



The Search for Gluon Saturation in pA Collisions

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Deep into low-x region

The Search
for Gluon
Saturation in
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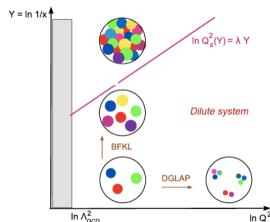
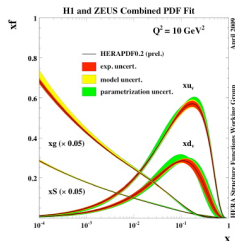
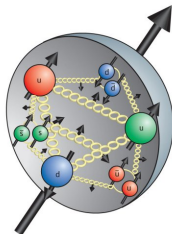
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Introduction

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Summary



- Partons in the low- x region is dominated by **gluons**. See **HERA** data.
- **BFKL equation** \Rightarrow Resummation of the $\alpha_s \ln \frac{1}{x}$.
- When too many gluons squeezed in a confined hadron, gluons start to overlap and recombine \Rightarrow **Non-linear dynamics** \Rightarrow **BK (JIMWLK) equation**
- Use $Q_s(x)$ to separate the **saturated dense** regime from the **dilute** regime.
- Core ingredients: **Multiple interactions** + **Small- x (high energy) evolution**
- Related theory talks: [Monday: Iancu, Lappi, Beuf; Tuesday: Ramnath, Jackson, Mantysaari; Thursday: Kovner, Lublinsky]



Saturation physics (Color Glass Condensate)

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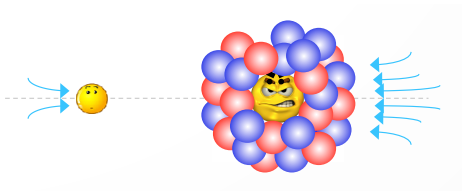
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Saturation physics describes high density parton distributions at high energy limit.

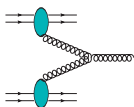


- **Saturation** is an **inevitable** consequence of QCD dynamics at high energy.
- Eminent question: What are the smoking guns? At what energy scale?
- Using AA collisions to search for saturation is too hard due to factorization issues: **Finding a needle in a haystack**
- The search for parton saturation is much easier in dilute-dense scatterings.
 - 1. single hadron (**pA and eA**);
 - 2. dijet (dihadron) correlation (**pA and eA**).



k_t factorization vs Dilute-Dense factorizations

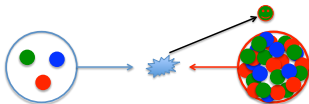
k_t factorization for single inclusive gluon productions in hadron-hadron collision:



$$\frac{d\sigma}{d^2p_T dy} = \frac{2\alpha_s}{C_F p_T^2} \times \int d^2k_{A,T} f_A(x_A, k_{A,T}) f_B(x_B, p_T - k_{A,T}).$$

- Factorization and NLO correction? **Only proved for DY and Higgs !**
- For dijet processes in pp, AA collisions, no k_t factorization [Collins, Qiu, 08], [Rogers, Mulders; 10].

Dilute-Dense factorizations



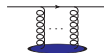
$$\begin{aligned} \text{projectile: } x_1 &\sim \frac{p_\perp}{\sqrt{s}} e^{+y} \sim 1 & \text{valence} \\ \text{target: } x_2 &\sim \frac{p_\perp}{\sqrt{s}} e^{-y} \ll 1 & \text{gluon} \end{aligned}$$

- Protons and virtual photons are **dilute** probes of the **dense** target hadrons.
- For dijet productions in **forward** pA collisions, effective k_t factorization:

$$\frac{d\sigma^{pA \rightarrow ggX}}{d^2P_\perp d^2q_\perp dy_1 dy_2} = x_p g(x_p, \mu) x_A g(x_A, q_\perp) \frac{1}{\pi} \frac{d\hat{\sigma}}{d\hat{t}}.$$

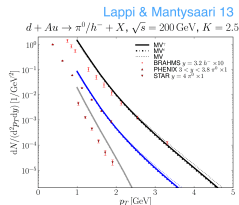
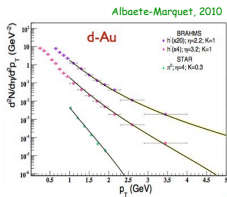
Forward hadron production in pA collisions

[Dumitru, Jalilian-Marian, 02] Inclusive forward hadron production in pA collisions

