

Azimuthal Jet Flavor Tomography via CUJET with Running Coupling in 2+1D Viscous QGP Fluids

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Quick Summary

- CUJET2.0 = Running Coupling DGLV + 2+1D Viscous Hydro
 - Pion R_{AA} from CUJET2.0 fits both RHIC and LHC at 0-5% and 20-30% centrality with 2 sets of parameters $(\alpha_{max}, f_E, f_M) = (0.25, 1, 0)$ & $(0.40, 2, 0)$.
 - Non-HTL Debye screening mass works $\rightarrow$ Introduce Lattice QCD coupling and potential in the future.
 - Simultaneous fit of R_{AA}^{in} and R_{AA}^{out} is explored.
- Robust level crossing of flavor dependent RAA $\rightarrow$ Heavy flavor, in particular B meson spectrum, is the key constraint on the model parameter space.
- In rcDGLV + VISH framework, absolute jet transport parameter $\hat{q}/T^3$ has non-trivial dependence on E and T.

DGLV model

$$x \frac{dN^{(n)}}{dx d^2\mathbf{k}} = \frac{C_R \alpha_s}{\pi^2} \frac{1}{n!} \int \prod_{i=1}^n \left(d^2\mathbf{q}_i \frac{L}{\lambda_g(i)} [\bar{v}_i^2(\mathbf{q}_i) - \delta^2(\mathbf{q}_i)] \right) \times$$

$$\times \left(-2 \tilde{\mathbf{C}}_{(1,\dots,n)} \cdot \sum_{m=1}^n \tilde{\mathbf{B}}_{(m+1,\dots,n)(m,\dots,n)} \left[\cos \left(\sum_{k=2}^m \Omega_{(k,\dots,n)} \Delta z_k \right) - \cos \left(\sum_{k=1}^m \Omega_{(k,\dots,n)} \Delta z_k \right) \right] \right)$$

Opacity series expansion $\rightarrow \left(\frac{L}{\lambda}\right)^n$

Soft Radiation ($E \gg \omega, x \ll 1$)
Soft Scattering ($E \gg q, \omega \gg k_T$)

Radiation antenna $\rightarrow$ Cascade terms $\tilde{\mathbf{C}}_{(i_1 i_2 \dots i_m)} = \frac{(\mathbf{k} - \mathbf{q}_{i_1} - \mathbf{q}_{i_2} - \dots - \mathbf{q}_{i_m})}{(\mathbf{k} - \mathbf{q}_{i_1} - \mathbf{q}_{i_2} - \dots - \mathbf{q}_{i_m})^2 + m_g^2 + M^2 x^2},$

$$\tilde{\mathbf{B}}_{(i_1 i_2 \dots i_m)(j_1 j_2 \dots j_n)} = \tilde{\mathbf{C}}_{(i_1 i_2 \dots j_m)} - \tilde{\mathbf{C}}_{(j_1 j_2 \dots j_n)}.$$

Gunion-Bertsch $\tilde{\mathbf{B}}_i = \tilde{\mathbf{H}} - \tilde{\mathbf{C}}_i,$

Hard $\tilde{\mathbf{H}} = \frac{\mathbf{k}}{\mathbf{k}^2 + m_g^2 + M^2 x^2},$

LPM effect $\rightarrow \Omega_{(m,\dots,n)} = \underbrace{\frac{(\mathbf{k} - \mathbf{q}_m - \dots - \mathbf{q}_n)^2}{2xE}}_{\text{Inverse formation time}} + \underbrace{\frac{m_g^2 + M^2 x^2}{2xE}}_{\text{Mass effects}}$

Inverse formation time *Mass effects*

Scattering center distribution $\rightarrow \Delta z_k = z_k - z_{k-1} \sim L/(n+1)$

CUJET1.0: Basics

- **Geometry**
 - Glauber model
 - Bjorken longitudinal expansion
- **Energy loss**
 - DGLV – MD Radiative energy loss model
 - Energy loss fluctuations (Poisson expansion)
 - Full dynamical computation:

$$\begin{aligned}
 - \frac{dN_g}{dx}(x_\perp, \phi) = & \frac{C_R \alpha_s}{\pi} \int d\tau \frac{d^2 k}{\pi} \frac{d^2 q}{\pi} \frac{1}{x} \frac{\frac{9}{2} \pi \alpha^2}{q^2 (q^2 + \mu^2(\tau))} \times \frac{2(k+q)}{(k+q)^2 + \chi(\tau)} \left(\frac{(k+q)}{(k+q)^2 + \chi(\tau)} - \frac{k}{k^2 + \chi(\tau)} \right) \times \\
 & \left(1 - \cos \left[\frac{(k+q)^2 + \chi(\tau)}{2xE} \tau \right] \right) \rho_{QGP}(x_\perp + \hat{\phi} \tau, \tau)
 \end{aligned}$$

$$\mu(\tau) = gT(x_\perp + \hat{\phi} \tau, \tau)$$

$$\chi(\tau) = M^2 x^2 + m_g^2(\tau)(1 - x)$$

- Detailed convolution over initial production spectra (LO pQCD CTEQ5, NLO/FONLL)
- In vacuum Fragmentation Functions (KKP, Peterson, VOGT)

CUJET1.0: Basics

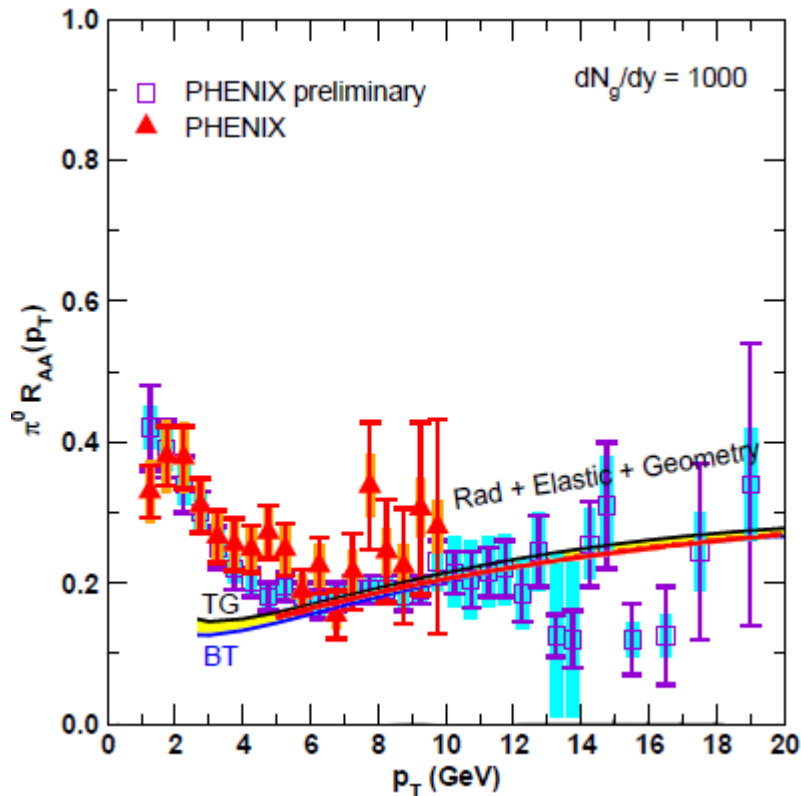
- **Geometry**
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Possibility to evaluate systematic theoretical uncertainties such as sensitivity to formation and decoupling phases of the QGP evolution, local running coupling and screening scale variations, and other effects out of reach with analytic approximations;

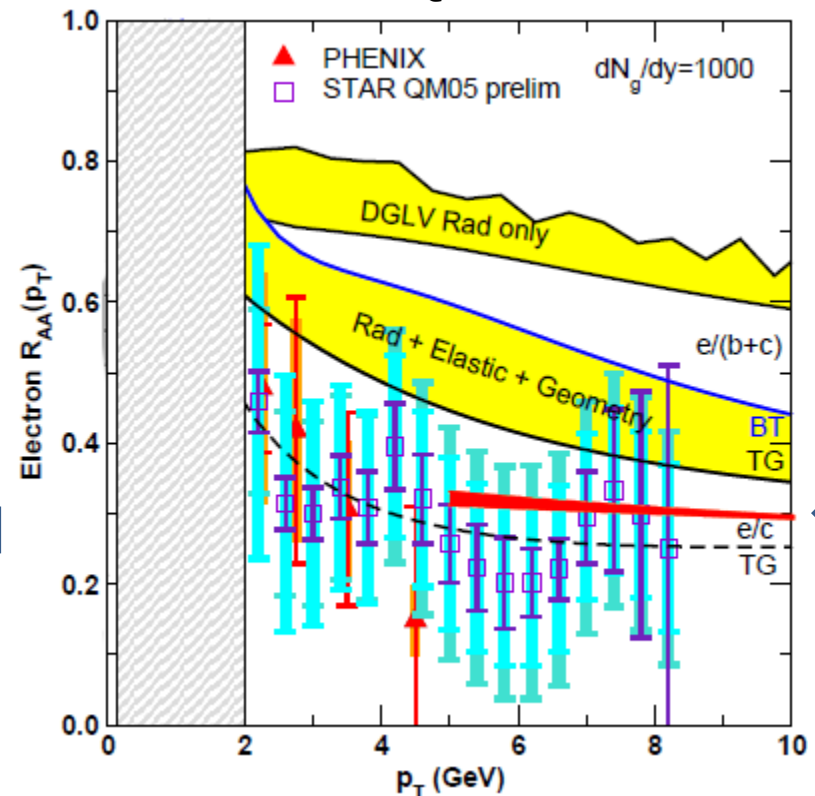
- **Detailed convolution over initial production spectra (LO pQCD CTEQ5, NLO/FONLL)**
- **In vacuum Fragmentation Functions (KKP, Peterson, VOGT)**

CUJET1.0: Pions and Electrons at RHIC

LIGHT QUARKS



HEAVY QUARKS

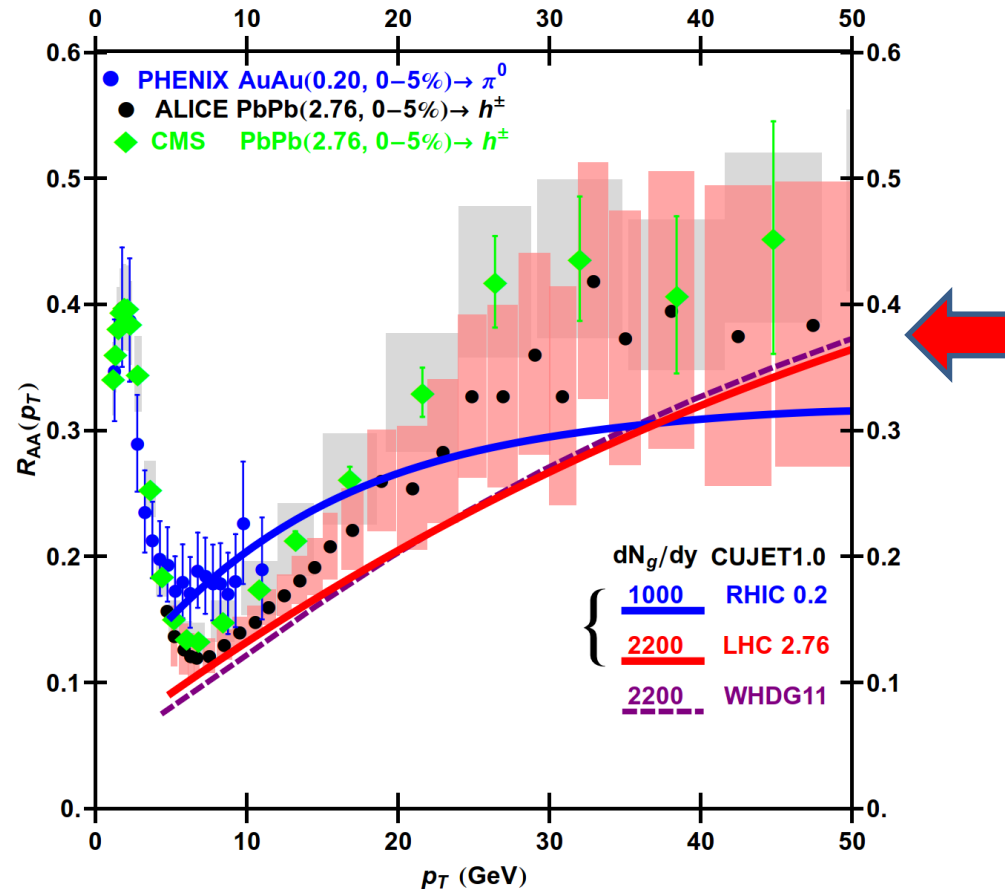


Wicks, Horowitz, Djordjevic, Gyulassy / NPA (2007)

CUJET1.0 solves the Heavy Quark puzzle...

CUJET1.0: Pions at LHC

A. Buzzatti and M. Gyulassy, Phys. Rev. Lett. 108, 0223101 (2012); See also B. Betz and M. Gyulassy, arXiv:1201.02181



Pion RAA from Fixed Coupling CUJET at LHC is overquenched $\rightarrow$ cannot explain the surprising transparency

Extended energy range probed at LHC $\rightarrow$ Running Coupling

Running Coupling DGLV model $\alpha(q^2)^2$

$$x \frac{dN^{(n)}}{dx d^2\mathbf{k}} = \frac{C_R \alpha_s}{\pi^2} \frac{1}{n!} \int \prod_{i=1}^n \left(d^2\mathbf{q}_i \frac{L}{\lambda_g(i)} [\bar{v}_i^2(\mathbf{q}_i) - \delta^2(\mathbf{q}_i)] \right) \times$$

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Inverse formation time *Mass effects*

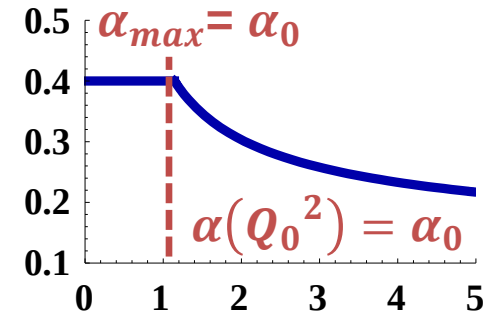
Scattering center distribution $\rightarrow \Delta z_k = z_k - z_{k-1} \sim L/(n+1)$

Multi-scale Running Coupling

- Introduce one-loop alpha running

$$\alpha_s \rightarrow \alpha(Q^2) = \begin{cases} \alpha_0, & \text{if } Q \leq Q_0 \\ \frac{2\pi}{9 \log(Q/\Lambda)}, & \text{if } Q > Q_0 \end{cases}$$

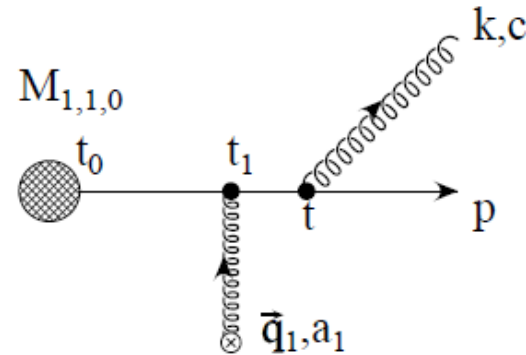
B. G. Zakharov, JETP Lett. 88 (2008) 781-786



$$Radiative = \begin{cases} \alpha(q^2)^2 \\ \alpha\left(\frac{k_{\perp}^2}{x(1-x)}\right) \\ \mu = g(\alpha(4T^2))T \end{cases}$$

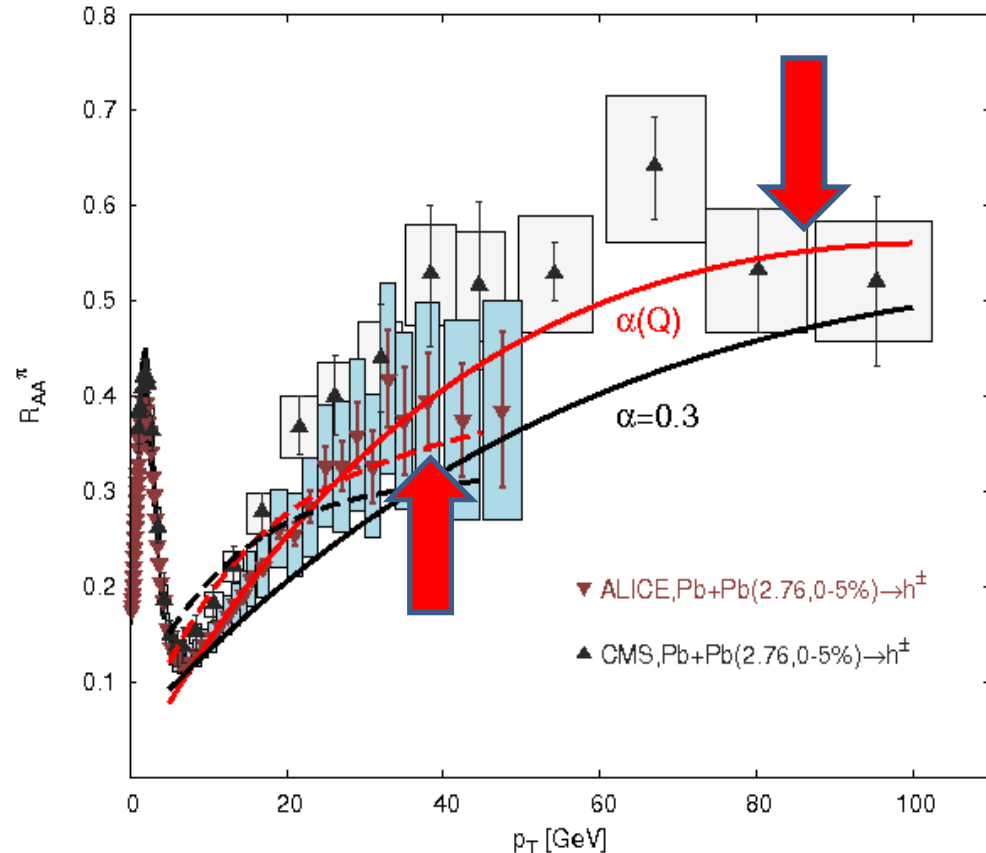
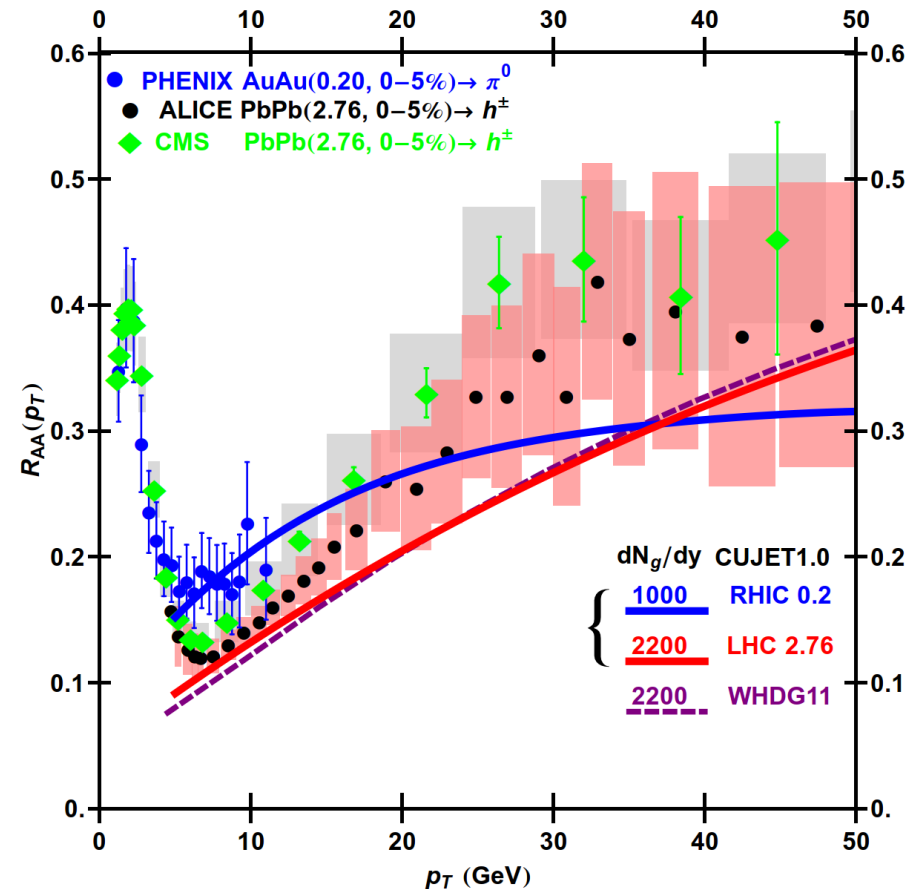
$$Elastic = \begin{cases} \alpha(4ET) \\ \alpha(\mu^2) \end{cases}$$

