Two-loop renormalisation in UED models

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Motivation

In particle physics one of the major issues is to explain the fermion mass hierarchy and their mixings

A completely satisfactory theory addressing this is certainly lacking at present

Models which can be used to explain this (as well as EWSB, gauge hierarchies etc.), are those with extra spatial dimensions

To explore the physics at a high energy scale we use RGEs to probe the momentum dependence of couplings

So we can consider one goal of the LHC as uncovering any new dynamics in the TeV range, where models with XDs may bring down the unification to a much lower energy scale.

But after already investigating the 1-loop RGEs, why study to 2-loops?

As we observed a power law running, this may be unstable to higher-orders

With the addition of higher-orders, our definition of model cut-offs will also change

The UED model

Here we will study the minimal UED model (one extra dimension compactified on a circle of radius R with a Z_2 orbifolding)

The 5D KK expansions of the fields are (where the corresponding coupling constants among the KK modes are simply equal to the SM couplings up to normalization):

$$H(x,y) = \frac{1}{\sqrt{\pi R}} \left\{ H(x) + \sqrt{2} \sum_{n=1}^{\infty} H_n(x) \cos\left(\frac{ny}{R}\right) \right\} \ ,$$

$$u(x,y) = \frac{1}{\sqrt{\pi R}} \left\{ u_R(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[u_R^n(x) \cos\left(\frac{ny}{R}\right) + u_L^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\} ,$$

$$Q(x,y) = \frac{1}{\sqrt{\pi R}} \left\{ q_L(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[Q_L^n(x) \cos\left(\frac{ny}{R}\right) + Q_R^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\} ,$$

etc.

The zero modes are the 4D SM fields the complex scalar field H is Z_2 even field, and there is a left- and a right-handed KK mode for each SM chiral fermion

Note that in models with UED momentum conservation in the XDs, we are led to the conservation of KK number at each vertex in the interactions of the 4D eff. theory

The gauge couplings $g_3^{(5D)}$, $g_2^{(5D)}$ and $g_1^{(5D)}$ are related to the 4D SM couplings

