Likelihood Analysis of Higgs Anomalous Couplings

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Motivation

- An exploration study for the possibilities of electron-positron colliders.
- Study the differences between what we expect to see from the Standard Model and Beyond Standard Model of Higgs couplings.
- Interest in such studies comes with the concept design of the LHeC and ILC which allow for more than just p-p collisions but also e-p and $e^+ - e^-$ respectively.

Background

- Higgs mass(126GeV), spin 1 excluded from y-y channel production .

Decay rates and branching ratios (so far) compatible with SM.

Spin-0 tentatively confirmed, however spin-2 still a possibility.

- Difficulty investigating with LHC

Higgs production dependant on many BSM couplings, so hard to identify the responsible vertex.

Symmetric initial states, backscatter and forward scatter cannot be discriminated.

- LHeC

Produces electron beams for the LHC tunnel.

LHeC allows for more than just p-p collisions. e-p collisions now accessible.

e-p collisions only one Feynman diagram involving HWW vertex.

COM energy 1-1.5TeV

- ILC

Collides electrons and positrons.

COM energy 500-1000GeV

Produce copious Higgs events

Clean Higgs Production

Lagrangian

- In the SM the couplings of the Higgs to massive electroweak gauge bosons are: $g_{HVV} \propto g M_V V_\mu V^\mu$
- However the most general form of the vertex can be parameterised in the form: $i\Gamma^{\mu\nu}(p,q)\epsilon_\mu(p)\epsilon^*_\nu(q)$
- So the most general form of the vertex will be:

$$\Gamma_{\mu\nu}(p,q) = \Gamma^{\mathrm{SM}}_{\mu\nu} + \Gamma^{\mathrm{BSM}}_{\mu\nu}(p,q)$$

$$\begin{aligned}\Gamma^{\mathrm{SM}}_{\mu\nu} &= -g M_V\, g_{\mu\nu} \\ \Gamma^{\mathrm{BSM}}_{\mu\nu}(p,q) &= \frac{g}{M_V}\left[\lambda\left(p\cdot q\, g_{\mu\nu} - p_\nu q_\mu\right) + \lambda'\, \epsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma\right]\end{aligned}$$

Lagrangian

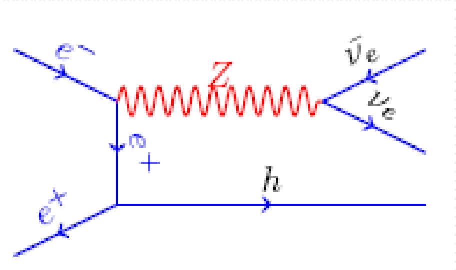
- λ and λ' being the effective coupling strengths for CP-even and CP-odd operators respectively.
- Such factors may also complex valued.
- λ and λ' shall be referred to as lambda1 and lambda2 respectively.

Method

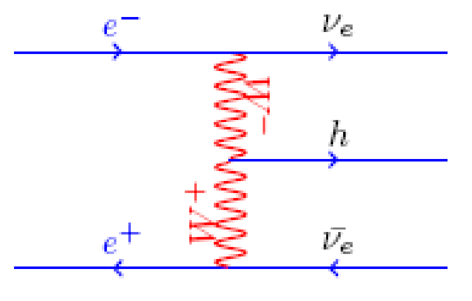
- Generate model with FeynRules
- Code for model obtained from Kirtimaan Mohan
- Vary parameters to generate different models
- Ad hoc choice of lambda1/2=+-1
- Generate 1M events in MadGraph (Beam energies between 125 and 150 GeV)(Higgs mass=126 GeV)(No Parton Distribution Function) (Next to Leading Order)
- Extract data, plot and inspect for different outcomes.

Process

- The process $e^+ + e^- > \nu_e + \tilde{\nu}_e + h$ consists of two channels.
- S-Channel:



- T-Channel:

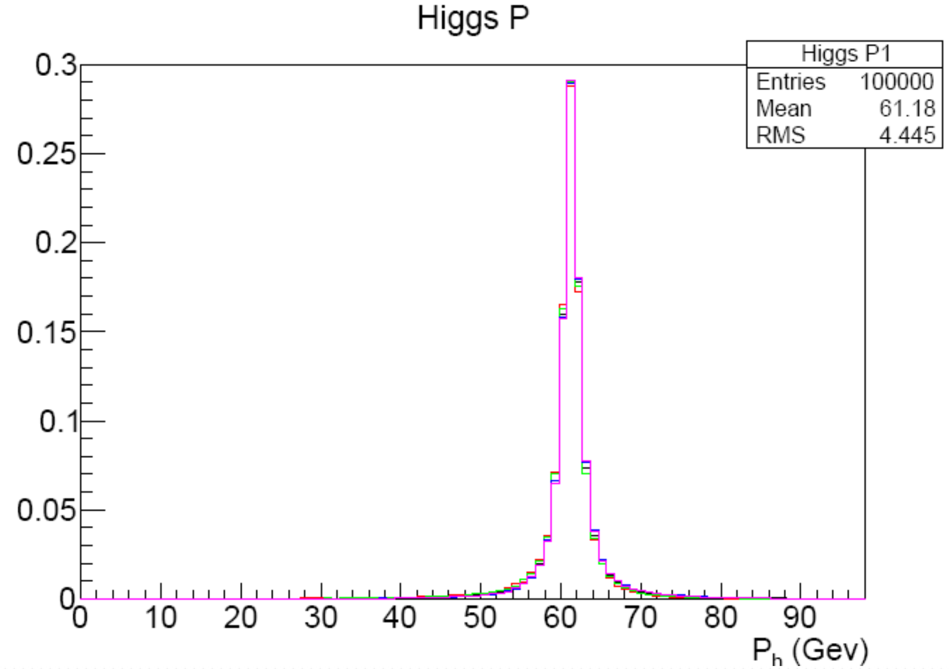


Insensitivity of S-Channel

- The S-Channel is unaffected by the differing BSM parameters.
- For example: Histogram of the Higgs momentum for colliding electron/positron beams each of 125 GeV

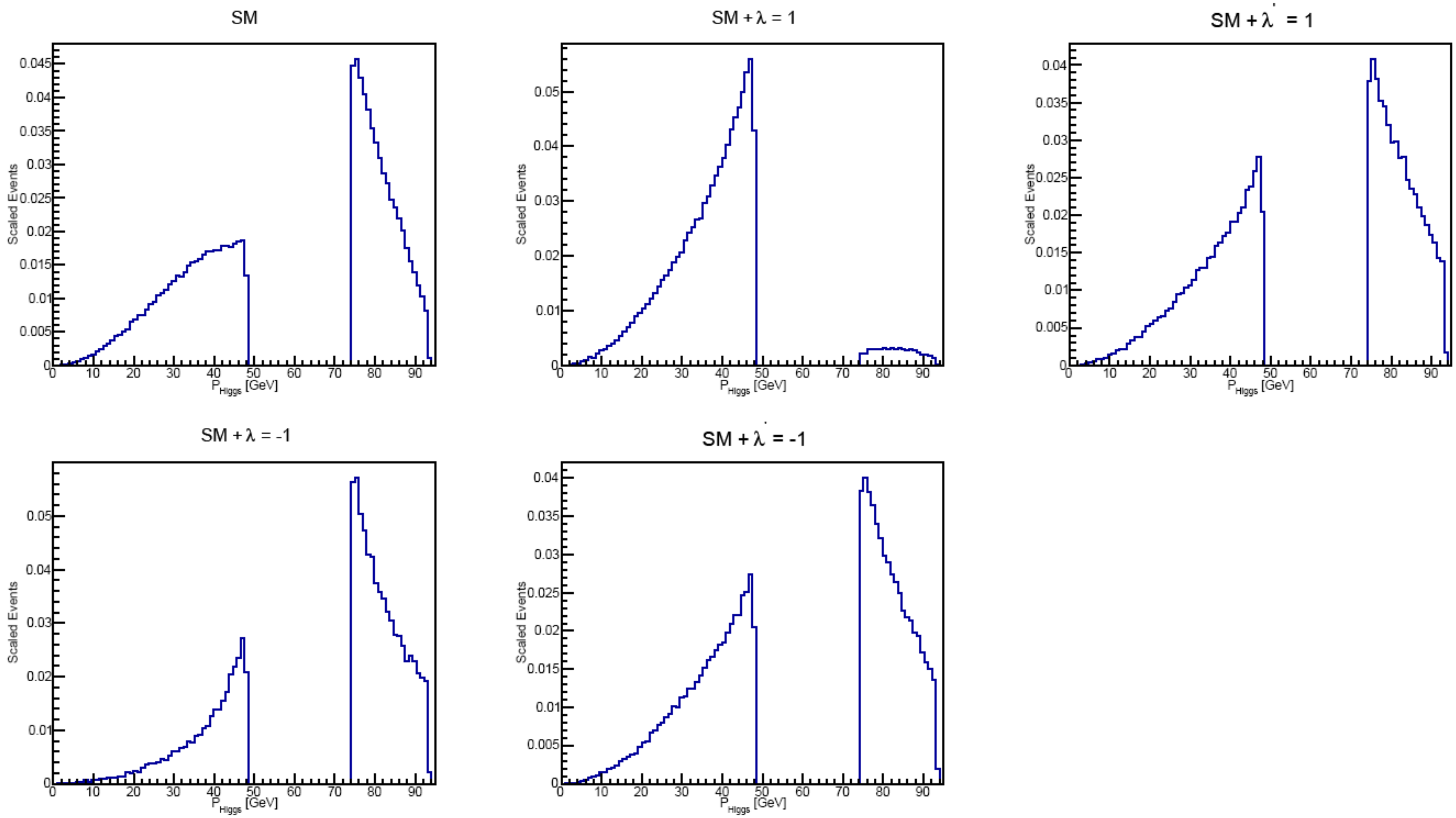
The SM is a black line

The BSM models are arbitrarily coloured.

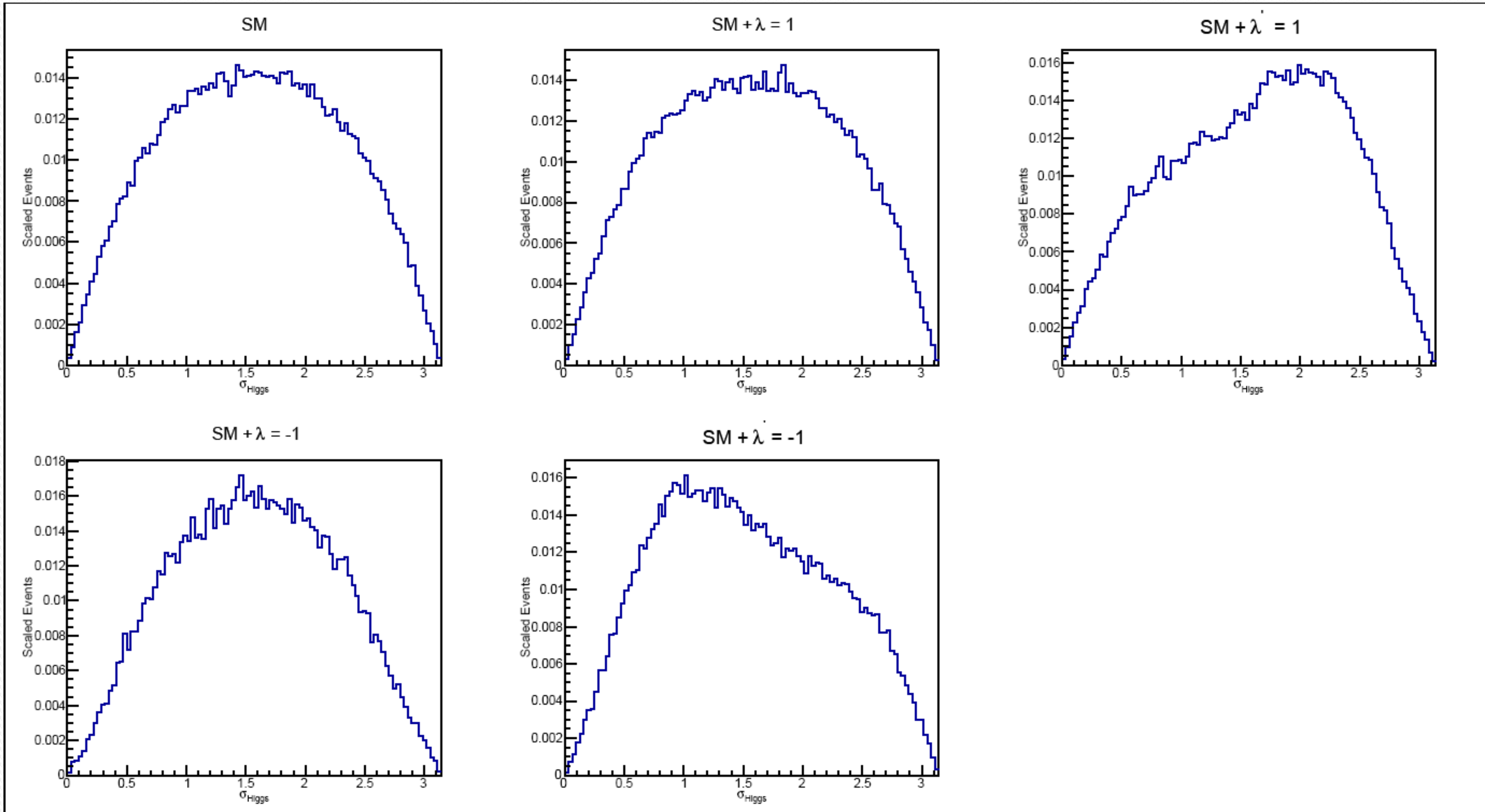


Comparative Graphs

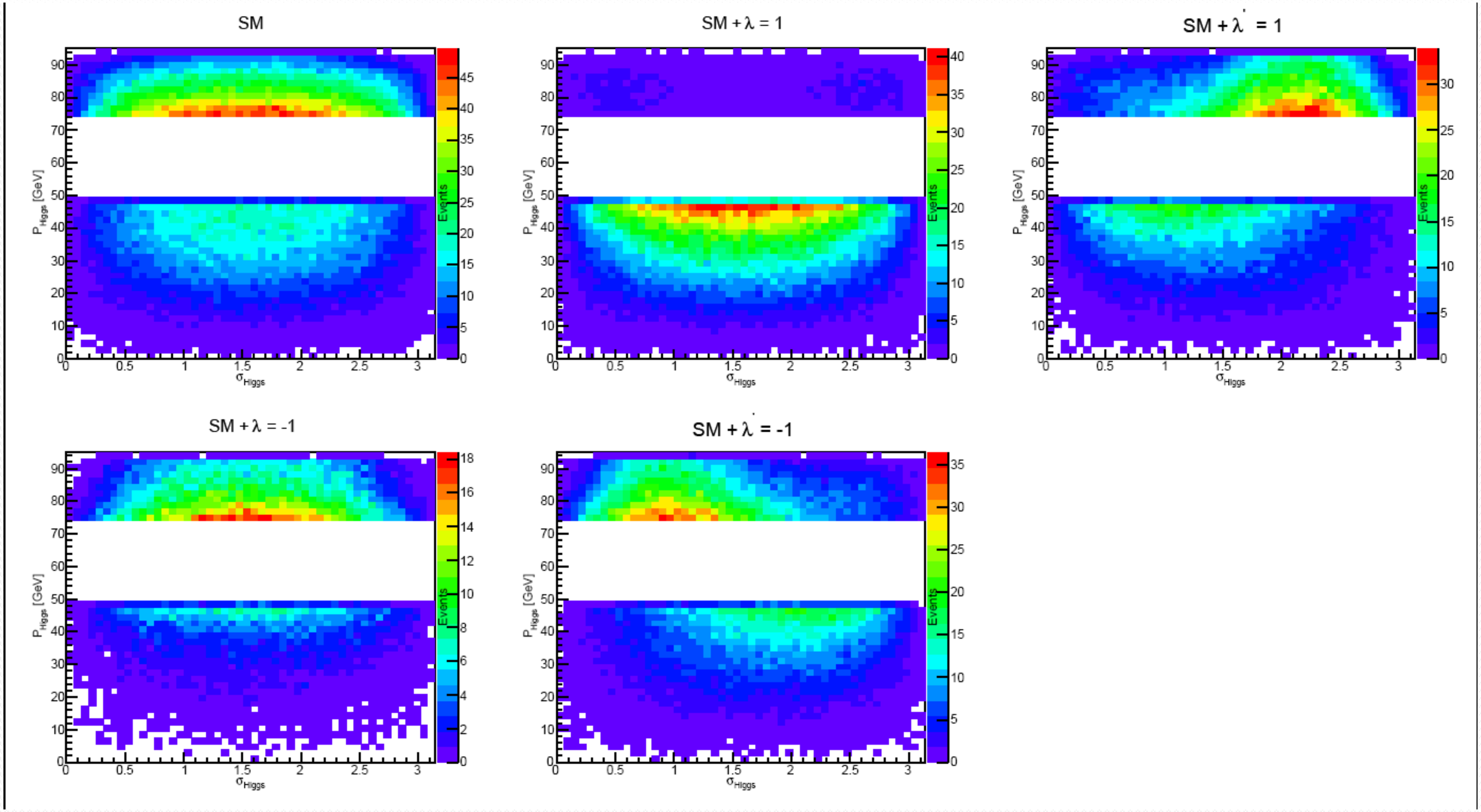
Momentum – 125GeV



Theta – 125GeV



Momentum vs Theta– 125GeV



Likelihoods and Test Statistics

- Likelihoods: the likelihood of a set of parameter values given specified outcomes is equal to the probability of those observed outcomes given those parameter values.

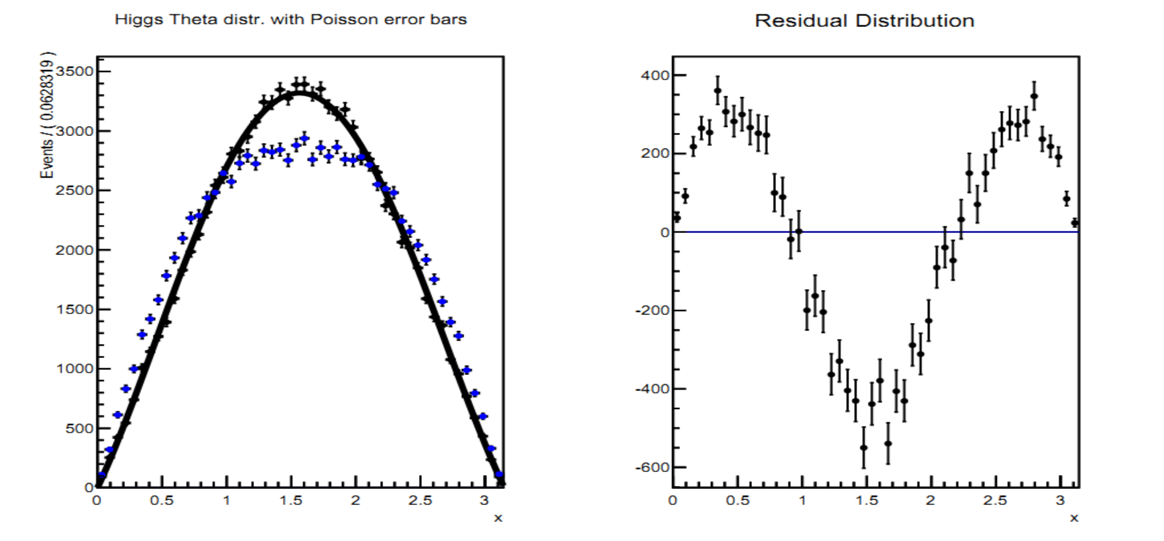
That is to say, the likelihood of which die (weighted or not) was used based on a list of the die roll outcomes.

- Test Statistics: Used for a Significance testing of the likelihood result.

A measure of how confident the likelihood is. 10000 die rolls says a lot more about which die was used than just one toss. Calculated using randomly generated results assuming one of the models to be true and forming log likelihoods and ratios.

Likelihood Tests

- Using the SM cross sections and three different luminosities, 1,5 and 10fb^-1, the number of 'toys' or random events fitting to the models is generated.
- Generating these toys again and again we can compare to the different models getting likelihood ratios.
- Test statistics can be made which use the likelihood ratios to determine the significance of the result.