S. Peigne and A. Peshier, Phys.Rev. D77 (2008) 114017
W. Horowitz, Y. Kovchegov Nucl. Phys. A849 (2011) 72



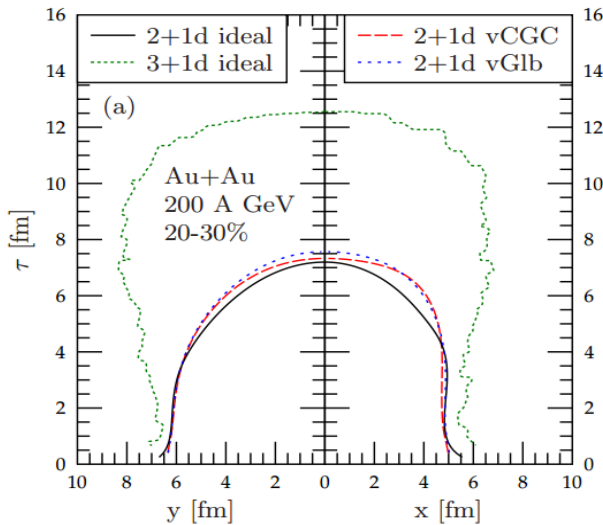
CUJET1.0: Pions at LHC

A. Buzzatti and M. Gyulassy, Phys. Rev. Lett. 108, 0223101 (2012); See also B. Betz and M. Gyulassy, arXiv:1201.02181

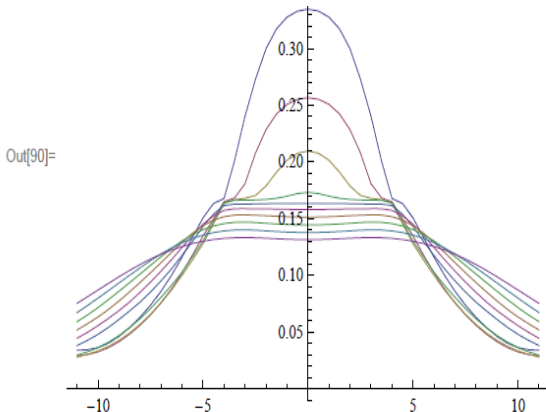


Running Coupling CUJET explains the surprising transparency at LHC, but Glauber + Bjorken longitudinal expansion background can be more realistic.

CUJET2.0 = rcDGLV + 2+1D Viscous Hydro Background



T. Renk, H. Holopainen, U. Heinz and C. Shen, PRC 83, 014910 (2011)
 C. Shen, U. Heinz, P. Huovinen and H. Song, PRC 82, 054904 (2010)
 H. Song and U. Heinz, PRC 78, 024902 (2008)



- Couple rcDGLV to VISH 2+1D expanding QGP fluid fields ($T(x,t), v(x,t)$)
- RHIC Au+Au 200 A GeV
 - Equation of State: SM-EOS Q
 - Initial Condition: MC-Glauber
 - $\eta/s=0.08$
 - Initial Time: 0.6 fm/c
 - Freeze-out temperature: 130 MeV
- LHC Pb+Pb 2.76 A TeV
 - Equation of State: s95p-PCE (P. Huovinen and P. Petreczky, NPA 837, 26 (2010))
 - Initial Condition: MC-Glauber
 - $\eta/s=0.08$
 - Initial Time: 0.6 fm/c
 - Freeze-out temperature: 130 MeV
- Initial conditions for the hydro evolution are adjusted to roughly reproduce experimental pion and proton spectra from the 5% most central 200 A GeV Au+Au collisions

CUJET2.0: More Realistic Effective Potential

Static potential (DGLV)

$$|\bar{v}_i(q_i)|^2 = \frac{1}{\pi} \frac{\mu(z_i)^2}{(q^2 + \mu(z_i)^2)^2}$$

- Static scattering centers
- Color-electric screened Yukawa potential (Debye mass)
- Full opacity series

Dynamical potential (MD)

$$|\bar{v}_i(q_i)|^2 = \frac{1}{\pi} \frac{\mu(z_i)^2}{q^2 (q^2 + \mu(z_i)^2)}$$

- Scattering centers recoil
- Includes not screened color-magnetic effects (HTL gluon propagators)
- Only first order in opacity

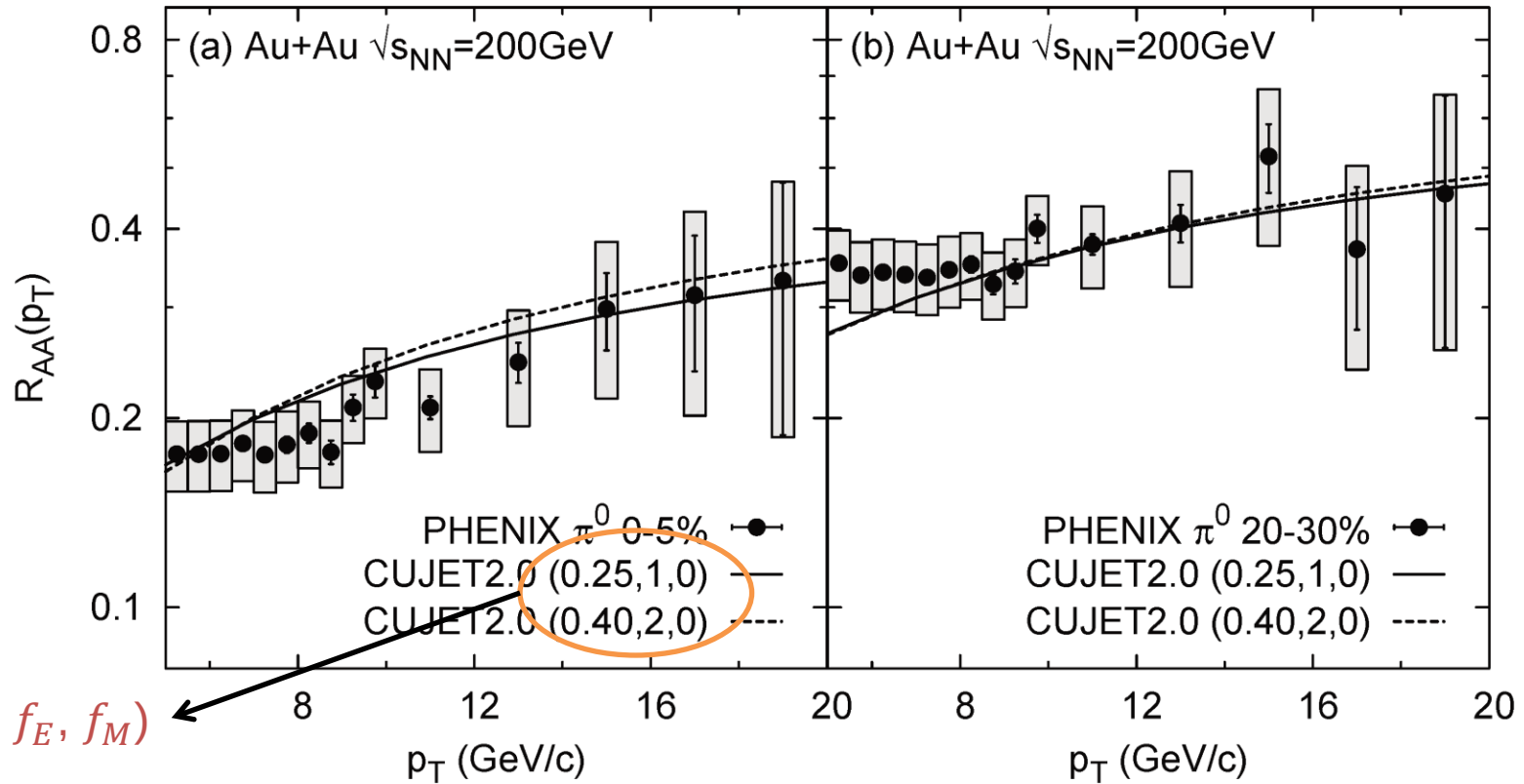
Interpolating potential (CUJET)

$$|\bar{v}_i(q_i)|^2 = \frac{\mathcal{N}(\mu_m)}{\pi} \frac{\mu_e(z_i)^2}{(q^2 + \mu_e(z_i)^2)(q^2 + \mu_m(z_i)^2)}$$

$\mu_e = f_E \mu$
 $\mu_m = f_M \mu$

- Introduces effective Debye color-magnetic mass
- Add f_E and f_M allows one to interpolate between the static and HTL dynamical limits, and further explore Non-HTL regime
- Magnetic screening allows full opacity series

CUJET2.0 Preliminary Result: R_{AA} at RHIC

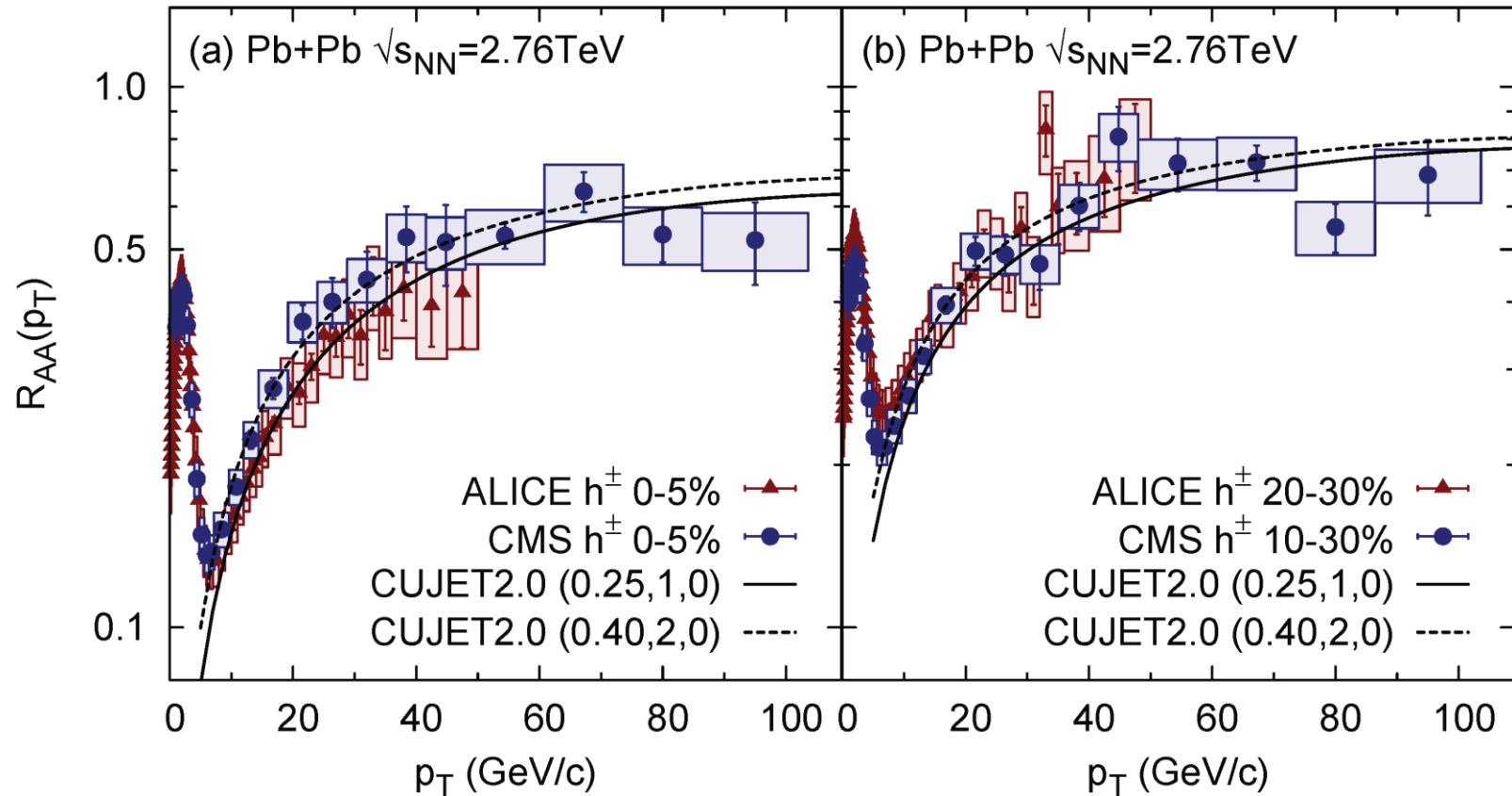


CUJET2.0: $\pi^0 R_{AA}$; 0-5% Centrality $\rightarrow b=2.4\text{fm}$, 20-30% Centrality $\rightarrow b=7.5\text{fm}$; $\tau_0 = 0.6\text{ fm/c}$, $dN/dy=1000$; (α_{max}, f_E, f_M) is adjusted to fit RHIC central $\pi^0 R_{AA}$ at $p_T = 19\text{ GeV/c}$

We explored higher $\alpha_{max} = 0.4$ with Non-HTL Debye screening masses and found that doubling HTL f_E (as per LQCD) allows to fit central RHIC RAA with this larger α_{max} .

PHENIX $\pi^0 R_{AA}$ Data: [arXiv:1208.2254](https://arxiv.org/abs/1208.2254), Phys. Rev. C 87, 034911 (2013)

CUJET2.0 Preliminary Result: R_{AA} at LHC



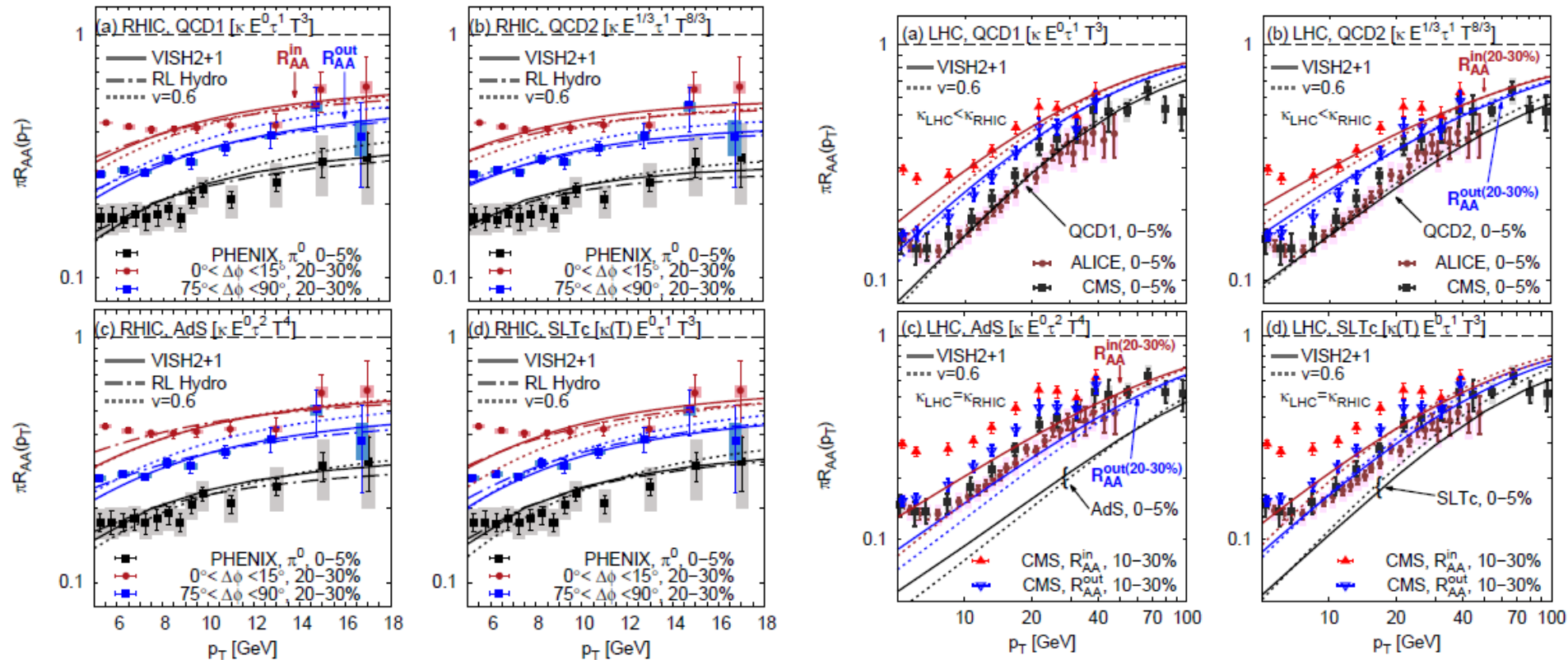
CUJET2.0: πR_{AA} ; 0-5% Centrality $\rightarrow b=2.4\text{fm}$, 10/20-30% Centrality $\rightarrow b=7.5\text{fm}$; $\tau_0 = 0.6$ fm/c, $dN/dy=2200$

Doubling HTL f_E (as per LQCD) also allows to fit central LHC RAA with this larger α_{max} .

