Introduction to the Standard Model

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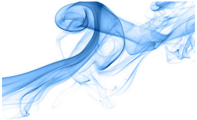
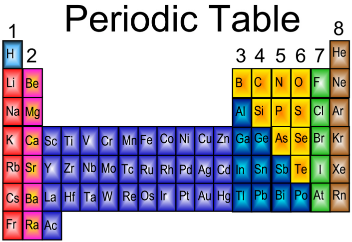
The *Fundamental* Building Blocks?

Earth, Fire, Wind, Water...





Particle Physics

The base hypotheses of *particle physics* (in my opinion):

- the universe is made of a limited number of *fundamental*[1] *particles* responsible for all physical phenomena
- there is a limited number of forces which interact with matter, these forces are manifestations of *one universal force*
- interactions evolve in a *4-dimensional vaccum* which is not only a means to parameterize the interactions, but also as an active participant in the interactions
- all conservation laws reflect an underlying *symmetry of nature*

[1] fundamental (or elementary) means the particle has no internal structure

From Lagrangian to Particles & Interactions

Classical Mechanics

- define the Lagrangian
  $L = \frac{1}{2}mv^2 - V(x)$
- use the Euler-Lagrange equation
  $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$
- get the equation of motion
  $F = ma$
- you get *Newton's Law*

Quantum Field Theory

- define the Lagrangian (density)
  $\mathcal{L}_f = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi$
- use the Euler-Lagrange equation
  $\partial^\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial^\mu\psi)}\right) - \frac{\partial\mathcal{L}}{\partial\psi} = 0$
- get the 'equation of motion'
  $(i\gamma^\mu\partial_\mu - m)\psi = 0$
- you get the *Dirac Equation*

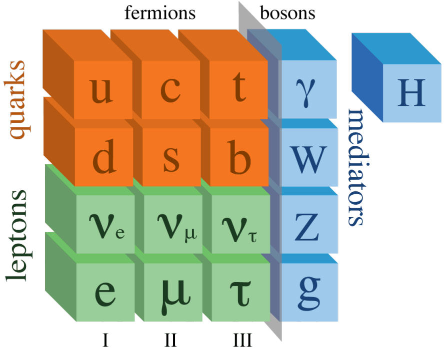
The Standard Model is defined by a Lagrangian:

- the Lagrangian determines the particles and their interactions

The Standard Model Lagrangian

$$
\begin{gathered}
\mathcal{L} = -\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \tfrac{1}{2}ig_s^2(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \\
\partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \tfrac{1}{2}\partial_\mu H \partial_\mu H - \tfrac{1}{2}m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \\
M^2\phi^+\phi^- - \tfrac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \tfrac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h[\tfrac{2M^2}{g^2} + \tfrac{2M}{g}H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)] + \tfrac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - \\
A_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - \tfrac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \tfrac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + \\
g^2c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - \\
2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \tfrac{1}{8}g^2\alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2] - \\
gMW^+_\mu W^-_\mu H - \tfrac{1}{2}g\tfrac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \tfrac{1}{2}ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \tfrac{1}{2}g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - \\
W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)] + \tfrac{1}{2}g\tfrac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\tfrac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_wMA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - \\
ig\tfrac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \tfrac{1}{4}g^2W^+_\mu W^-_\mu[H^2 + (\phi^0)^2 + 2\phi^+\phi^-] - \tfrac{1}{4}g^2\tfrac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2 + \\
(\phi^0)^2 + 2(2s_w^2 - 1)^2\phi^+\phi^-] - \tfrac{1}{2}g^2\tfrac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \tfrac{1}{2}ig^2\tfrac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \tfrac{1}{2}g^2s_wA_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) + \tfrac{1}{2}ig^2s_wA_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\tfrac{s_w}{c_w}(2c_w^2 - 1)Z^0_\mu A_\mu\phi^+\phi^- - g^1 s_w^2 A_\mu A_\mu\phi^+\phi^- - \bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \\
\bar{u}^\lambda_j(\gamma\partial + m_u^\lambda)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + m_d^\lambda)d^\lambda_j + igs_wA_\mu[-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)] + \tfrac{ig}{4c_w}Z^0_\mu[(\bar{\nu}^\lambda\gamma^\mu(1 + \gamma^5)\nu^\lambda) + \\
(\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(\tfrac{4}{3}s_w^2 - 1 - \gamma^5)u^\lambda_j) + (\bar{d}^\lambda_j\gamma^\mu(1 - \tfrac{8}{3}s_w^2 - \gamma^5)d^\lambda_j)] + \tfrac{ig}{2\sqrt{2}}W^+_\mu[(\bar{\nu}^\lambda\gamma^\mu(1 + \gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(1 + \\
\gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \tfrac{ig}{2\sqrt{2}}W^-_\mu[(\bar{e}^\lambda\gamma^\mu(1 + \gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\lambda\kappa}\gamma^\mu(1 + \gamma^5)u^\lambda_j)] + \tfrac{ig}{2\sqrt{2}}\tfrac{m_e^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda(1 - \gamma^5)e^\lambda) + \phi^-(\bar{e}^\lambda(1 + \gamma^5)\nu^\lambda)] - \\
\tfrac{g}{2}\tfrac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) + i\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda)] + \tfrac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar{u}^\lambda_j C_{\lambda\kappa}(1 - \gamma^5)d^\kappa_j) + m_u^\lambda(\bar{u}^\lambda_j C_{\lambda\kappa}(1 + \gamma^5)d^\kappa_j] + \tfrac{ig}{2M\sqrt{2}}\phi^-[m_d^\lambda(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1 + \\
\gamma^5)u^\kappa_j) - m_u^\kappa(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1 - \gamma^5)u^\kappa_j] - \tfrac{g}{2}\tfrac{m_u^\lambda}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \tfrac{g}{2}\tfrac{m_d^\lambda}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2}\tfrac{m_u^\lambda}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \tfrac{ig}{2}\tfrac{m_d^\lambda}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \bar{X}^+(\partial^2 - \\
M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \tfrac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_wW^+_\mu(\partial_\mu\bar{X}^0X^- - \partial_\mu\bar{X}^+X^0) + igs_wW^+_\mu(\partial_\mu\bar{Y}X^- - \\
\partial_\mu\bar{X}^+Y) + igc_wW^-_\mu(\partial_\mu\bar{X}^-X^0 - \partial_\mu\bar{X}^0X^+) + igs_wW^-_\mu(\partial_\mu\bar{X}^-Y - \partial_\mu\bar{Y}X^+) + igc_wZ^0_\mu(\partial_\mu\bar{X}^+X^+ - \partial_\mu\bar{X}^-X^-) + \\
igs_wA_\mu(\partial_\mu\bar{X}^+X^+ - \partial_\mu\bar{X}^-X^-) - \tfrac{1}{2}gM[\bar{X}^+X^+H + \bar{X}^-X^-H + \tfrac{1}{c_w^2}\bar{X}^0X^0H] + \tfrac{1-2c_w^2}{2c_w}igM[\bar{X}^+X^0\phi^+ - \bar{X}^-X^0\phi^-] + \\
\tfrac{1}{2c_w}igM[\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-] + igMs_w[\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-] + \tfrac{1}{2}igM[\bar{X}^+X^+\phi^0 - \bar{X}^-X^-\phi^0]
\end{gathered}
$$

The Standard Model



- The Standard Model is the theory of all 'known' fundamental particles and their interactions.
- It explains almost all known physical phenomena.
- It is a *Quantum Field Theory* (QFT).