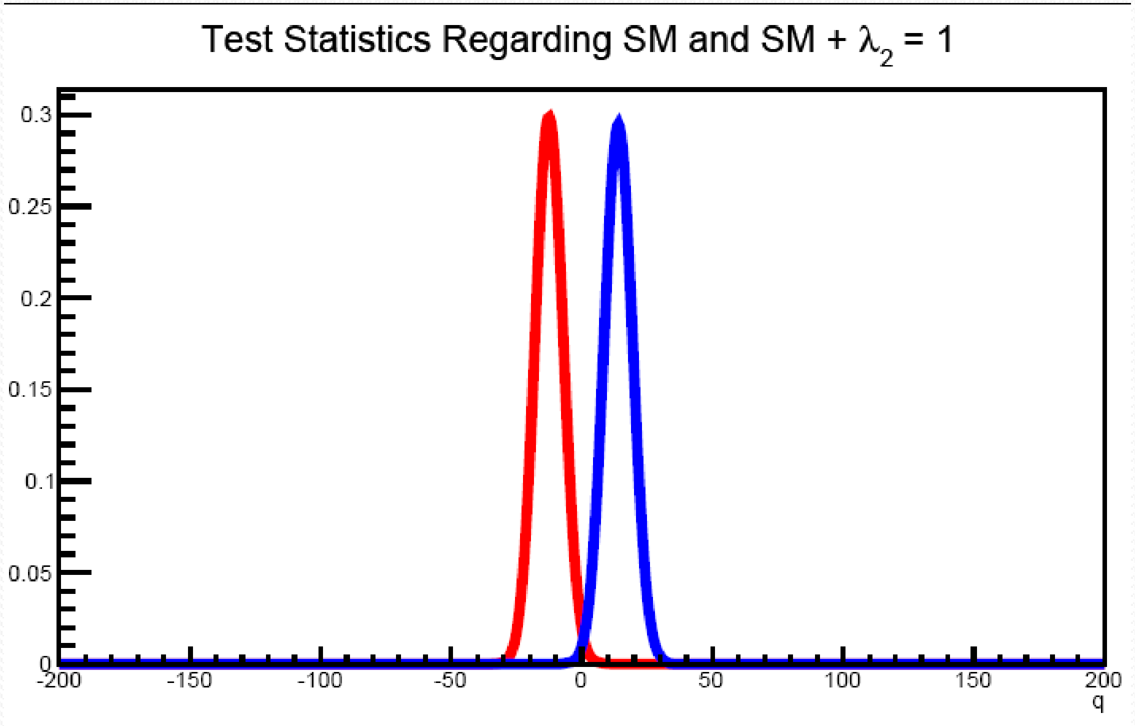


Test Statistic Generation

- The test statistic is the log likelihood ratio:

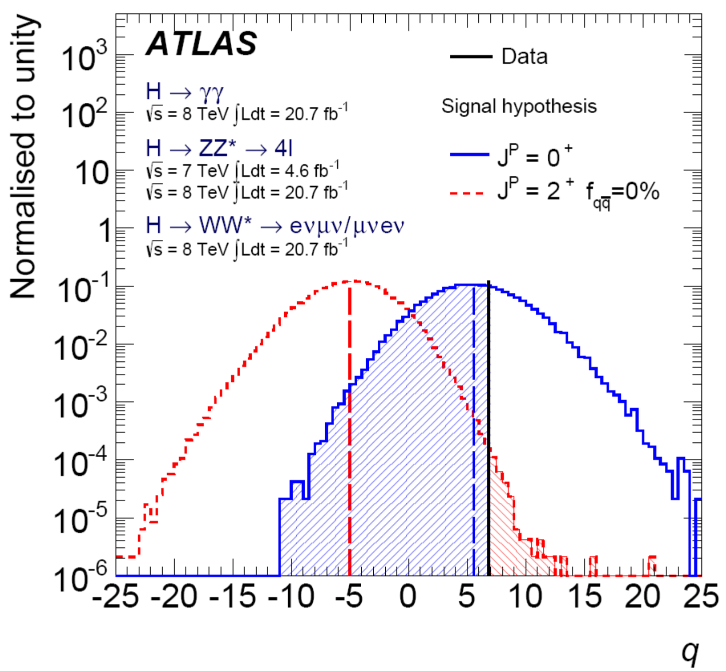
q1 = -ln[L(SM)w.r.t BSM / L(SM)w.r.t SM] IN BLUE

q2= -ln[L(BSM)w.r.t BSM / L(BSM)w.r.t SM] IN RED



Test statistics of competing models

- Taking measurements of the overlap we can determine the significance to which we can say one model better explains the data than another.
- The following represents only 100 000 test statistics and subsequent analysis to determine the significance at which one can discriminate between these different models.



Toy Number Determination

ENERGY	SM MAGRAPH Cross-sec.(pb)	Fraction Surviving Momentum Cut	Effective Cross-sec.(fb)	Toy No(1fb^-1)	Toy No(5fb^-1)	Toy No(10fb^-1)
125	0.02252	0.18659	12.6060204	12.6060204	63.030102	126.060204
130	0.02324	0.19521	13.6100412	13.6100412	68.050206	136.100412
135	0.02399	0.233351	16.79427147	16.79427147	83.97135735	167.9427147
140	0.02488	0.281477	21.00944328	21.00944328	105.0472164	210.0944328
145	0.02595	0.326968	25.4544588	25.4544588	127.272294	254.544588
150	0.02722	0.383965	31.3545819	31.3545819	156.7729095	313.545819

- The MadGraph cross-section data was for the process of $e^+ + e^- > \nu_e + \tilde{\nu}_e + h$. The true cross-section for the reaction is that given by MadGraph times by three to account for the tau and muon neutrinos.
  That times the fraction of events surviving the momentum range cut result in the effective cross-section.

- What follows is the results with the following naming convention used:

“Model 1”: Standard Model

“Model 2”: Standard Model + lambda 1=1

“Model 3”: Standard Model + lambda 2=1

“Model 4”: Standard Model + lambda 1=-1

“Model 5”: Standard Model + lambda 2=-1

1fb^-1

Energy	Process	1DTheta		1DMomentum		2D	
		P-val	Significance	P-val	Significance	P-val	Significance
125	12	0.47052	**0.07396**				
	13	0.33578	**0.42401**	0.28533	**0.56708**	0.06510	**1.51331**
	14	0.33056	**0.43837**	0.05593	**1.58989**	0.03945	**1.75710**
	15	0.36578	**0.34305**	0.28323	**0.57327**	0.06628	**1.50412**
130	12	0.46278	**0.09343**	0.00028	**3.45028**	0.00009	**3.74555**
	13	0.36247	**0.35186**	0.25409	**0.66167**	0.07121	**1.46684**
	14	0.32538	**0.45271**	0.01517	**2.16562**	0.01304	**2.22502**
	15	0.40457	**0.24154**	0.24721	**0.68330**	0.06695	**1.49890**
135	12	0.46295	**0.09300**	0.00625	**2.49771**	0.00592	**2.51682**
	13	0.32487	**0.45412**	0.21482	**0.78981**	0.05828	**1.56938**
	14	0.33236	**0.43341**	0.00765	**2.4252**	0.00401	**2.65123**
	15	0.30243	**0.51742**	0.20118	**0.83741**	0.05365	**1.61045**
140	12	0.48166	**0.04599**	0.04761	**1.66848**	0.04562	**1.68889**
	13	0.21728	**0.78141**	0.16196	**0.98643**	0.04501	**1.69529**
	14	0.32355	**0.45780**	0.00521	**2.56157**	0.00391	**2.65974**
	15	0.21542	**0.78776**	0.15979	**0.99532**	0.04202	**1.72771**
145	12	0.44583	**0.13620**	0.20626	**0.81947**	0.17324	**0.94144**
	13	0.15910	**0.99816**	0.14507	**1.05781**	0.03872	**1.76574**
	14	0.33689	**0.42097**	0.0146	**2.18078**	0.00519	**2.56291**
	15	0.15965	**0.99590**	0.14428	**1.06129**	0.04141	**1.73455**
150	12	0.44538	**0.13734**	0.26202	**0.63713**	0.21717	**0.78179**
	13	0.12141	**1.16797**	0.12885	**1.13184**	0.04149	**1.73365**
	14	0.30954	**0.49716**	0.05535	**1.59505**	0.03025	**1.87713**
	15	0.09921	**1.28607**	0.1311	**1.12121**	0.03277	**1.84156**

5fb^-1

Energy	Process	1DTheta		1DMomentum		2D	
		P-val	Significance	P-val	Significance	P-val	Significance
125	12	0.39710	**0.26086**				
	13	0.15063	**1.03374**	0.09010	**1.34014**	0.0003	**3.43161**
	14	0.11763	**1.18692**	0.00039	**3.35980**	0.00007	**3.80817**
	15	0.17285	**0.94296**	0.08993	**1.34119**	0.00031	**3.42271**
130	12	0.38069	**0.30367**				
	13	0.21213	**0.79905**	0.06411	**1.52116**	0.00044	**3.32632**
	14	0.11218	**1.21502**				
	15	0.24600	**0.68713**	0.05591	**1.59007**	0.00038	**3.36697**
135	12	0.38329	**0.29685**				
	13	0.10743	**1.24031**	0.03207	**1.85121**	0.00018	**3.57517**
	14	0.13757	**1.0913**				
	15	0.11875	**1.18126**	0.02727	**1.92252**	0.00013	**3.66226**
140	12	0.37716	**0.31295**	0.00010	**3.71902**	0.00008	**3.77501**
	13	0.03339	**1.83315**	0.01037	**2.31268**	0.00009	**3.74555**
	14	0.13927	**1.08360**				
	15	0.03876	**1.76526**	0.01012	**2.32187**	0.00002	**4.10748**
145	12	0.33183	**0.43487**	0.02766	**1.91635**	0.01405	**2.19589**
	13	0.00767	**2.42425**	0.00807	**2.40573**	0.00005	**3.89059**
	14	0.16854	**0.95995**				
	15	0.00886	**2.37142**	0.00630	**2.49488**	0.00004	**3.95984**
150	12	0.32894	**0.44285**	0.06732	**1.49605**	0.03178	**1.85525**
	13	0.00258	**2.79750**	0.00491	**2.58210**	0.00003	**4.01281**
	14	0.1388	**1.08573**	0.00006	**3.84613**		
	15	0.00203	**2.87424**	0.00401	**2.65123**		

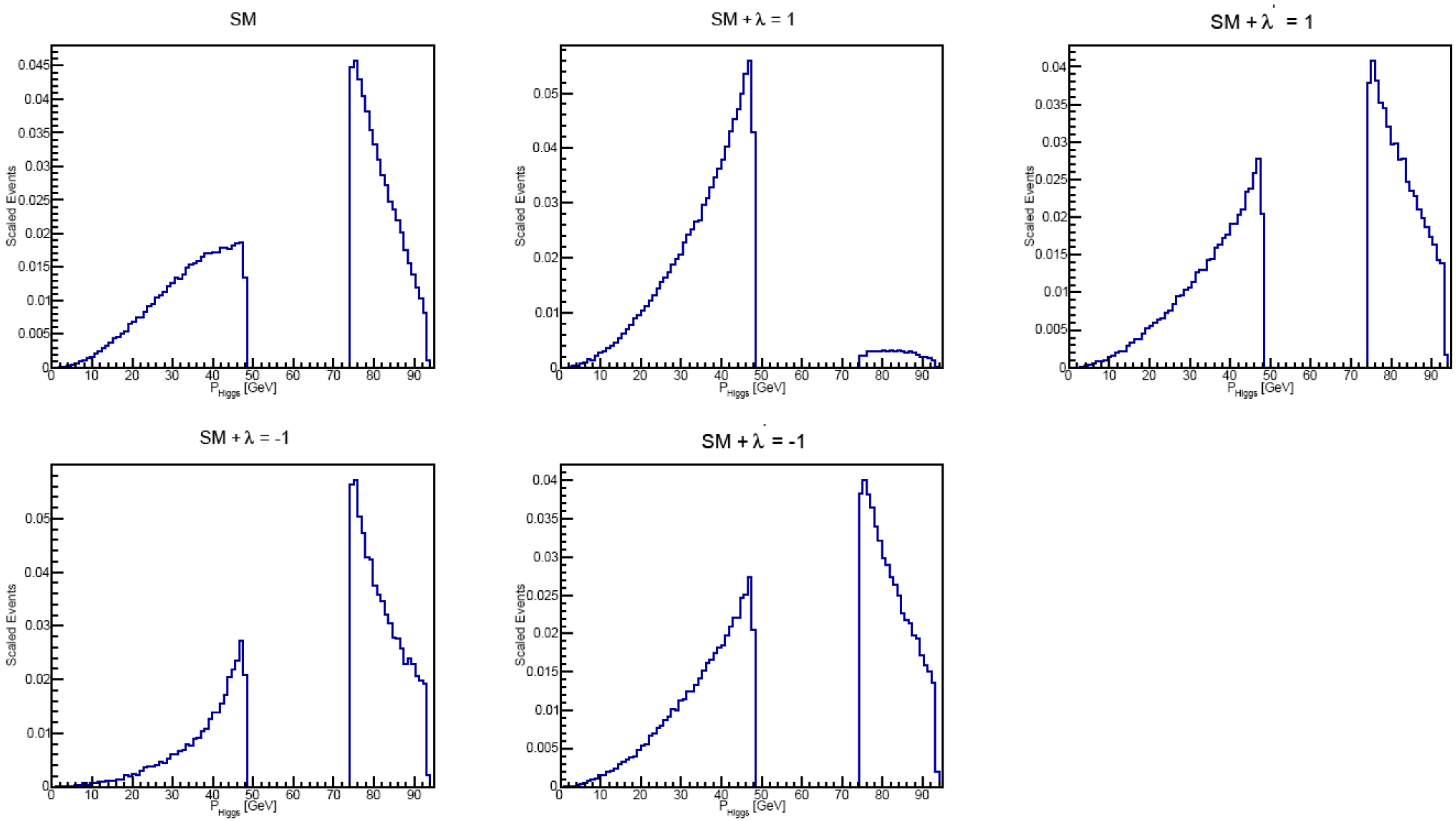
10fb^-1

Energy	Process	1DTheta		1DMomentum		2D	
		P-val	Significance	P-val	Significance	P-val	Significance
125	12	0.36176	**0.35376**				
	13	0.06913	**1.48230**	0.02790	**1.91259**		
	14	0.04749	**1.66969**				
	15	0.08330	**1.38321**	0.02770	**1.91573**		
130	12	0.33518	**0.42565**				
	13	0.13003	**1.12625**	0.01507	**2.16825**		
	14	0.04199	**1.72805**				
	15	0.15860	**1.00023**	0.01302	**2.22561**		
135	12	0.34745	**0.39221**				
	13	0.04202	**1.72771**	0.00344	**2.70260**		
	14	0.05657	**1.58424**				
	15	0.04679	**1.67681**	0.00277	**2.77383**		
140	12	0.31271	**0.48818**				
	13	0.00444	**2.61664**	0.00043	**3.33272**		
	14	0.05884	**1.56459**				
	15	0.00559	**2.53702**	0.00049	**3.29621**		
145	12	0.26315	**0.63366**	0.00314	**2.73279**	0.00049	**3.29764**
	13	0.00023	**3.50888**	0.00019	**3.55360**		
	14	0.08196	**1.39199**				
	15	0.0003	**3.43161**	0.00020	**3.54008**		
150	12	0.26304	**0.63400**	0.01554	**2.15605**	0.00374	**2.67468**
	13	0.00004	**3.9444**	0.00013	**3.65220**		
	14	0.05698	**1.58064**				
	15	0.00001	**4.26489**	0.00013	**3.65220**		

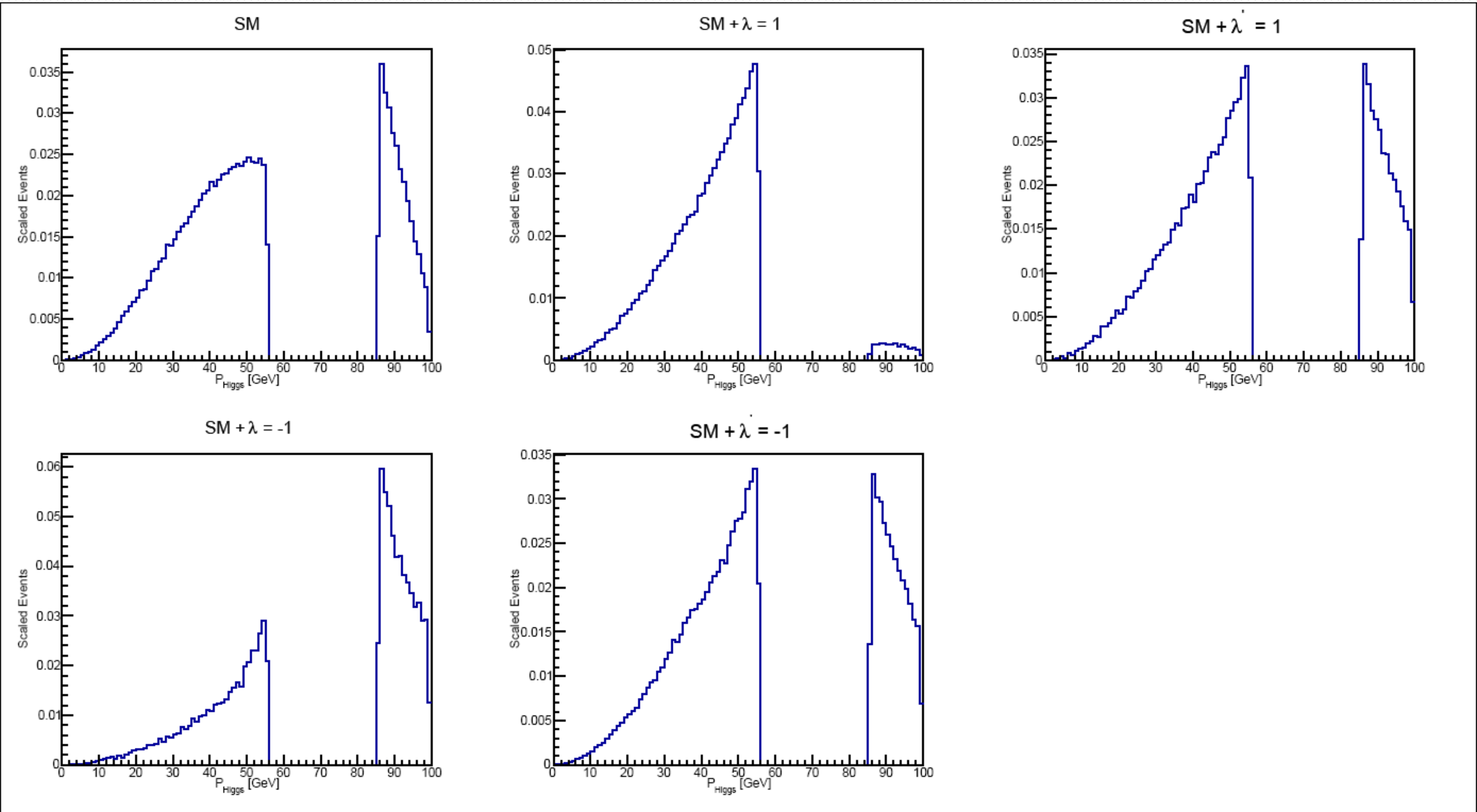
- It seems that in order to distinguish between BSM and SM models for lambda1/2 =+-1 only 5fb^-1 at beam energies of 125GeV is required to get a significance of greater than three.
- The same study could, in principle, with more advanced computing, determine the required amount of data to collect for differing beam energies to distinguish SM and BSM models of varied lambda1 and 2 values. Alternatively such a study can be performed on collected data to exclude ranges for the values of lambda1 and lambda2.