ALICE $h^\pm R_{AA}$ Data: arXiv:1208.2711, Phys. Lett. B 720, 52 (2013)

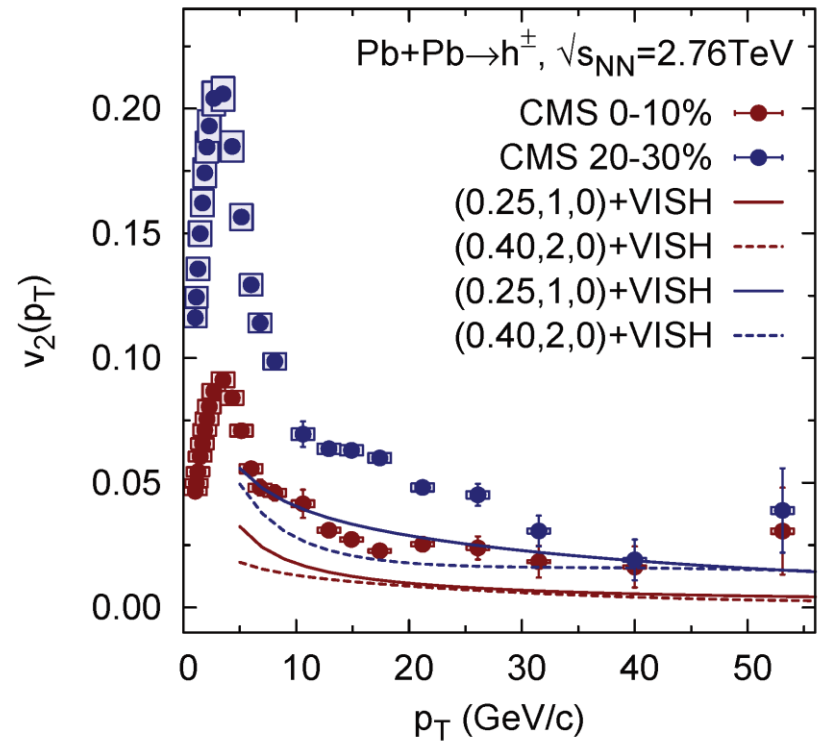
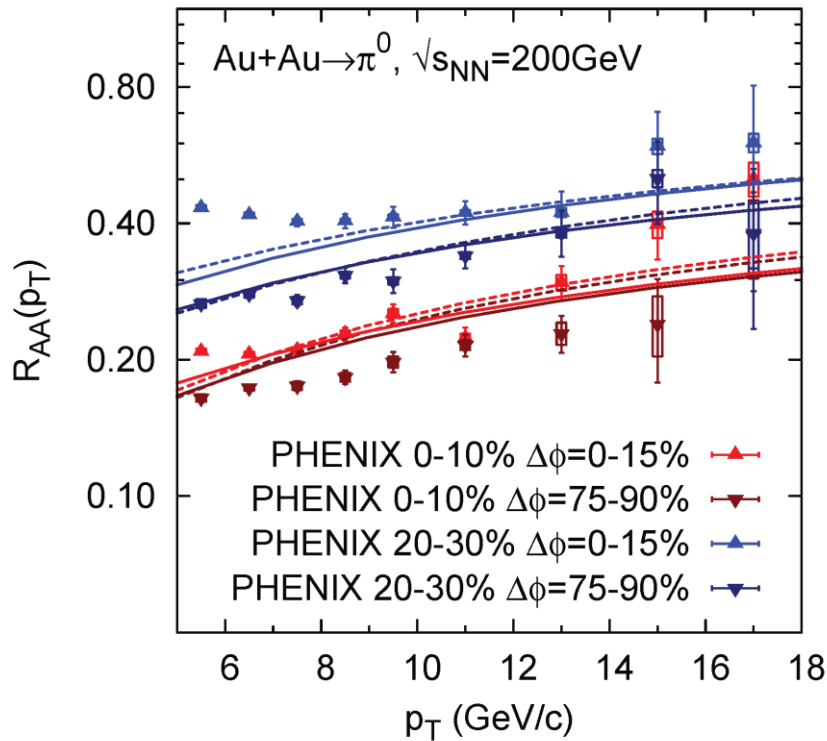
CMS $h^\pm R_{AA}$ Data: arXiv:1202.2554, Eur. Phys. J. C 72, 1945 (2012)

Puzzle: simultaneously fit R_{AA}^{in} and R_{AA}^{out}



- **B. Betz and M. Gyulassy, arXiv:1305.6458**
 - QCD1 → **rcCUJET1.0**; QCD2 → **fcCUJET1.0**; AdS → fixed t'Hooft conformal falling string; SLTc → Shurya-Liao assuming T_c dominated
 - VISH2+1, Shen, Heinz, Song; RL-Romatschke, Luzum
- **D. Molnar and S. Deke, arXiv:1305.1046**
 - R_{AA} and v_2 cannot be satisfied with the same set of parameters in MPC+GLV

CUJET2.0 Preliminary Result: v_2 at RHIC and LHC



v_2 is 30% lower at both RHIC and LHC

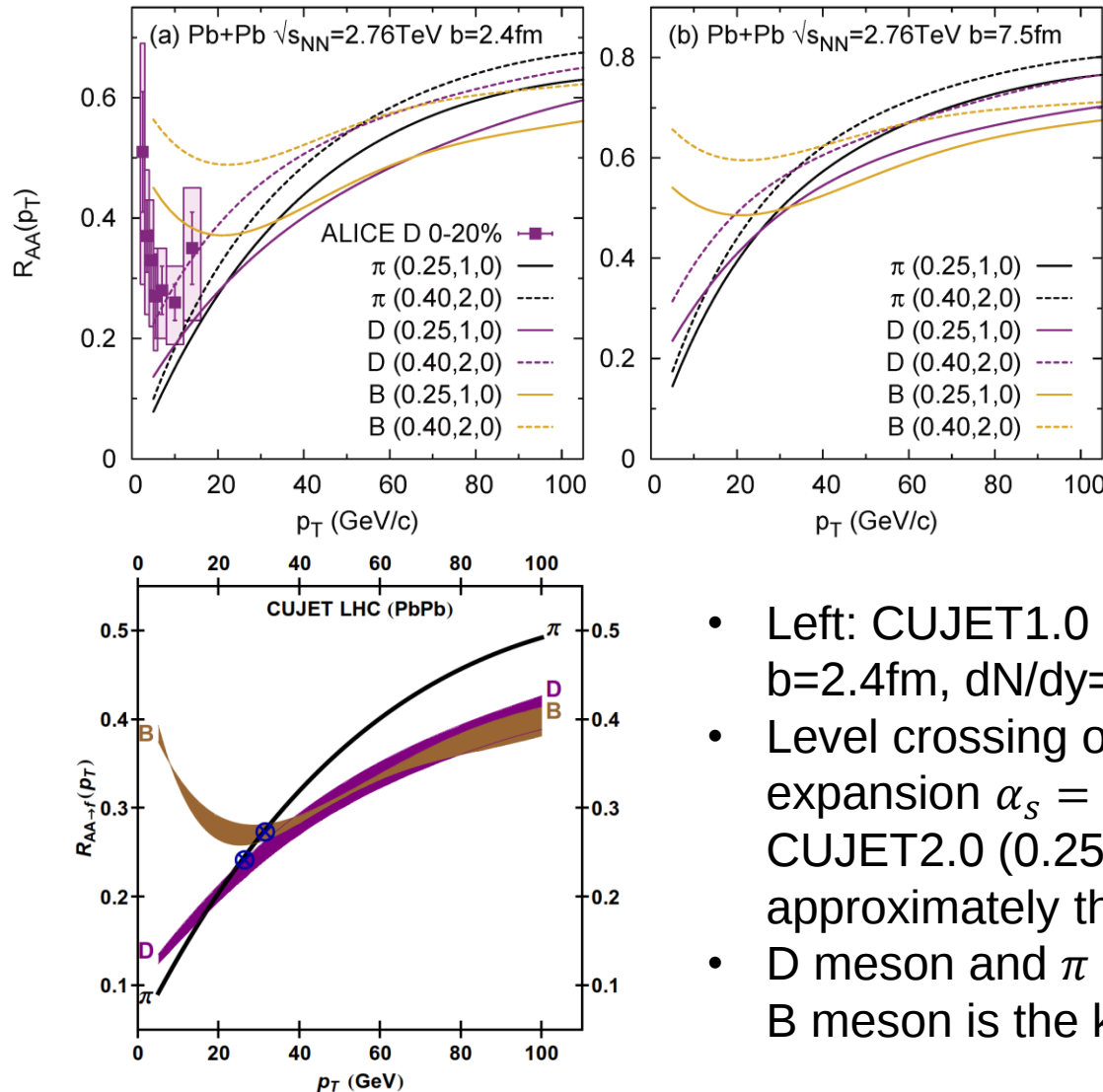
- 1) Azimuthal Asymmetry $\approx (\text{dE/dx Model})/2 + (\text{spacetime bulk hydro 2+1D flow})/2$
- 2) Depend on a complex interplay between details of microscopic $p_T > 10$ jet dE/dx (abc) and details of the spacetime evolution of the bulk soft $p_T < 2$ GeV sQGP (IC, eta/s, tau)
- 3) Azimuthal averaged RAA is less sensitive to this hard+soft convolution

Long standing puzzle of simultaneous fit of RAA and v_2 correlations remains!

PHENIX $\pi^0 R_{AA}$ Data: arXiv:1208.2254, Phys. Rev. C 87, 034911 (2013)

CMS $h^\pm v_2$ Data: arXiv:1204.1850, Phys. Rev. Lett. 109, 022301 (2012)

CUJET2.0 Preliminary Result: Heavy Flavor at LHC

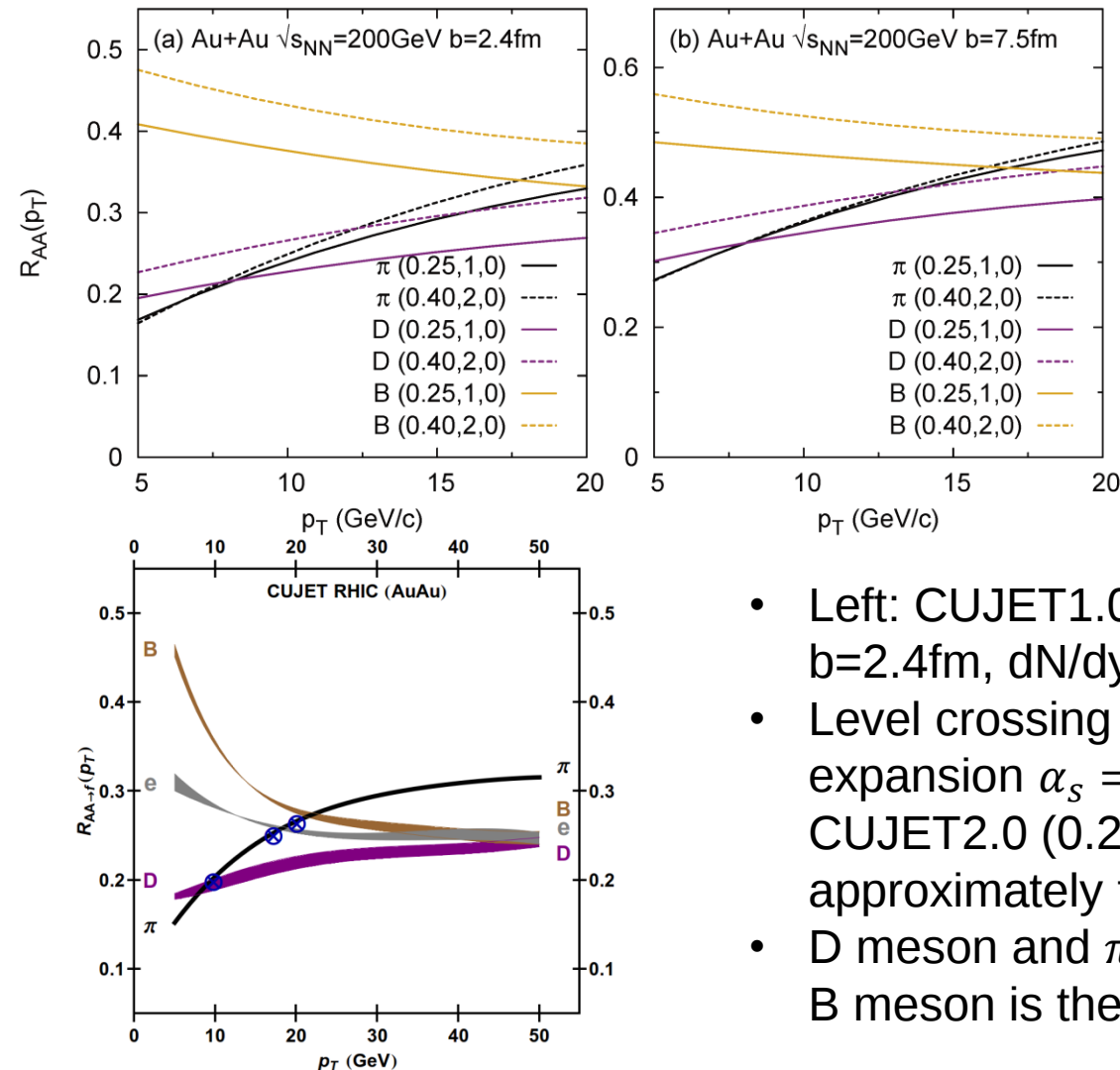


- D meson and π mixed together, B meson is the key constraint on the mode space.
- ALICE D meson R_{AA} Data: arXiv:1203.2160, JHEP 09, 112 (2012)

- Left: CUJET1.0 result for Pb+Pb 2.76A TeV $b = 2.4$ fm, $dN/dy = 2200$
- Level crossing of fixed coupling Bjorken expansion $\alpha_s = 0.3$ CUJET1.0 result and CUJET2.0 (0.25, 1, 0) result occurs at approximately the same p_T
- D meson and π R_{AA} almost overlap at low $p_T \rightarrow$ B meson is the key constraint for the model

A. Buzzatti and M. Gyulassy, Phys. Rev. Lett. 108, 0223101 (2012)

CUJET2.0 Preliminary Result: Heavy Flavor at RHIC



- D meson and π mixed together at the p_T range 5 GeV/c to 20 GeV/c, B meson is **STILL** the key constraint on the mode space.
- ALICE D meson R_{AA} Data: arXiv:1203.2160, JHEP 09, 112 (2012)

- Left: CUJET1.0 result for Au+Au 200A GeV $b=2.4$ fm, $dN/dy=1000$
- Level crossing of fixed coupling Bjorken expansion $\alpha_s = 0.3$ CUJET1.0 result and CUJET2.0 (0.25, 1, 0) result occurs at approximately the same p_T
- D meson and π R_{AA} almost overlap at low $p_T \rightarrow$ B meson is the key constraint on model space

A. Buzzatti and M. Gyulassy, Phys. Rev. Lett. 108, 0223101 (2012)

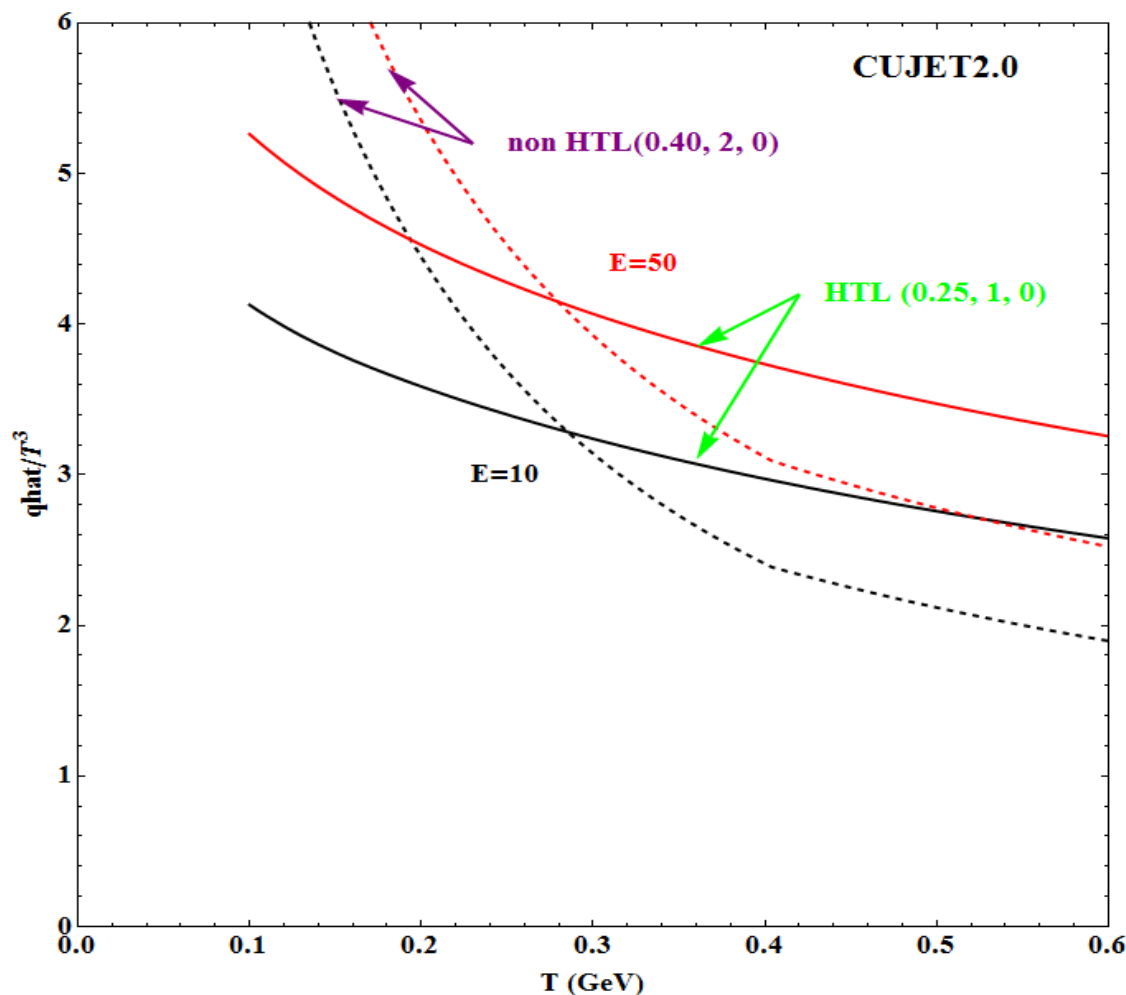
CUJET2.0 Preliminary Result: Jet Transport Parameter

$\hat{q}(E,T)/T^3$ vs T for jet $E=10$ black, **50 red**

Running Coupling CUJET2.0 solutions

(1) ($\alpha_{\max}=0.25, f_E=1, f_M=0$) solid

(2) ($\alpha_{\max}=0.4, f_E=2, f_M=0$) dashed



- In DGLV + VISH2.1 framework, absolute jet transport parameter $\hat{q}$ varies with E and T
- This issue could be fundamental in a transverse expanding medium where temperature and density evolves position with time
- This problem still needs to be tested with heavy flavor and peripheral collisions

Summary

Conclusions

- Two CUJET2.0 Models in the parameter space produce satisfactory results when considering the flavor and density dependence of R_{AA}
- Long standing puzzle of [simultaneously fit of RAA and \$v_2\$](#) correlations remains!
- [Heavy flavor](#) is the key constraint on model space.
- [Nontrivial jet transport parameter](#) in the DGLV + VISH2.1 framework.