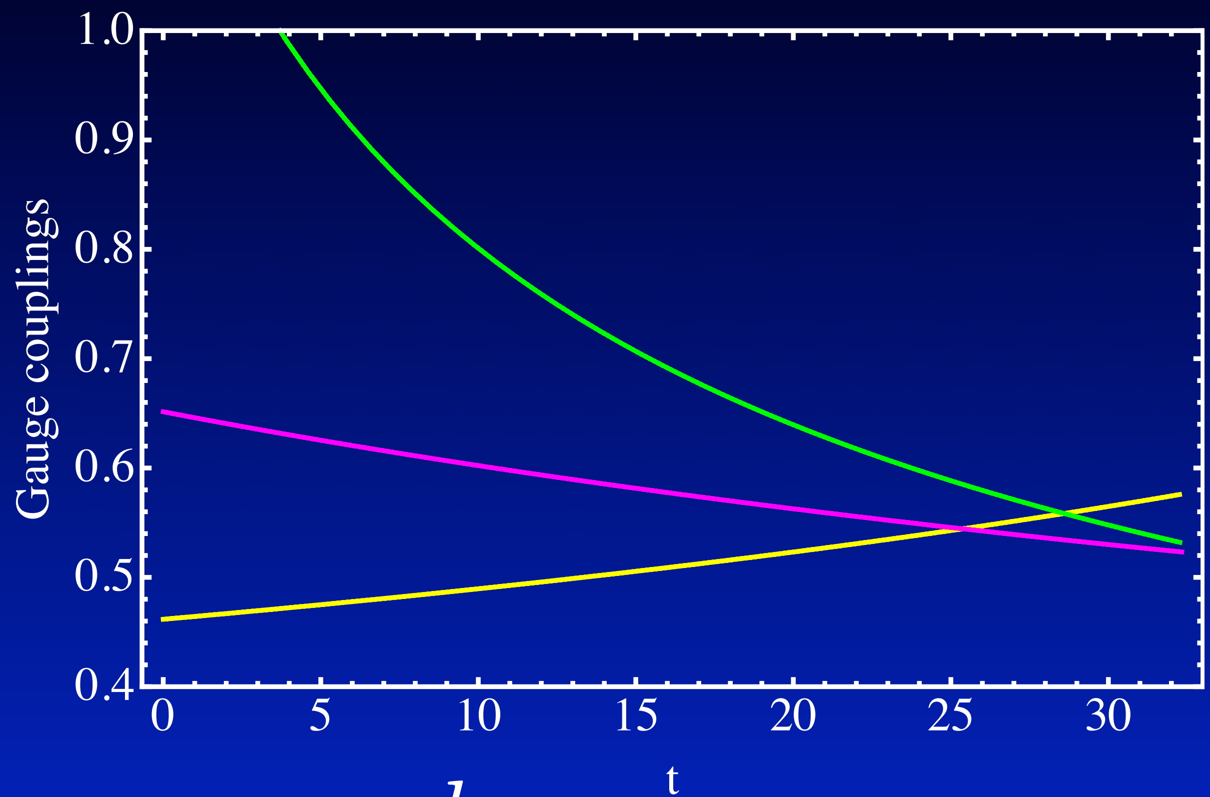
$$g_i = g_i^{(5D)}/\sqrt{\pi R}$$

Note that g_1 is chosen to follow the conventional $SU(5)$ normalization

After integrating out the compactified dimension, the 4D effective Lagrangian has interactions involving the zero mode and the KK modes.

However, these KK modes cannot affect EW processes at tree level, and only contribute to higher order EW processes

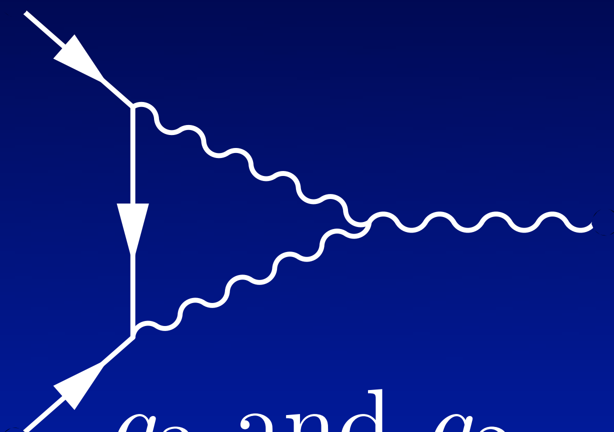
Gauge coupling constants



$$16\pi^2 \frac{dg_i}{dt} = b_i^{SM} g_i^3 \ ,$$

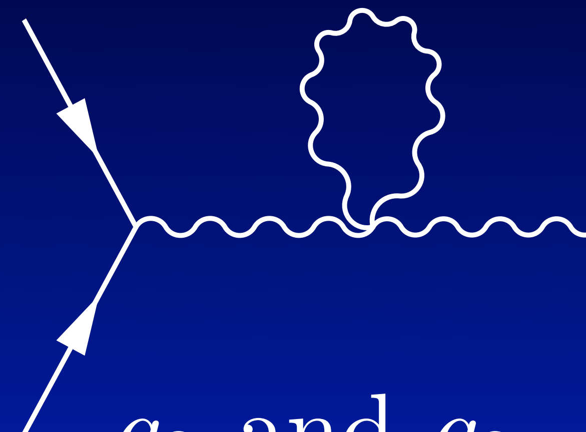
where $b_i^{SM} = (\frac{41}{10}, -\frac{19}{6}, -7), t = ln(\mu/M_Z)$.
This leads to unification around 10^{14} GeV.