BACKUP SLIDES FOLLOW

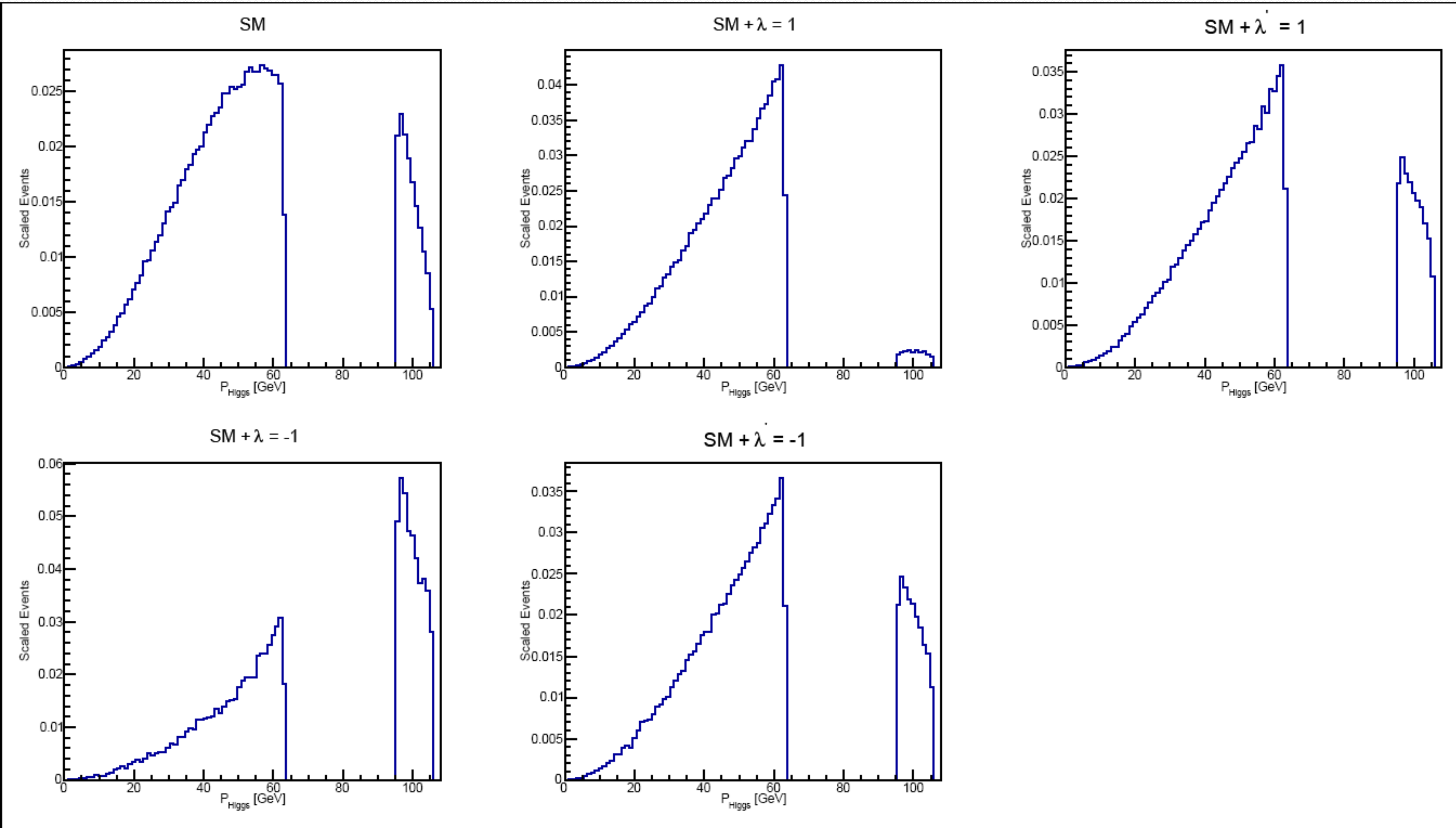
Momentum – 125GeV



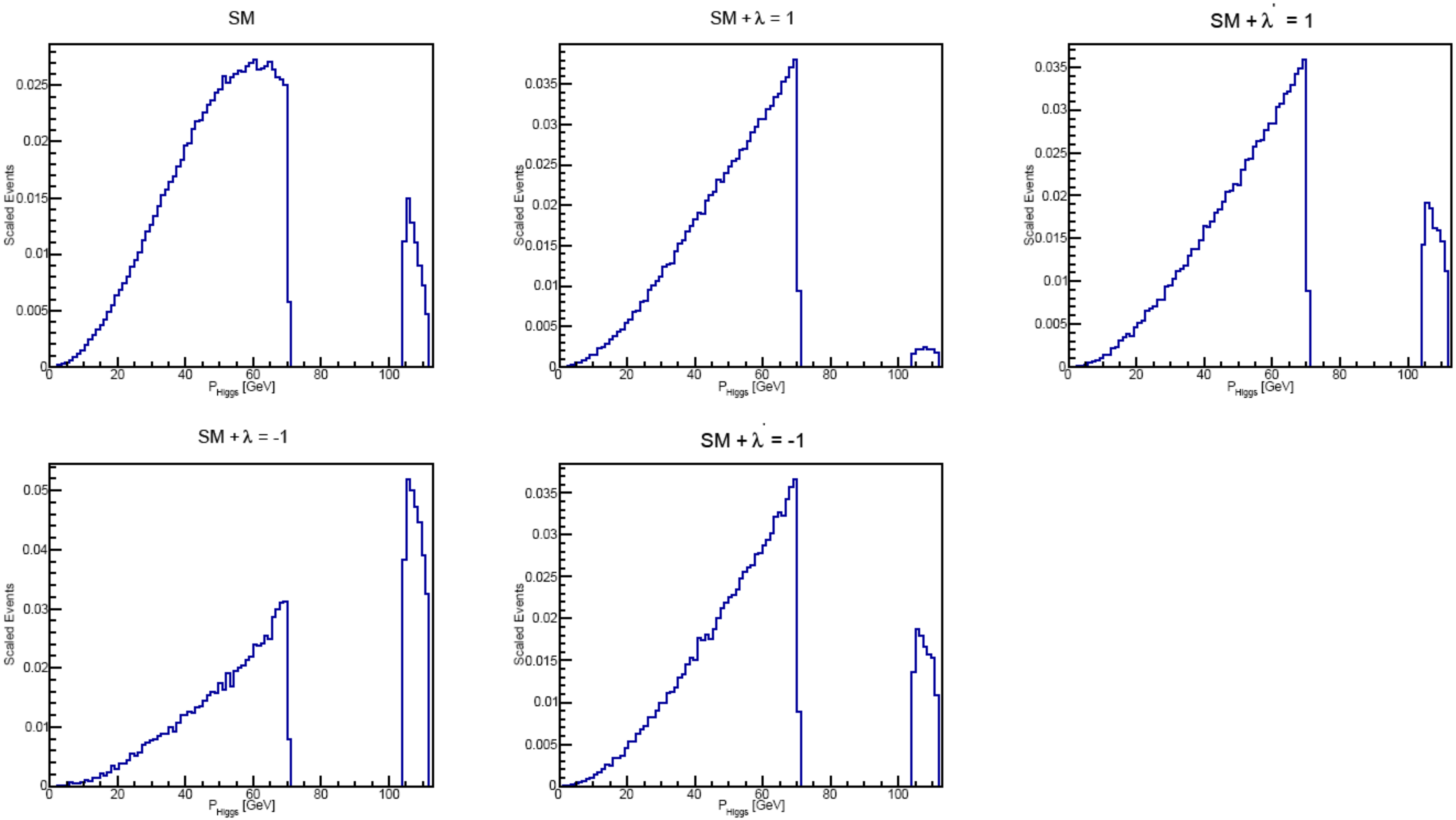
Momentum – 130GeV



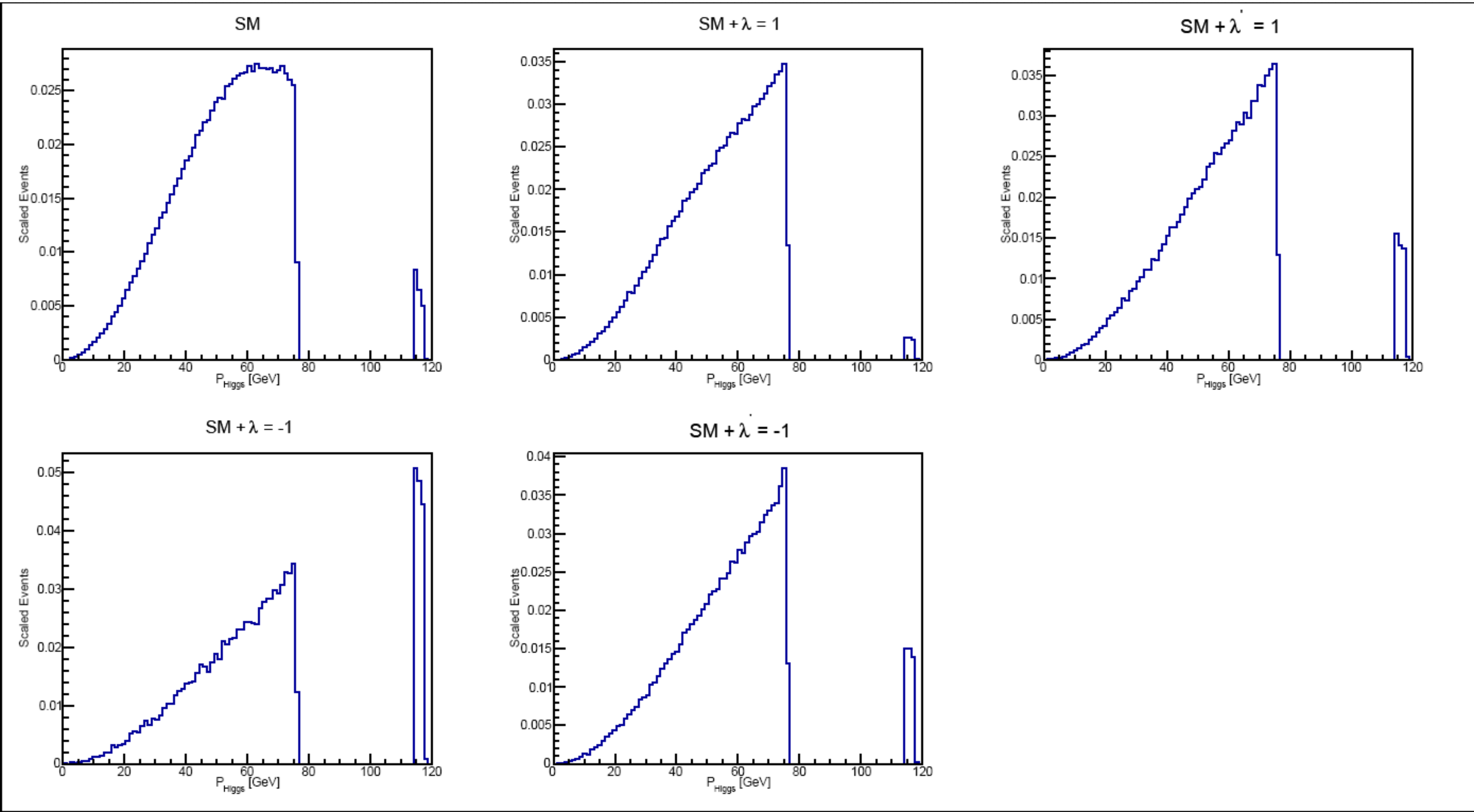
Momentum – 135GeV



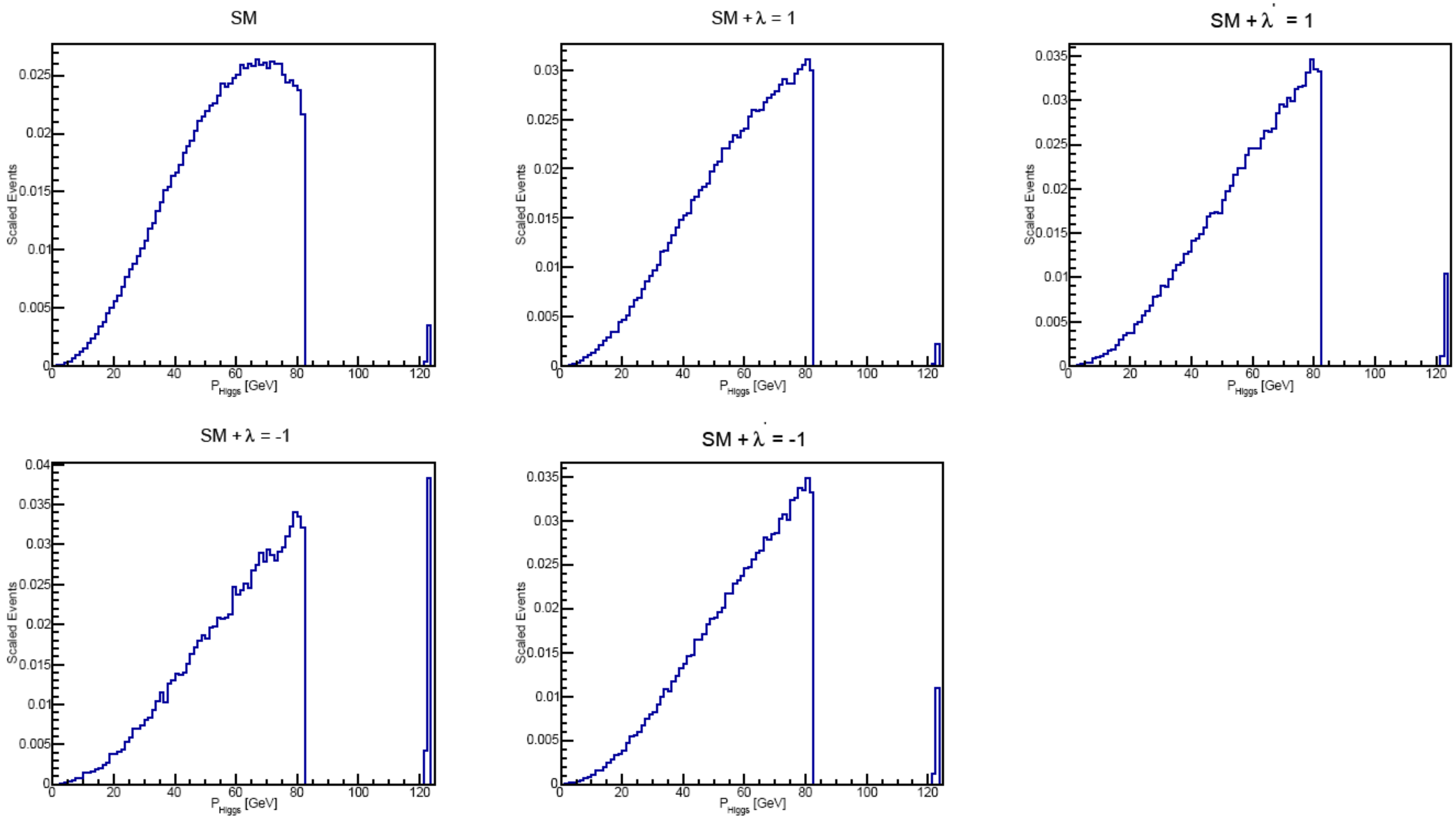
Momentum – 140GeV



Momentum – 145GeV

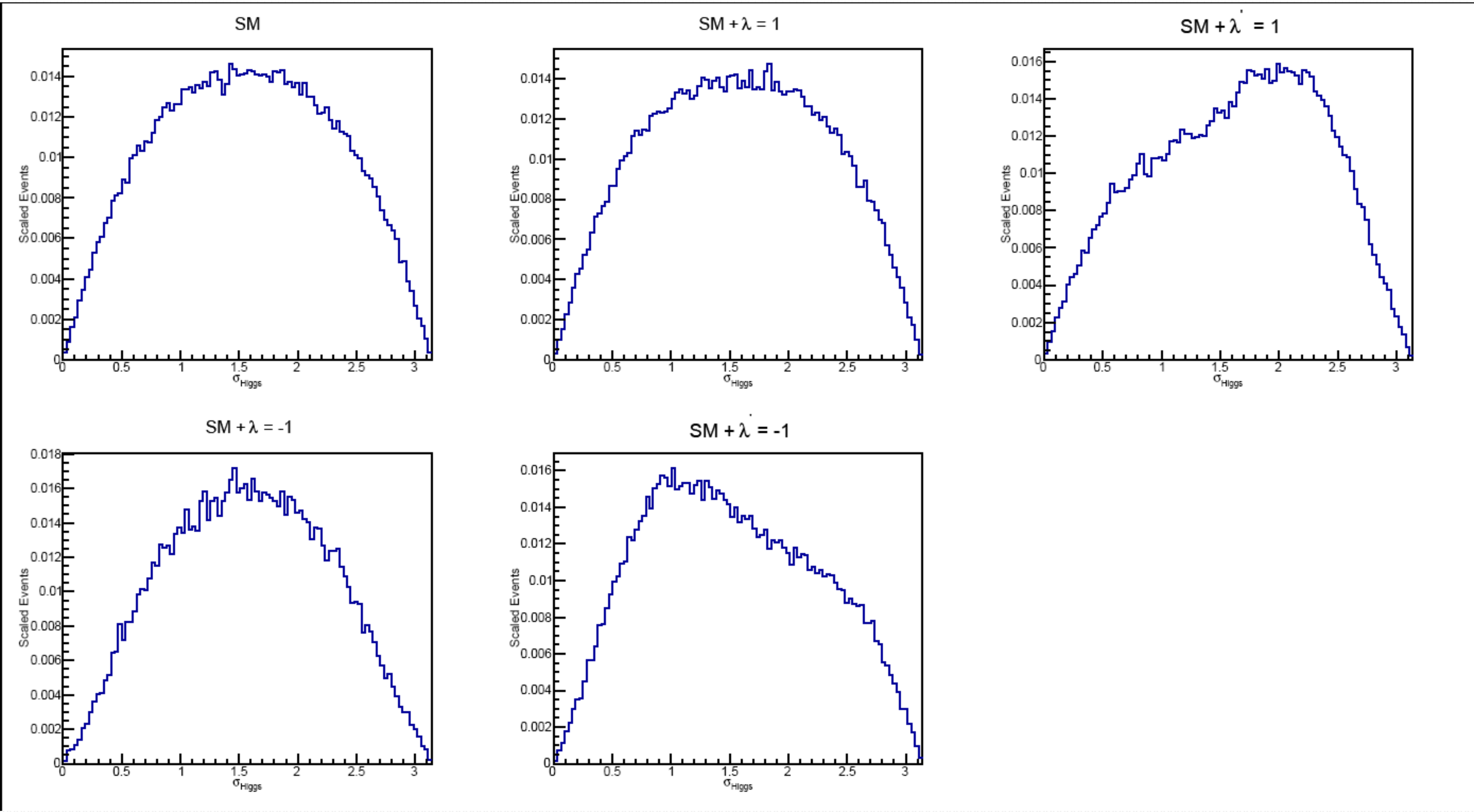


Momentum – 150GeV

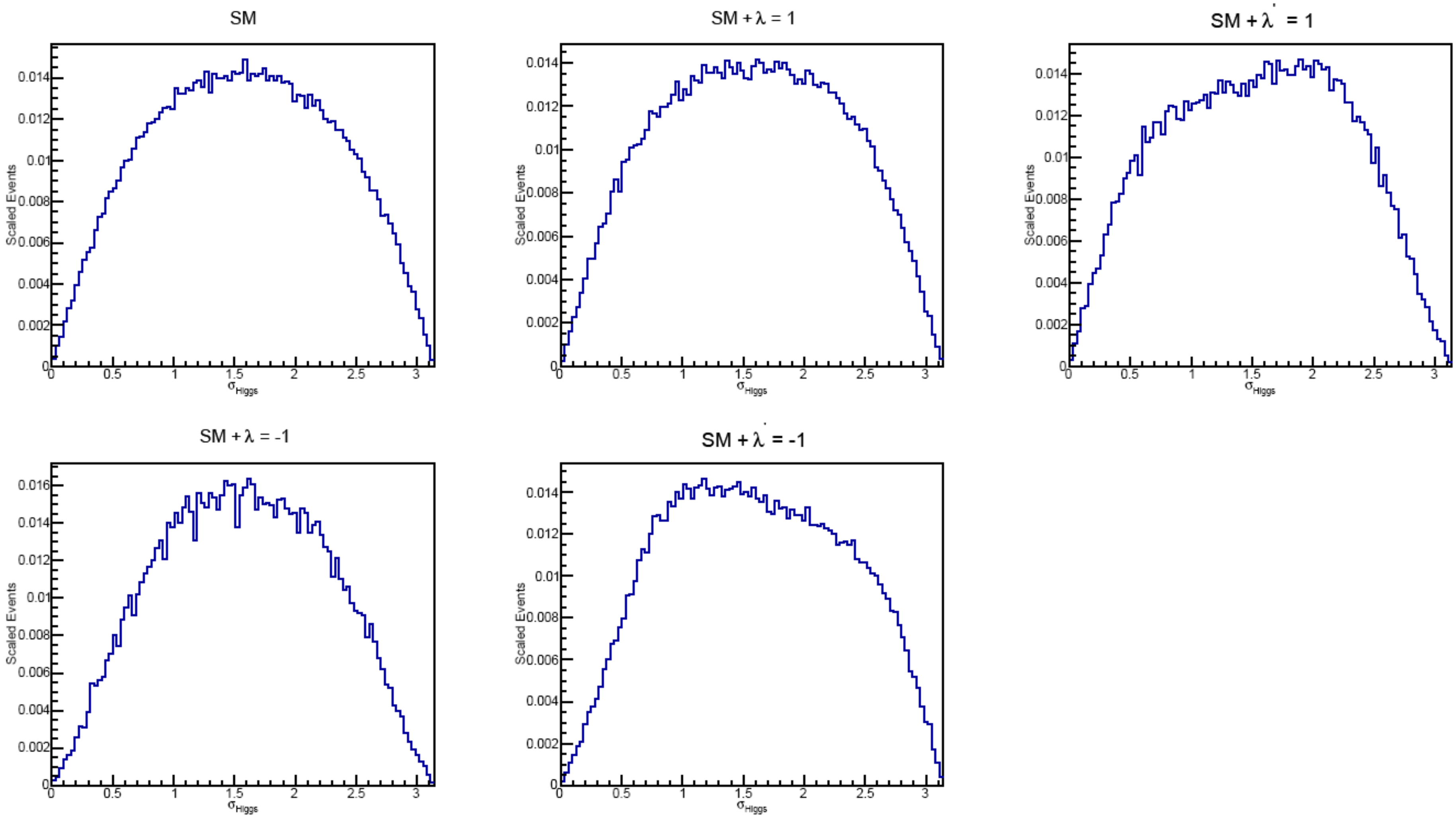


Histograms of Theta Distribution

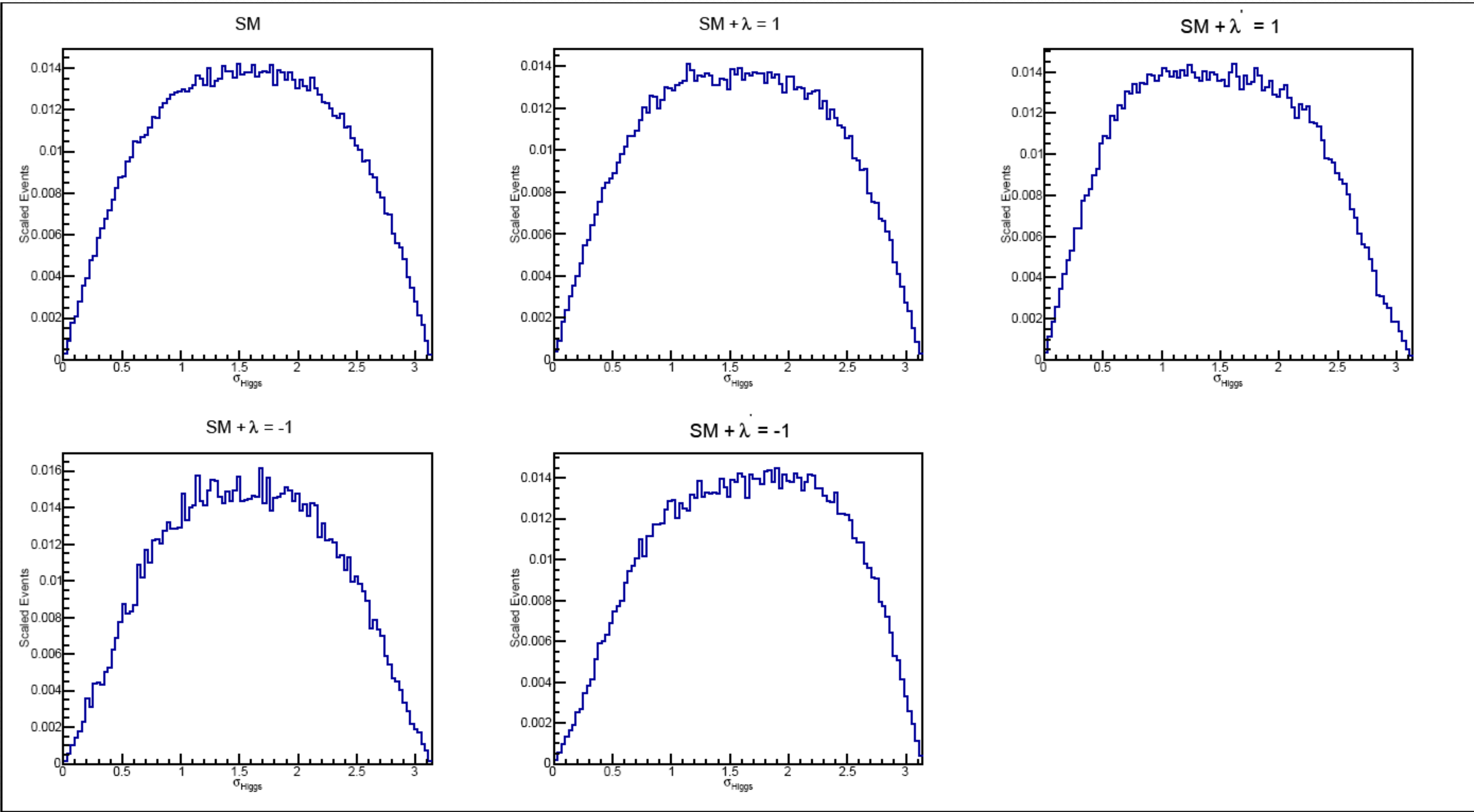
Theta – 125GeV



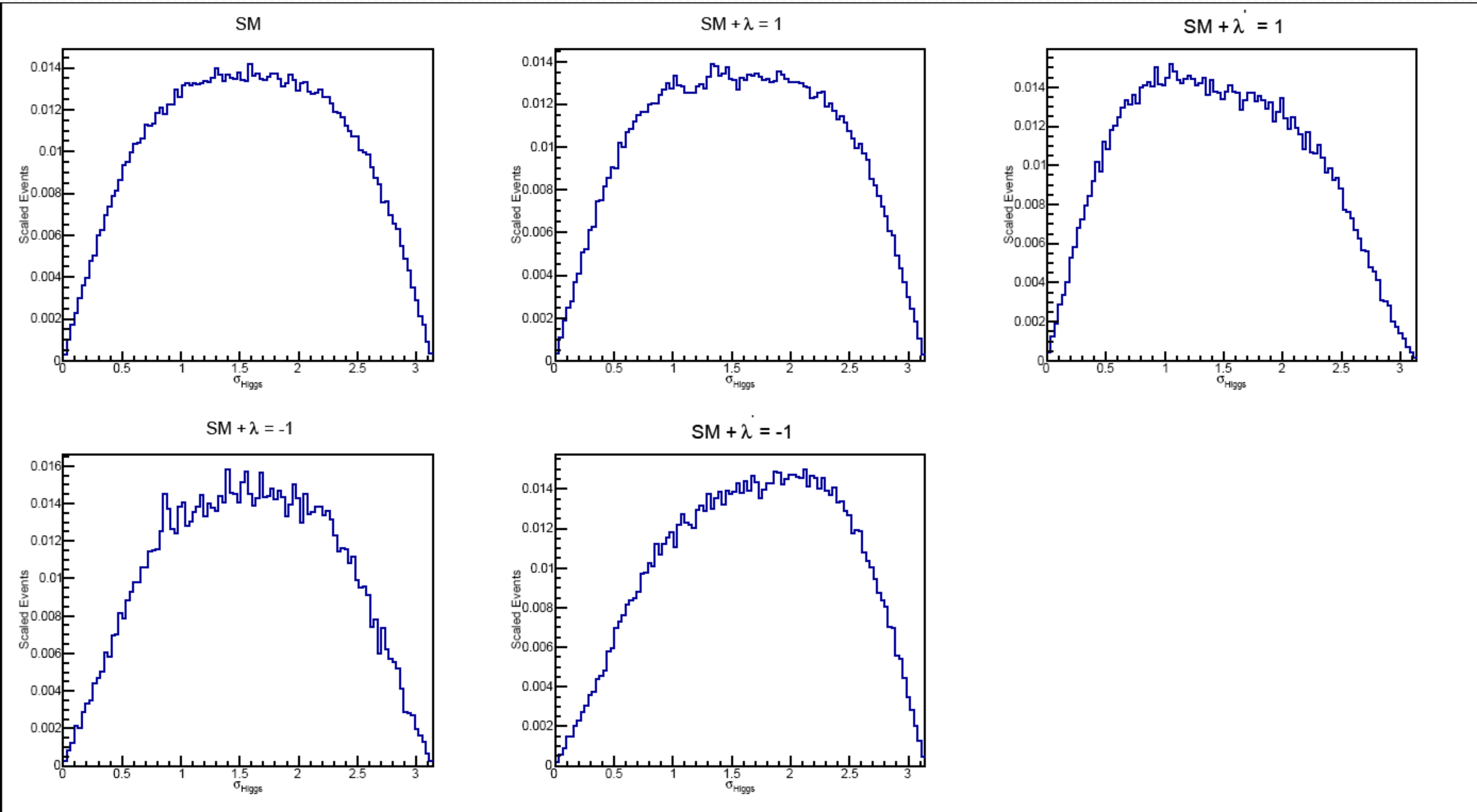
Theta – 130GeV