Prospects

- CUJET offers a reliable and flexible model which is able to compute leading hadron jet energy loss and compare directly with data.
 - Results are satisfactory when considering the flavor and density dependence of R_{AA} .
 - Possible to study systematic theoretical uncertainties.
 - Easy to improve.
- New RHIC electron predictions now consistent with uncertainties of data (Heavy Quark puzzle).
- Strong prediction of novel level crossing pattern of flavor dependent R_{AA} .
- Running alpha strong coupling constant is in [simultaneous agreement](#) with RHIC and LHC data.

Future Works

- Extend 2.0 $\rightarrow$ 2.n to test [lattice QCD](#) data on non-perturbative $V(r,T)$ and answer the question: Does lattice QCD predict correct non-conformal jet medium physics near T_c ?
- Test CUJET2.0 predicted jet flavor and azimuthal tomography against [all RHIC vs LHC](#) data systematics.

Thank you!

BACKUP

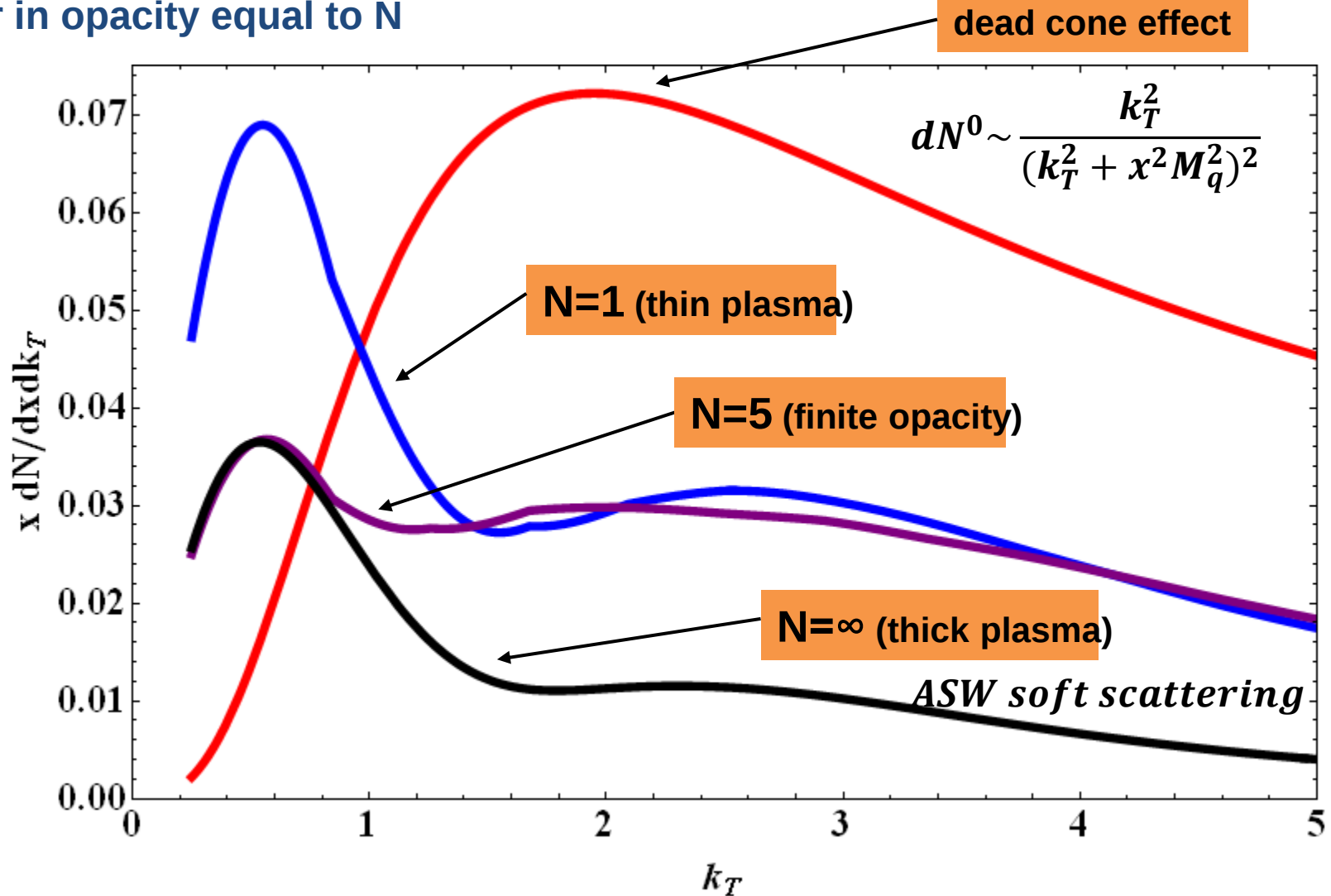
Systematic errors

- Opacity order expansion
 - Choice of interaction potential
 - Pre-equilibrium phase
 - **ALSO:**
 - pp Spectra
 - Running coupling scales
- 

1. One free parameter in the model: α_s^{eff}
2. Fit α_s^{eff} to 10GeV RHIC Pion data $\alpha_s^{eff} \approx 0.3 \pm 10\%$
3. All other predictions are fully constrained

k_T distribution

En=20GeV, x=0.25, bottom quark, plasma thickness 5fm
Order in opacity equal to N



Beyond first order in opacity

Interpolate between DGLV and MD with a new effective potential

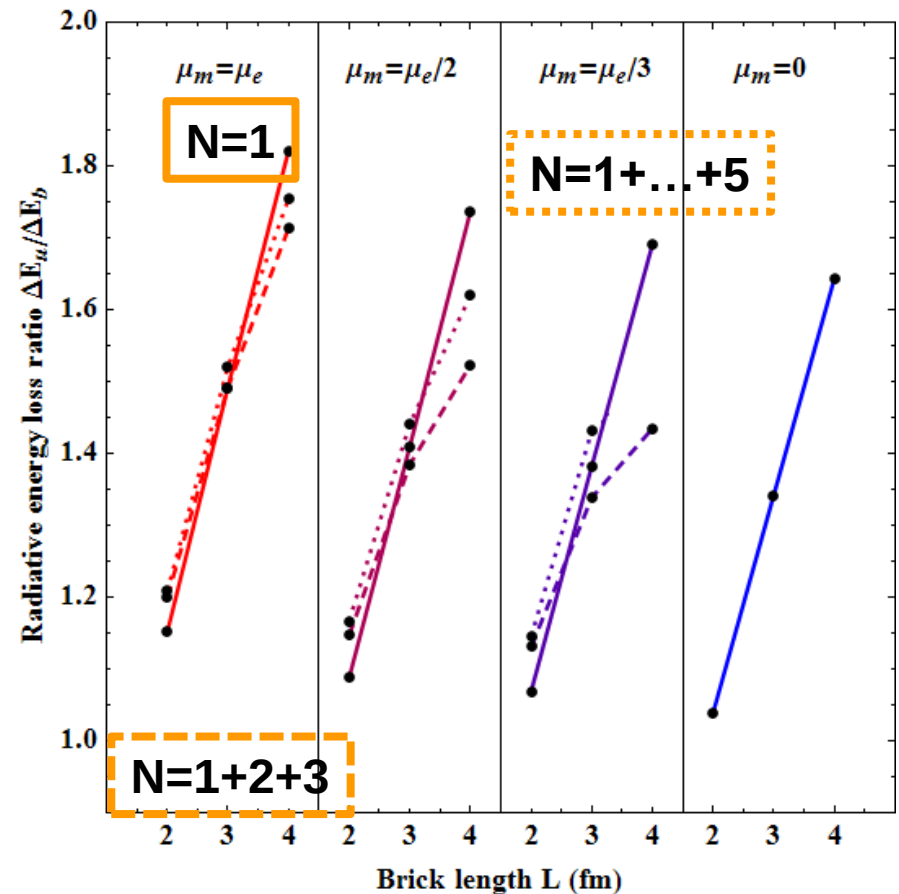
$$\frac{1}{(q^2 + \mu^2)^2} \xleftarrow{\text{DGLV}} \frac{1}{(q^2 + \mu_m^2)(q^2 + \mu_e^2)} \xrightarrow{\text{MD}} \frac{1}{q^2(q^2 + \mu^2)}$$

It is possible to study the limit $\mu_m \rightarrow 0$ for values of $\mu_m \gtrsim \mu_e/3$

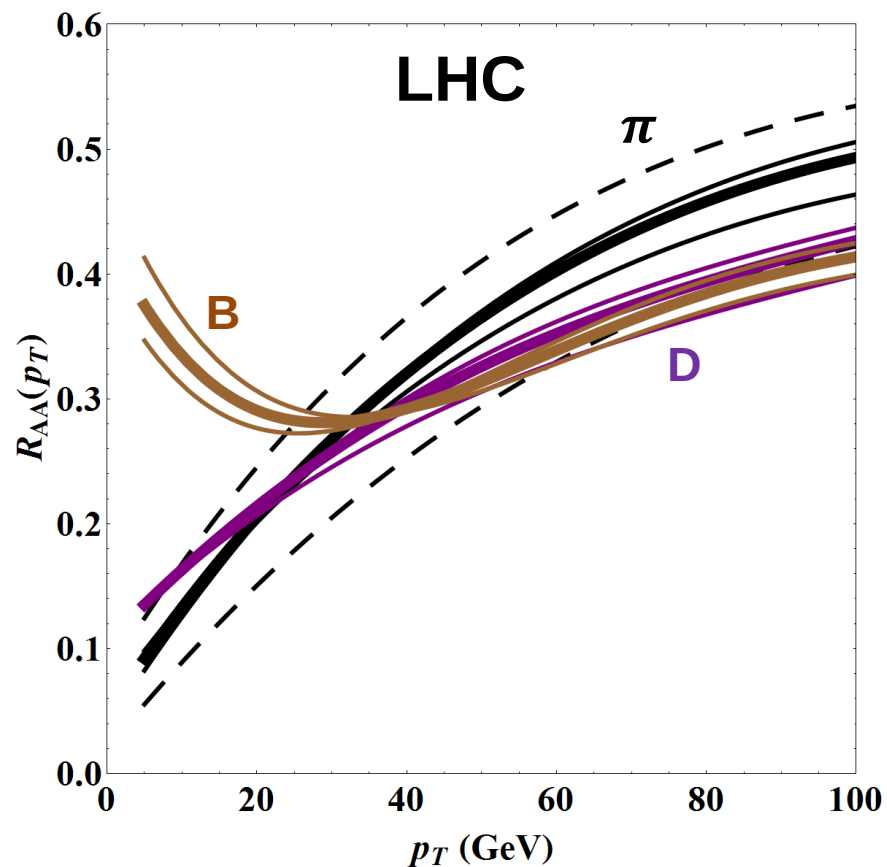
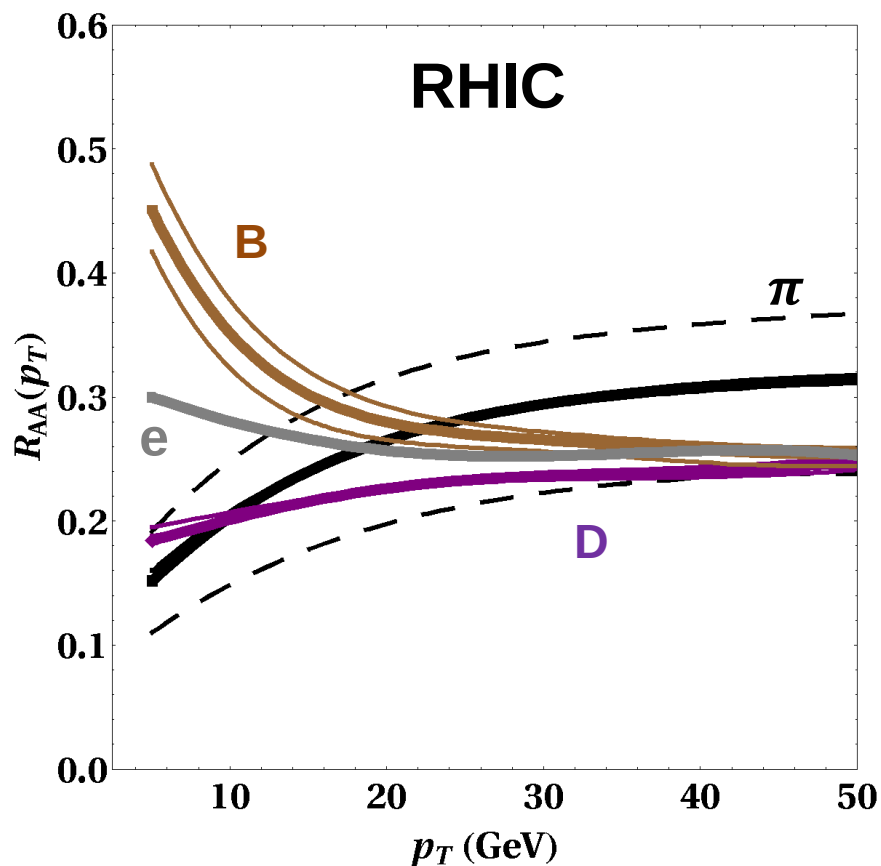
- The mean free path $\frac{1}{\lambda} =$

$\int d\mathbf{q} \frac{d\sigma}{dq} \rho$ is divergent for $\mu_m = 0$

$\left(\frac{\Delta E_u}{\Delta E_b}\right)$ ratio improves for $N > 1$ and $\mu_m \rightarrow 0$, but likely not enough.



τ_0 sensitivity



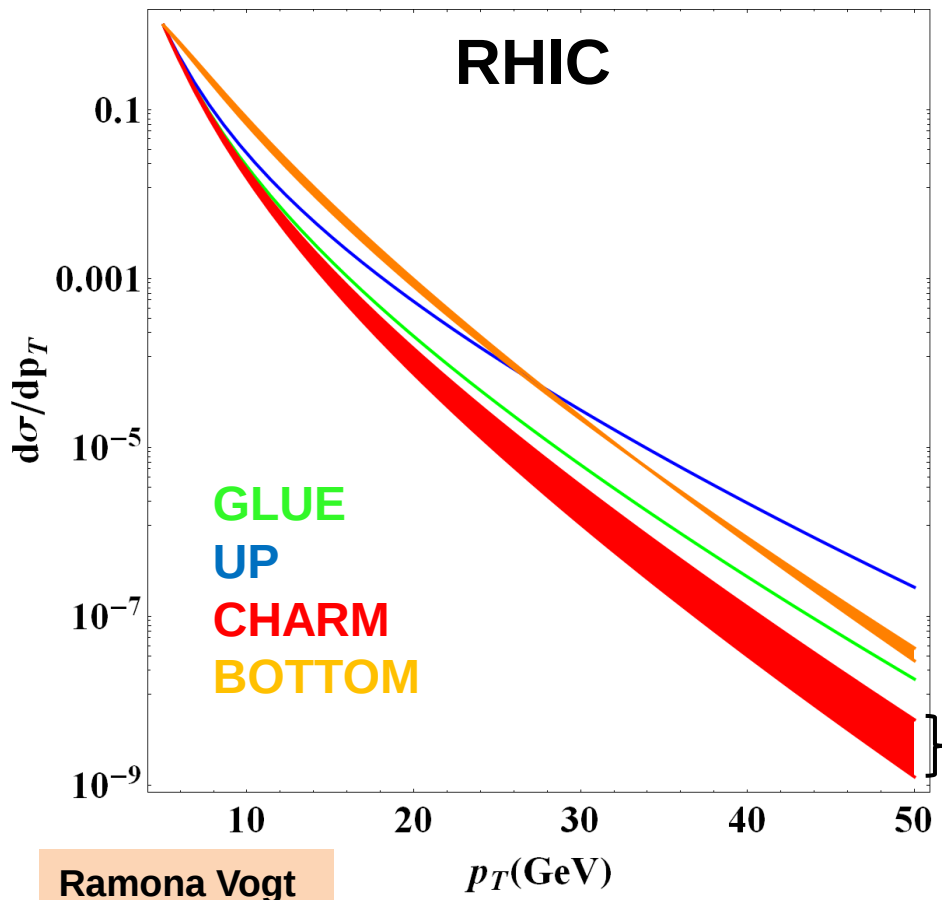
THICK: *Linear* with $\alpha_s = 0.3$

THIN: *Divergent* with $\alpha_s = 0.27$ or *Freestreaming* with $\alpha_s = 0.325$

