft, quasi–democratic, branchings leading to turbulent flow (di-jet asymmetry)
  - probabilistic picture, well suited for Monte Carlo implementations
  - fully 3+1–dim simulations possible $\Longrightarrow$ jet shapes
  - large radiative corrections to $\hat{q}$ which are under control in pQCD
- Many open problems: proper interplay with 'vacuum' radiation, matching with the lattice result for $\hat{q}$, fully 3+1–dim simulations, extensive phenomenology ...
- All that was possible due to the LHC fantastic work and results ! THANKS !

Energy transport at large angles

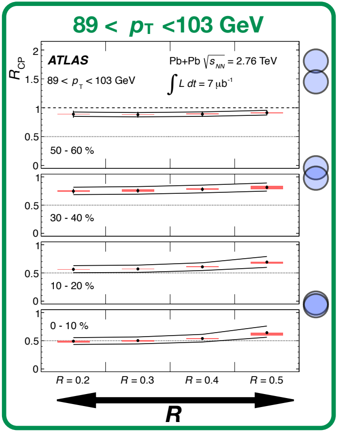
- Just a little fraction of the 'missing energy' is recovered when gradually increasing the jet opening : most of the energy is lost at large angles



ATLAS, arXiv:1208.1967



- Even for R as large as $R = 0.8$, a large fraction of the missing energy lies still outside the cone and is carried by very soft hadrons



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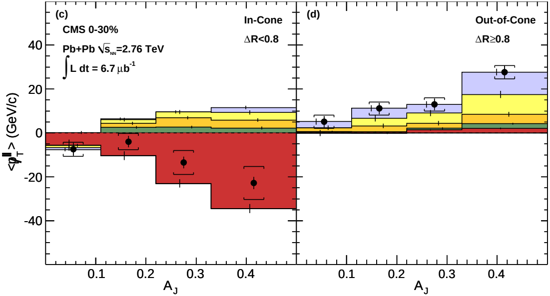
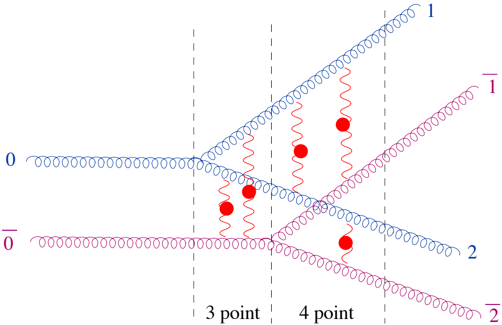


CMS, arXiv:1102.1957; cf. talk by J. Velkovska

- What is the mechanism for energy transport at large angles ?

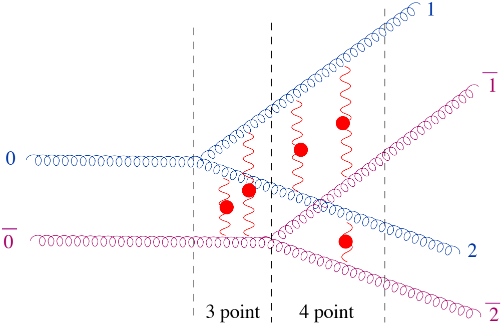
- Quantum emission: amplitude × the complex conjugate amplitude



- 'Medium' = randomly distributed scattering centers (Gaussian)
  - Coulomb scattering with Debye screening
  - multiple scattering in eikonal approximation (one Wilson line per gluon)
  - $1 \to 2$ gluon branching ⇒ 3–p and 4–p functions of the Wilson lines

A few words on the formalism

- Quantum emission: amplitude $\times$ the complex conjugate amplitude



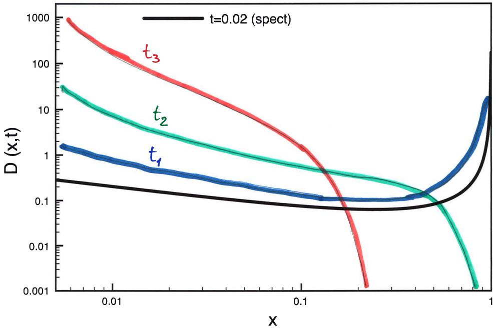
- Contains and extends the original BDMPSZ/AMI formalisms already at the level of a single medium–induced emission ...

  ▷ transverse momentum dependence for the emission vertex, correct inclusion of single scattering, color (de)coherence after emission ...

- Permits the treatment of interference & multiple branchings

A fake DGLAP–like scenario

- Via successive branchings, gluons fall at smaller and smaller values of x
- At any t, the energy remains in the spectrum: $\int_0^1 \mathrm{d}x\, D(x,t) \,=\, 1$



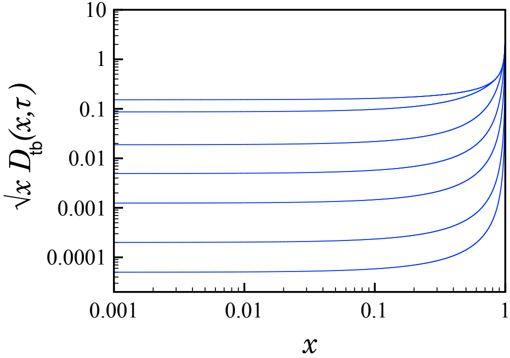
- The spectrum becomes steeper and steeper at small x, yet the total energy stored in the bins with $x \ll 1$ remains small

  ▷ very little energy can be lost in this way at large angles

The usual turbulence set-up

- Steady source at $x = 1$ and sink at $x = x_{\rm th}$ (here $x_{\rm th} = 0$)

$$D_{\rm tb}(x,t) \;=\; \frac{1}{2\pi\sqrt{x(1-x)}}\;\left(1 - {\rm e}^{-\pi\frac{t^2}{1-x}}\right)$$



- The spectrum approaches a steady shape when $\pi t^2 \gtrsim 1$