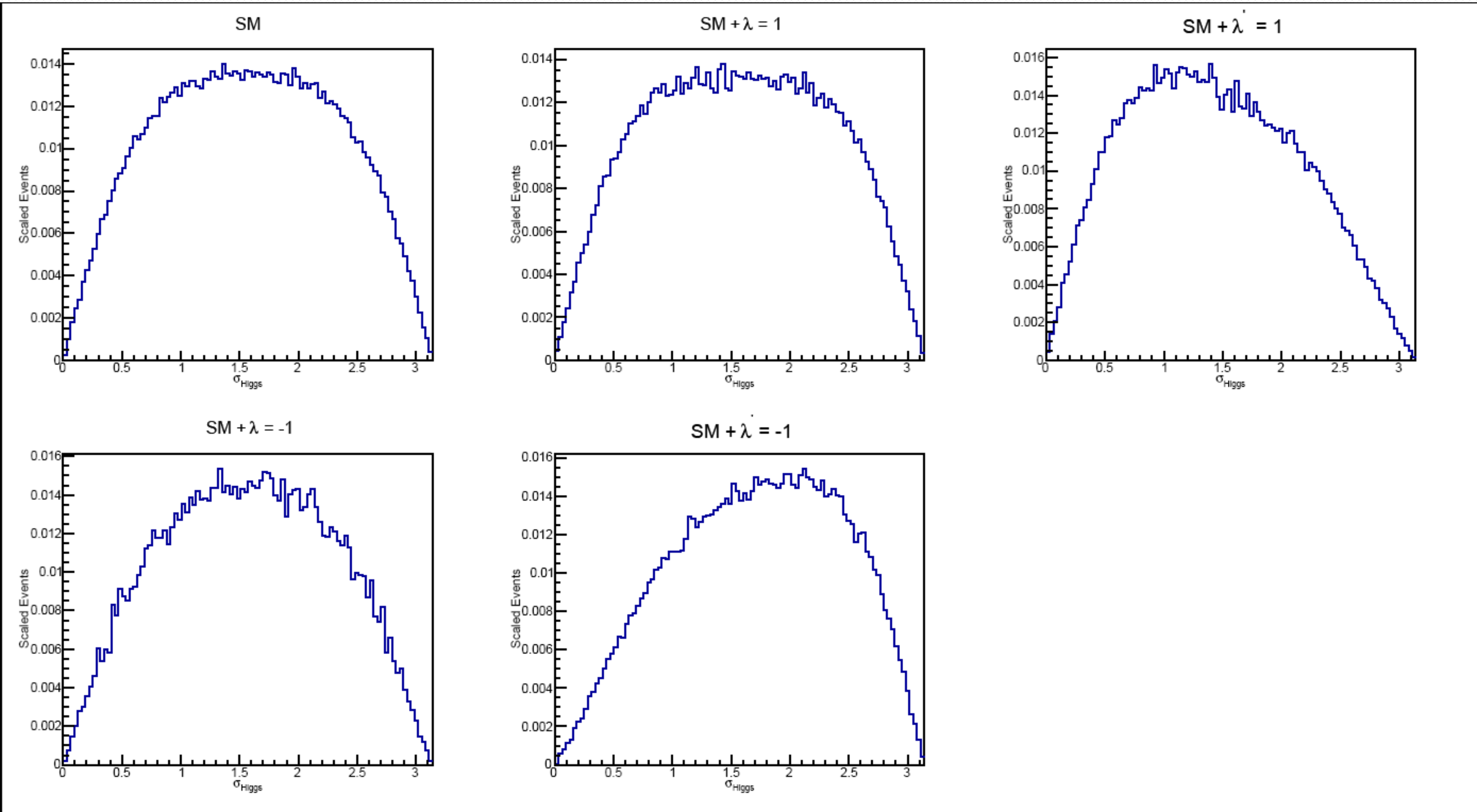
Theta – 135GeV



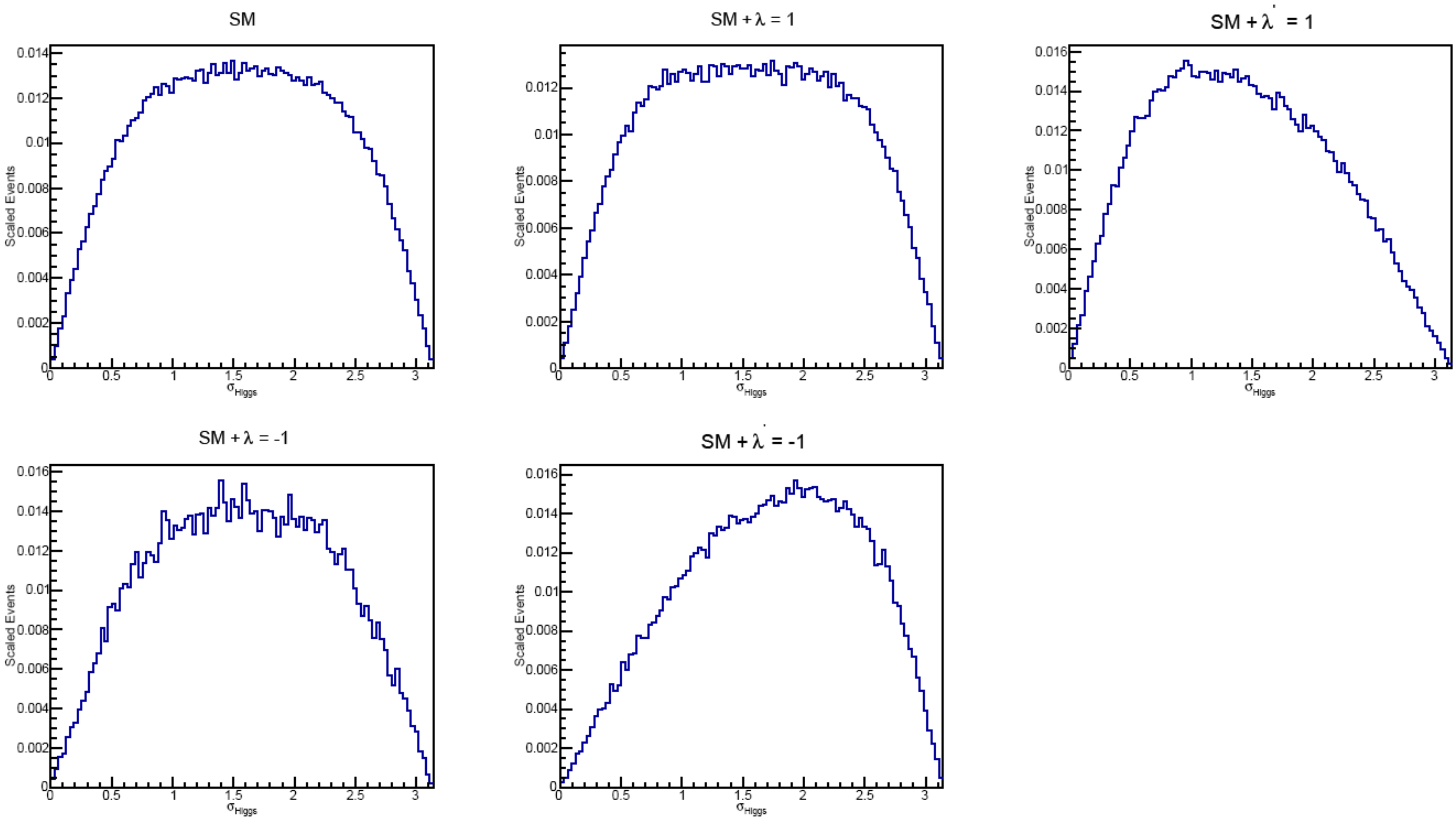
Theta – 140GeV



Theta – 145GeV



Theta – 150GeV



2 Dimensional Plots of Model Data

- Y-axis: Momentum
- X-axis: Theta distribution (convention of 0 to Pi used)

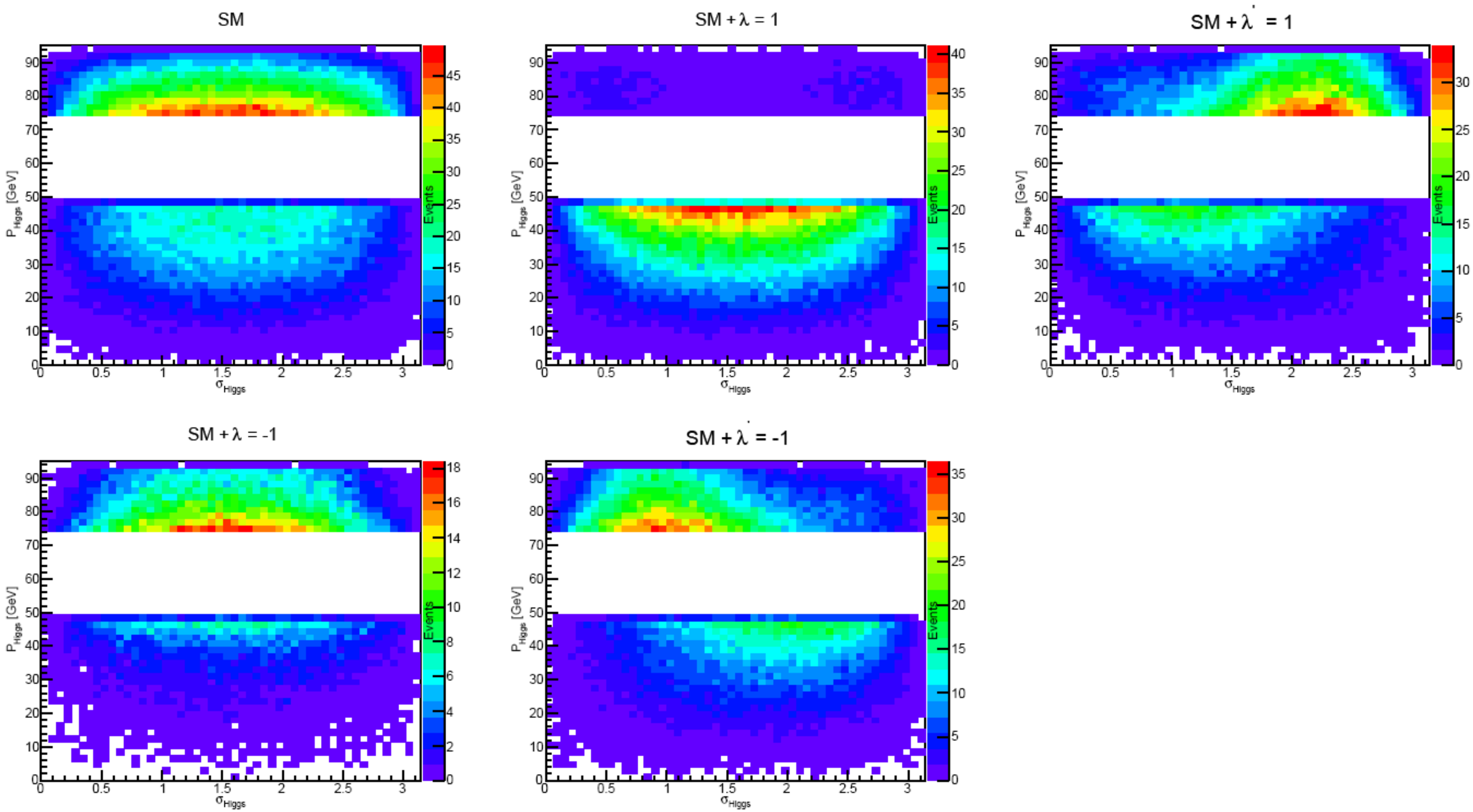
S-Channel momentum range of 2sigma cut out.

This is the S-Channel of the SM used.

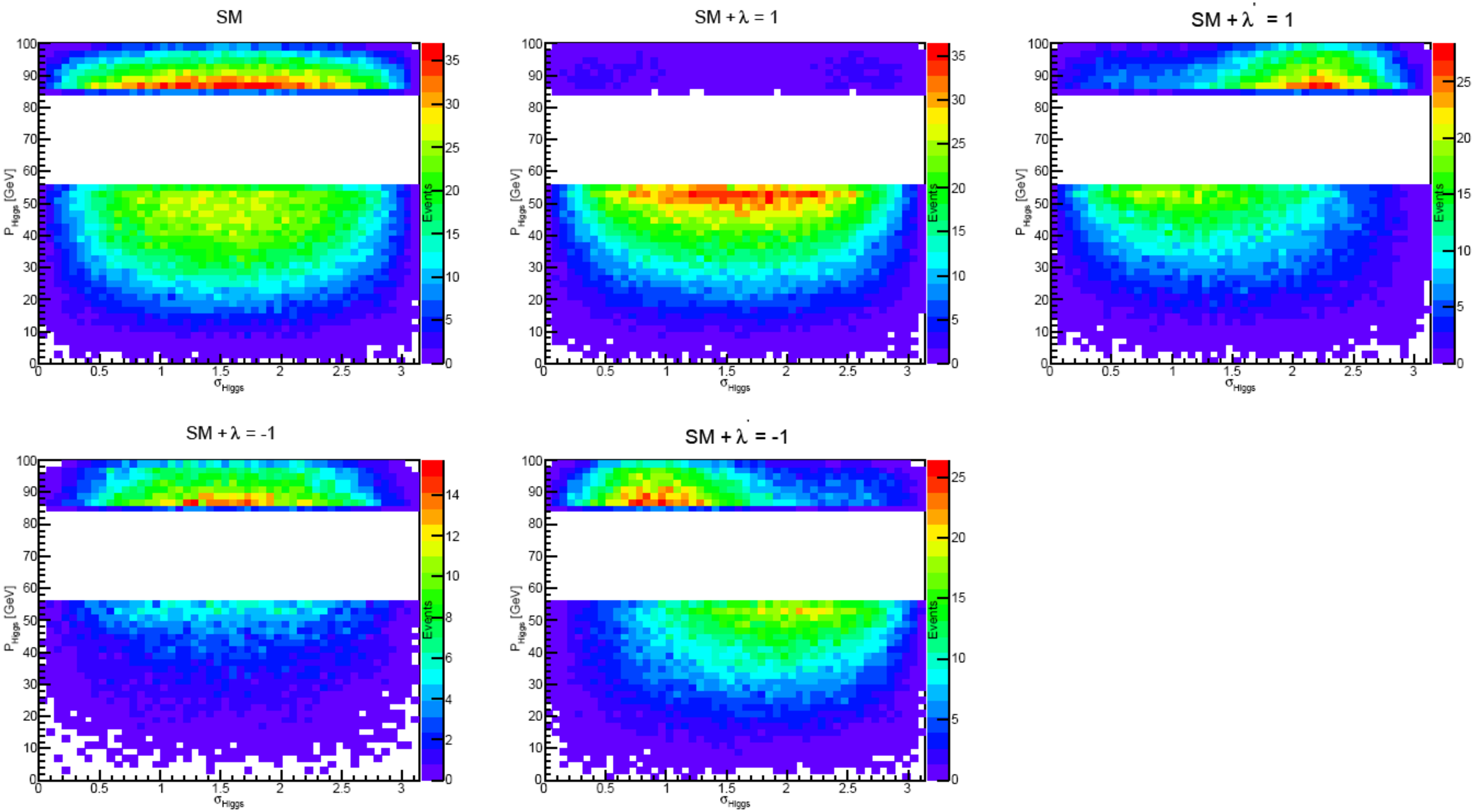
Little variation in the S-channel momentum mean and standard deviation in any BSM model compared to SM.