DAHSED: *Divergent* or *Freestreaming* with $\alpha_s = 0.3$

Initial pQCD spectra

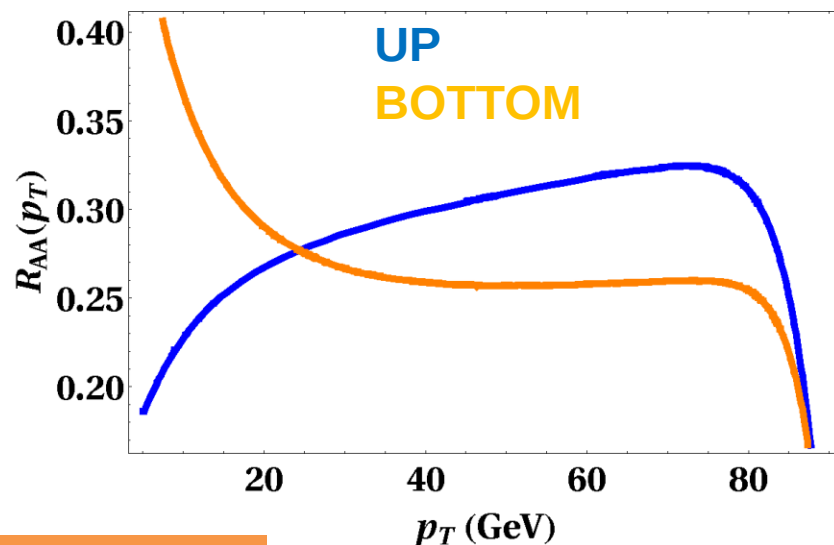
Initial quark production spectra



NLO-FONLL uncertainty

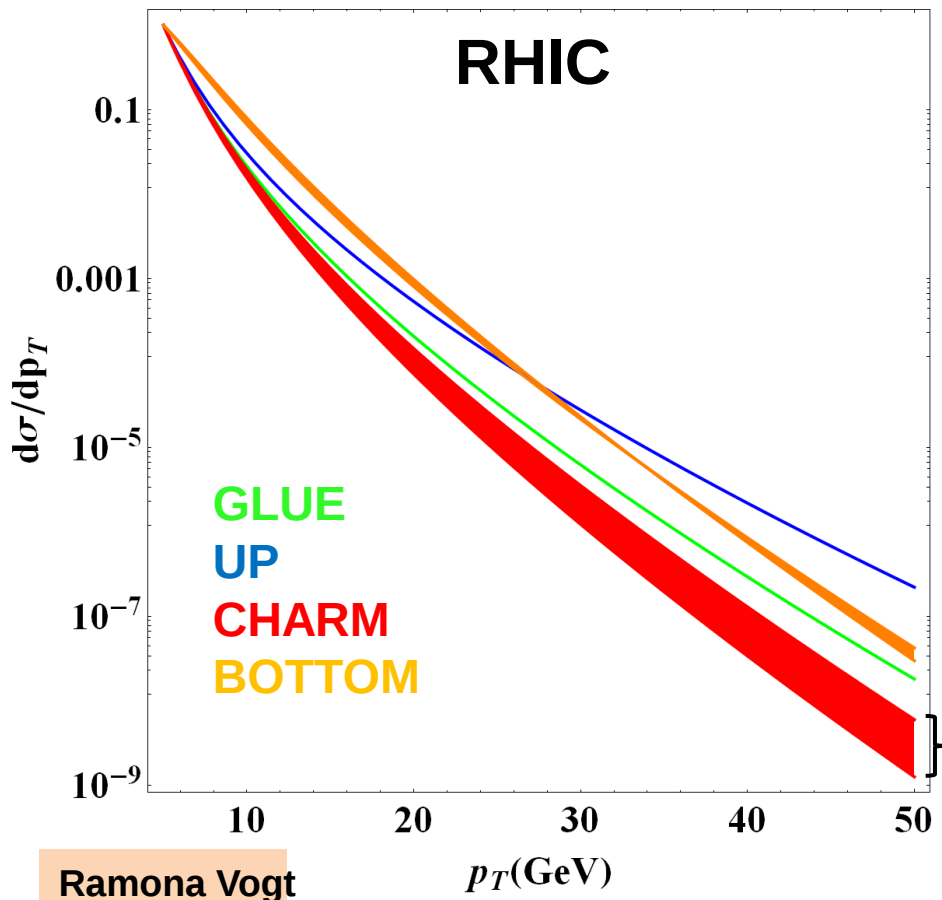
Competing effects between increased density and harder production spectra

- RHIC density and spectra
- LHC density, RHIC spectra
- LHC density and spectra



Initial pQCD spectra

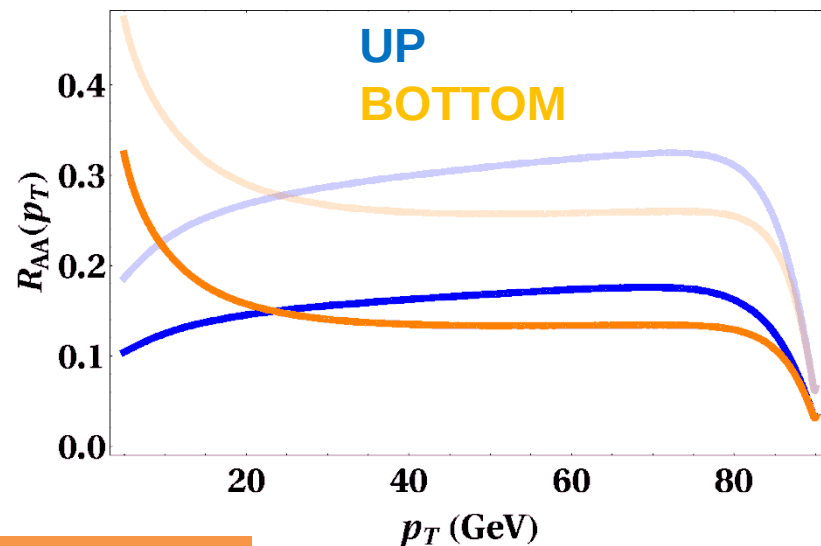
Initial quark production spectra



NLO-FONLL uncertainty

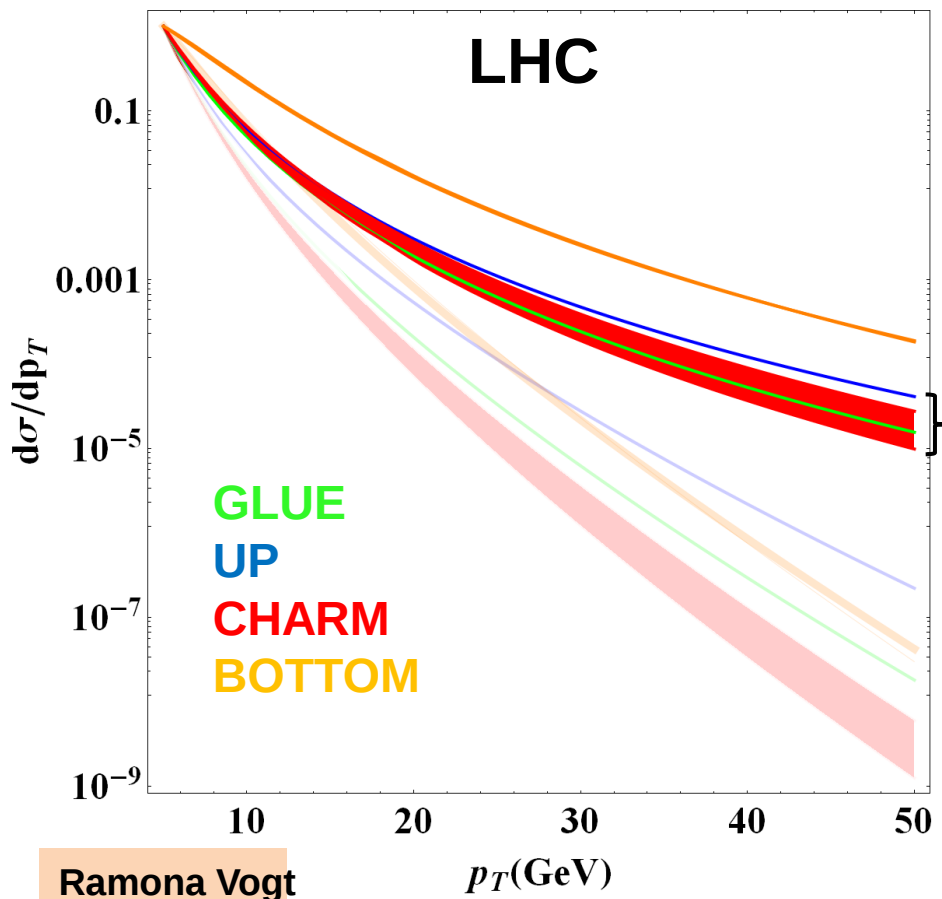
Competing effects between increased density and harder production spectra

- RHIC density and spectra
- LHC density, RHIC spectra
- LHC density and spectra



Initial pQCD spectra

Initial quark production spectra

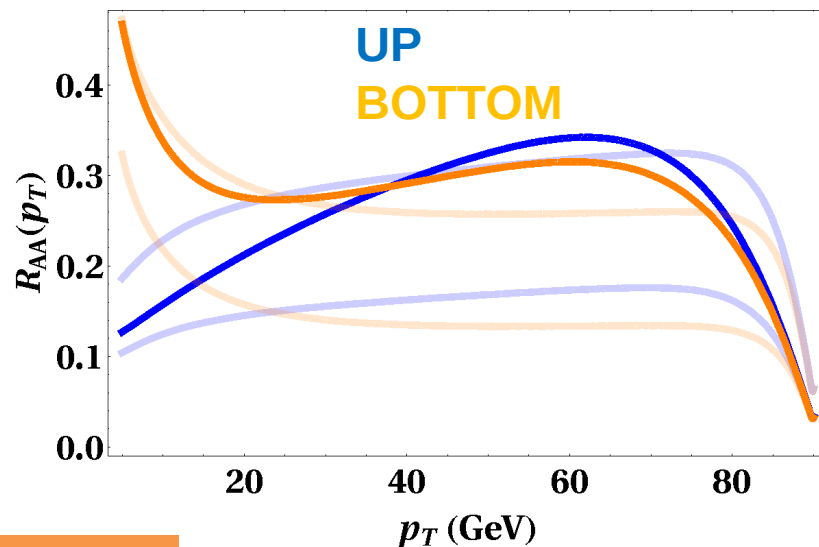


Ramona Vogt

NLO-FONLL uncertainty

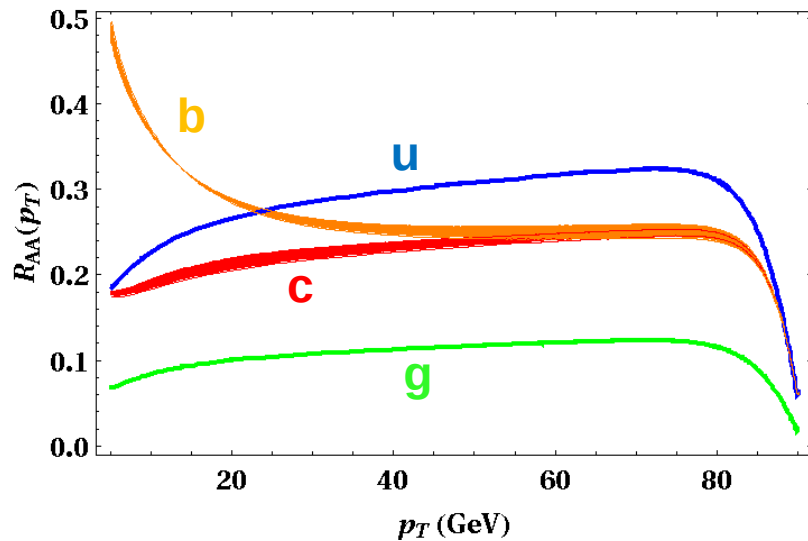
Competing effects between increased density and harder production spectra

- RHIC density and spectra
- LHC density, RHIC spectra
- LHC density and spectra

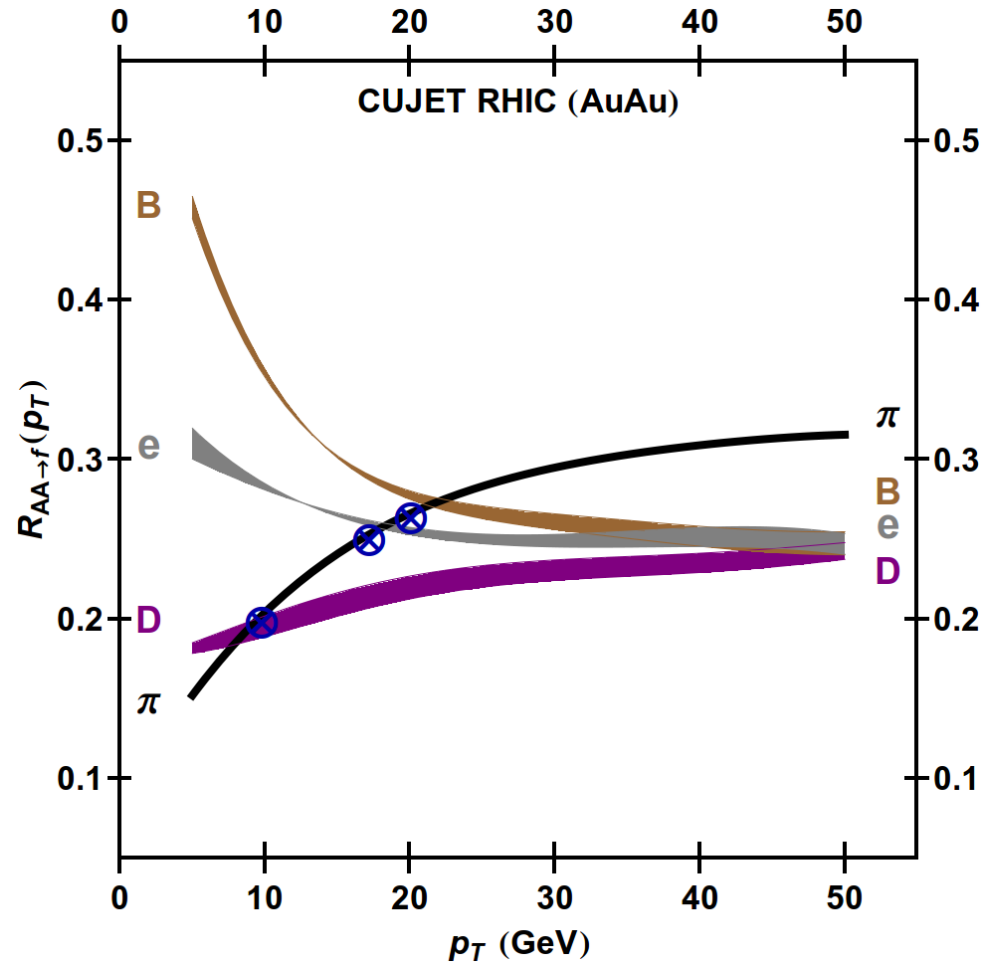


RHIC Results

0 – 5% centrality, $dNdy = 1000$, $\alpha_s = 0.3$, $\tau_0 = 1fm/c$



Inversion of R_{AA} flavor hierarchy at sufficiently high p_t



AB and M. Gyulassy, Phys. Rev. Lett. 108, 0223101 (2012)

LHC Results

Parameters constrained by RHIC
 $dNdy = 2200$

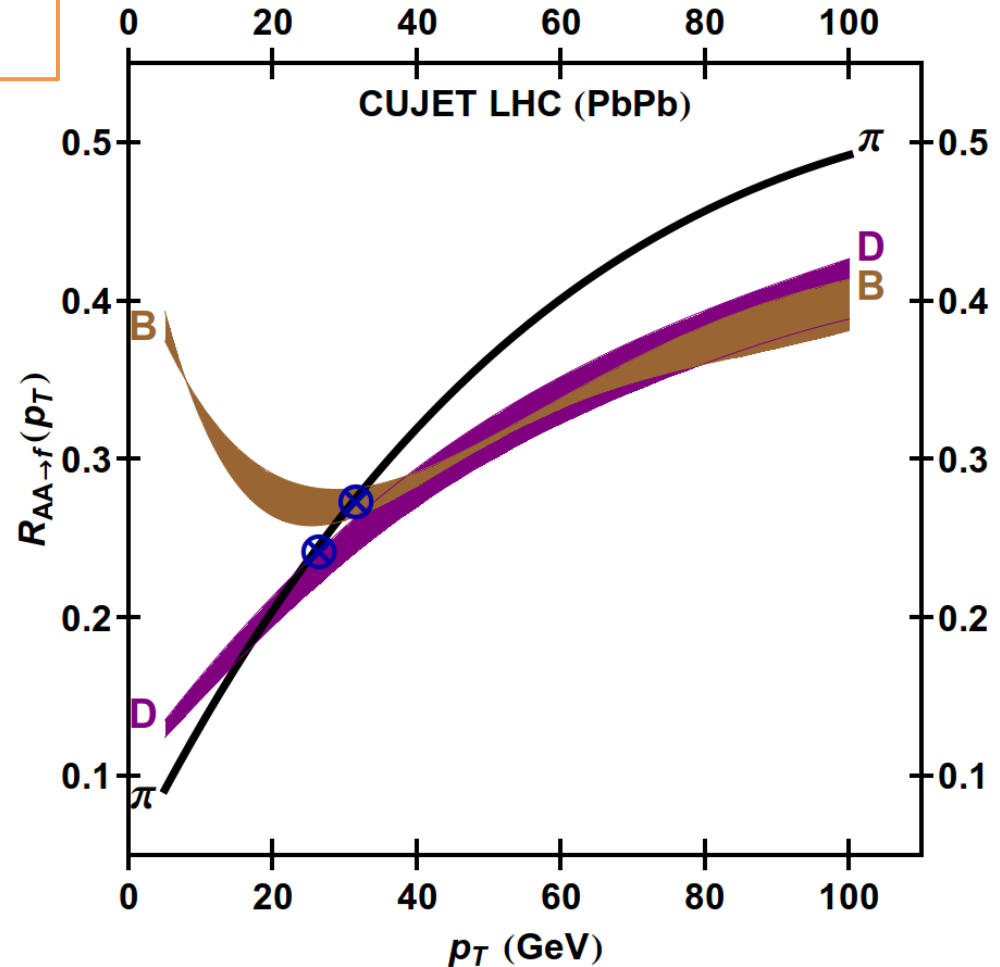
Competing effect between
Energy loss ordering...

