The Energy Momentum Tensor associated with Hard Parton production in Finite Time

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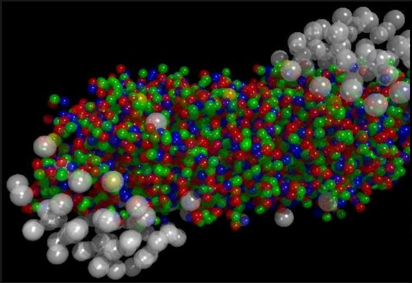
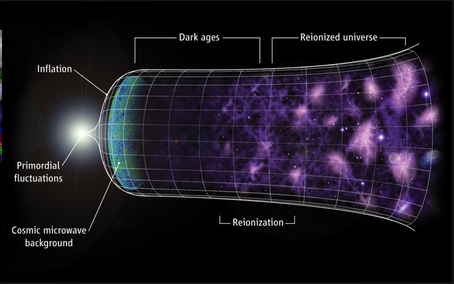
1 - 5 Dec, 2014

SA CERN

What is the Quark Gluon Plasma?

- A new Fundamental state of matter, that we don't yet fully understand
- Dominated the universe at $\sim 10^{-34}$ seconds after the big bang





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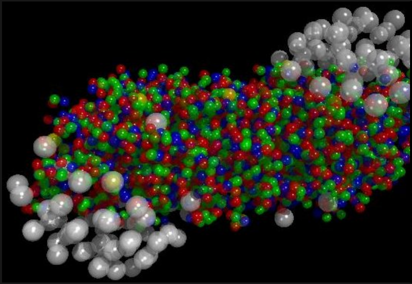

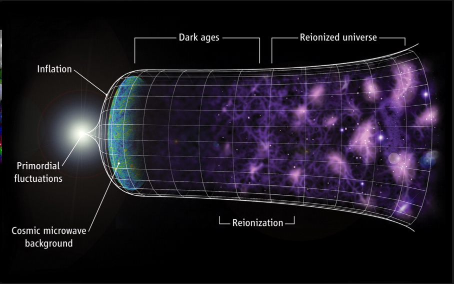

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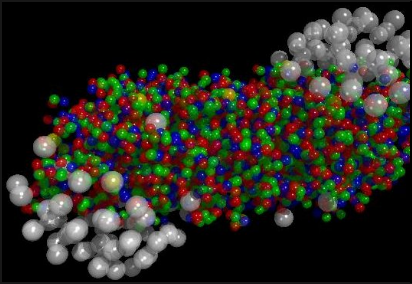

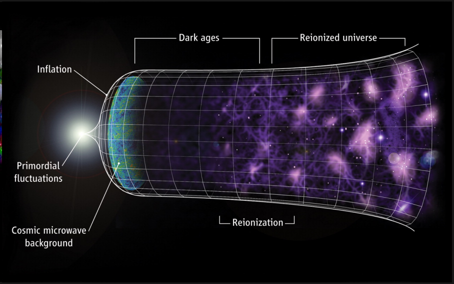

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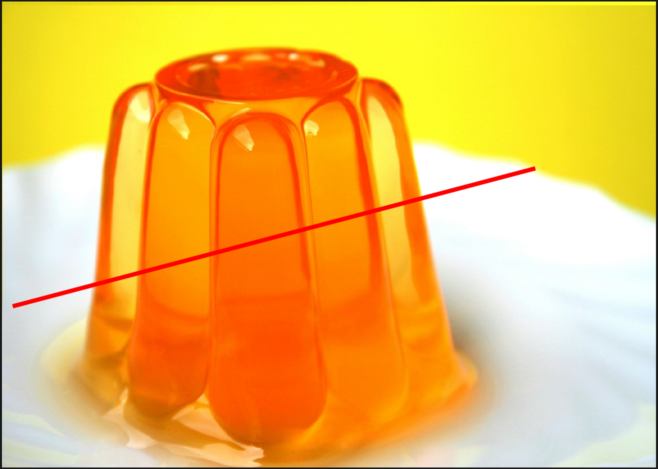