What is Force?

Force is transmitted by an exchange of particles.

The Four Forces

Force is transmitted by an exchange of particles.

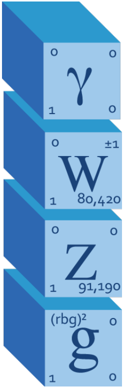
There are 4 known fundamental forces:

Force	Strength	Theory	Mediator
Strong	10	Quantum Chromodynamics (QCD)	gluon
Electromagnetic	10^{-2}	Quantum Electrodynamics (QED)	photon
Weak	10^{-13}	Glashow-Weinberg-Salam (GSW)	W and Z
Gravitation	10^{-42}	General Relativity	graviton

The Standard Model is a quantum field theory:

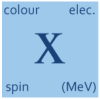
- QED, QCD, and GSW are all quantum field theories
- General relativity is <u>not</u> a quantum field theory

Therefore, gravity is <u>not</u> part of the Standard Model

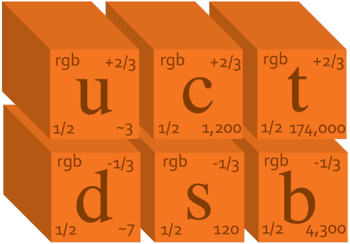


- photon (γ): transmits the *electromagnetic force* between 'electrically' charged $(+, -)$ particles, it has zero mass and carries 1 unit of spin
- W & Z: transmit the *weak force* between 'weakly' charged particles, they have masses around 100 times the mass of the proton and carry 1 unit of spin
- gluon (g): transmits the *strong force* between 'strongly' charged particles, it has zero mass and carries 1 unit of spin

All mediators are spin-1, so they are called *bosons*.

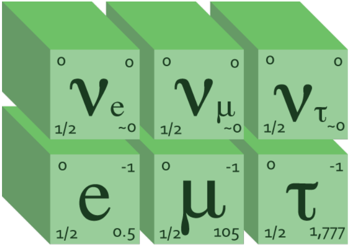


The Matter Particles



Quarks

- electrically, weakly, and strongly charged
- u and d form nuclear matter: proton $= (u, u, d)$

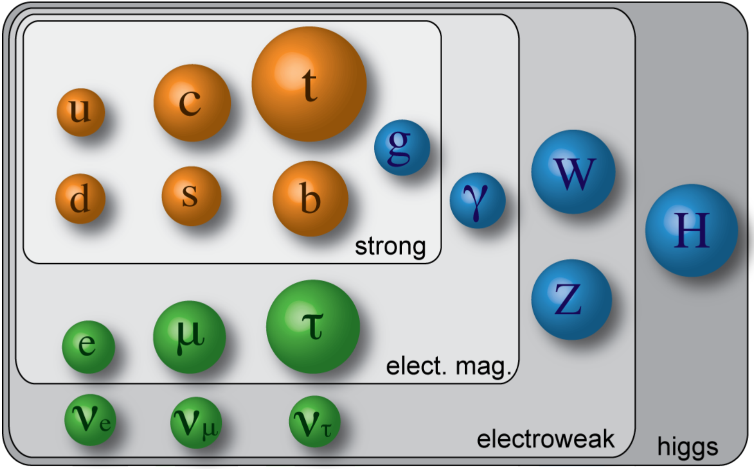


Leptons

- electrically and weakly charged
- e is our well known friend the electron

All matter particles are *fermions* (spin-$\frac{1}{2}$)

Standard Model Summary



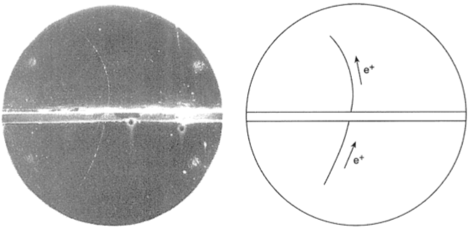
*the photon and gluon do not directly couple to the Higgs...

The Small Matter of Antimatter

All matter particles have a corresponding *antiparticle*:

- an oppositely charged 'twin' with exactly the same mass and quantum properties

Predicted by Dirac in 1928, observed by Anderson in 1932.



- denoted by their the charge for charged leptons(e^+), or an overbar for neutrinos ($\bar{\nu}_e$) and quarks ($\bar{u}$)

Prediction of Antimatter

The Dirac equation, $(i\gamma^\mu \partial_\mu - m)\psi = 0$, is the equation of motion for spin-$\frac{1}{2}$, where ψ is a 4-component *"spinor"*: $\psi = \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$

Finding the simplest stationary solutions leads to[2]:

$$\psi_A(t) = e^{-i\,m\cdot t} \begin{pmatrix} \psi_1(0) \\ \psi_2(0) \end{pmatrix} \quad \text{and} \quad \psi_B(t) = e^{+i\,m\cdot t} \begin{pmatrix} \psi_3(0) \\ \psi_4(0) \end{pmatrix}$$

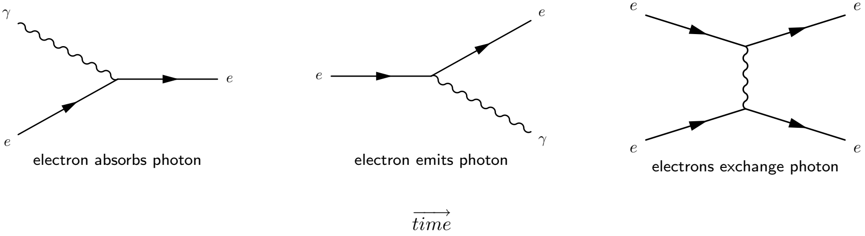
How do you interpret $e^{+i\,m\cdot t}$?

- mathematically, anti-particles are either *negative energy (mass) particles* or particles moving *backward in time*

[2] if interested, see Griffith's Chap. 8 for details

Particle Interactions

Force is transmitted when a fermion emits or absorbs a boson:

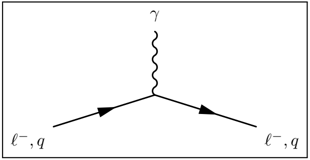


These are called *Feynman diagrams*.