Histograms of Momentum vs Theta Distribution

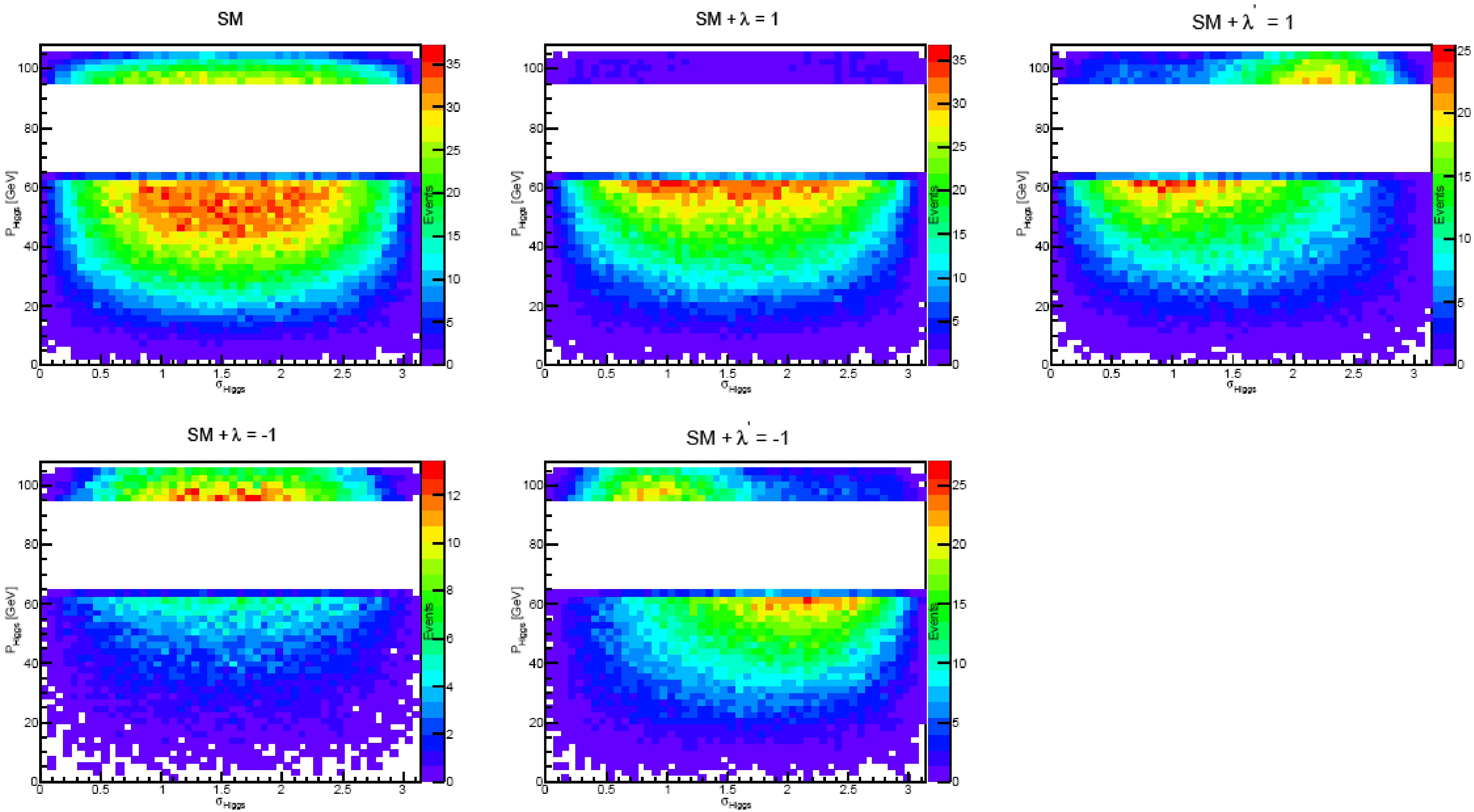
2D – 125GeV



2D – 130GeV

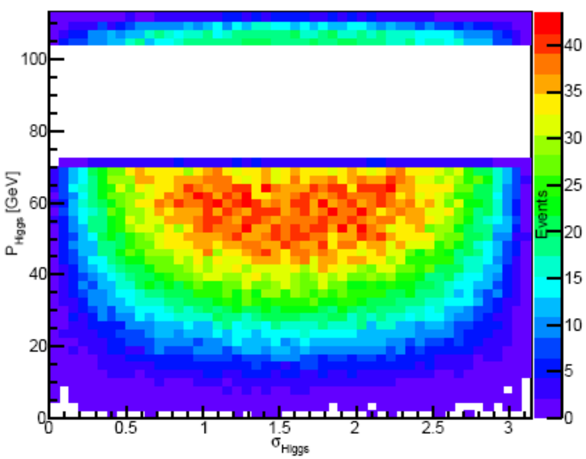


2D – 135GeV

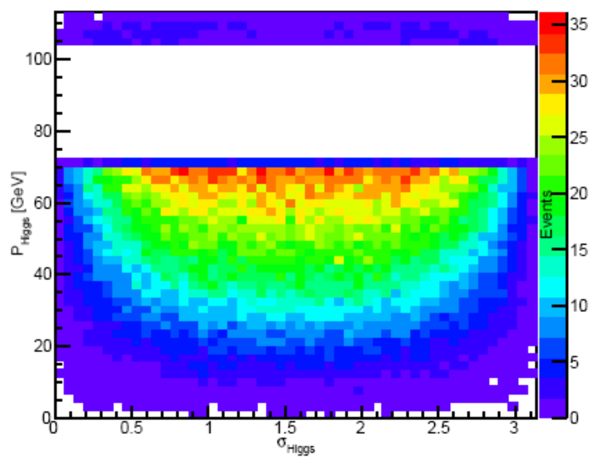


2D – 140GeV

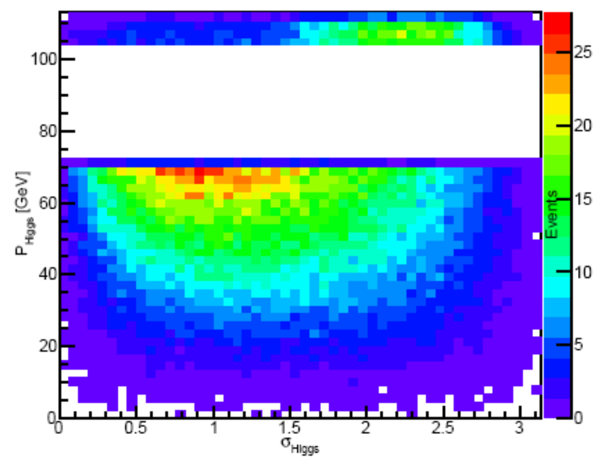
SM



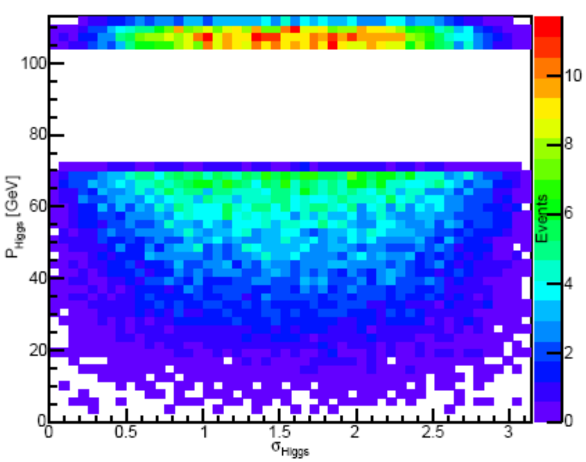
SM + λ = 1



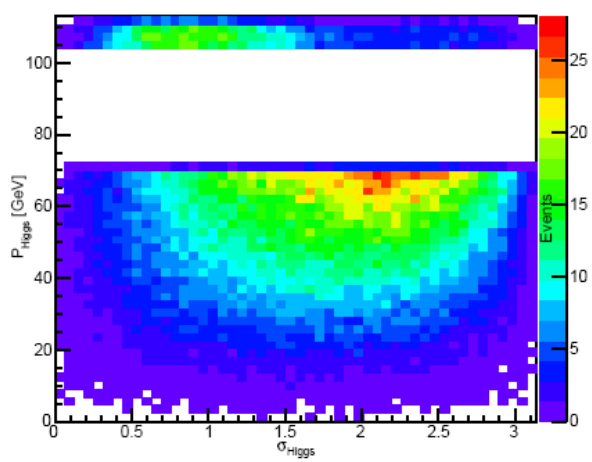
SM + λ' = 1



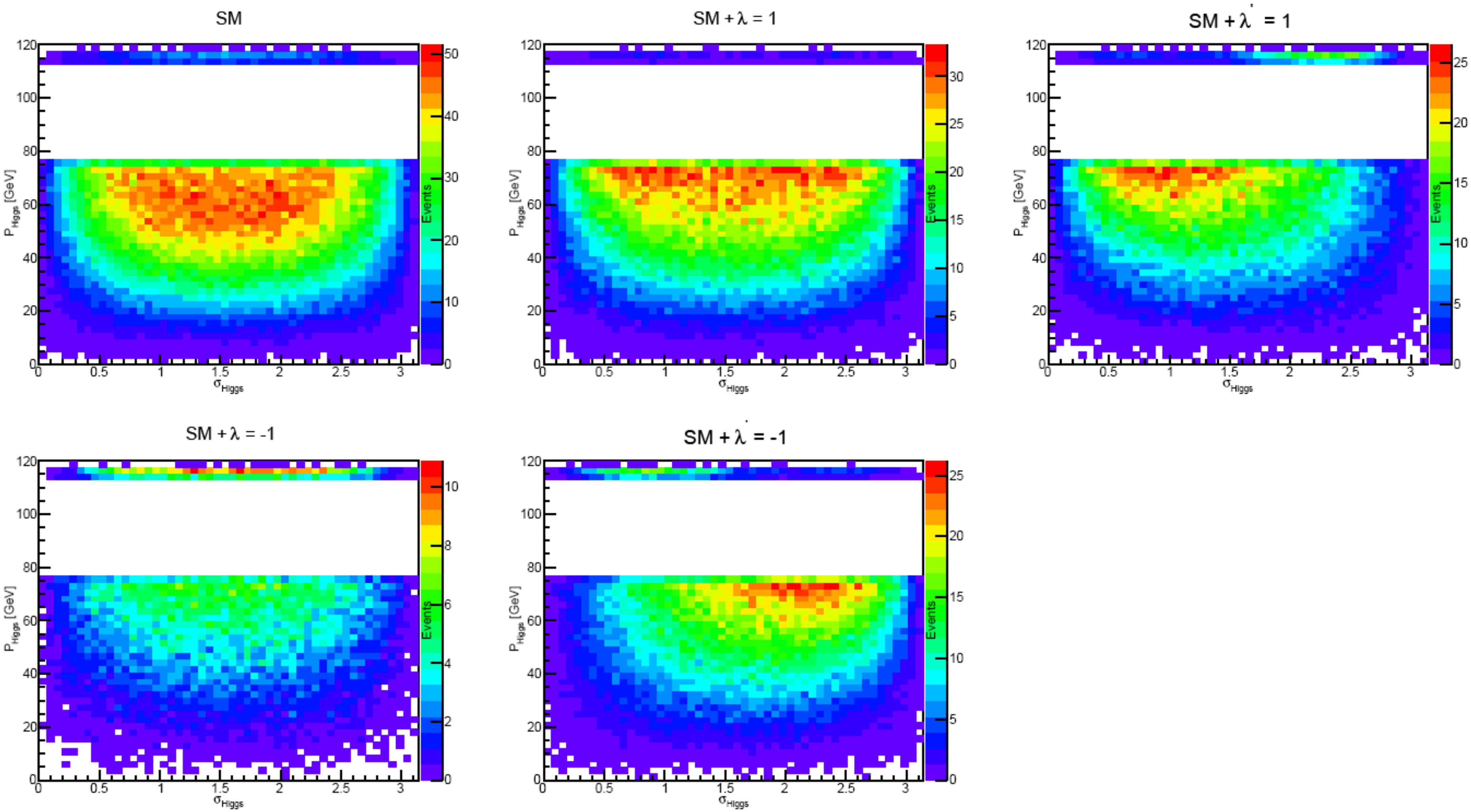
SM + λ = -1



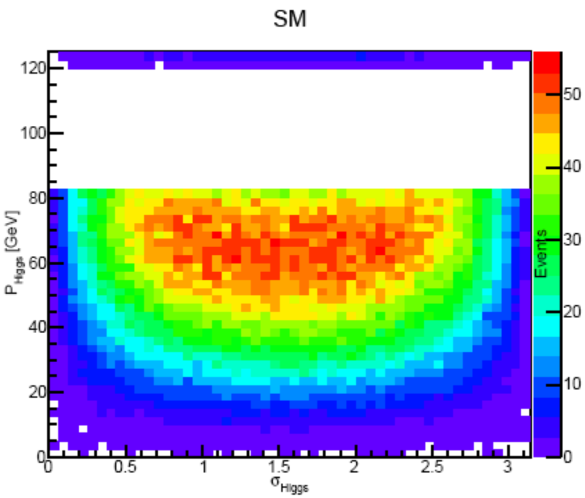
SM + λ' = -1

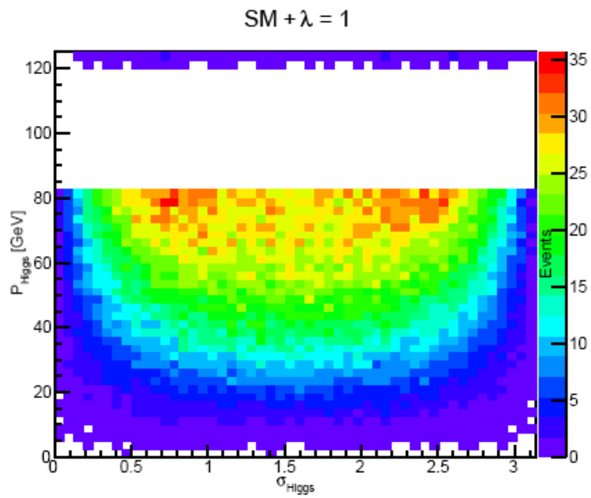


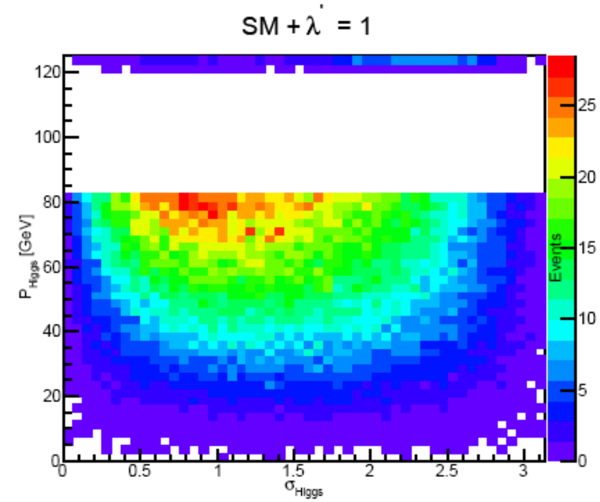
2D – 145GeV

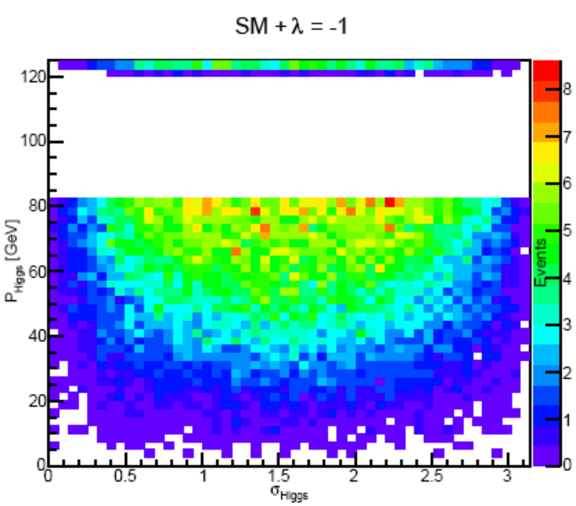


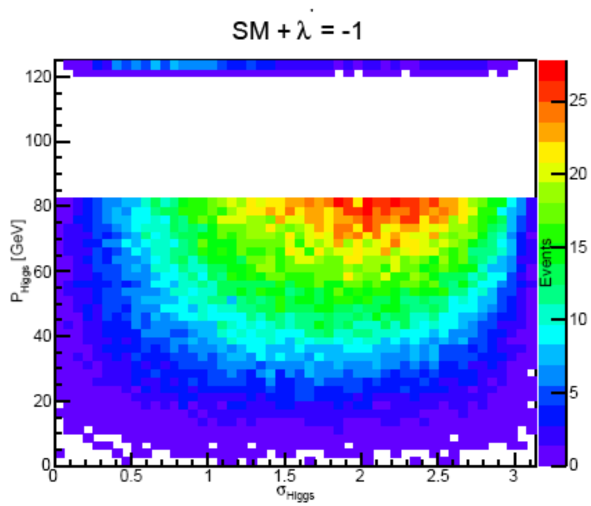