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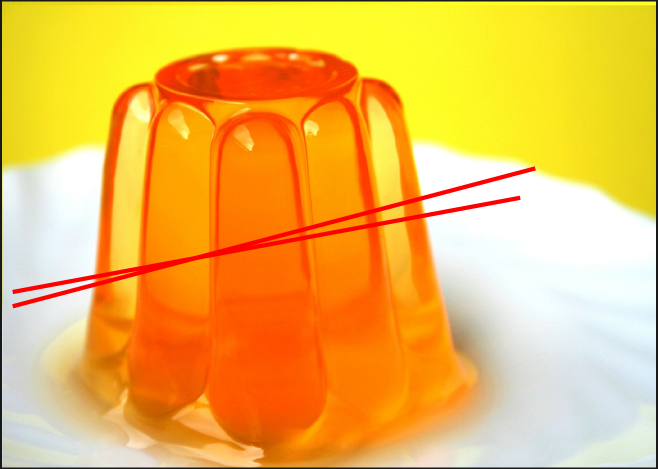

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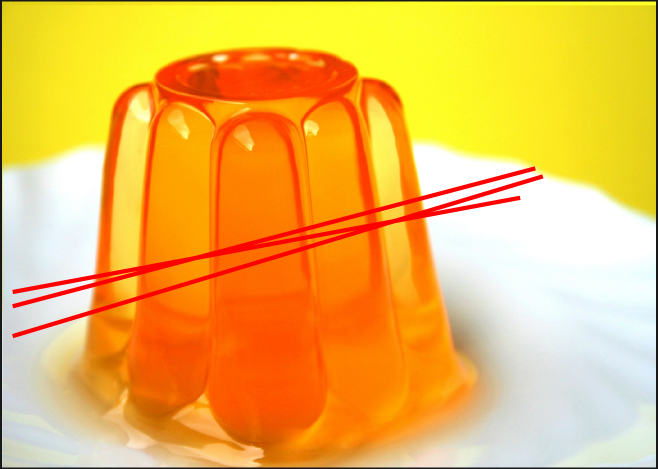

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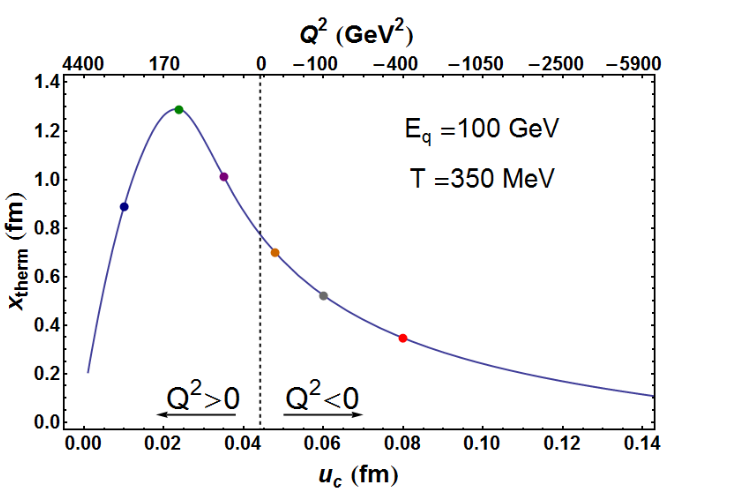

Figure: Stopping Distance as a function of radial u_c. R. Morad & W A Horowitz (1409.7545)

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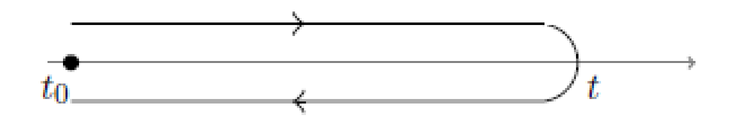

Figure: Schwinger-Keldysh Contour

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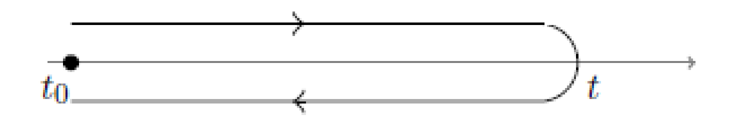