- time flows left to right *(caution! not a universal convention)*
- arrow denotes fermion (forward) or anti-fermion (backward)
- the vertical axis has no physical meaning

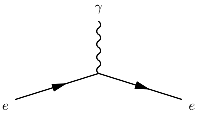
Quantum Electrodynamics (QED)

Electromagnetism mediated by the photon and described by QED.
Every QED interaction is based on this *vertex*:

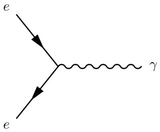


- the solid line is any quark (q) or electrically charged lepton (ℓ^-)
- the squiggly line is a photon (γ)
- the *coupling constant* is $\alpha = \frac{1}{137}$

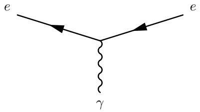
The vertex can be rotated to give other processes:



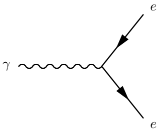
e^- scatter



e^+e^- annihilation

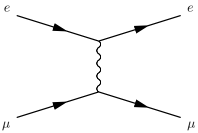


e^+ scatter

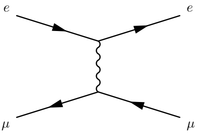


e^+e^- pair production

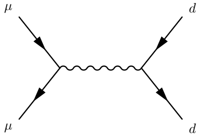
Some Examples of QED Interactions



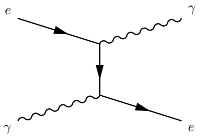
$e^- \mu^- \to e^- \mu^-$



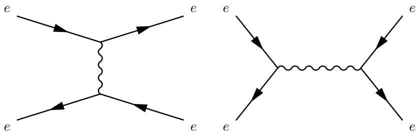
$e^- \mu^+ \to e^- \mu^+$



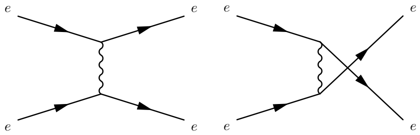
$\mu^+ \mu^- \to d\bar{d}$



$e^- \gamma \to e^- \gamma$



$e^+ e^- \to e^+ e^-$

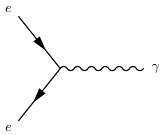


$e^- e^- \to e^- e^-$

Virtual Particles

The particle that transmits the force is called the *propagator*.

Look at the first half of the $e^+e^- \to e^+e^-$ diagram:



conservation of momentum is not possible unless $m_\gamma > 0$!

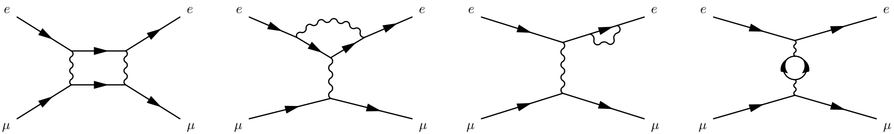
Particles that are *off mass shell* are called *virtual* particles.

- propagators are virtual
- initial and final state particles are *real*

Higher Order Diagrams

The previous examples are the *lowest order (LO)* diagrams for the processes. Every process has *higher order* diagrams.

Next-to-leading order (NLO) diagrams for $e^- \mu^- \to e^- \mu^-$ are:

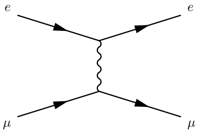


Higher order diagrams are constructed by adding additional *internal lines* without adding *external lines*.

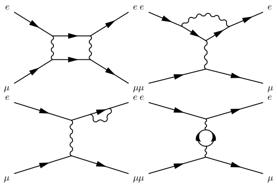
Note that each diagram is constructed of the fundamental QED vertex, each vertex with a 'strength' proportional to α.

Perturbation Theory

To calculate what happens in an interaction like $e^- \mu^- \rightarrow e^- \mu^-$, one must add the diagrams at every order:



$+$



$+ \cdots$

$\mathcal{O}(\alpha^2) \quad + \quad \mathcal{O}(\alpha^4) \quad + \cdots$

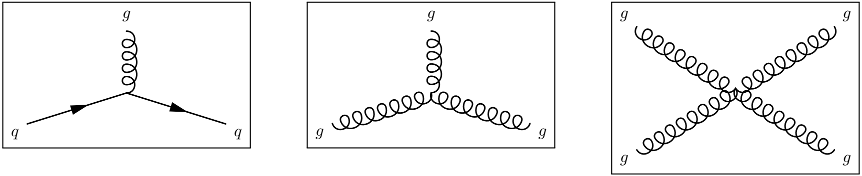
Because $\alpha < 1$, each higher order contributes a smaller amount to the result. Phew!

Quantum Chromodynamics (QCD)

QCD describes the strong interaction mediated by the gluon

- the charge of the strong interaction is *colour*
- colour comes in 3 types: *red, green, blue*
- only quarks and gluons carry colour charge

There are 3 fundamental QCD vertices:



The strong *coupling constant* is $\alpha_s \gtrsim 1$

Freedom and Confinement

Asymptotic Freedom:

- gluon carries colour, the photon does not carry electric charge
- coupling constants: $\alpha_s \gtrsim 1$, while $\alpha < 1$
- thankfully, at small distances, α_s becomes < 1, so perturbation theory can be used for some QCD calculations
- this is called *asymptotic freedom*

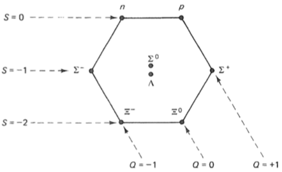
Confinement:

- no naturally occurring particles carry colour
- quarks are *confined* to colourless bound states of 2 or 3 quarks

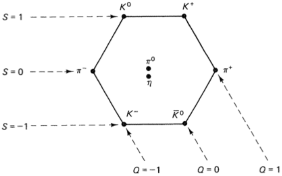
Hadrons and the Strong Interaction

Before the strong interaction was understood:

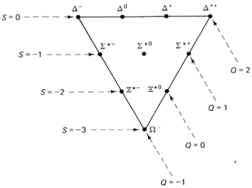
- many 'fundamental' particles were observed ($m \lesssim 2$ GeV)
- the particles were arranged in patterns, Gell-Mann called it the "The Eightfold Way"



baryon octet



meson octet

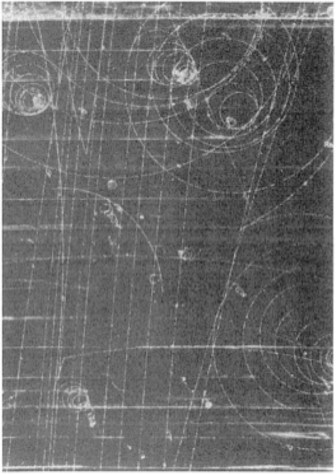


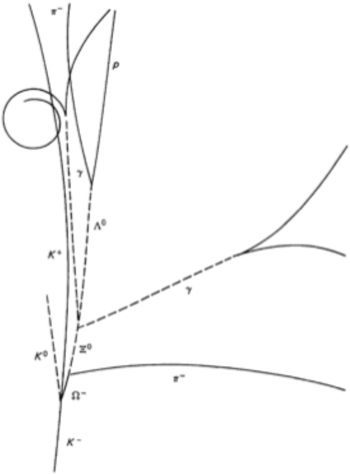
baryon decuplet

The symmetry indicates that hadrons are composite particles.