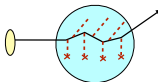


$$p + A \rightarrow H + X.$$

$$\frac{d\sigma_{LO}^{pA \rightarrow hX}}{d^2p_{\perp} dy_h} = \int_{\tau}^1 \frac{dz}{z^2} \left[\sum_f x_p q_f(x_p, \mu) \mathcal{F}(k_{\perp}) D_{h/q}(z, \mu) + x_p g(x_p, \mu) \tilde{\mathcal{F}}(k_{\perp}) D_{h/g}(z, \mu) \right].$$



- **Caveats:** arbitrary choice of the renormalization scale μ and K factor.
- NLO correction? [Dumitru, Hayashigaki, Jalilian-Marian, 06; Altinoluk, Kovner 11] [Chirilli, Xiao and Yuan, 12]



Why do we need NLO calculations?

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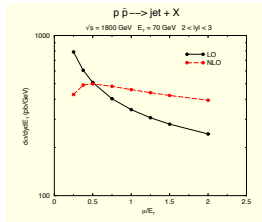
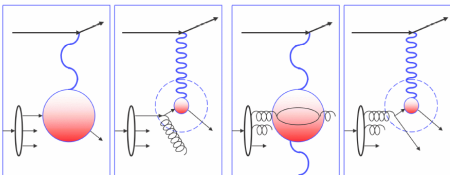
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Summary

Large x : valence quarks

Small x : Gluons, sea quarks



- Due to quantum evolution, PDF and FF changes with scale. This introduces **large theoretical uncertainties** in $xf(x)$ and $D(z)$. Choice of the scale at LO requires information at NLO.
- LO cross section is always a monotonic function of μ , thus it is just **order of magnitude estimate**.
- NLO calculation significantly reduces the scale dependence. More reliable.
- $K = \frac{\sigma_{LO} + \sigma_{NLO}}{\sigma_{LO}}$ is not a good approximation.
- NLO is vital in establishing **the QCD factorization in saturation physics**.



NLO Calculation and Factorization

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- Factorization is about separation of **short distant physics** (perturbatively calculable **hard factor**) from **large distant physics** (Non perturbative).

$$\sigma \sim xf(x) \otimes \mathcal{H} \otimes D_h(z) \otimes \mathcal{F}(k_{\perp})$$

- NLO (1-loop) calculation always contains various kinds of **divergences**.
 - Some divergences can be absorbed into the corresponding **evolution equations**.
 - The rest of divergences should be cancelled.

- Hard factor**

$$\mathcal{H} = \mathcal{H}_{\text{LO}}^{(0)} + \frac{\alpha_s}{2\pi} \mathcal{H}_{\text{NLO}}^{(1)} + \dots$$

should always be finite and free of divergence of any kind.

- NLO vs NLL **Naive α_s expansion sometimes is not sufficient!**

| | LO | NLO | NNLO | ... |
|-----|----|--------------|-------------------------|-----|
| LL | 1 | $\alpha_s L$ | $(\alpha_s L)^2$ | ... |
| NLL | | α_s | $\alpha_s (\alpha_s L)$ | ... |
| ... | | | ... | ... |

- Evolution \rightarrow Resummation of large logs.
LO evolution resums LL; NLO \Rightarrow NLL.



Factorization for single inclusive hadron productions

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Systematic factorization for the $p + A \rightarrow H + X$ process [Chirilli, BX and Yuan, 12]

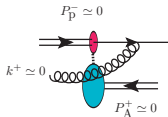
$$\frac{d^3\sigma^{p+A \rightarrow h+X}}{dyd^2p_\perp} = \sum_a \int \frac{dz}{z^2} \frac{dx}{x} \xi_a f_a(x, \mu) D_{h/c}(z, \mu) \int [dx_\perp] S_{a,c}^Y([x_\perp]) \mathcal{H}_{a \rightarrow c}(\alpha_s, \xi, [x_\perp] \mu)$$

Collinear divergence: pdfs

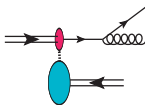
Collinear divergence: fragmentation functs

Rapidity divergence: BK evolution

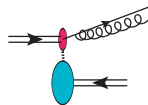
Finite hard factor



Rapidity Divergence



Collinear Divergence (P)



Collinear Divergence (F)

- The NLO correction arises after subtracting off divergence according to convention and \overline{MS} scheme.
- In principle, by invoking the NLO evolutions, we can promote this NLO calculation up to **NLL**.