2D – 150GeV


SM
P_Higgs [GeV]
σ_Higgs
Events


SM + λ = 1
P_Higgs [GeV]
σ_Higgs
Events

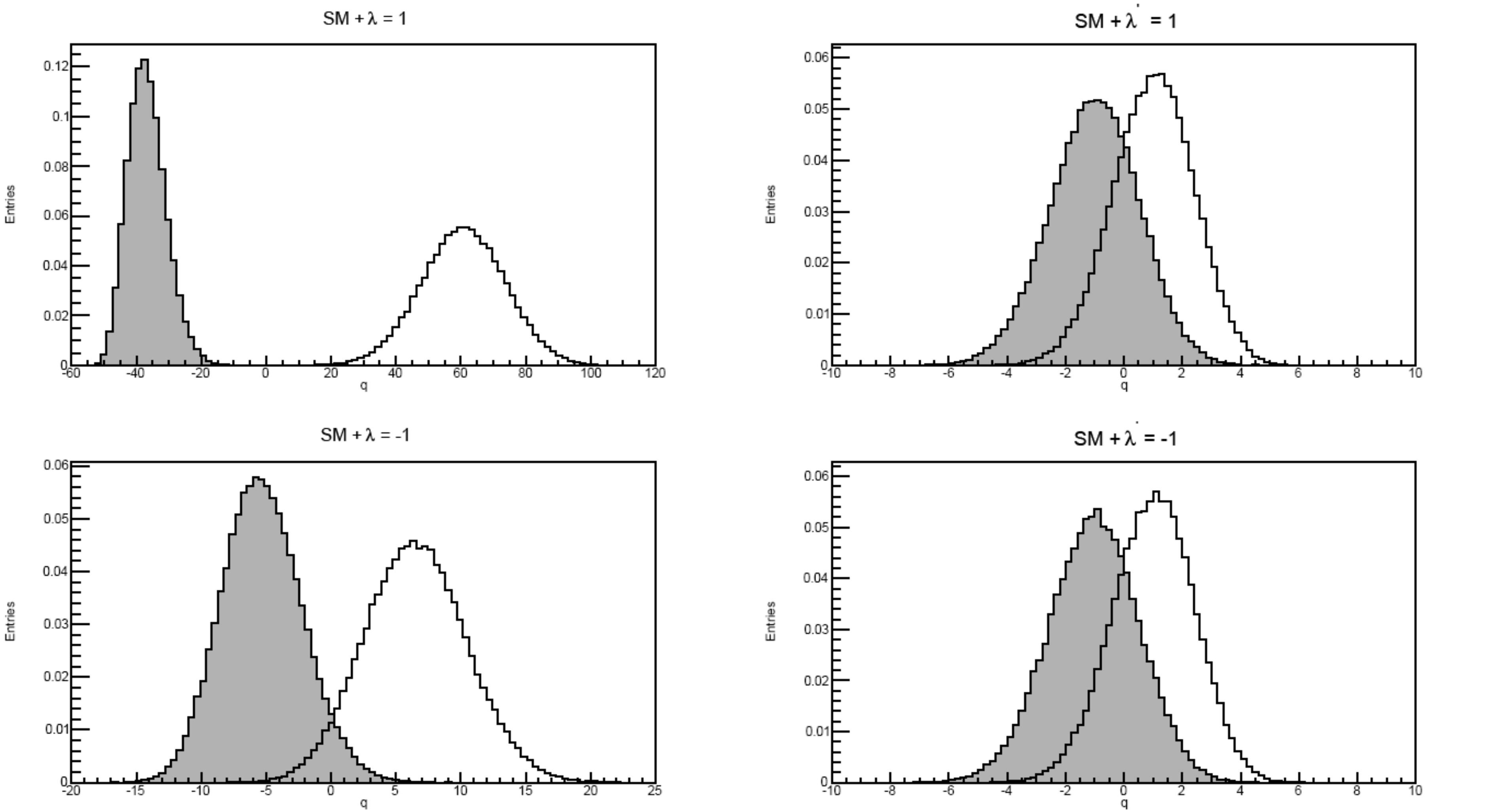

SM + λ' = 1
P_Higgs [GeV]
σ_Higgs
Events


SM + λ = -1
P_Higgs [GeV]
σ_Higgs
Events

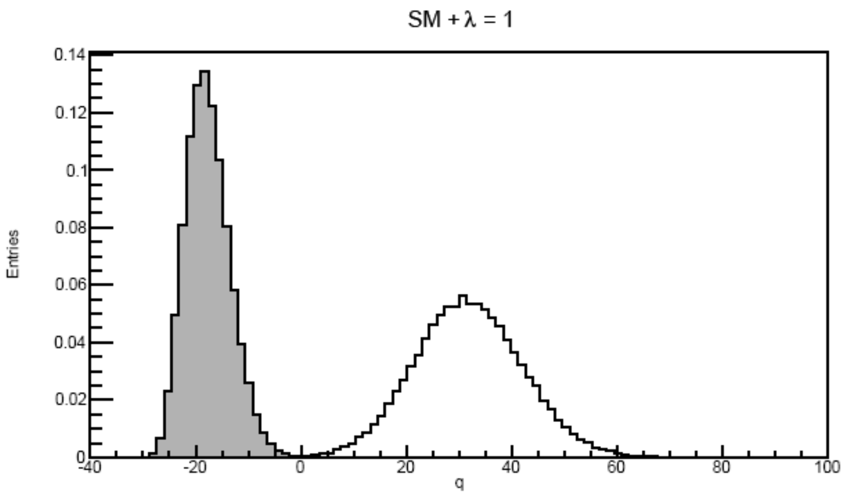

SM + λ' = -1
P_Higgs [GeV]
σ_Higgs
Events

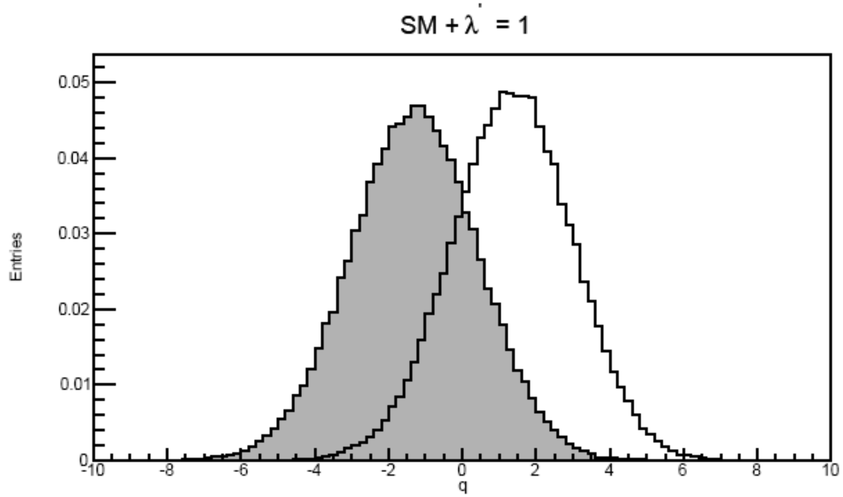
Test Statistic Comparisons

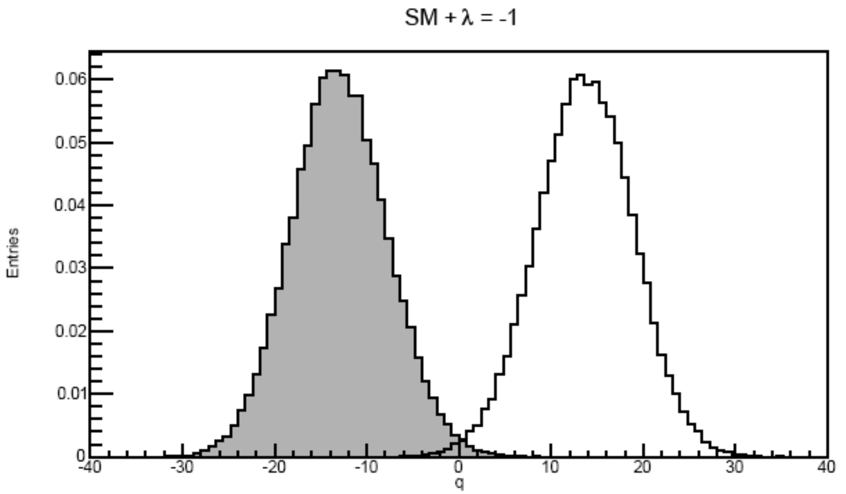
Momentum – 125GeV

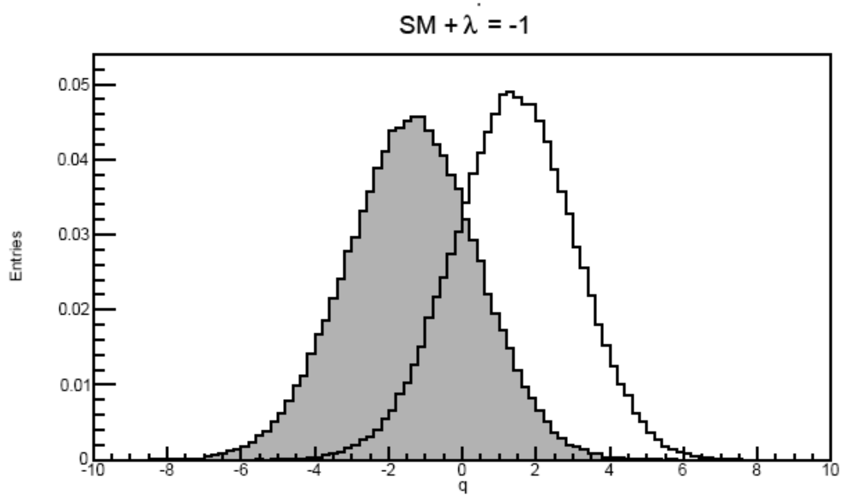


Momentum – 130GeV

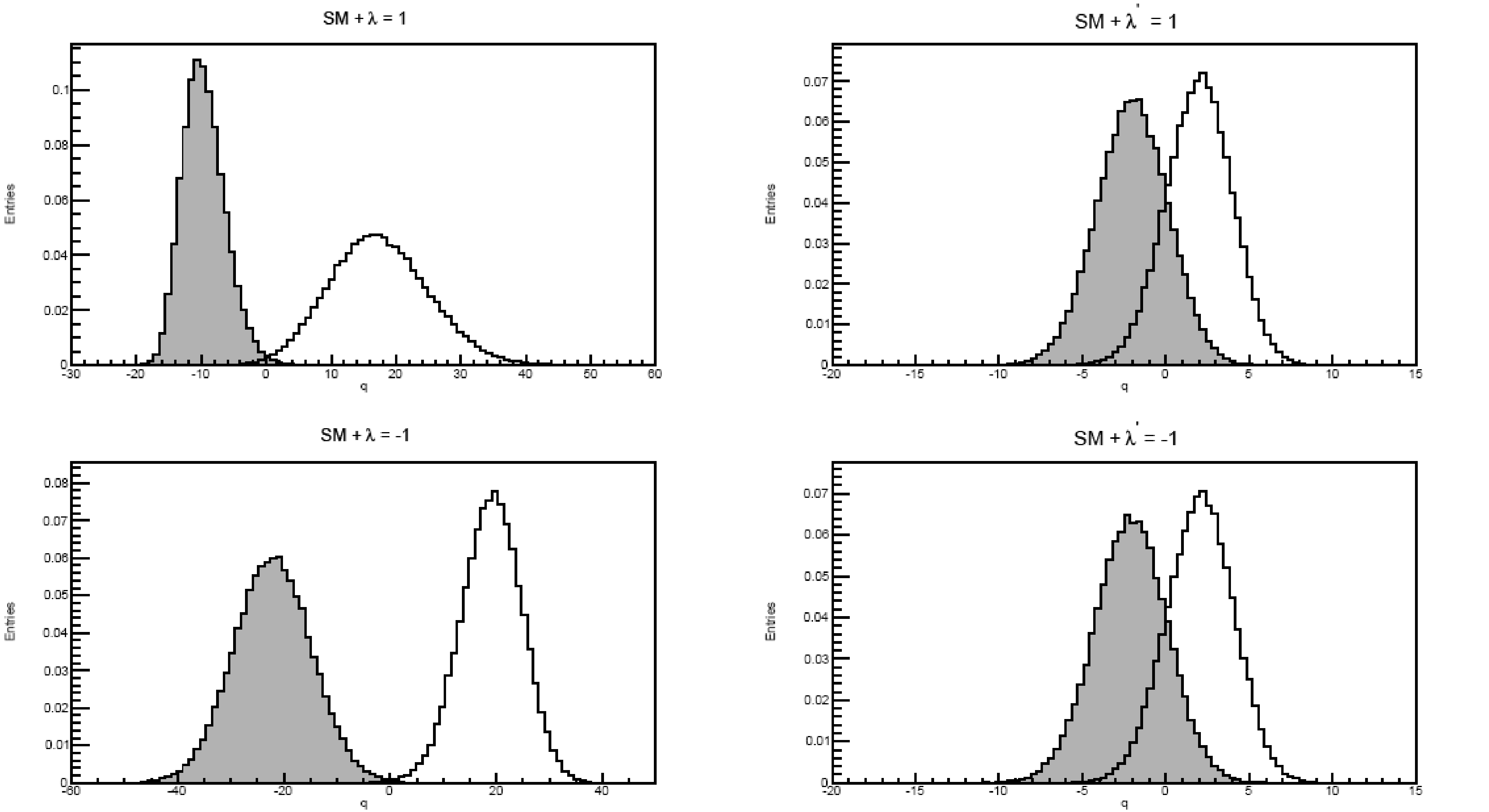




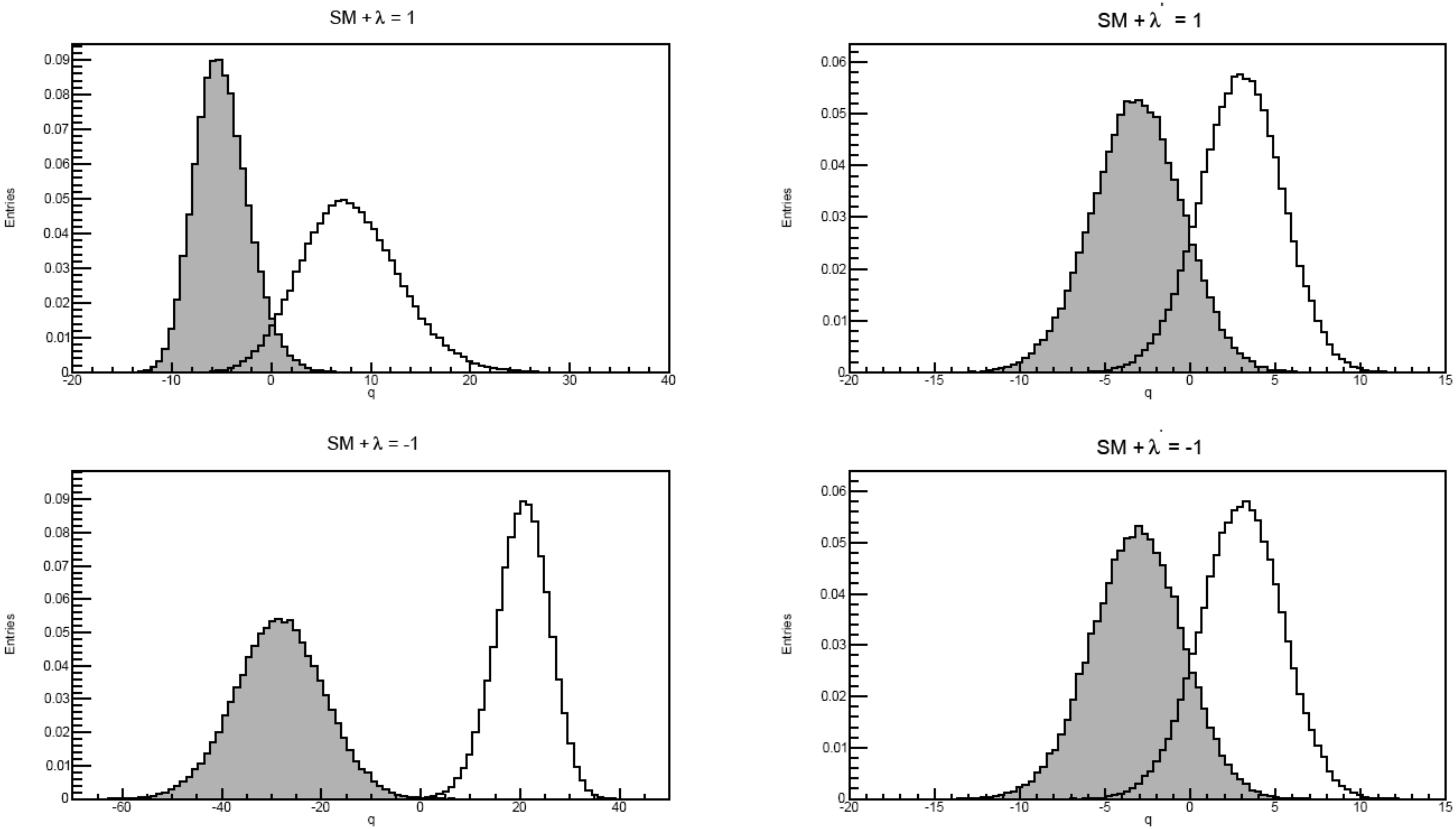




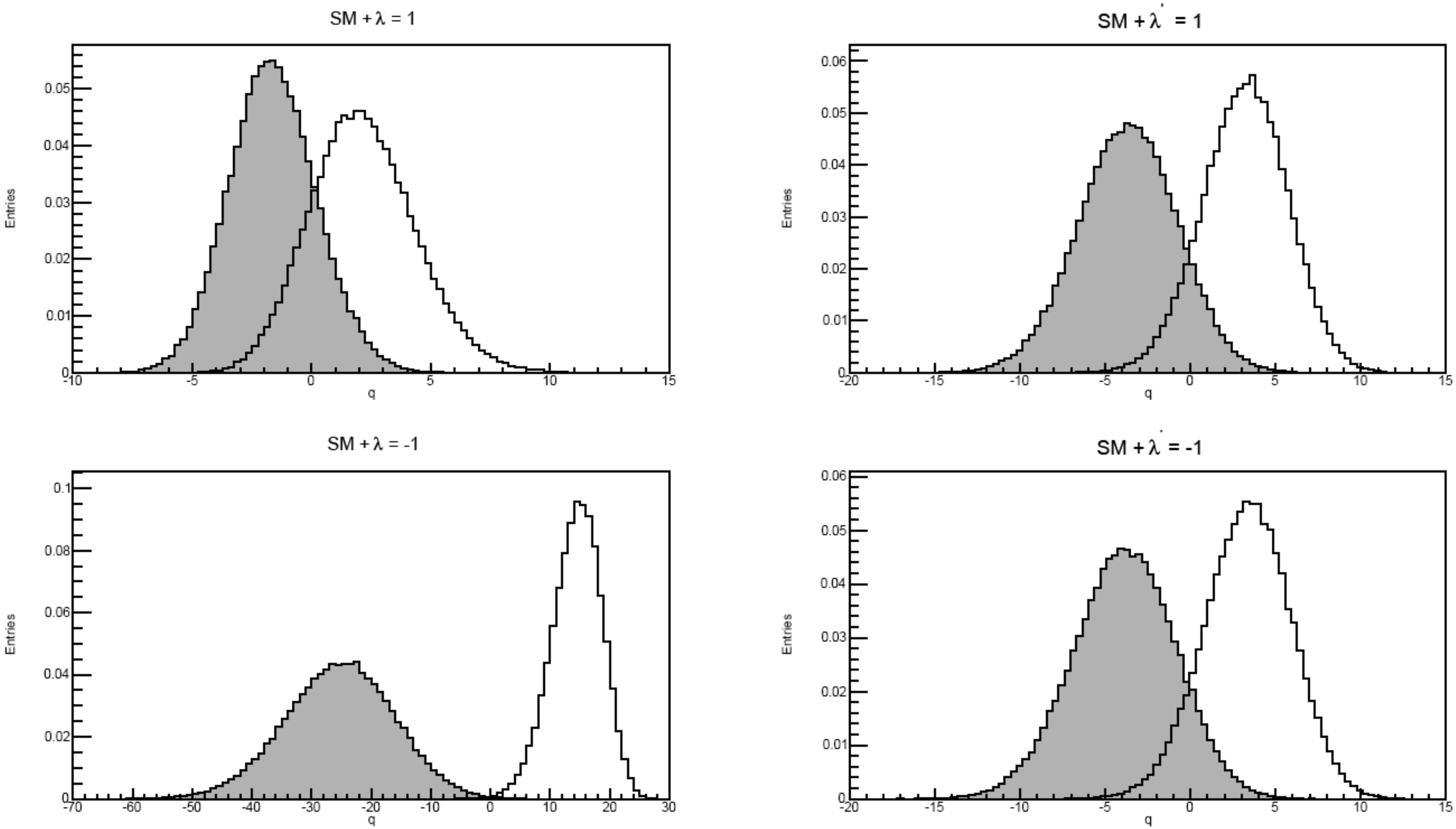
Momentum – 135GeV



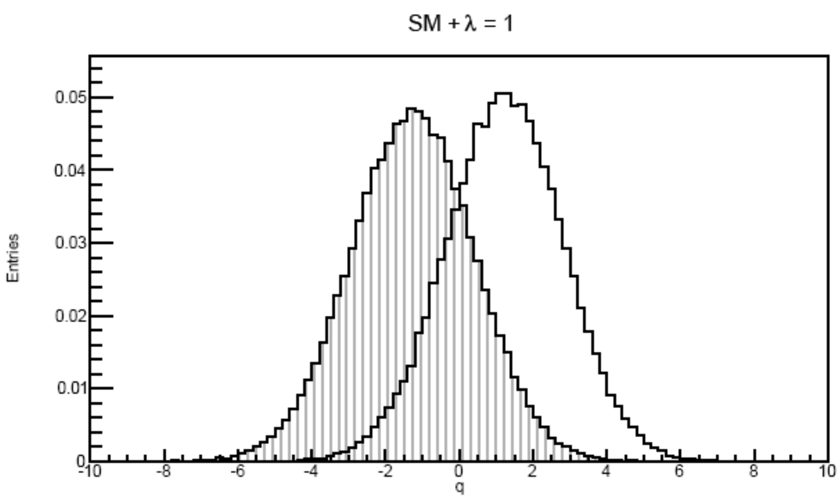