The Omega Minus

Based on the baryon decuplet, Gell-Mann predicted the Ω^-





What about the Pauli exclusion principle?

Hadron Classification

Hadron: a particle made from a bound state of quarks

- *Meson:* a hadron made of a quark-antiquark pair
- *Baryon:* a hadron made of three quarks or three antiquarks

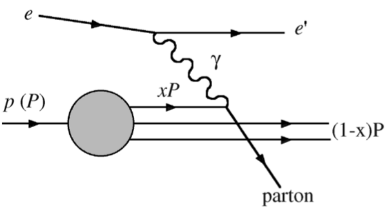
Examples:

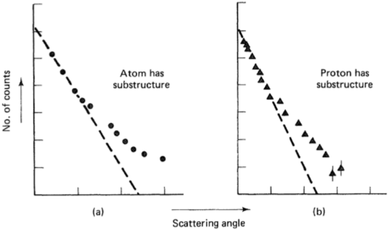
	Quark Content	Spin	Charge	Mass (MeV)
Baryon				
p	uud	$1/2$	$+1$	938
$\bar{p}$	$\bar{u}\bar{u}\bar{d}$	$1/2$	-1	938
n	udd	$1/2$	0	939
Σ^0	uds	$1/2$	0	1192
Δ^{++}	uuu	$3/2$	2	1232
Meson				
$\pi^{\pm}$	$u\bar{d}$, $d\bar{u}$	0	± 1	140
$\rho^{\pm}$	$u\bar{d}$, $d\bar{u}$	1	± 1	775
$K^{\pm}$	$u\bar{s}$, $s\bar{u}$	0	± 1	494
$D^{\pm}$	$c\bar{d}$, $d\bar{c}$	0	± 1	1869
$B^{\pm}$	$u\bar{b}$, $b\bar{u}$	0	± 1	5279
ψ	$c\bar{c}$	1	0	3097
Υ	$b\bar{b}$	1	0	9460

Evidence for Quarks: Lepton-Nucleon Scattering

Similar to Rutherford's discovery of the nucleus

- bombard protons and neutrons with electron 'probes'
- if nucleons are made of *partons* the resulting differential cross section will show the internal structure

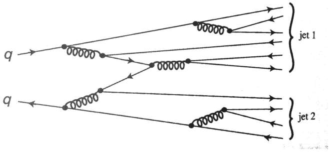


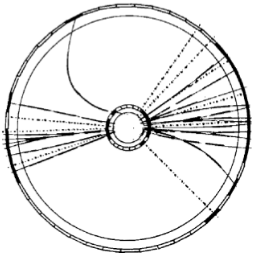


Evidence for Quarks: Jet Production

Jets (a columnated flow of hadrons) are observed in electron-positron collisions.

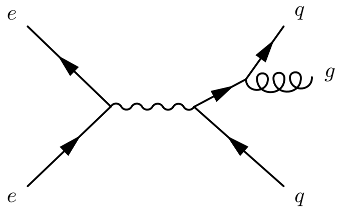
- underlying process $e^+ + e^- \to q + \bar{q}$
- outgoing quarks form hadrons due to confinement, this is called *hadronization*

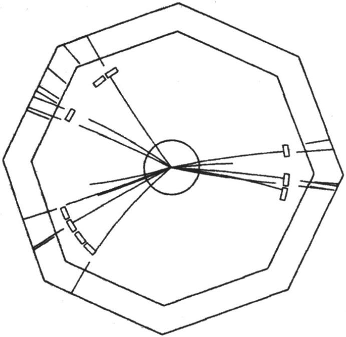




Gluons

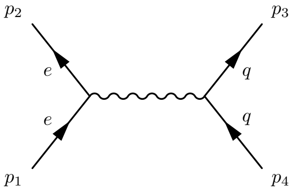
Gluons can also be produced, in e^+e^- collisions:





Colour Charge

Most direct evidence of colour comes from $R \equiv \frac{\sigma(ee \to hadrons)}{\sigma(ee \to \mu\mu)}$.



$\rightarrow$

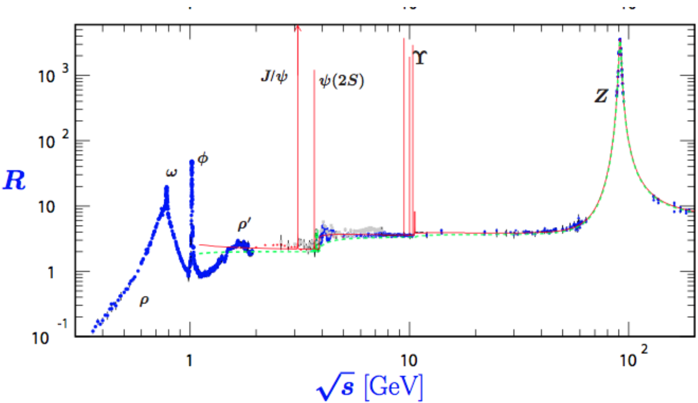
$$\sigma = \frac{\pi}{3}\left(\frac{Q\alpha}{E}\right)^2$$

where Q is the charge in units of e ($\frac{2}{3}$ for u, c, t and $-\frac{1}{3}$ for d, s, b)

- if $E < m_q$, quark production is kinematically forbidden
- σ increases when heavier quarks are energetically allowed

If we assume quarks carry 3 colours: $R(E) = 3\sum Q_i^2$

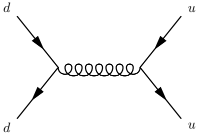
$$R \rightarrow \underbrace{3\left[(\tfrac{2}{3})^2 + 2(-\tfrac{1}{3})^2\right]}_{2 \text{ for } E<m_c} \rightarrow \underbrace{3\left[2(\tfrac{2}{3})^2 + 2(-\tfrac{1}{3})^2\right]}_{3.33 \text{ for } E<m_b} \rightarrow \underbrace{3\left[2(\tfrac{2}{3})^2 + 3(-\tfrac{1}{3})^2\right]}_{3.67 \text{ for } E<m_t}$$



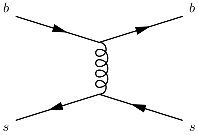
R does not describe hadronic resonances, but:

- the factor of 3 is clearly needed to describe data
- strong evidence of quarks carrying 3 colours