Figure: Schwinger-Keldysh Contour

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi + \partial_\mu\psi\partial_\nu\psi - g_{\mu\nu}\mathcal{L}$$
$$\mathcal{L} = \frac{1}{2}\left(\partial\psi\right)^2 - \frac{1}{2}m_\psi^2\psi^2 + \frac{1}{2}\left(\partial\phi\right)^2 - \frac{1}{2}m_\phi^2\phi^2 - g\psi\phi\psi$$

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$$\Theta_{\mu\nu} = T_{\mu\nu} - \frac{1}{4}\frac{(n-2)}{(n-1)}\left(\partial_\mu\partial_\nu - g_{\mu\nu}\right)\phi^2$$

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$$\langle\phi(x)\phi(x)\rangle = \langle\phi(x)\rangle^2 + (\Delta\phi(x))^2$$
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So we find $\langle\phi(x)\rangle$ and plug in

$$\langle T_{\mu\nu}(x)\rangle \approx T_{\mu\nu}\left(\langle\phi(x)\rangle\right)$$

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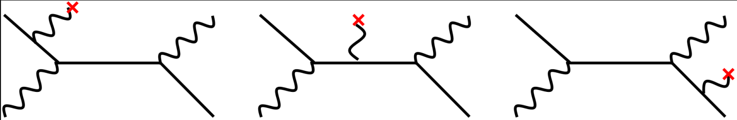

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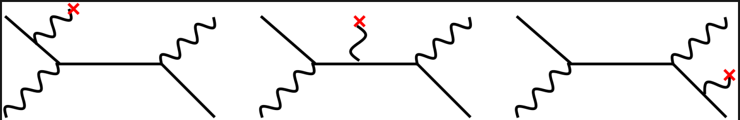

Figure: Typical Diagrams for $\langle \phi(x) \rangle$ for $|\text{in}\rangle = |\psi\phi\rangle$

$$\langle \phi(x) \rangle = -ig \int d^4z \, D_R(x-z) \langle \text{in} | \psi(z)\psi(z) | \text{in} \rangle$$

(Generalizes for arbitrary H_{int})

Approximation: Calculate $\langle \phi(x) \rangle$ and Plug into $T_{\mu\nu}$

Choosing gaussian smeared wavepackets for the $|\text{in}\rangle = |\psi\rangle$ state with momentum width $1/\alpha$. We take $m_\psi \to \infty$

$$\boxed{\langle \phi(x) \rangle = \frac{g}{m_\psi} \int d^3z \left(\frac{e^{-m_\phi |\vec{z}|}}{|\vec{z}|} \right) \frac{e^{-\frac{(\vec{x}-\vec{z})^2}{\alpha}}}{\sqrt{4\pi\alpha}^3}}$$

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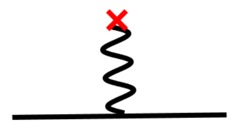
$\partial_\alpha\langle\phi(x,\alpha)\rangle = \partial^2_{\vec{x}}\langle\phi(x,\alpha)\rangle$



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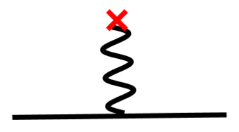
Solution to the Diffusion equation
$\partial_\alpha \langle \phi(x, \alpha) \rangle = \partial^2_{\vec{x}} \langle \phi(x, \alpha) \rangle$

with initial condition
$\phi(x, 0) = \frac{e^{-m_\phi |\vec{x}|}}{|\vec{x}|}$.



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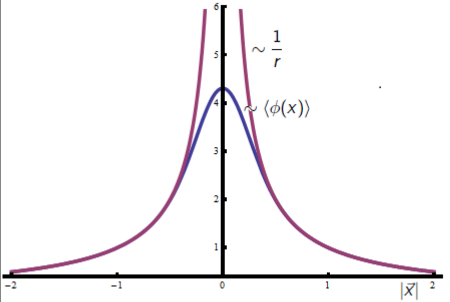

Conditional Expectation Value

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- We want to refine our result by specifying the final state

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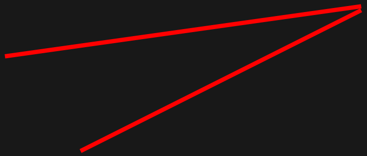