$$\Delta E(\text{light}) \approx \Delta E(c) > \Delta E(b)$$

...and pp Production spectra

$d\sigma(c, b)$ harder than $d\sigma(\text{light})$

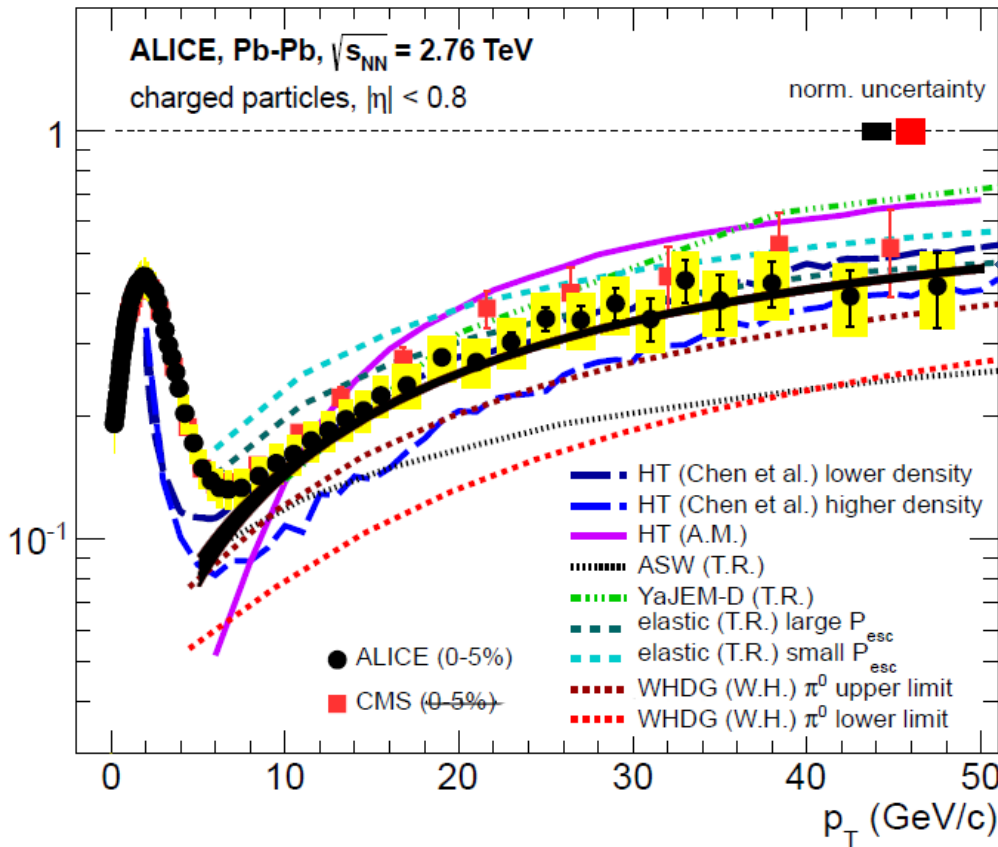
$$RAA \sim (1 - \Delta E/E)^{n-2}$$



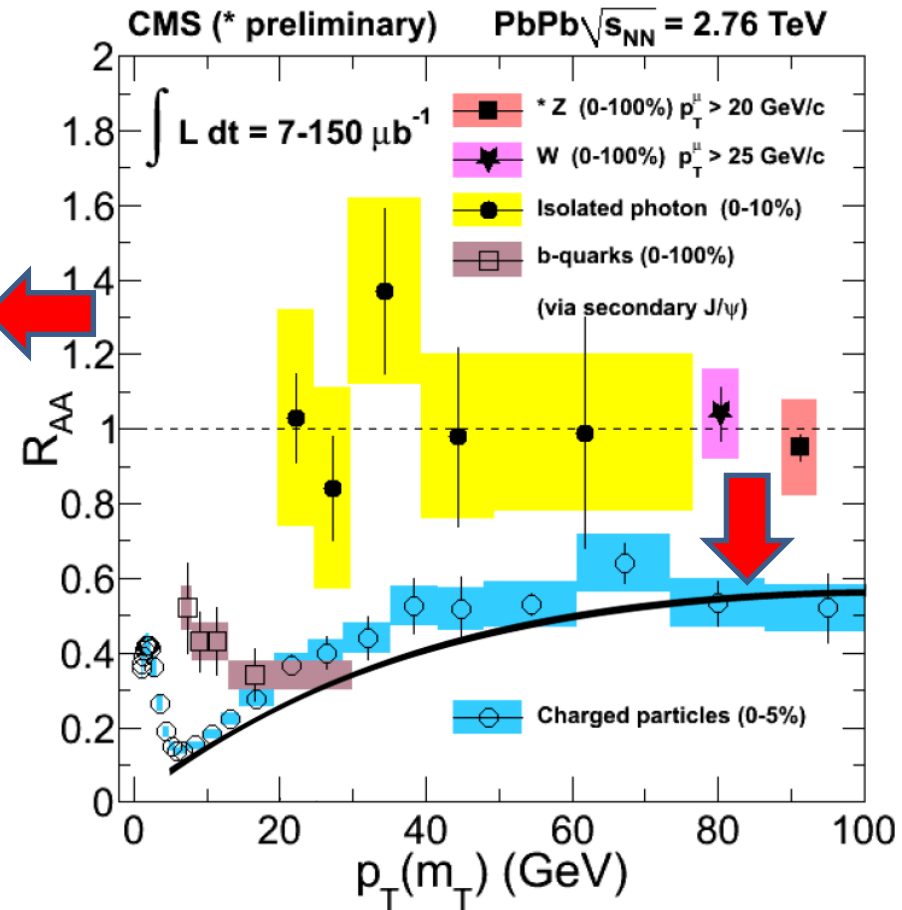
AB and M. Gyulassy, Phys. Rev. Lett. 108, 0223101 (2012)

ALICE and CMS Pions

ALICE Collaboration

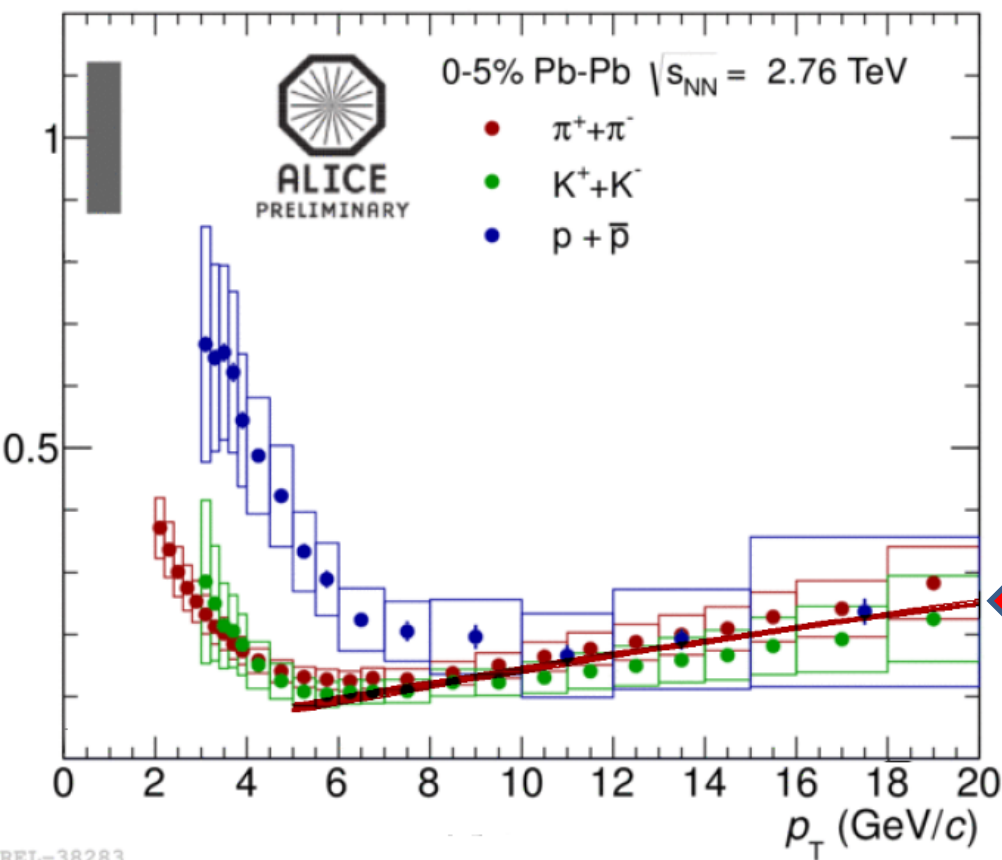


CMS Collaboration

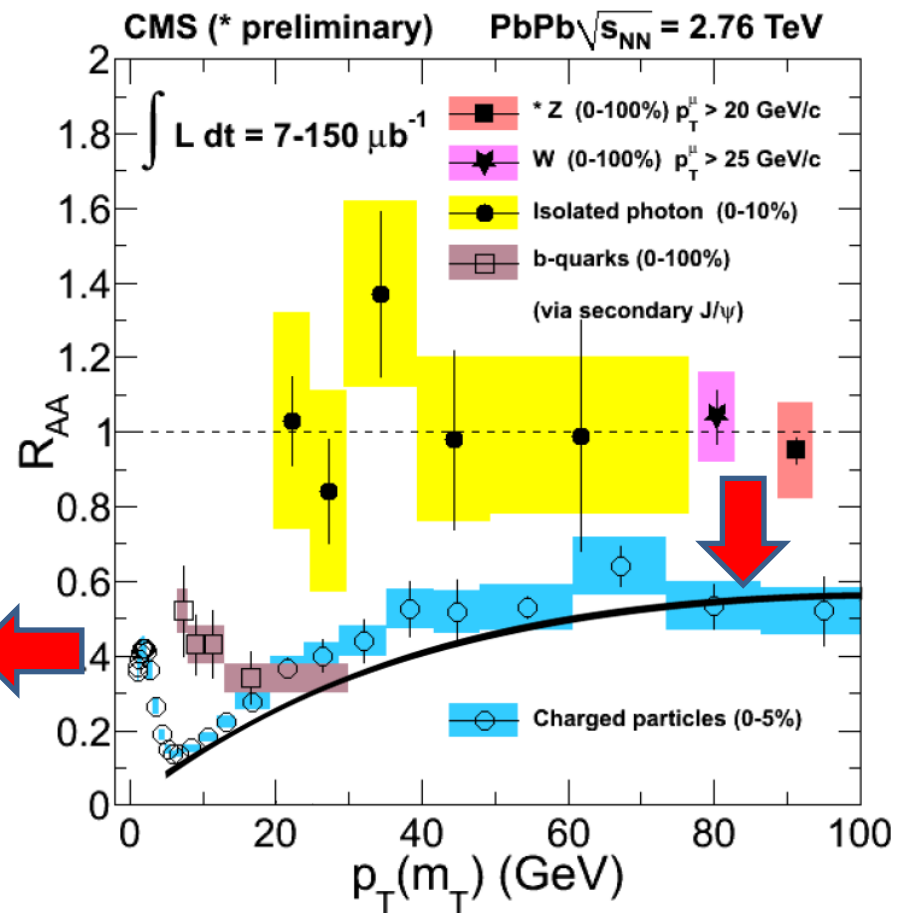


ALICE and CMS Pions

ALICE Collaboration

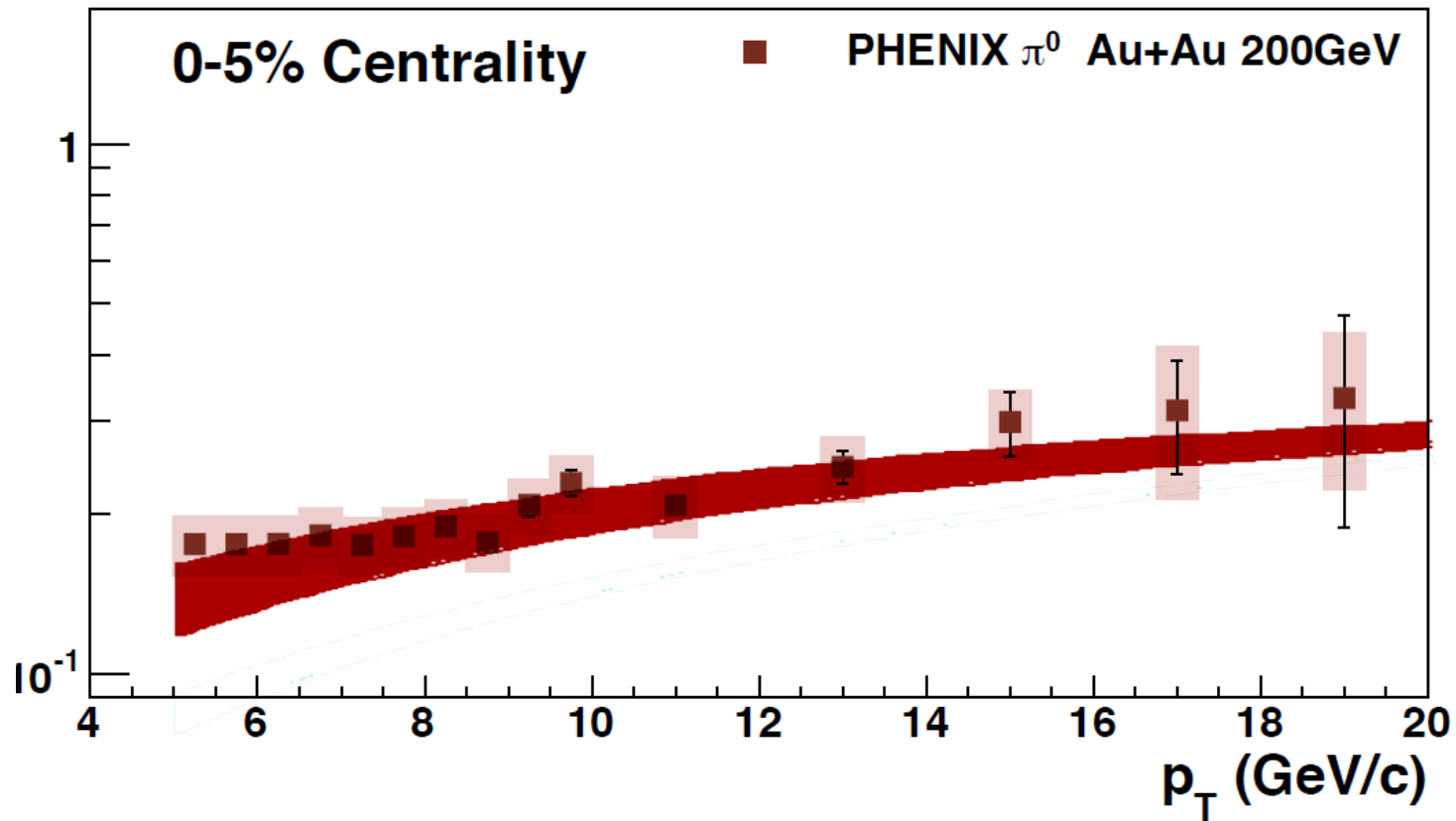


CMS Collaboration



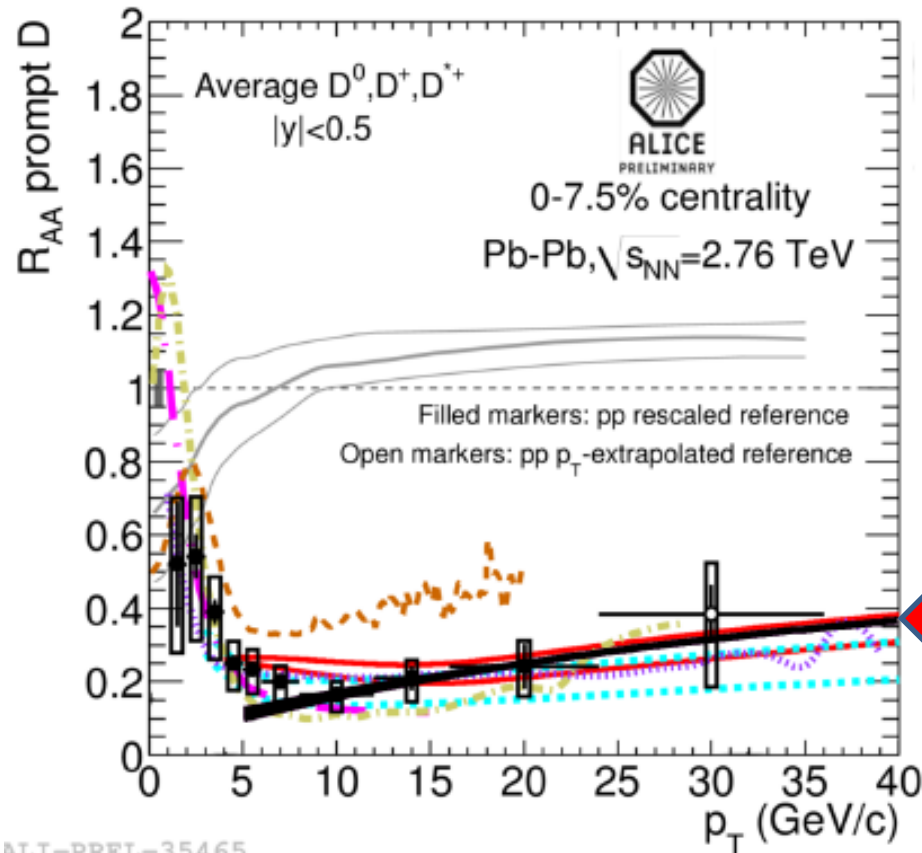
PHENIX Pions

PHENIX Collaboration



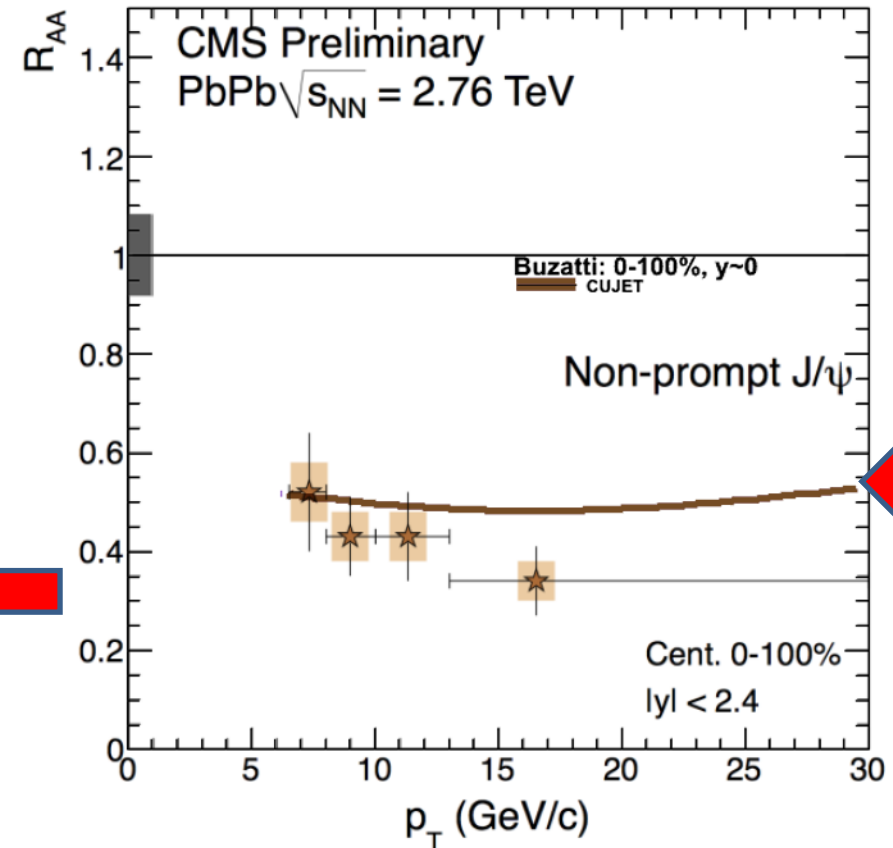
ALICE and CMS Heavy Flavors

ALICE Collaboration



CMS Collaboration

CMS-PAS HIN-12-014



Elastic energy loss and Fluctuations

Bjorken elastic collisions

- Soft scattering
- Thoma-Gyulassy model

$$\frac{dE}{dx} = -C_R \pi \alpha^2 T^2 \log[B]$$

$$\rightarrow B_{TG} = \frac{4pT}{E-p+4T} / \mu$$

Energy loss fluctuations

- The probability of losing a fractional energy $\varepsilon = \frac{\Delta E}{E}$ is the convolution of Radiative and Elastic contributions

$$P(\varepsilon) = \int dx P_{rad}(\varepsilon) P_{el}(x - \varepsilon)$$

- Radiative: $P_{rad}(\varepsilon) = P_0 \delta(\varepsilon) + \tilde{P}(\varepsilon)|_0^1 + P_{stop} \delta(1 - \varepsilon)$