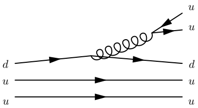
QCD Examples



$d\bar{d} \to u\bar{u}$



$b\bar{s} \to b\bar{s}$

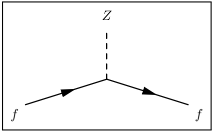


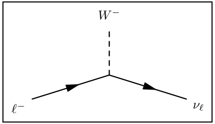
$\Delta^+ \to p + \pi^0$

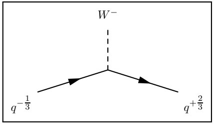
Weak Interactions

Weak interactions are mediated by W and Z

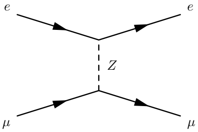
- the weak charge is rather complex...
- all fermions carry weak charge
- W boson couples charged leptons to neutrinos
- W boson can also change quark flavour



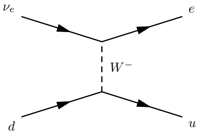




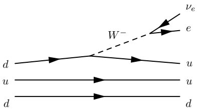
Weak Interaction Examples



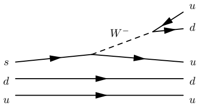
$$e^- + \mu^- \rightarrow e^- + \mu^-$$



$$d + \nu_e \rightarrow u + e^-$$



$$n \rightarrow p + e^- + \bar{\nu_e}$$



$$\Lambda \rightarrow p + \pi^-$$

The Cabibbo-Kobayashi-Maskawa (CKM) Matrix

The CKM matrix gives the relative strength of the coupling for quarks in a W interaction:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.974 & 0.227 & 0.004 \\ 0.227 & 0.973 & 0.042 \\ 0.008 & 0.042 & 0.999 \end{pmatrix}$$

- transitions within a *generation* are most probable

There may be a similar (but much closer to unitary) matrix for the leptons due to *neutrino oscillations*.

Conservation Laws

Every particle decays to lighter particles unless prevented by some conservation law. Particles that don't decay are:

- photon: *conservation of energy/momentum* ($m_\gamma = 0$)
- electron: *conservation of charge* (e is lightest charged particle)
- proton: *conservation of baryon number* (p is lightest baryon)
- neutrino: *conservation of lepton number* (ν's is lightest lepton)
- gluon: *conservation of colour* (g is lightest coloured particle)

Symmetry

Symmetry: an operation that leaves a system *invariant*

Static symmetry:

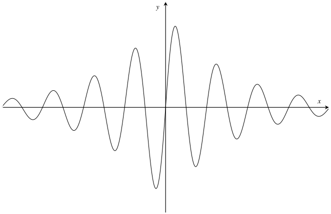
- ie. *shape*

Dynamical symmetry:

- ie. *motion*

We are interested in *dynamical symmetries* manifest in the *equations of motion* (*or Lagrangian*) of a system.

Symmetry allows you to say something about a system even when it's full description is not available.



an odd function (ie. $f(-x) = -f(x)$):

$$[f(-x)]^2 = [f(x)]^2 \quad ; \quad \int_{-3}^{+3} f(x)dx = 0$$

Noether's Theorem

Every *symmetry* in nature reflects a *conservation law*;
every *conservation law* reflects and underlying *symmetry*.

Symmetry		**Conservation Law**
Translation in Time	$\leftrightarrow$	Energy
Translation in Space	$\leftrightarrow$	Momentum
Rotation in Space	$\leftrightarrow$	Angular Momentum
Gauge Transformation	$\leftrightarrow$	Charge

Example: space$\leftrightarrow$momentum with $L = \frac{1}{2}m\dot{x}^2 - V(x)$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0 \quad \rightarrow \quad \frac{dp}{dt} = -\frac{\partial V}{\partial x}$$

Momentum is conserved if V (and thus L) does not depend on x

Group Theory

A *group* is a set of symmetry operations (R) that have:

- *Closure:* The *product* of elements, $R_k = R_i R_j$, is an element in the set.
- *Identity:* There is an *identity* element I such that $IR_i = R_i I = R_i$
- *Inverse:* There is an *inverse*, R_i^{-1}, such that $R_i R_i^{-1} = R_i^{-1} R_i = I$
- *Associativity:* All element are *associative*, $R_i(R_j R_k) = (R_i R_j)R_k$

If all elements *commute*, $R_i R_j = R_j R_i$, the group is called *'Albelian'*
Note: the rotation group is non-Albelian.

Groups can be *continuous* (eg. rotations), or *discrete* (eg. reflections). Continuous groups are called *'Lie'* groups.

Matrix Representation of Groups

Most groups can be represented by *matrices*.
Example: the Lorentz group is a set of 4×4 Λ matrices.

Sets of *unitary* matrices with dimension $n \times n$ are labeled *'U'*
Sets of *real unitary* matrices with dimension $n \times n$ are labeled *'O'*
Sets of *special* matrices with determinant of 1 are labeled *'S'*

Group Name	**Dimension**	**Matrices in Group**
$U(n)$	$n \times n$	unitary $(U^\dagger U = 1)$
$SU(n)$	$n \times n$	unitary, determinant 1
$O(n)$	$n \times n$	orthogonal, $(\tilde{O}O = 1)$
$SO(n)$	$n \times n$	orthogonal, determinant 1

Example: Rotational symmetry in 3 dimensional space is described by the group $SO(3)$.

Angular Momentum

Rotational symmetry $SO(3) \leftrightarrow$ Conservation of angular momentum.

A system's *total angular momentum* is conserved: $\mathbf{J} = \mathbf{L} + \mathbf{S}$

Orbital momentum $(\mathbf{L})$:

- 'motion' of quarks
- $\mathbf{L}^2 \to \ell(\ell+1)$
  $\ell = 0, 1, 2, ...$
- $L_z \to m_\ell$
  $m_\ell = -\ell, -\ell+1, ..., \ell$

Spin $(\mathbf{S})$:

- intrinsic property
- $\mathbf{S}^2 \to s(s+1)$
  $s = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, ...$
- $S_z \to m_s$
  $m_s = -s, -s+1, ..., s$

States of **L**, **S**, and **J** represented by: $|\ell, m_\ell\rangle$, $|s, m_s\rangle$, and $|j, m\rangle$

Addition of Total Angular Momentum

A meson example: A $q\bar{q}$ bound state with $\mathbf{L} = 0$ gives:

$\mathbf{J} = \mathbf{S} = \frac{1}{2} + \frac{1}{2} = 1$, the 'vector' mesons ($\rho$,$K^*$,$\phi$,...)

$\mathbf{J} = \mathbf{S} = \frac{1}{2} - \frac{1}{2} = 0$, the 'pseudoscalar' mesons (π,K,η,...)

A baryon example: A qqq bound state with $\mathbf{L} = 0$ gives:

$\mathbf{J} = \mathbf{S} = (\frac{1}{2} + \frac{1}{2}) + \frac{1}{2} = \frac{3}{2}$, the baryon 'decuplet' (Δ,Σ^*,Ω^-,...)

$\mathbf{J} = \mathbf{S} = (\frac{1}{2} + \frac{1}{2}) - \frac{1}{2} = \frac{1}{2}$, the baryon 'octet' ($p$,$n$,$\Lambda$,...)