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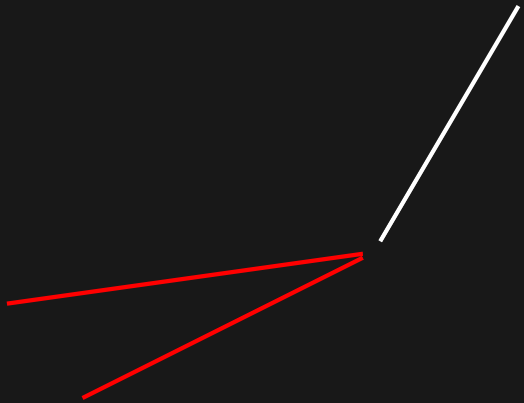

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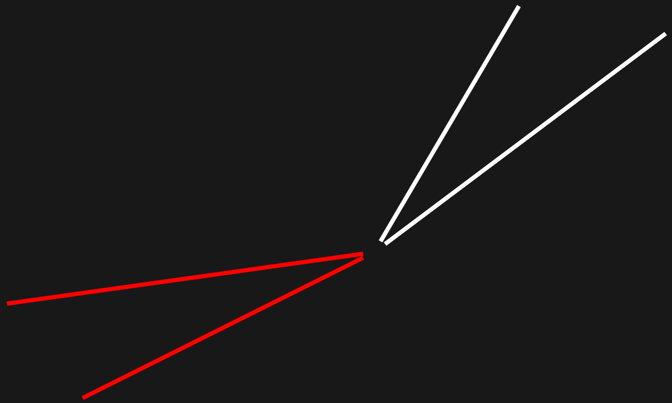

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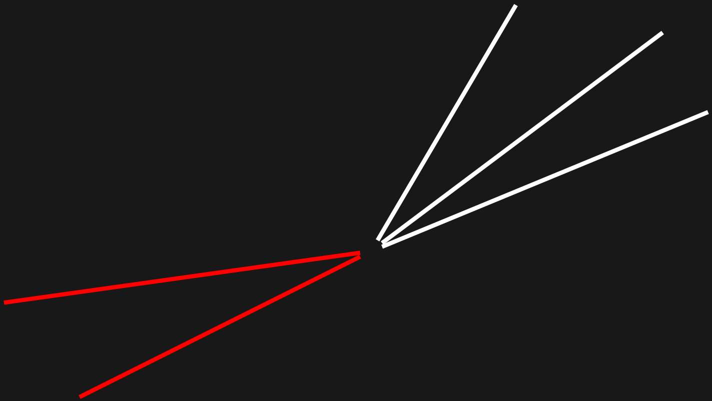

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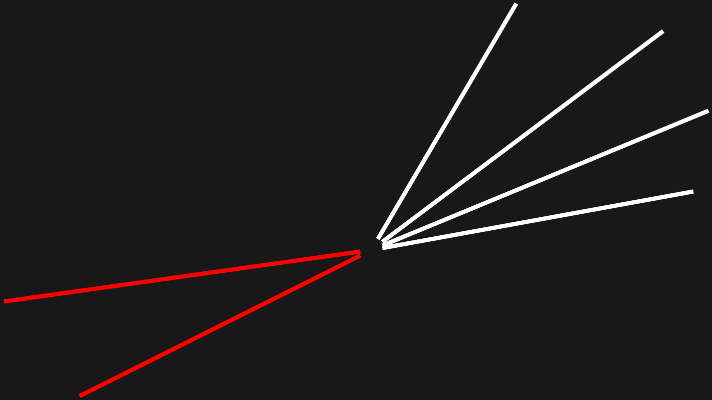