Numerical implementation of the NLO result

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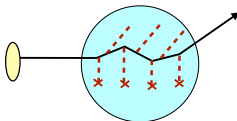
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Single inclusive hadron production up to NLO

$$\begin{aligned} d\sigma &= \int x f_a(x) \otimes D_a(z) \otimes \mathcal{F}_a^{xg}(k_\perp) \otimes \mathcal{H}^{(0)} \\ &+ \frac{\alpha_s}{2\pi} \int x f_a(x) \otimes D_b(z) \otimes \mathcal{F}_{(N)ab}^{xg} \otimes \mathcal{H}_{ab}^{(1)}. \end{aligned}$$



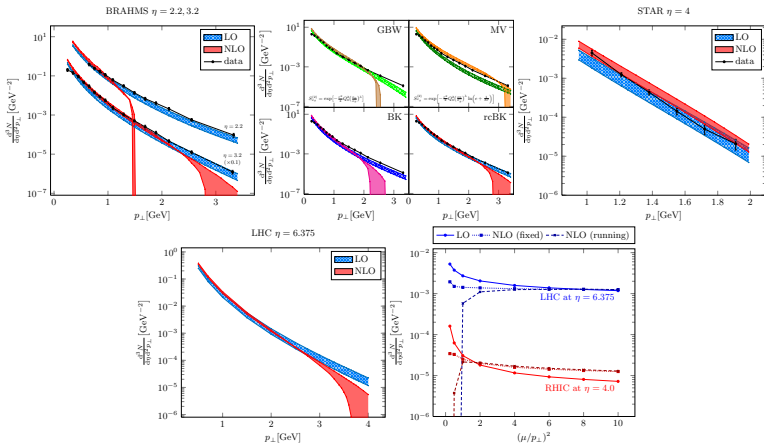
Consistent implementation should include all the NLO α_s corrections.

- **NLO parton distributions.** (MSTW or CTEQ)
- **NLO fragmentation function.** (DSS or others.)
- **Use NLO hard factors.** (11 in total)
- **Use the one-loop approximation for the running coupling**
- **rcBK evolution equation for the dipole gluon distribution** [Balitsky, Chirilli, 08; Kovchegov, Weigert, 07]. Full NLO BK evolution not available.
- **The first most complete NLO results.** [Stasto, Xiao, Zaslavsky, 13]



Numerical implementation of the NLO result

[Albacete, Dumitru, Fujii, Nara, 12] [Stasto, Xiao, Zaslavsky, 13]



- Agree with data for low p_\perp , and reduced scale dependence, no K factor.
- For large p_\perp , NLO correction dominates and becomes **negative**.
- **Additional resummation** ? or **Kinematic constraints** ? [G. Beuf]



Two Different Gluon Distributions

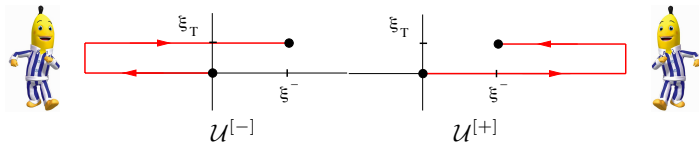
[F.Dominguez, C.Marquet, BX and F. Yuan, 11]

I. **Weizsäcker Williams** gluon distribution: Gauge Invariant definitions

$$xG^{(1)} = 2 \int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - ik_\perp \cdot \xi_\perp} \text{Tr} \langle P | F^{+i}(\xi^-, \xi_\perp) \mathcal{U}^{[+] \dagger} F^{+i}(0) \mathcal{U}^{[+]} | P \rangle.$$

II. **Color Dipole** gluon distributions: Gauge Invariant definitions

$$xG^{(2)} = 2 \int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - ik_\perp \cdot \xi_\perp} \text{Tr} \langle P | F^{+i}(\xi^-, \xi_\perp) \mathcal{U}^{[-] \dagger} F^{+i}(0) \mathcal{U}^{[+]} | P \rangle.$$



- The WW gluon distribution is the **conventional gluon distributions**.
Quadrupole \Rightarrow Direct measurement: DIS dijet, etc.
- The dipole gluon distribution has no such interpretation.
Dipole $\Rightarrow \gamma$ -jet correlation in pA.