Poisson expansion
of the number of
INCOHERENTLY
emitted gluons

- Elastic: $P_{el}(\varepsilon) = e^{-\langle N_c \rangle} \delta(\varepsilon) + N e^{-\frac{(\varepsilon - \bar{\varepsilon})^2}{4T\bar{\varepsilon}}}$

Gaussian fluctuations

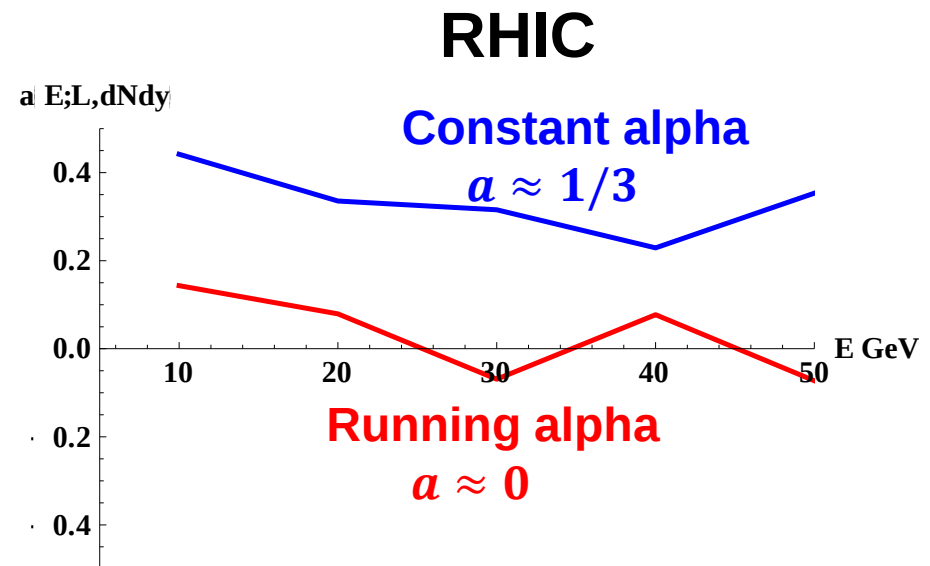
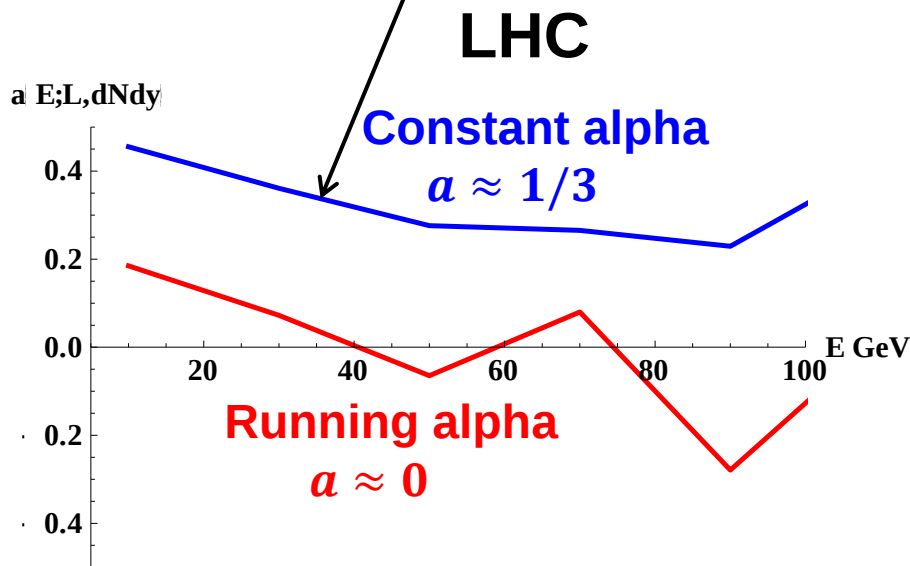
Energy loss

- Consider a simplified power law model for Energy loss:

$$\frac{\Delta E}{E} = \kappa E^{a-1} L^b \rho^c$$

W. A. Horowitz and M. Gyulassy, arXiv:1104.4958

B. Betz and M. Gyulassy, arXiv:1201.0218



Bjorken expansion

- The local thermal equilibrium is established at τ_0

$$s(\tau) = s_0 \frac{\tau_0}{\tau} \quad (\text{entropy equation})$$

$$s_0 \approx 3.6 \rho_0 = 3.6 \frac{1}{\pi R^2 \tau_0} \frac{dN}{dy} \quad \left(\frac{dN}{dy} \text{ is the observed rapidity density} \right)$$

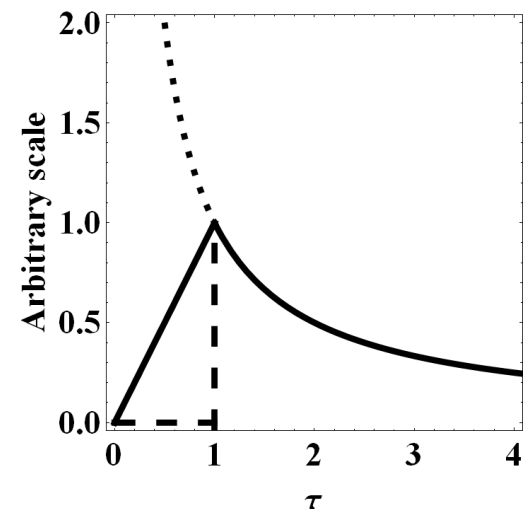
$$\rho_{QGP}(x_{\perp}, \tau) = \frac{1}{\tau_0} \frac{\rho_{part}(x_{\perp})}{N_{part}} \frac{dN}{dy} f\left(\frac{\tau}{\tau_0}\right)$$

MONOTONIC density dependence

- Before equilibrium

Temporal envelopes: linear, divergent, freestreaming

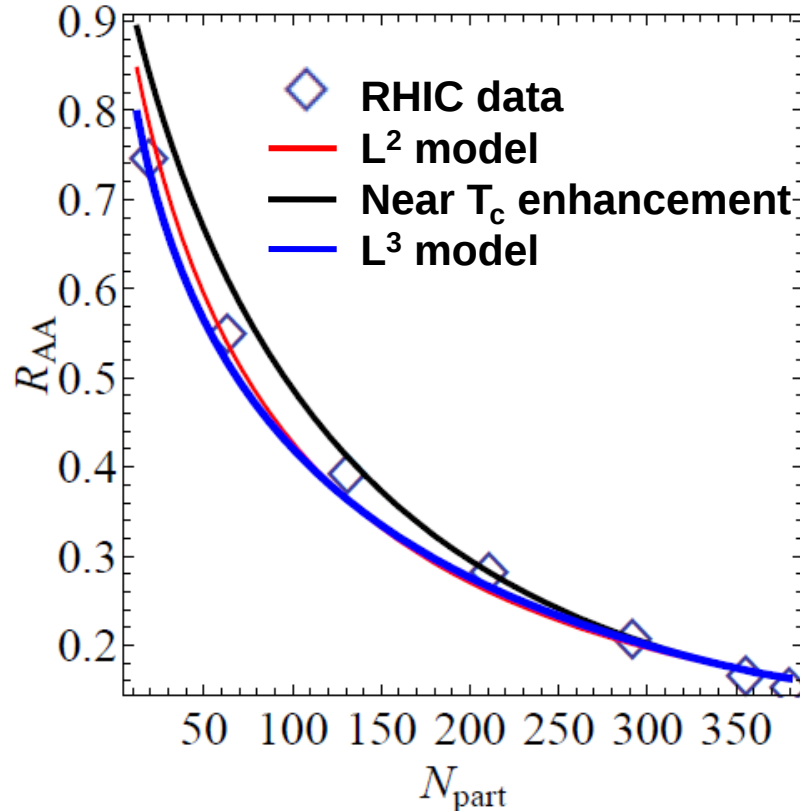
$$f\left(\frac{\tau}{\tau_0}\right) = \begin{cases} \frac{\tau}{\tau_0}, \frac{\tau_0}{\tau}, 0 & (\tau < \tau_0) \\ \frac{\tau_0}{\tau} & (\tau > \tau_0) \end{cases}$$



Magnetic monopoles

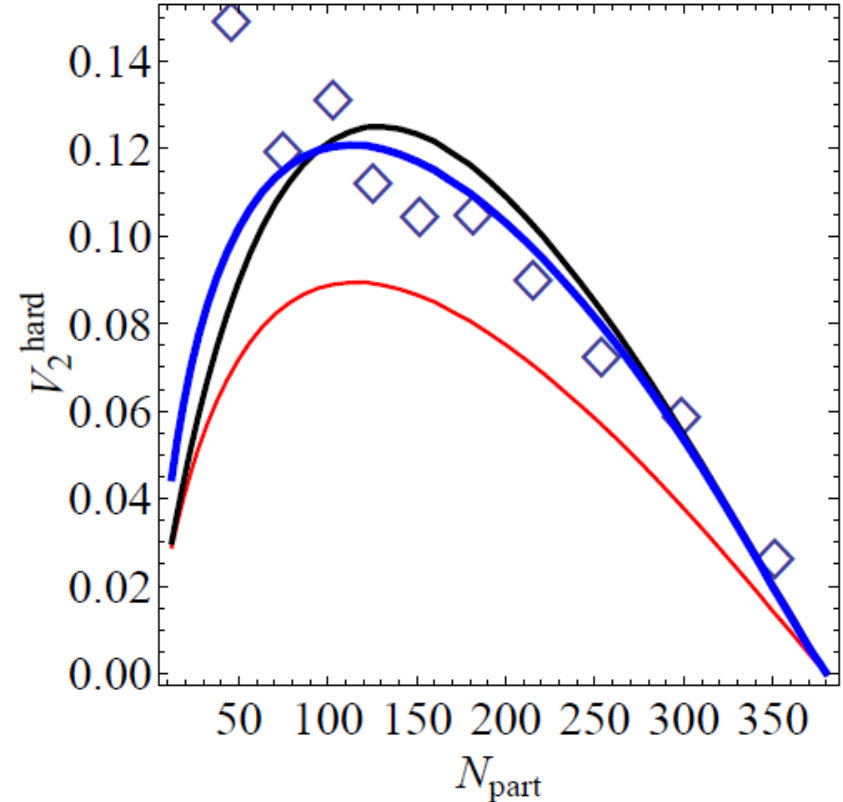
Magnetic monopole enhancement

- Nonlinear density dependence near T_c

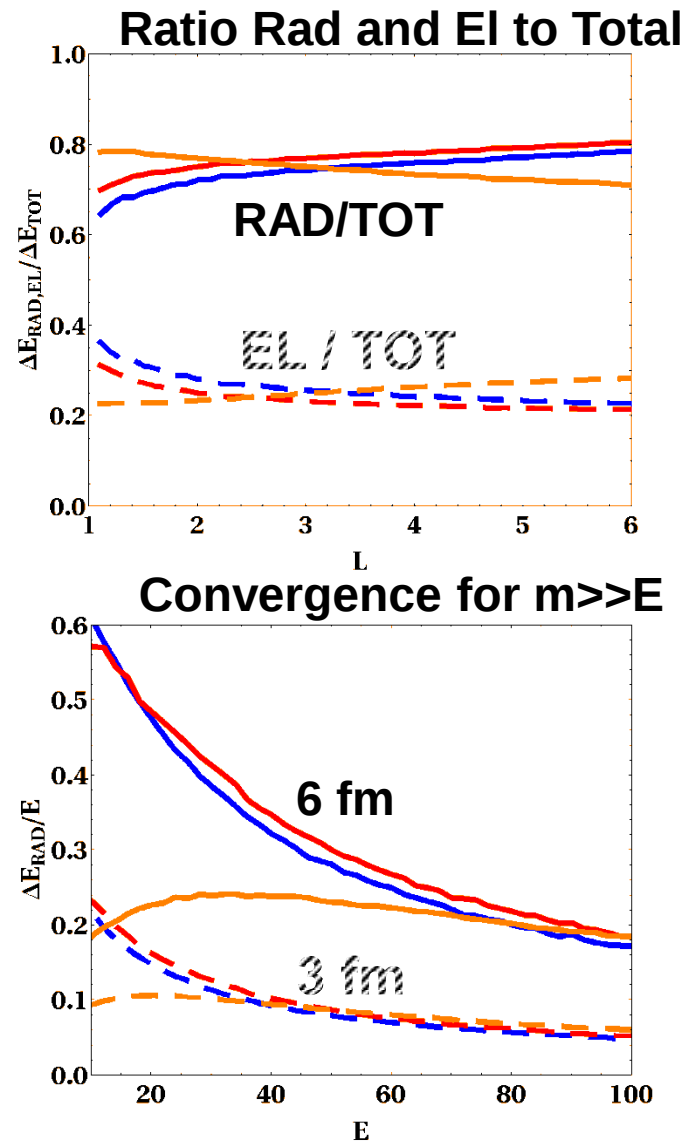
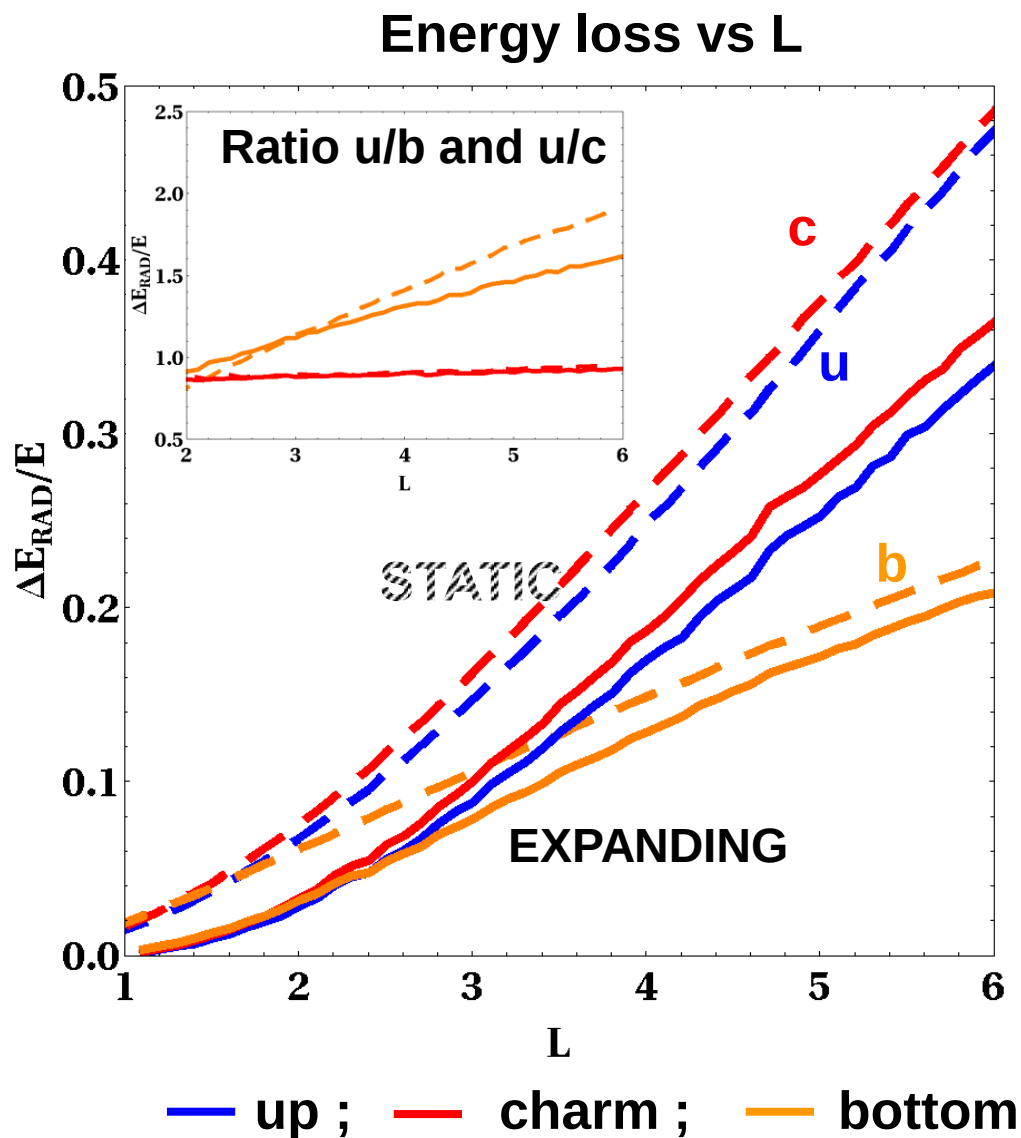