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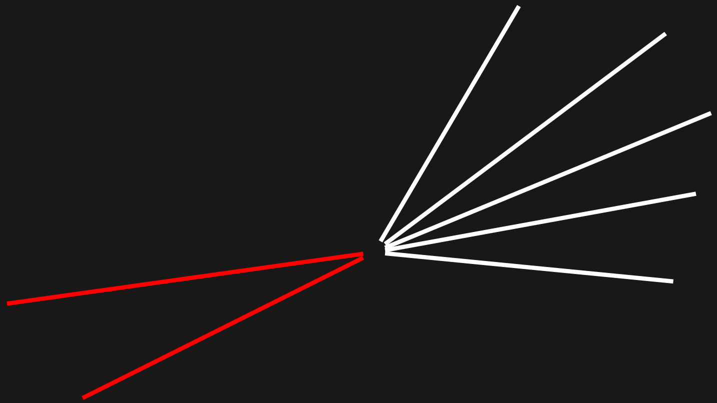

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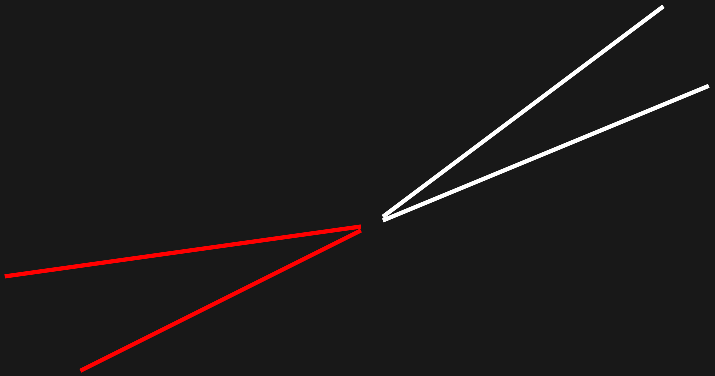

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$$E[\hat{O}(x)|\ |\text{in}\rangle, |\text{out}\rangle] = \sum_i O_i P(|q_i\rangle|\ |\text{in}\rangle, |\text{out}\rangle)$$

$$= \frac{\langle\text{in}|\hat{\Theta}_M \hat{O}(x) \hat{\Theta}_M|\text{in}\rangle}{\langle\text{in}|\hat{\Theta}_M|\text{in}\rangle}$$

where $\hat{\Theta}_M = |\text{out}\rangle\langle\text{out}|$ is the projection operator built from the out states.

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This result is actually a generalization of Baye's Theorem.

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The End.

Back up Slides

Data for QGP

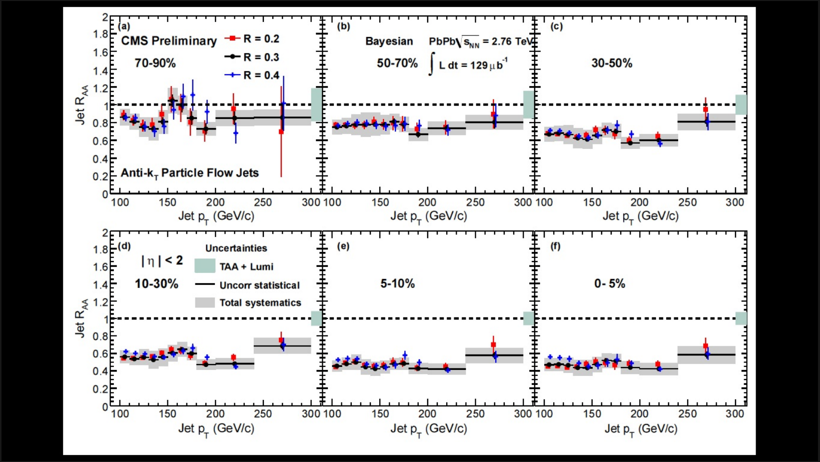

Figure: CMS Preliminary data for Jet R_{AA}. Taken from (1409.7545)

Conditional Expectation Value: Example

For $|\text{in}\rangle = |\psi\rangle$, $|\text{out}\rangle = |\psi'\rangle$ we find

$$E[\phi(x)|\ |\psi\rangle, |\psi'\rangle] = \langle\psi'|\phi(x)|\psi'\rangle + 2i\text{Im}\left(\frac{\langle\psi'|\phi(x)|\psi\rangle}{|\langle\psi'|\psi\rangle|}\right)$$

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This says that if $|\psi\rangle \neq |\psi'\rangle$, the expectation of the field is complex.