Momentum – 140GeV

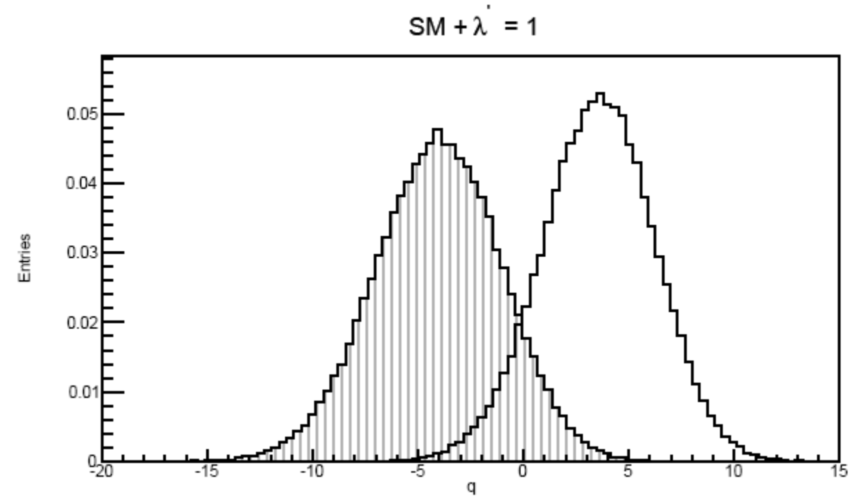


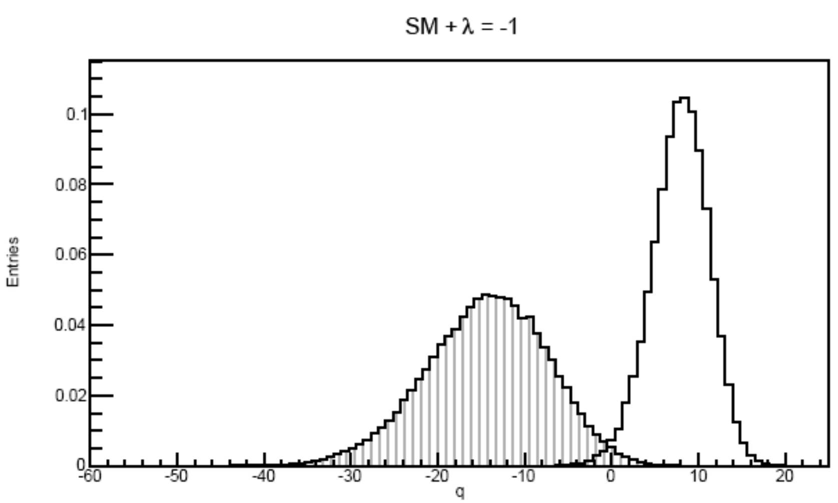
Momentum – 145GeV

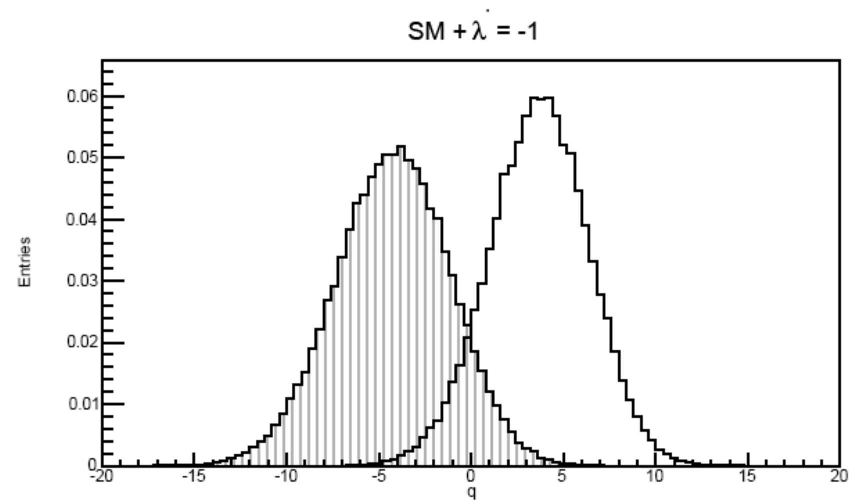


Momentum – 150GeV

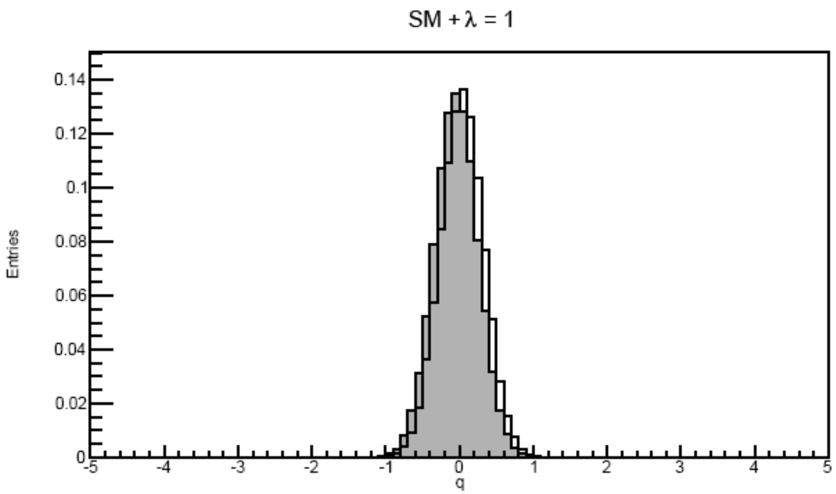


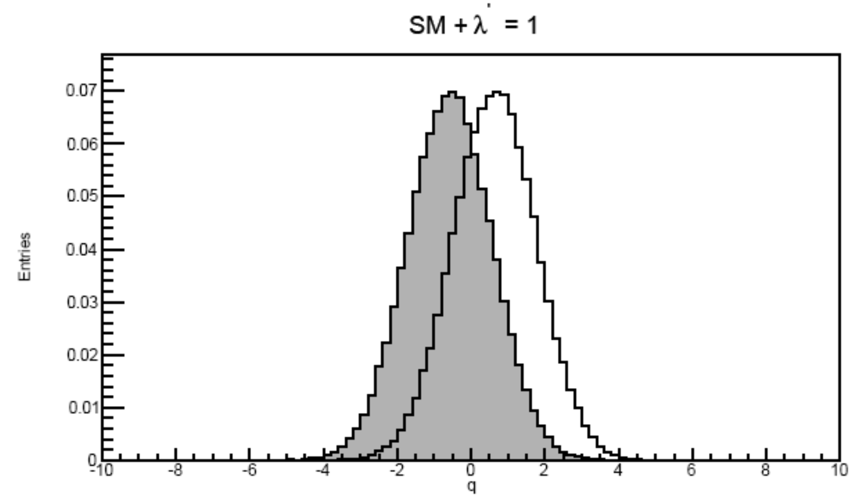


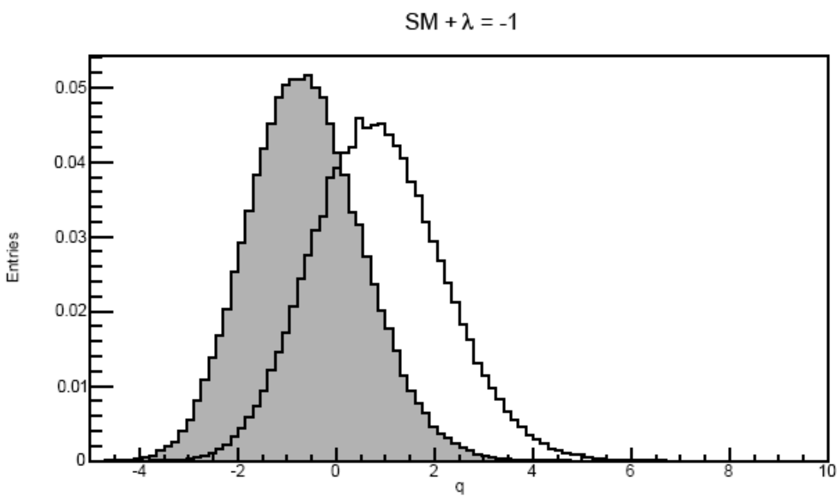


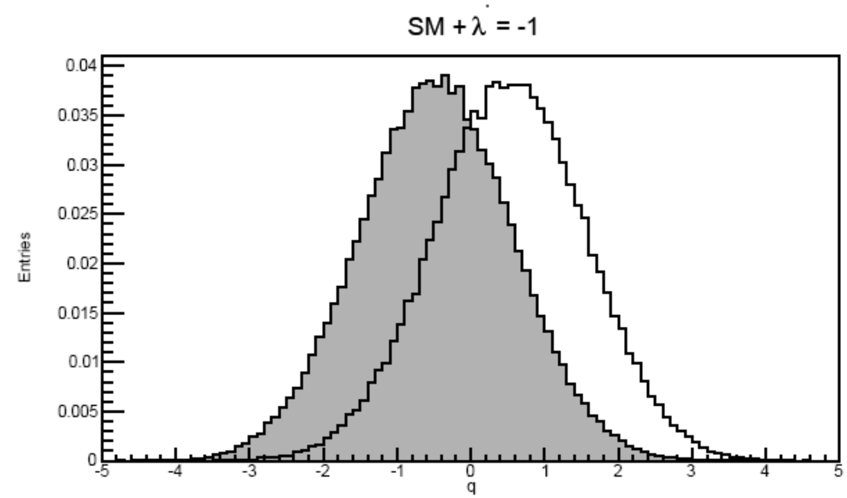


Theta – 125GeV

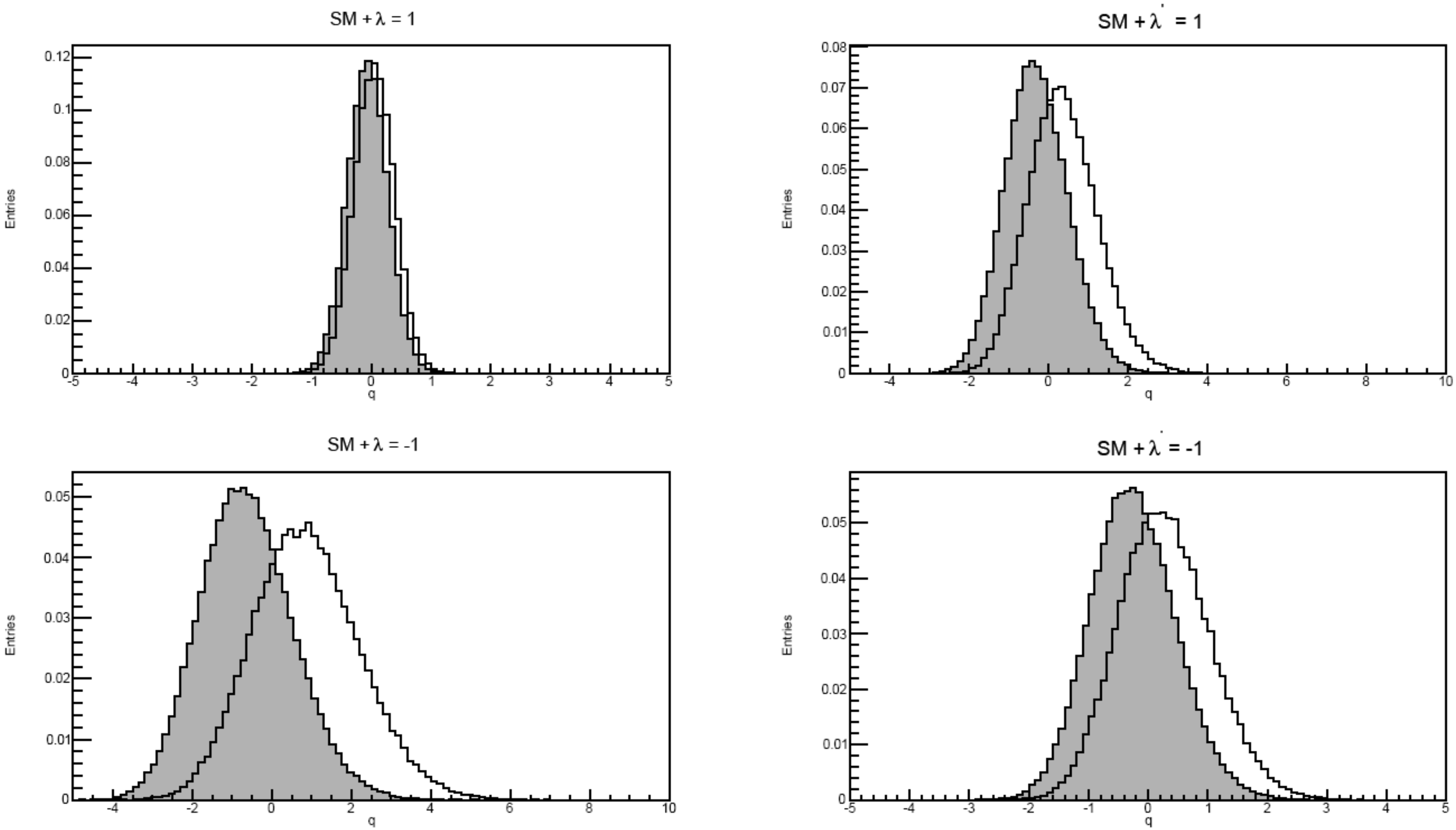




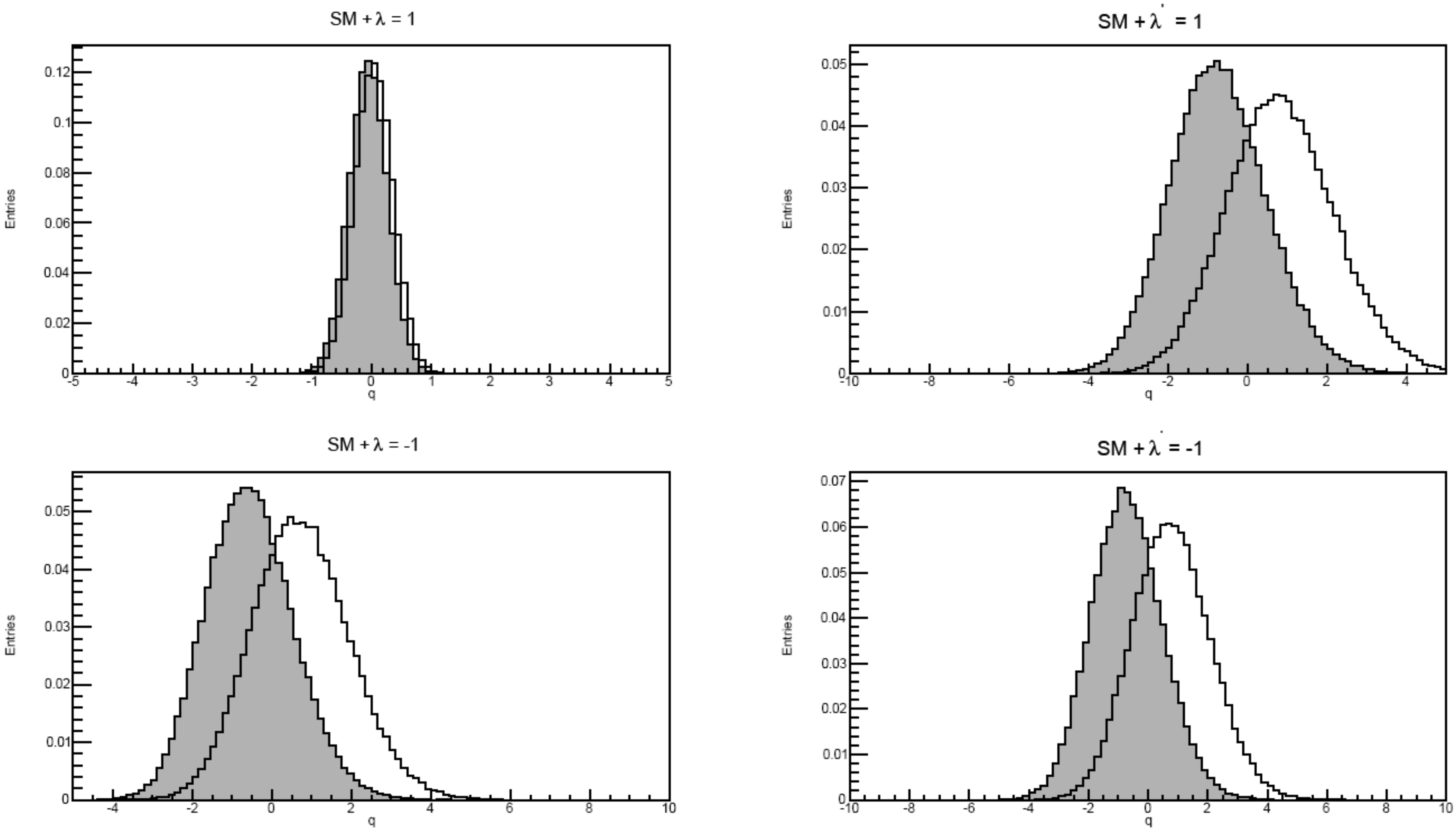




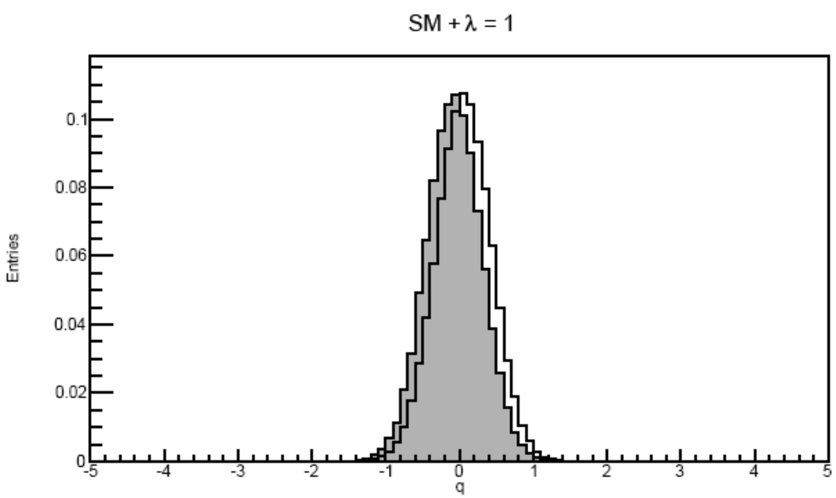
Theta – 130GeV

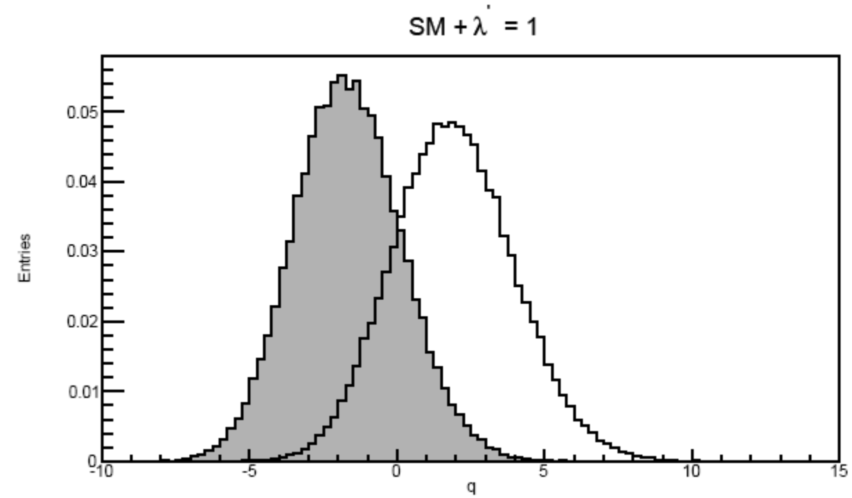


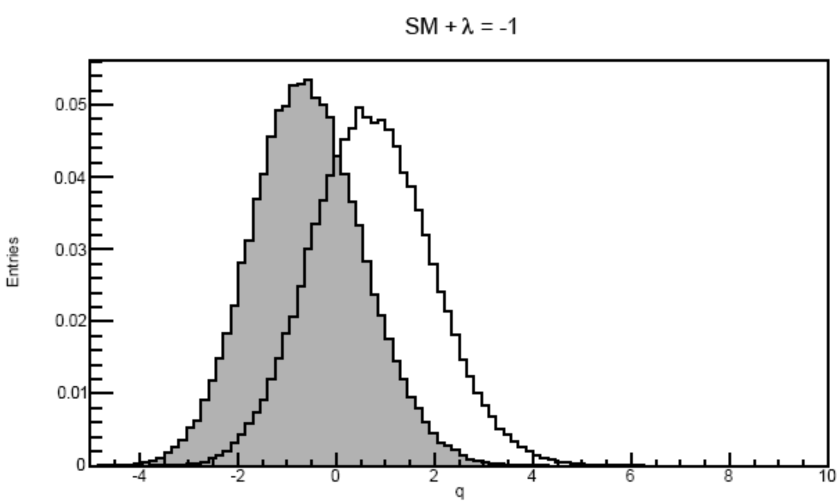
Theta – 135GeV

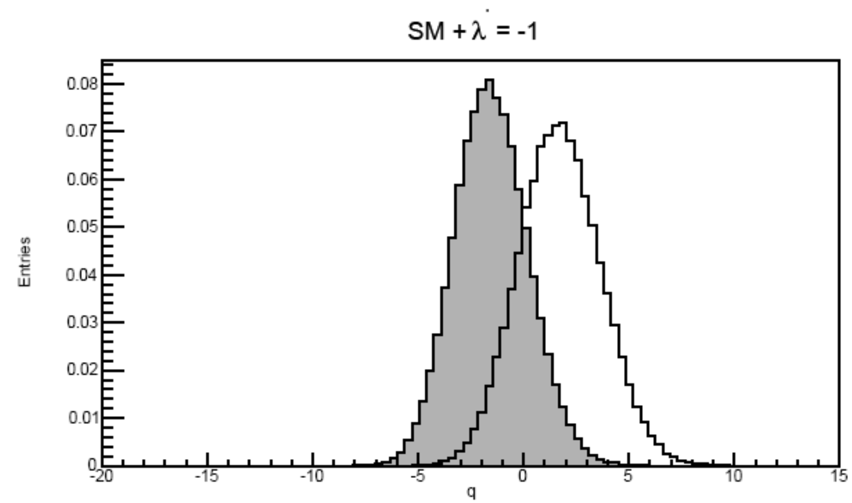


Theta – 140GeV