$\mathbf{J} = \mathbf{S} = (\frac{1}{2} - \frac{1}{2}) + \frac{1}{2} = \frac{1}{2}$, also the baryon 'octet'

(Note that the $\frac{1}{2}$ state can be achieved in two ways.)

Particles classified by spin (**S**):

Bosons		**Fermions**	
Spin 0	**Spin 1**	**Spin $\frac{1}{2}$**	**Spin $\frac{3}{2}$**
-	Mediators	Quarks/Leptons	-
pseudoscalar mesons	vector mesons	baryon 'octet'	baryon 'decuplet'

Adding Components of Angular Momentum

How do $\mathbf{J}_1 = |j_1, m_1\rangle$ and $\mathbf{J}_2 = |j_2, m_2\rangle$ add to get $\mathbf{J} = |j, m\rangle$?

z component just adds:

- $m = m_1 + m_2$

$magnitude$ component gives:

- $j = |j_1 - j_2|, ..., |j_1 + j_2|$

You could work out all possible $|\, j, m\rangle$ states yourself...

or look up in the *Clebsh-Gordon coefficients*, $C^{j,j_1,j_2}_{m,m_1,m_2}$:

$$|\, j_1, m_1\rangle \,|\, j_2, m_2\rangle = \sum_{j=|j_1-j_2}^{(j_1+j_2)} C^{j,j_1,j_2}_{m,m_1,m_2} \,|\, j, m\rangle$$

$|C^{j,j_1,j_2}_{m,m_1,m_2}|^2$ is the probability of getting the $|\, j, m\rangle$ state from a system composed of $|\, j_1, m_1\rangle$ and $|\, j_2, m_2\rangle$.

The Clebsh-Gordon Table

Problem: Find the *decomposition* of the $\mathbf{L} = 0$ mesons $(\frac{1}{2} \times \frac{1}{2})$

Solution: We found earlier that total angular momentum is 0 or 1. For decomposition into possible $|j, m\rangle$ (or $|J, M\rangle$) states, see the Clebsh-Gordon table (PDG), here is the $1/2 \times 1/2$ entry.

Notation:

		J	J	...
		M	M	...
m_1	m_2			
m_1	m_2	Coefficients		
.	.			
.	.			

1/2×1/2

		1			
		+1			
+1/2	+1/2	1			

		1	0
		0	0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

		1
		−1
−1/2	−1/2	1

$$\left.\begin{aligned} |1,1\rangle &= |\tfrac{1}{2},\tfrac{1}{2}\rangle\,|\tfrac{1}{2},\tfrac{1}{2}\rangle \\ |1,0\rangle &= \tfrac{1}{\sqrt{2}}\left[|\tfrac{1}{2},\tfrac{1}{2}\rangle\,|\tfrac{1}{2},-\tfrac{1}{2}\rangle + |\tfrac{1}{2},-\tfrac{1}{2}\rangle\,|\tfrac{1}{2},\tfrac{1}{2}\rangle\right] \\ |1,-1\rangle &= |\tfrac{1}{2},-\tfrac{1}{2}\rangle\,|\tfrac{1}{2},-\tfrac{1}{2}\rangle \end{aligned}\right\} symmetric(1 \leftrightarrow 2)$$

$$|0,0\rangle = \tfrac{1}{\sqrt{2}}\left[|\tfrac{1}{2},\tfrac{1}{2}\rangle\,|\tfrac{1}{2},-\tfrac{1}{2}\rangle - |\tfrac{1}{2},-\tfrac{1}{2}\rangle\,|\tfrac{1}{2},\tfrac{1}{2}\rangle\right] \Big\} antisymmetric(1 \leftrightarrow 2)$$

Favour Symmetry (Isospin)

Because u and d quarks have almost equal mass, they can be viewed as two states of the same particle *by the strong force.*

Let's call this 'space' *isospin*, $\mathbf{I} = |I, I_3\rangle$, give u and d the values:

$$u = |\tfrac{1}{2}, \tfrac{1}{2}\rangle\,, \qquad d = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle \quad \text{and} \quad \bar{u} = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle\,, \qquad \bar{d} = |\tfrac{1}{2}, \tfrac{1}{2}\rangle$$

Isospin addition is analogous to *spin*, some possible states include:

$$\begin{array}{l} \pi^+(u\bar{d}) = |\tfrac{1}{2}, \tfrac{1}{2}\rangle\,|\tfrac{1}{2}, \tfrac{1}{2}\rangle = |1, 1\rangle \\ \pi^-(d\bar{u}) = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle\,|\tfrac{1}{2}, -\tfrac{1}{2}\rangle = |1, -1\rangle \end{array} \qquad \pi^0 \left\{ \begin{array}{l} (u\bar{u}) = |\tfrac{1}{2}, \tfrac{1}{2}\rangle\,|\tfrac{1}{2}, -\tfrac{1}{2}\rangle = |1, 0\rangle \\ (d\bar{d}) = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle\,|\tfrac{1}{2}, \tfrac{1}{2}\rangle = |1, 0\rangle \end{array} \right.$$

$$p(uud) = |\tfrac{1}{2}, \tfrac{1}{2}\rangle\,, \quad n(udd) = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$$

$$\Delta^{++}(uuu) = |\tfrac{3}{2}, \tfrac{3}{2}\rangle\,,\ \Delta^{+}(uud) = |\tfrac{3}{2}, \tfrac{1}{2}\rangle\,,\ \Delta^{0}(uuu) = |\tfrac{3}{2}, -\tfrac{1}{2}\rangle\,,\ \Delta^{-}(uuu) = |\tfrac{3}{2}, -\tfrac{3}{2}\rangle$$

What is the difference between Λ and Σ^0?

Parity

Parity (P): the inversion of the 3 spatial dimensions.

$$P(x, y, z) \equiv (-x, -y, -z)$$

Does $P(\mathbf{x})$ always give $-\mathbf{x}$? No!

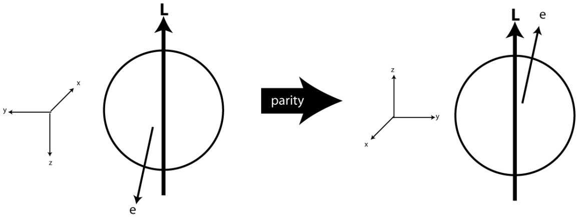
Name	**Behaviour**
Scalar	$P(s) = s$
Pseudoscalar	$P(p) = -p$
Vector (or polar vector)	$P(\mathbf{v}) = -\mathbf{v}$
Pseudovector (or axial vector)	$P(\mathbf{a}) = \mathbf{a}$

Angular momentum (classic and quantum) is one pseudovector.

$$\begin{aligned} \mathbf{L}_{classical} &= \mathbf{r} \times \mathbf{p} \\ P(\mathbf{L}_{classical}) &= P(\mathbf{r} \times \mathbf{p}) = P(\mathbf{r}) \times P(\mathbf{p}) = (-1)^2 \, \mathbf{r} \times \mathbf{p} = \mathbf{L}_{classical} \end{aligned}$$

Parity Violation

Is 'nature' left/right symmetric? Is parity a conserved quantity?
Wu's ^{60}Co experiment (recall: $\mathbf{L}$ is a pseudovector)



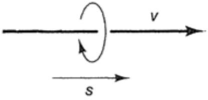
If electron emission is asymmetric, then parity is not conserved.