Dihadron correlations in dAu collisions (STAR and PHENIX)

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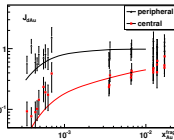
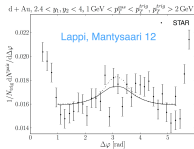
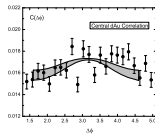
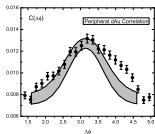
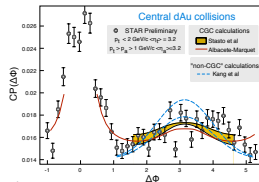
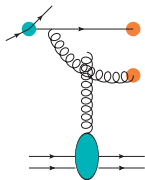
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Predicted by [C. Marquet]. Further calculated [Marquet, Albacete, 10]; [Stasto, BX, Yuan, 11] [Lappi, Mantysaari, 13] Need to have **both gluon distributions**.

$$C(\Delta\phi) = \frac{\int_{|p_{1\perp}|, |p_{2\perp}|} \frac{d\sigma^{pA \rightarrow h_1 h_2}}{dy_1 dy_2 d^2p_{1\perp} d^2p_{2\perp}}}{\int_{|p_{1\perp}|} \frac{d\sigma^{pA \rightarrow h_1}}{dy_1 d^2p_{1\perp}}} \quad J_{dA} = \frac{1}{\langle N_{\text{coll}} \rangle} \frac{\sigma_{dA}^{\text{pair}} / \sigma_{dA}}{\sigma_{pp}^{\text{pair}} / \sigma_{pp}}$$





Dijet processes at one-loop order

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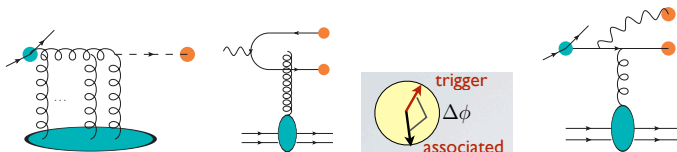
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Using various dijet processes to distinguish these gluon distributions.



Comments:

- What happens at one loop order for back-to-back dijet processes?
($P_{\perp} \equiv \frac{1}{2} |k_{1\perp} - k_{2\perp}| \gg q_{\perp} \equiv |k_{1\perp} + k_{2\perp}|$)
- Can we prove the factorization for dijet productions?
- Small- x $\left[\frac{\alpha_s N_c}{2\pi} \ln \frac{1}{x} \right]^n$ resummation vs Sudakov (CSS) $\left[\frac{\alpha_s C_R}{2\pi} \ln^2 \frac{Q_1^2}{Q_2^2} \right]^n$ resummation. Consistently resum both types of logarithms at the same time?
Unified description of the CSS and small- x evolution?

Sudakov factor for dijet productions in pA collisions and DIS

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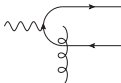
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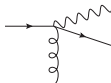
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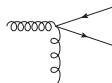
Consider the dijet productions in pA collisions:[A. Mueller, BX, F. Yuan, 13]



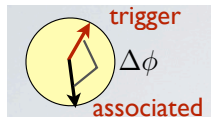
$$C_{q\bar{q}} = \frac{N_c}{2}$$



$$C_{q\gamma} = \frac{N_c}{2} + \frac{C_F}{2}$$



$$C_{g \rightarrow q\bar{q}} = N_c$$



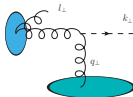
$$\frac{d\sigma}{dy_1 dy_2 dP_{\perp}^2 d^2k_{\perp}} \propto H(P_{\perp}^2) \int d^2x_{\perp} d^2y_{\perp} e^{ik_{\perp} \cdot R_{\perp}} e^{-S_{sud}(P_{\perp}, R_{\perp})} \tilde{W}_{xA}(x_{\perp}, y_{\perp}) .$$

Comments:

- For **back-to-back dijet** processes, $M_J^2 \sim P_{\perp}^2 \gg k_{\perp}^2$

$$\mathcal{S}_{sud} = \frac{\alpha_s C}{2\pi} \ln^2 \frac{P_{\perp}^2 R_{\perp}^2}{c_0^2} \quad \text{with} \quad R_{\perp} \sim \frac{1}{k_{\perp}} .$$

- Probability interpretation of the Sudakov factor (Parton shower)



- Competition between Sudakov and Saturation suppressions.



Conclusion

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- Inclusive forward hadron productions in pA collisions in the small- x saturation formalism at one-loop order. (More work).
- Towards the test of saturation physics beyond LL (More precise).
- Dijet (dihadron) correlation in pA collisions. (More striking)
- One-loop calculation for hard processes in pA collisions, Sudakov factor. (More complete)