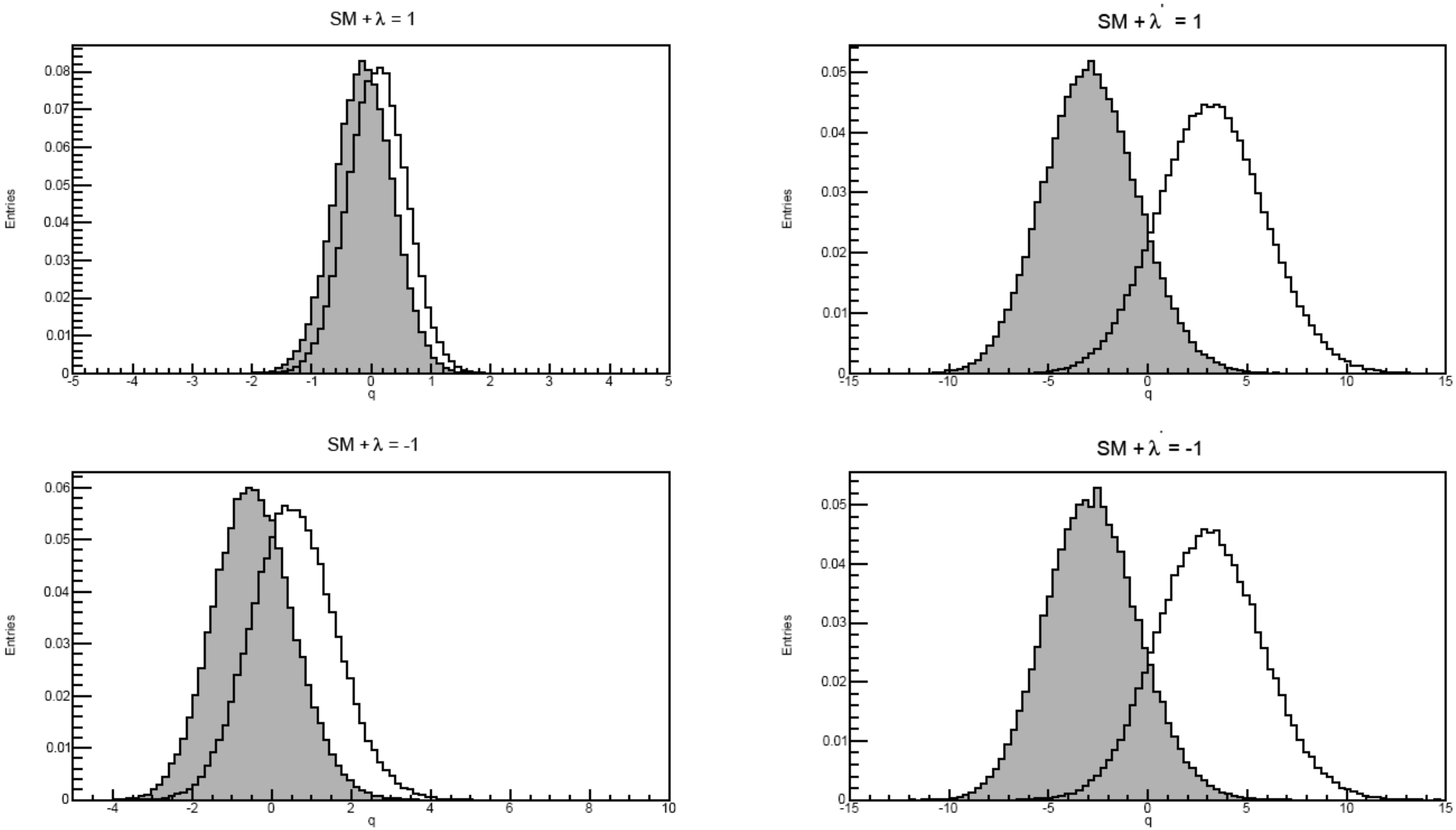




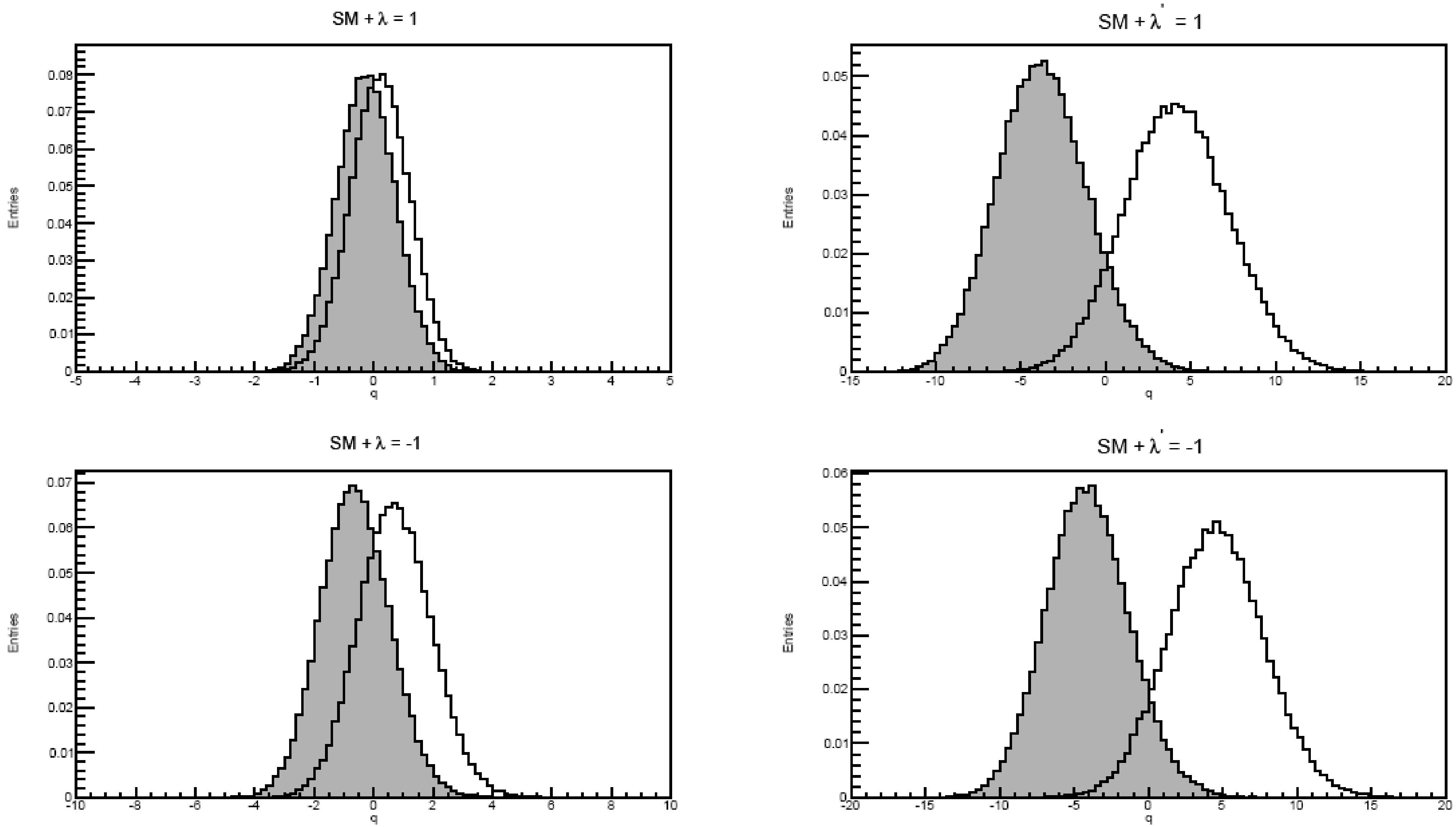




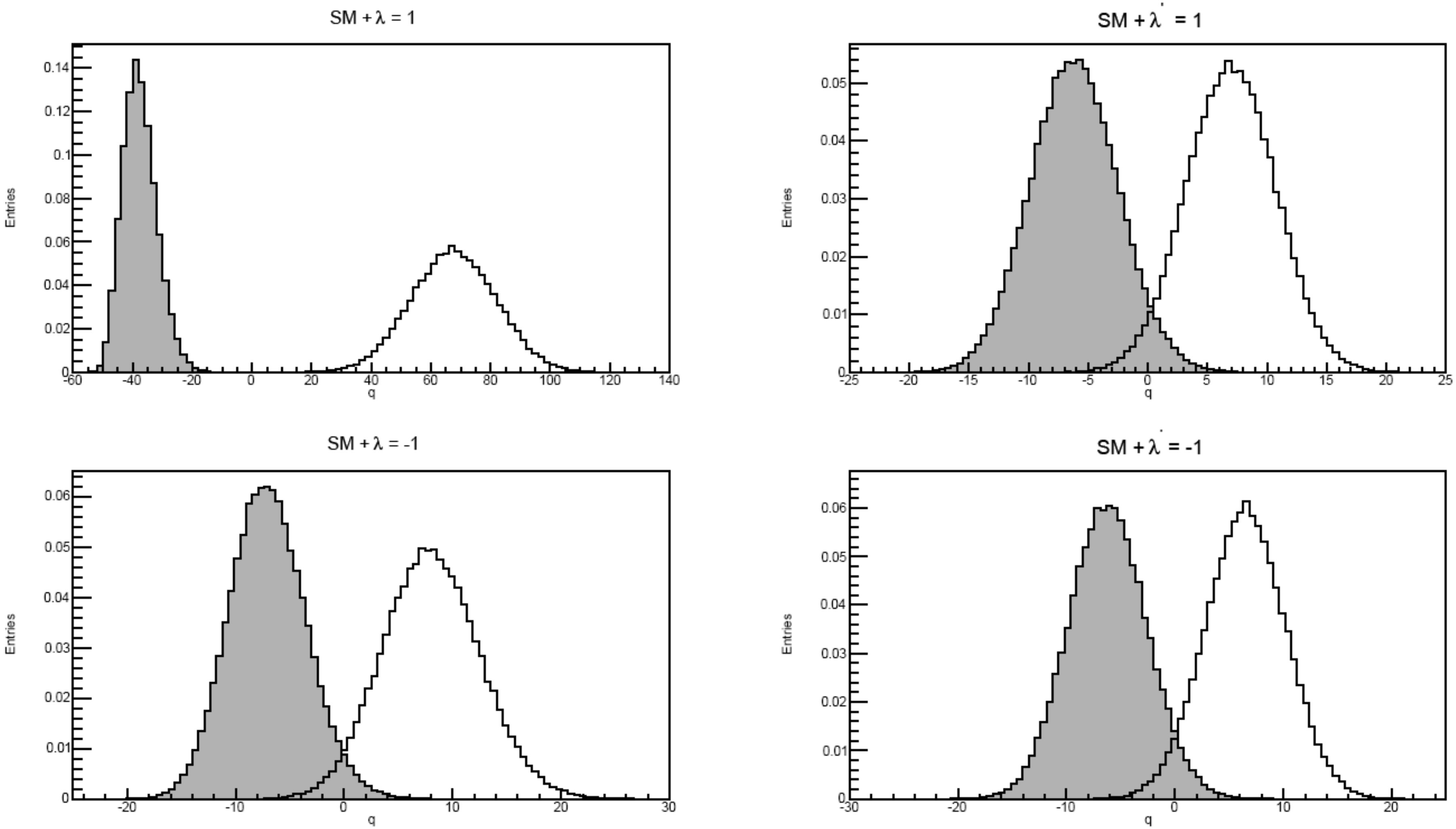
Theta – 145GeV



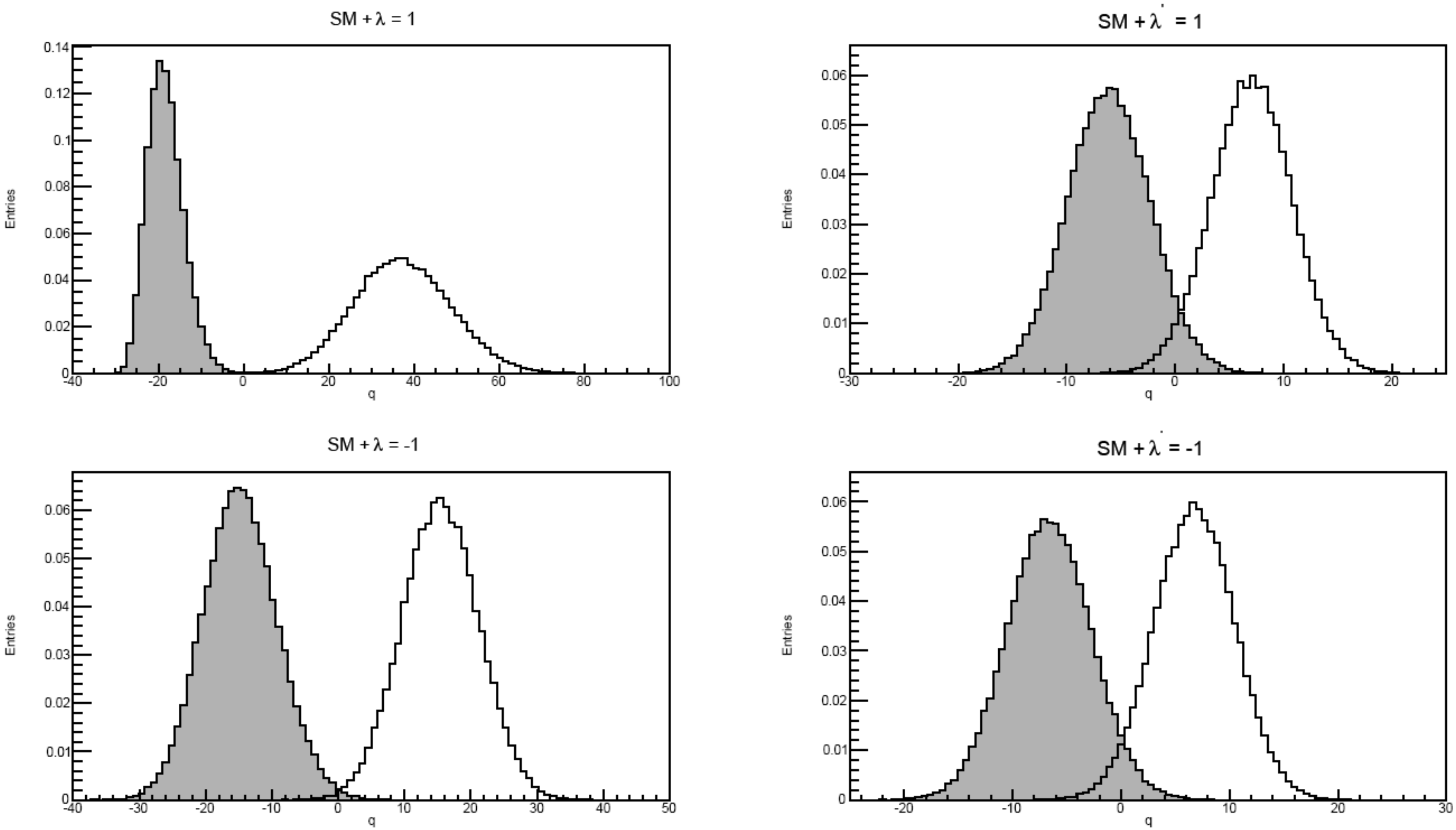
Theta – 150GeV



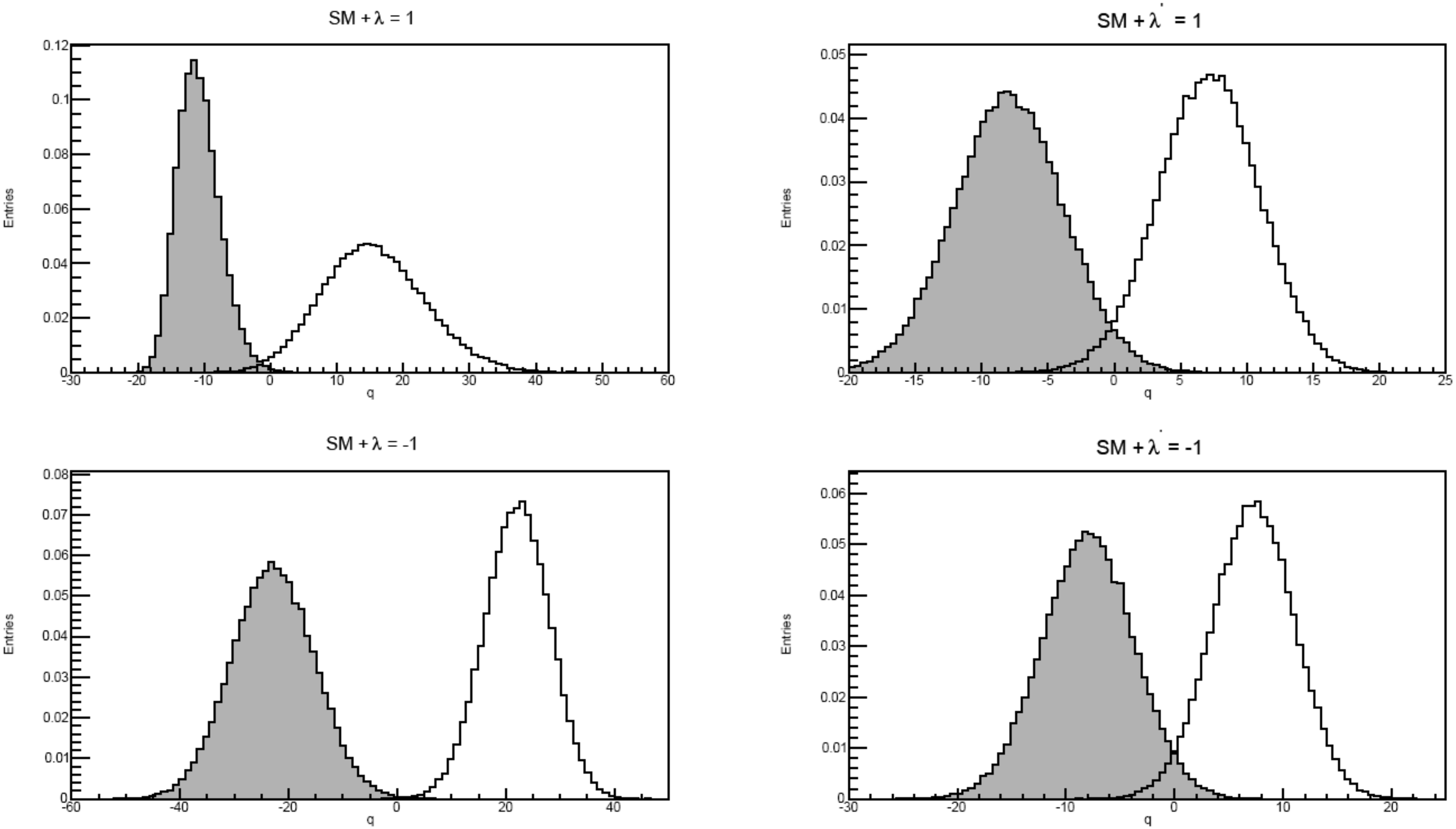
2D – 125GeV



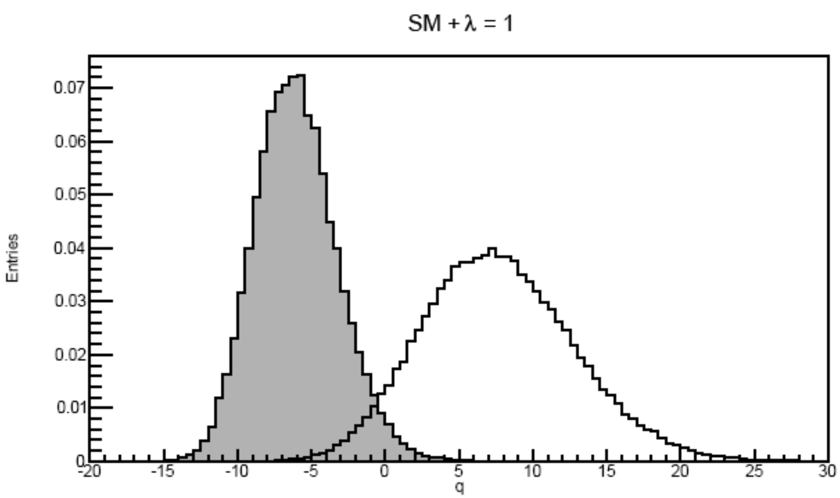
2D – 130GeV

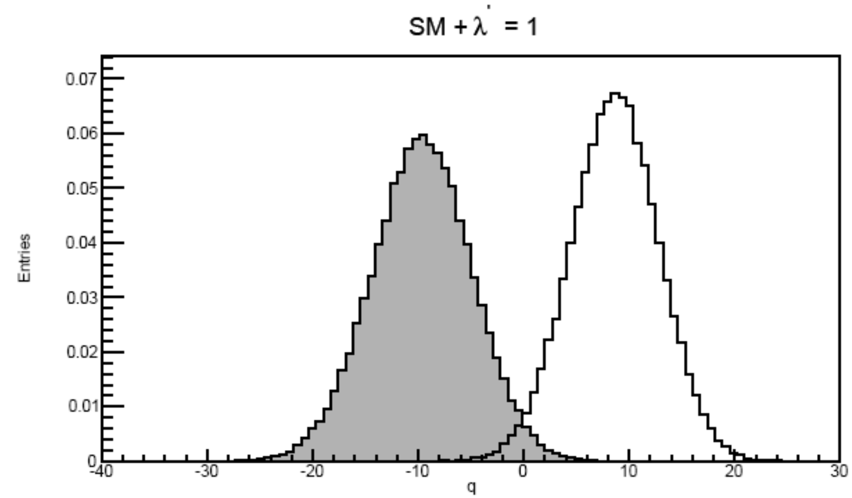


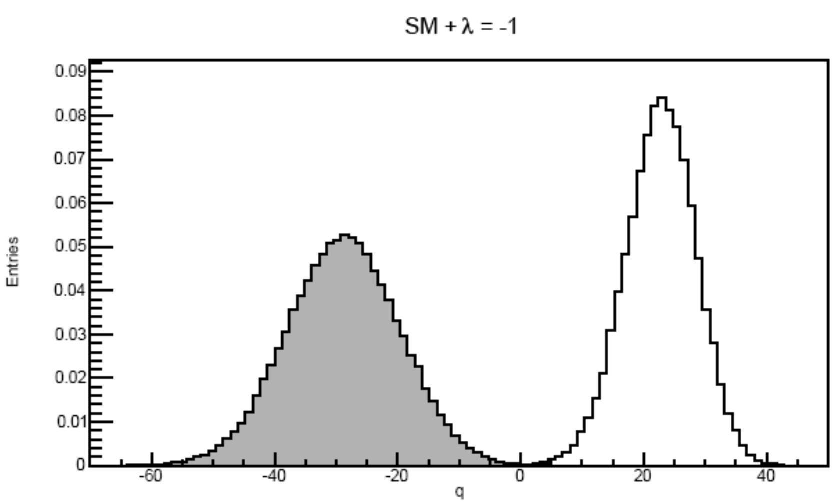
2D – 135GeV

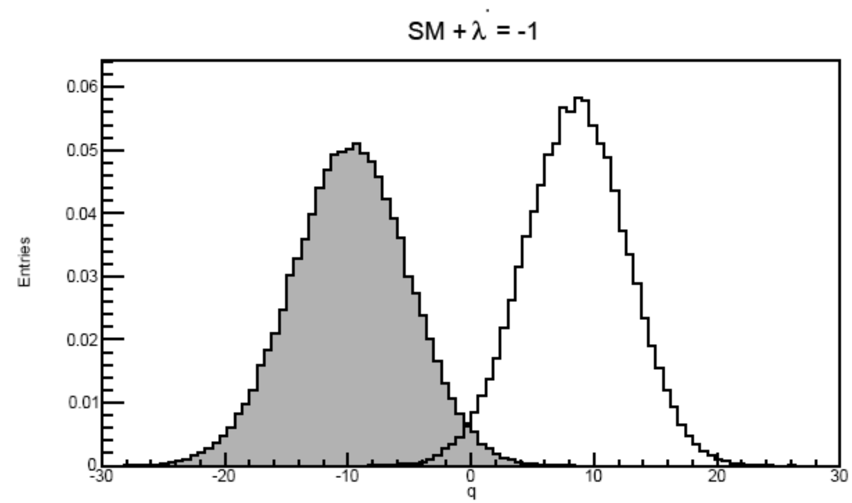


2D – 140GeV

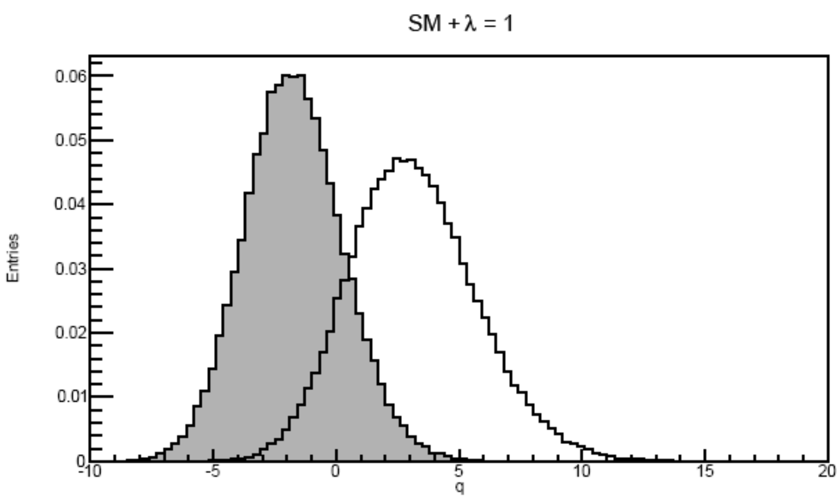


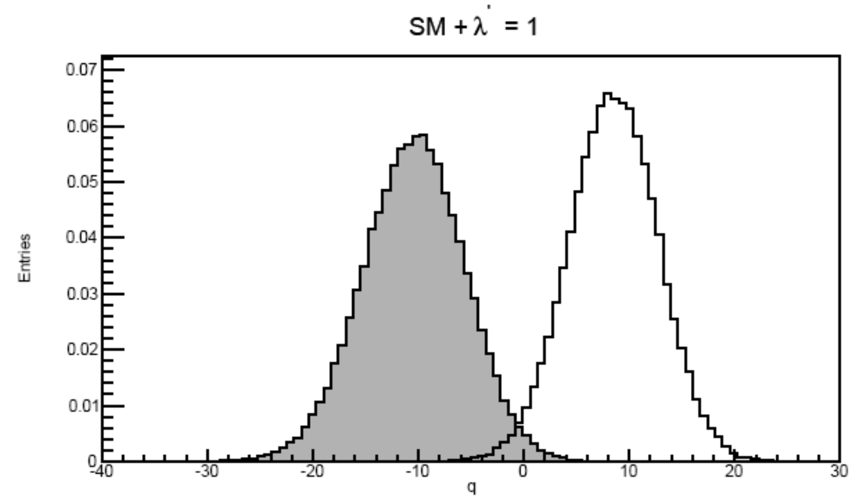


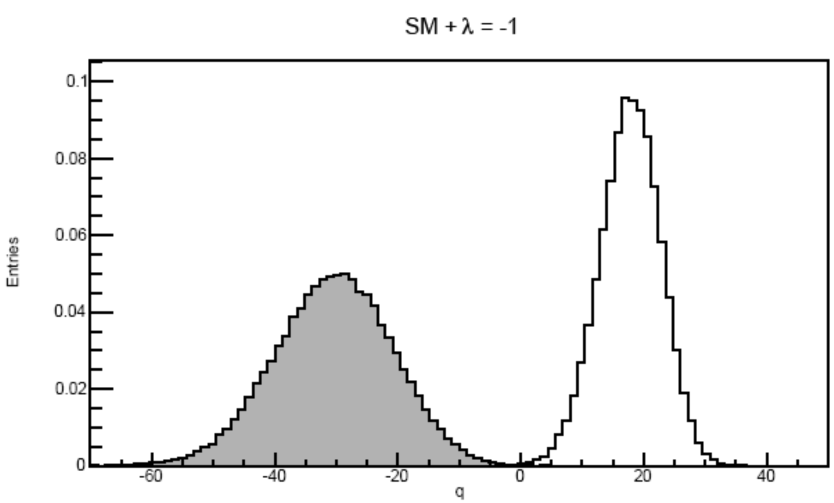


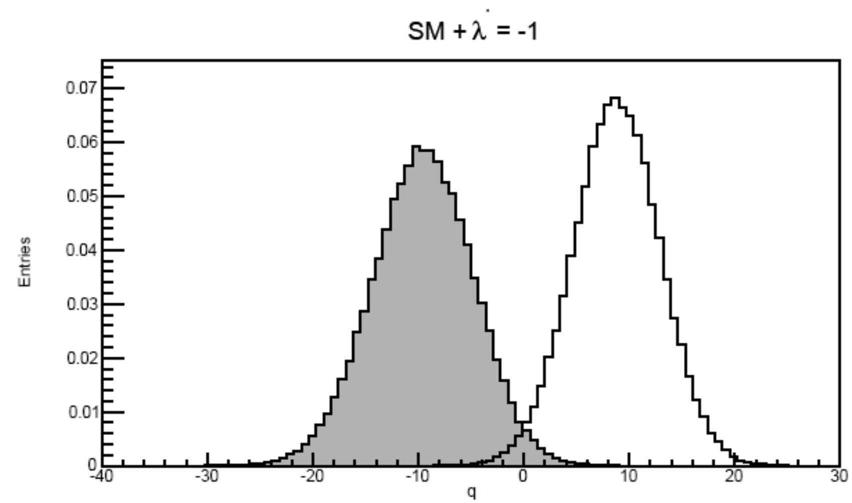


2D – 145GeV









2D – 150GeV