AdS/CFT

Jinfeng Liao, arXiv:1109.0271

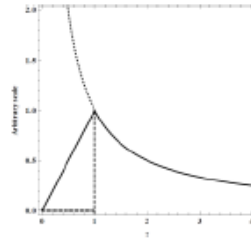


Energy loss

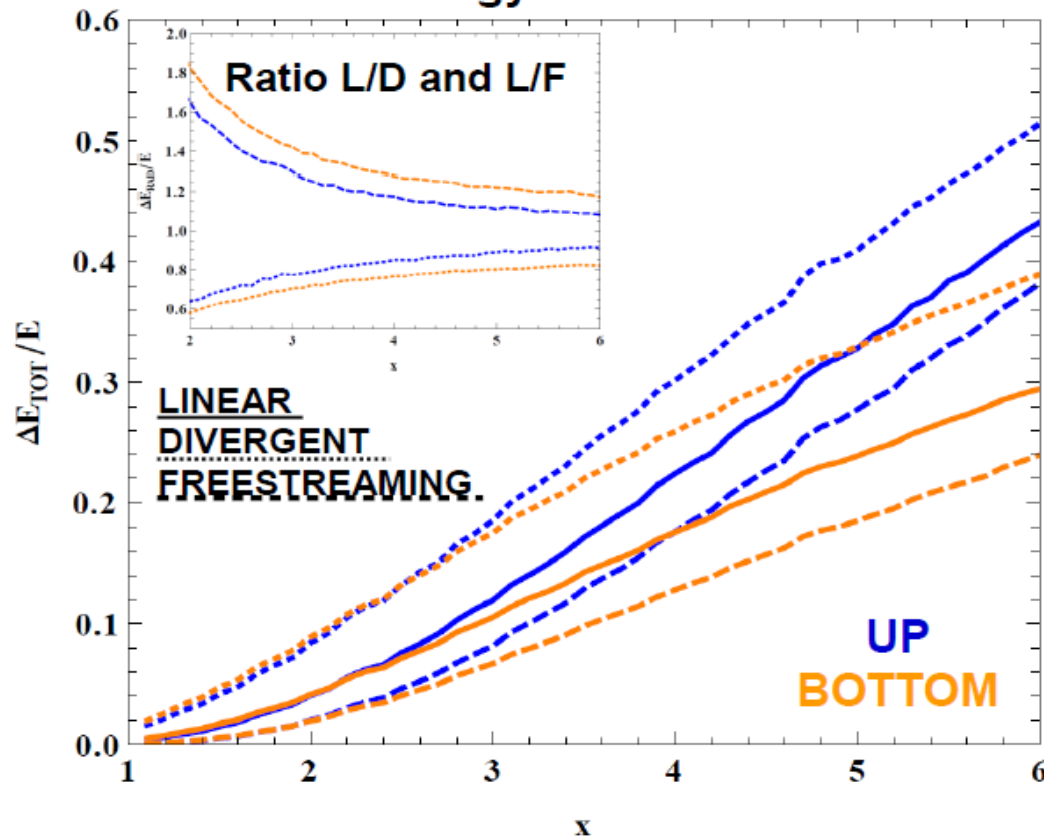


Temporal envelope

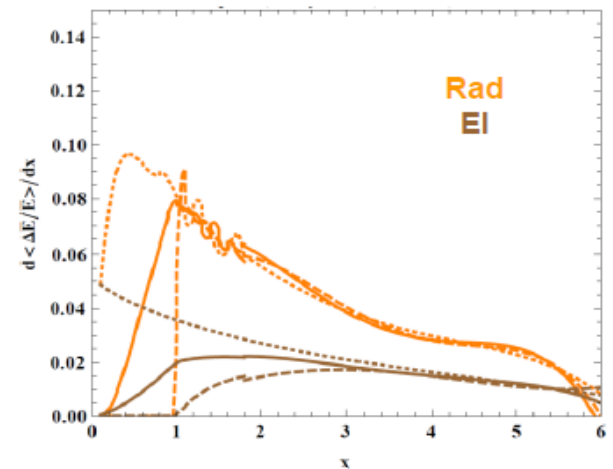
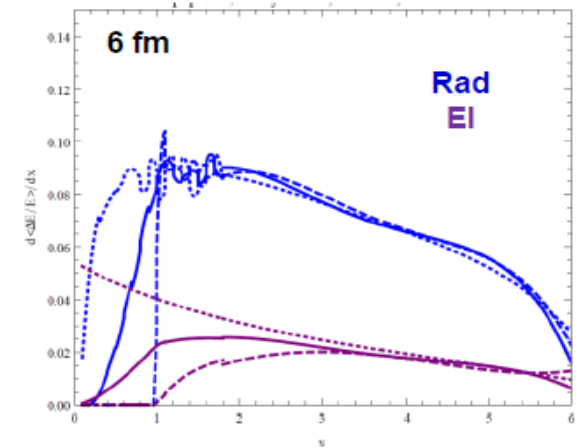
$$f\left(\frac{\tau}{\tau_0}\right) = \begin{cases} \frac{\tau}{\tau_0}, \frac{\tau_0}{\tau}, 0 & (\tau < \tau_0) \\ \frac{\tau_0}{\tau} & (\tau > \tau_0) \end{cases}$$



Energy loss vs L



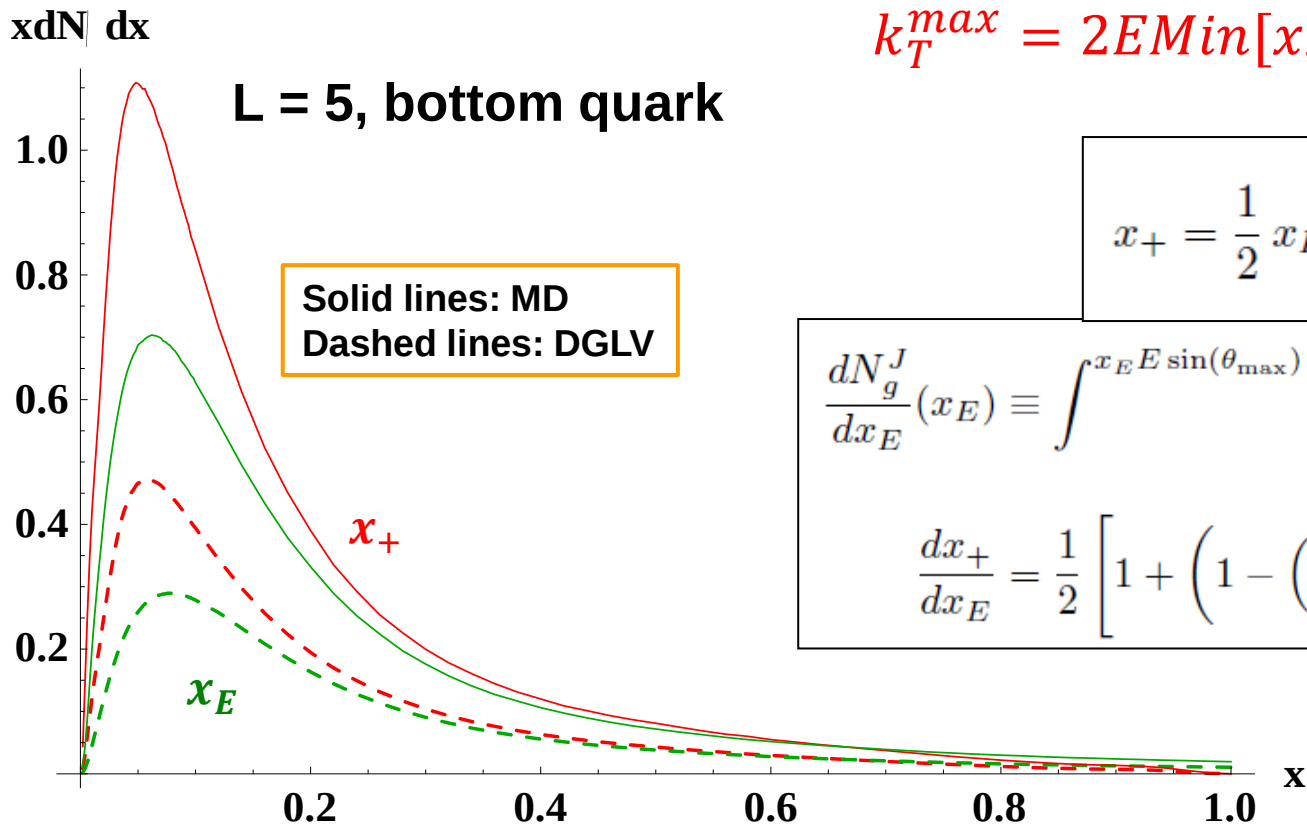
Differential Energy Loss $\frac{d\langle\Delta E/E\rangle}{dx}$



k_T sensitivity

- Collinear approximation: $x_E = x_+ \left(1 + O \left(\frac{k_T}{x_+ E} \right)^2 \right)$
 - DGLV formula has the same functional form for x_E or x_+
 - Different kinematic limits: $k_T^{max} = x_E E$

$$k_T^{max} = 2E \text{Min}[x_+, 1 - x_+]$$



$$x_+ = \frac{1}{2} x_E \left(1 + \sqrt{1 - \left(\frac{k_T}{x_E E} \right)^2} \right)$$

$$\frac{dN_g^J}{dx_E}(x_E) \equiv \int^{x_E E \sin(\theta_{\max})} dk_T \frac{dx_+}{dx_E} \frac{dN_g}{dx_+ dk_T}(x_+(x_E)),$$

$$\frac{dx_+}{dx_E} = \frac{1}{2} \left[1 + \left(1 - \left(\frac{k_T}{x_E E} \right)^2 \right)^{-1} \right].$$

Scaling violation

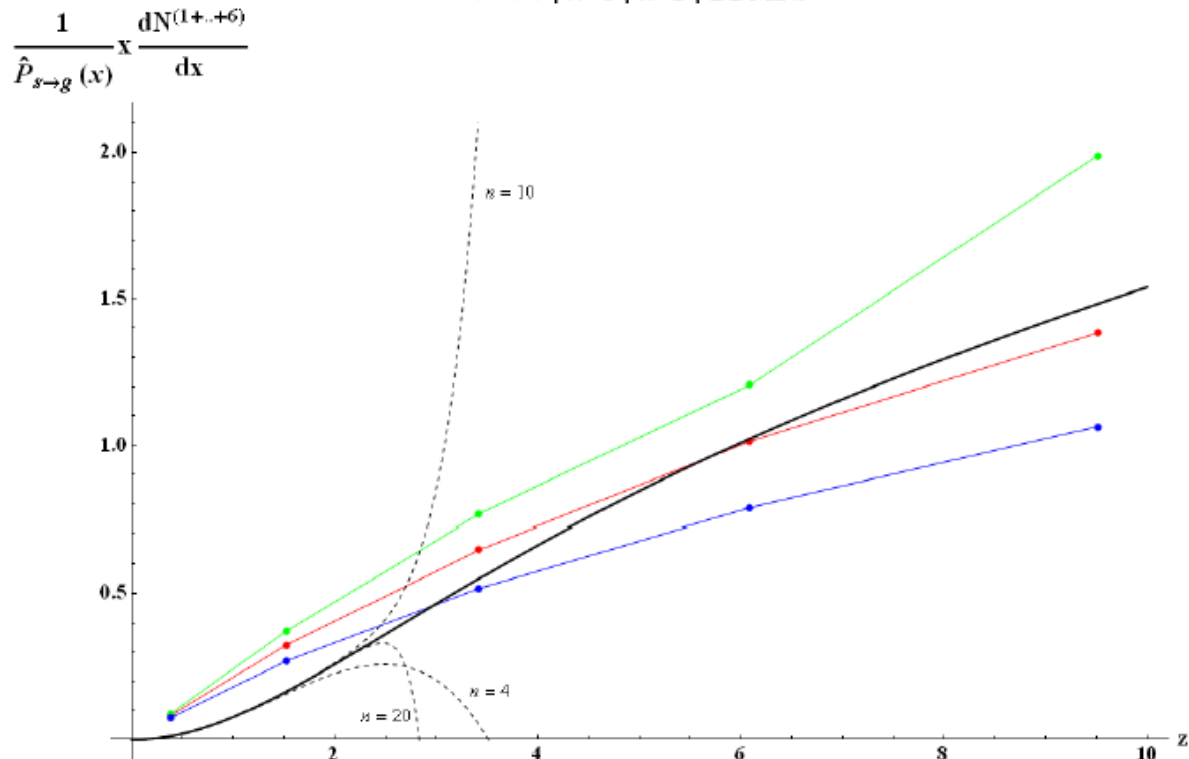
$$\hat{q} \sim \mu^2 / \lambda$$

- BDMPS predicts the spectrum with
$$z \equiv |\omega_0^2| L^2, \quad \omega_0^2 \equiv -i \frac{[(1-x)C_A + x^2 C_s] \hat{q}}{2x(1-x)E}$$

thru

$$E=100, x=0.05, M_q=0.25, \hat{q}=0.25, \mu=\sqrt{\hat{q} \lambda}, m_g=\mu/\sqrt{2}, L=1-5 \text{ (adj.)}$$

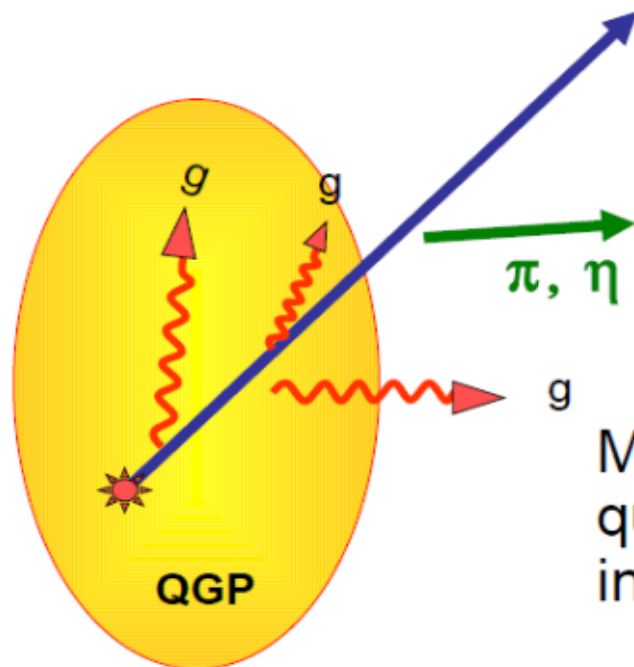
$\lambda=0.5$ | $\lambda=1$ | $\lambda=2$ | BDMPS



Jet Tomography

Jet Tomography: GLV, DGLV, WHDG, CUJET1.0

Gyulassy, Levai, Vitev, Djordjevic, Wicks, Horowitz, Buzzatti



Quark or Glue Jet probes:

$(\eta, p_T, \phi - \phi_{\text{reac}}, M_Q)_{\text{init}}$

Hadron jet fragments:

$(\eta, p_T, \phi - \phi_{\text{reac}})_{\text{final}}$

Measurements of hadronic/leptonic quenching patterns provides information about QGP density

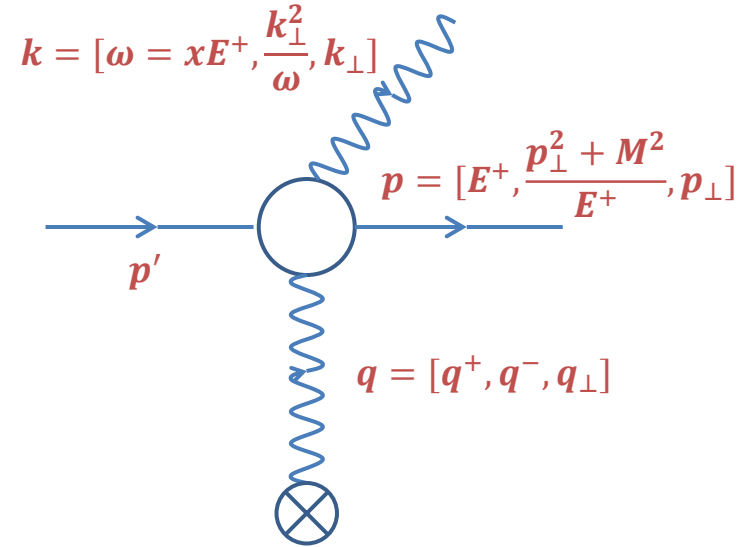
$$\Delta E^{\text{rad}} \propto \alpha_s^3 \int d\tau \tau \rho_{\text{QGP}}(\tau, \vec{r}(\tau)) \text{Log}\left(\frac{E_{\text{Jet}}}{T}\right)$$

$$\Delta E^{\text{elas}} \propto \alpha_s^2 \int d\tau \rho_{\text{QGP}}^{2/3}(\tau, \vec{r}(\tau)) \text{Log}\left(\frac{E_{\text{Jet}}}{T}\right)$$

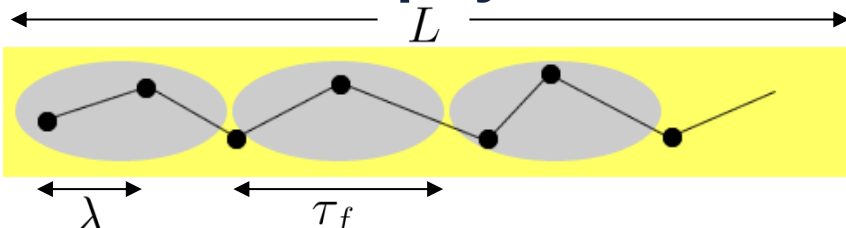
Radiative Energy Loss

Incoherent limit: Gunion-Bertsch

- $$\frac{dN}{dx dk_{\perp}} = \frac{1}{x} \frac{C_A \alpha_s}{\pi^2} \frac{q_{\perp}^2}{k_{\perp}^2 (q_{\perp} - k_{\perp})^2}$$
 - Incoming quark is on-shell and massless
 - The non-abelian nature of QCD alters the spectrum from the QED result
 - Multiple scattering amplitudes are summed incoherently



Formation time physics

- 

$$\tau_f \sim \frac{2\omega}{k_{\perp}^2}$$
 - $\tau_f < \lambda < L$ Incoherent multiple collisions
 - $\lambda < \tau_f < L$ LPM effect (radiation suppressed by multiple scatterings within one coherence length)
 - $\lambda < L < \tau_f$ Factorization limit (acts as one single scatterer)