That is, we have diagrams such as:

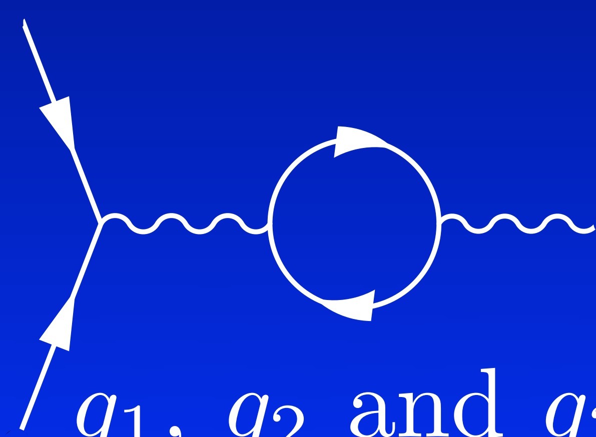


g_3 and g_2

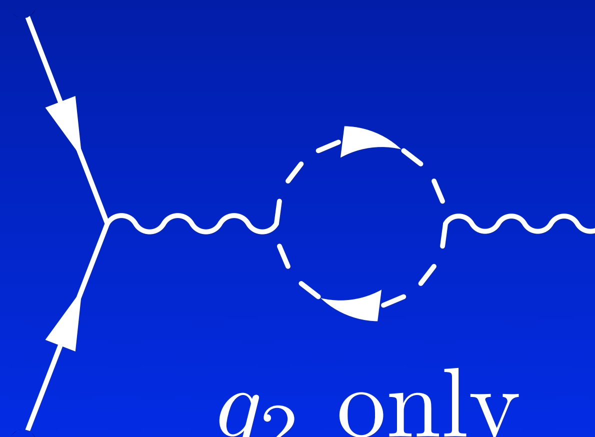
g_3 and g_2



g_3 and g_2



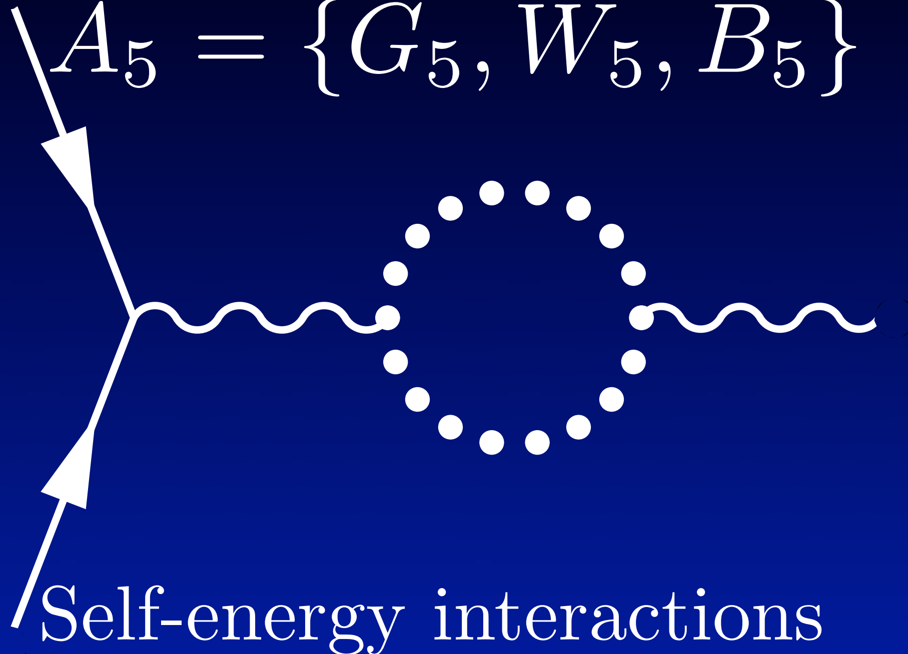
g_1, g_2 and g_3



g_2 only

One-loop

$$A_5 = \{G_5, W_5, B_5\}$$



Self-energy interactions with the A_5^n scalar fields

At each KK level the one-loop corrections exactly mirroring those of the SM ground states

Plus self-energy contributions from the fifth component of the gauge field A_M

$$A_5(x,y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} A_5^n(x) \sin\left(\frac{ny}{R}\right)$$

For the closed fermion loop diagrams, the contributions from both the left- and right-handed KK modes of each chiral fermion need to be counted

Between the R^{-1}, where the first KK states are excited, and the cutoff Λ, there are finite quantum corrections to the couplings from the ΛR KK states.