- Result: electron emission is anti-parallel to nuclear spin
- Conclusion: *weak interactions violate parity*

Helicity

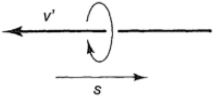
Helicity of a spin-$\frac{1}{2}$ particle is defined as the value of $\frac{m_s}{s}$ when $\hat{z}$ is defined as the direction of motion.

- in other words; is the spin parallel or anti-parallel to the direction of motion



(a) Right-handed

$\left(\frac{m_s}{s} = +1\right)$



(b) Left-handed

$\left(\frac{m_s}{s} = -1\right)$

Helicity is only Lorentz invariant for massless particles:

- for massive particles, you can always go to a frame which reverses the velocity, therefore also reverses the helicity

Neutrino Helicity

We can use the $\pi^- \to \mu^- + \bar{\nu}_\mu$ decay to measure the helicity of the antineutrino:

if μ right-handed then $\bar{\nu}$ must be right-handed

Observation: all μ's in π^- decays are right handed
Similarly for π^+ decays, all 'observed' ν_μ's are left handed

$$\boxed{\text{All } \nu\text{'s are left handed; All } \bar{\nu}\text{'s are right handed}}$$ [3]

Neutrino's violate parity!

[3] strictly true only if $m_\nu = 0$

The Parity Operator

Let P denote the *parity operator*.

- $P^2 = I$ (acting P twice gets you back where you started)
- P is *multiplicative*, so eigenvalues of P are ± 1
- scalars and pseudovectors have $P = +1$
- vectors and pseudoscalars have $P = -1$

Hadrons are eigenstates of parity.

- *quarks are defined as parity $= +1$*
- parity of composite system is *product* of constituent parities
- parity of fermion is *opposite* its corresponding antifermion
- parity of boson is *same as* its corresponding antiboson
- excited states carry and extra $(-1)^\ell$ parity contribution

Charge Conjugation

Charge Conjugation: convert particle antiparticle

$$C\,|p\rangle = |\bar{p}\rangle$$

Most particles are not eigenstates of C:

- only particles who are their own antiparticle are C eigenstates
- for example; π^0, η, ψ

Act twice and you get back to where you started:

$$C^2 = I$$

C is *multiplicative*, so eigenstates are ± 1.
For a $q\bar{q}$ state, $C = (-1)^{\ell+s}$. Photons have $C = -1$.

Determining C & P

Particles are often listed with their J^{PC} value.
Work out the J^{PC} for the following:

Orbital Momentum	Net Spin	J^{PC}	Examples
$\ell = 0$	$s = 0$	0^{-+}	π, K, η
	$s = 1$	1^{--}	ρ, ϕ, ω
$\ell = 1$	$s = 0$	1^{+-}	
	$s = 1$	0^{++}	
	$s = 1$	1^{++}	
	$s = 1$	2^{++}	

CP

Combination of C and P (almost) restores left/right symmetry

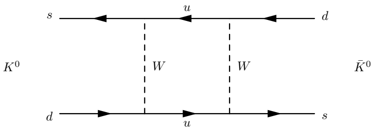
Example: act CP on $\pi^+ \to \mu_L^+ + \nu_\mu^L$ (I've explicitly noted the handedness)

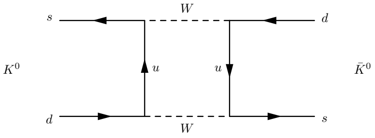
$$\begin{array}{rcl} CP(\pi^+) & \to & CP(\mu_L^+ + \nu_\mu^L) \\ C(\pi^+) & \to & C(\mu_R^+ + \nu_\mu^R) \\ \pi^- & \to & \mu_R^- + \bar{\nu}_\mu^R \end{array}$$

So, acting C and P together (almost) restores left-right symmetry if we consider the 'mirror image' of a right-handed particle to be the left-handed antiparticle.

Neutral Kaons

Neutral Kaons can change from their particle into their antiparticle: $K^0 \rightleftharpoons \bar{K}^0$





So Kaons are actually a linear combination of K^0 and $\bar{K}^0$

Also, K's are pseudoscalars, so K^0 and $\bar{K}^0$ are not CP eigenstates:

$$CP\,|K^0\rangle = -\,|\bar{K}^0\rangle\,, \quad CP\,|\bar{K}^0\rangle = -\,|K^0\rangle$$

CP is also violated in the Kaon system! (not covered here)

Time Reversal & TCP

Time reversal symmetry (T): interactions are equally likely in both time directions

Quantum field theory predicts the *TCP theorem* which states that the combined operation of *time reversal*, *charge conjugation*, and *parity* is an exact symmetry:

- if true, then T must not be conserved in weak interactions
- T violation has never been experimentally observed

Natural Units

$E^2 = p^2c^2 + m^2c^4$ and $\alpha = \frac{e^2}{\hbar c}$ $\rightarrow$ that's a lot of c's and $\hbar$'s...

$E^2 = p^2 + m^2$ and $\alpha = e^2$ $\rightarrow$ that's much prettier!

Particle physicists are a clever (or lazy) bunch,
we just say $c = \hbar = 1$ are *"Natural Units"*.

Implications:

- time is measured in centimeters
- we use *electron volts (eV)* for mass, energy, and momentum

A handy number is: $\hbar c = 197\ MeV \cdot fm$

Special Relativity

Space-time 4-vector x^μ:

$$x^\mu \equiv (x^0, x^1, x^2, x^3) = (t, x, y, z)$$

In another inertial frame,

$$x^{\mu'} = \sum_{\nu=0}^{3} \Lambda^\mu_\nu x^\nu \qquad \Lambda = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

with

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}} \quad \text{and} \quad \beta \equiv v$$

so

$$t' = \gamma t \quad \text{and} \quad L' = \frac{L}{\gamma}$$

Einstein Notation

Quantities that have the same value in *any inertial system*, are called *Lorentz Invariant*, for example:

$$I \equiv t^2 - x^2 - y^2 - z^2 = t'^{\,2} - x'^{\,2} - y'^{\,2} - z'^{\,2}$$

The negative spacial indices are so useful we define the *metric*:

$$\mathbf{g} \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \qquad \text{so} \qquad I = \sum_{\mu=0}^{3}\sum_{\nu=0}^{3} g_{\mu\nu} x^\mu x^\nu$$

To keep track of $-$'s, the *covariant* 4-vector is defined:

$$x_\mu \equiv g_{\mu\nu} x^\nu \qquad (x^\mu \text{ is called } \textit{contravariant})$$

So that using *Einstein summation notation*:

$$I \equiv x_\mu x^\mu$$

Energy-Momentum 4-vector

The *4-momentum* p^μ:

$$p^\mu \equiv (p^0, p^1, p^2, p^3) = (E, p_x, p_y, p_z) = (E, \mathbf{p})$$

$$E = \gamma m \quad \text{and} \quad \mathbf{p} = \gamma m \mathbf{v}$$

Using Einstein summation notation for p^μ:

$$p^\mu p_\mu \equiv E^2 - \mathbf{p}^2 = m^2$$

A particle's mass, m, is a *Lorentz Invariant*.

For massless particles, $E = |\mathbf{p}|$

Note: I will sometimes use the notation $p \cdot p$ to mean $p^\mu p_\mu$.

Conserved vs. Invariant

Invariant: the same in all inertial reference frames. A particle's mass is invariant, so it is sometimes called *invariant mass*:

$$p^\mu p_\mu = m^2$$

Conserved: the same before and after an interaction. In relativistic collisions[4], *4-momentum is conserved* in all particle collisions:

$$p_A^\mu + p_B^\mu = p_C^\mu + p_D^\mu$$

for a collision of $A + B \to C + D$.

[4] In classical collisions, mass, momentum, and energy are all independently conserved quantities - this is not true in relativistic collisions.

Collision Example: π^0 decay

A π^0 at rest decays to $\gamma + \gamma$, what is E_γ?

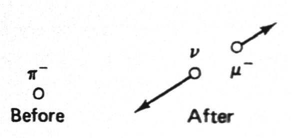
$$
\begin{aligned}
p_\pi^\mu &= p_{\gamma_1}^\mu + p_{\gamma_2}^\mu \quad \text{(conservation of 4-momentum)} \\
p_\pi^2 &= p_{\gamma_1}^2 + p_{\gamma_2}^2 + 2\, p_{\gamma_1} \cdot p_{\gamma_2} \quad \text{(I dropped } ^\mu \text{ index)} \\
m_\pi^2 &= m_{\gamma_1}^2 + m_{\gamma_2}^2 + 2\,(E_{\gamma_1} E_{\gamma_2} - \mathbf{p}_{\gamma_1} \cdot \mathbf{p}_{\gamma_2}) \\
&\rightarrow \; m_\gamma = 0,\; E_\gamma = |\mathbf{p}_\gamma|,\; \mathbf{p}_{\gamma_1} = -\mathbf{p}_{\gamma_2},\; E_{\gamma_1} = E_{\gamma_2} \\
E_\gamma &= \frac{m_{\pi^-}}{2} \simeq 70\ MeV
\end{aligned}
$$

When solving kinematic problems:

- start with conservation of 4-momentum
- use 4-vector notation and exploit $p^\mu p_\mu = m^2$
- think about where 0's will be useful

Collision Example: π^- decay

A π^- at rest decays to $\bar{\nu}_\mu + \mu^-$, what is E_ν and E_{μ^-}?



$$p^\mu_{\pi^-} = p^\mu_\nu + p^\mu_{\mu^-}$$

$$\begin{aligned} p_\nu &= p_{\pi^-} - p_{\mu^-} \\ m_\nu^2 &= m_{\pi^-}^2 + m_{\mu^-}^2 - 2E_\pi E_\mu \\ E_{\mu^-} &= \frac{m_{\pi^-}^2 + m_{\mu^-}^2}{2m_\pi} \end{aligned}$$

$$\begin{aligned} p_{\mu^-} &= p_{\pi^-} - p_\nu \\ m_{\mu^-}^2 &= m_{\pi^-}^2 + 0 - 2E_\pi E_\nu \\ E_\nu &= \frac{m_{\pi^-}^2 - m_{\mu^-}^2}{2m_\pi} \end{aligned}$$

Choose the right frame

Laboratory Frame: one particle is at rest. (also called *fixed target*)

Center-of-Momentum Frame: total 3-vector momentum is zero.

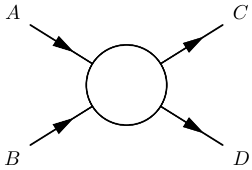
Example: What is the threshold energy for $p + p \rightarrow p + p + p + \bar{p}$ in a fixed target experiment?

Evaluate the invariants: $p^\mu p_\mu$ and $k^\mu k_\mu$ (p is total 4-momentum in lab frame, k is total 4-momentum in cm frame)

$$
\begin{aligned}
p &= (E + m, \mathbf{p}) \quad \text{(before collision)} \\
k &= (4m, 0) \quad \text{(after collision)} \\
p^\mu p_\mu &= k^\mu k_\mu \rightarrow (E + m)^2 - \mathbf{p}^2 = (4m)^2 \\
E &= 7m \quad \text{(after using } \mathbf{p}^2 = E^2 - m^2\text{)}
\end{aligned}
$$

Mandelstam Variables

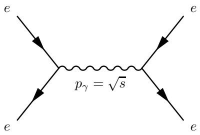
Mandelstam variables are Lorentz invariants in $2 \to 2$ interactions:



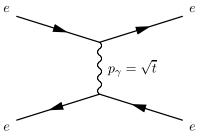
$$
\begin{aligned}
s &\equiv (p_A + p_B)^2 \\
t &\equiv (p_A - p_C)^2 \\
u &\equiv (p_A - p_D)^2
\end{aligned}
$$

Examples:

$e^+e^- \to e^+e^-$

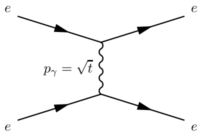


s-channel

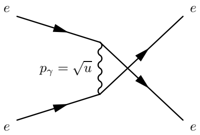


t-channel

$e^-e^- \to e^-e^-$



t-channel



u-channel

Collider vs. Fixed Target

Example: Find the CM energy ($\sqrt{s}$) for a fixed target vs colliding proton beam with energy is $E_b = 3.5\ TeV$.

fixed target:

$$\begin{aligned} s &= (p_b + p_t)^2 \\ &= p_b^2 + p_t^2 + 2\,p_b \cdot p_t \\ &= 2m_p^2 + 2\,E m_p \\ \sqrt{s} &= \sqrt{2m_p(E + m_p)} \\ &\simeq 83\ GeV \end{aligned}$$

colliding beam:

$$\begin{aligned} s &= (p_{b1} + p_{b2})^2 \\ &= p_{b1}^2 + p_{b2}^2 + 2\,p_{b1} \cdot p_{b2} \\ &= 2m_p^2 + 2\,(E_b^2 + |\mathbf{p}_b|^2) \\ \sqrt{s} &= 2E \\ &= 7000\ GeV \end{aligned}$$