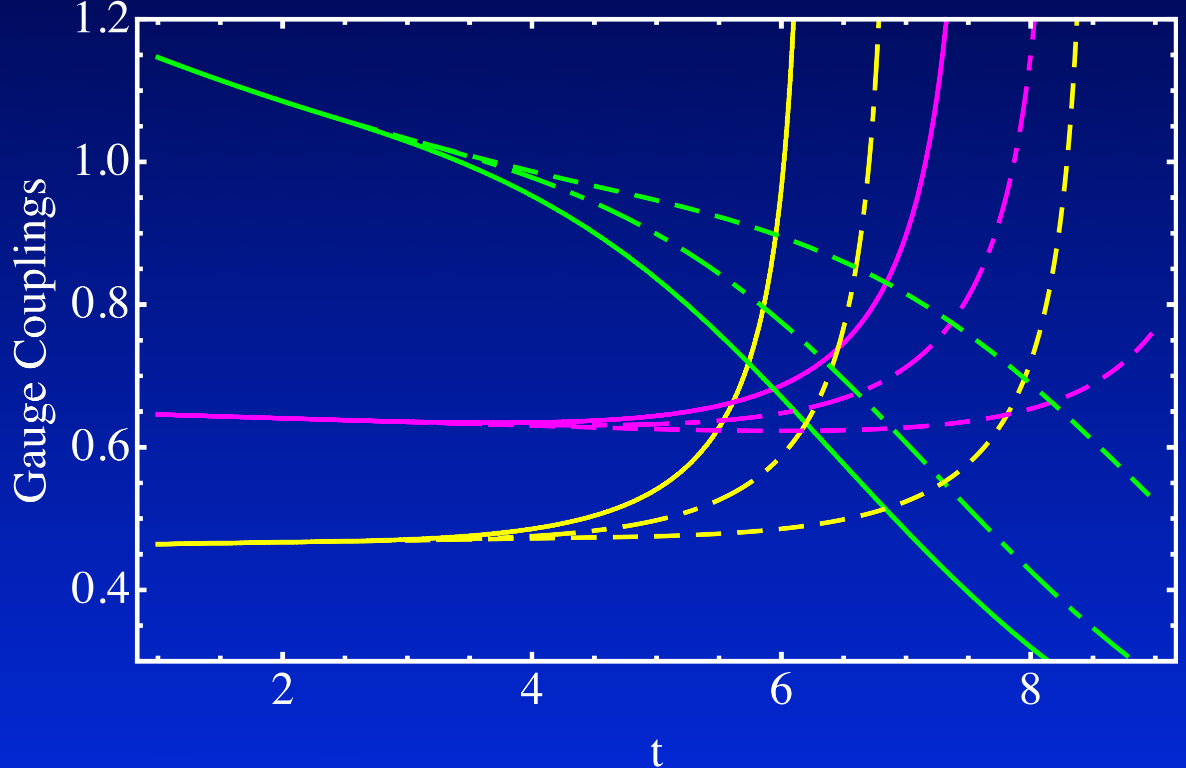
Up to R^{-1} the RG evolution is logarithmic, controlled by the SM β fns.

With increasing energy, new excitations come into play until the next threshold is reached. Therefore, the one-loop RGE becomes:

$$16\pi^2 \frac{dg_i}{dt} = \left[{b_i}^{SM} + (S(t) - 1)\tilde{b}_i \right] {g_i}^3 \ ,$$

where $S(t) = e^t M_Z R$, and $\tilde{b}_i = \left(\frac{81}{10}, \frac{7}{6}, -\frac{5}{2} \right)$

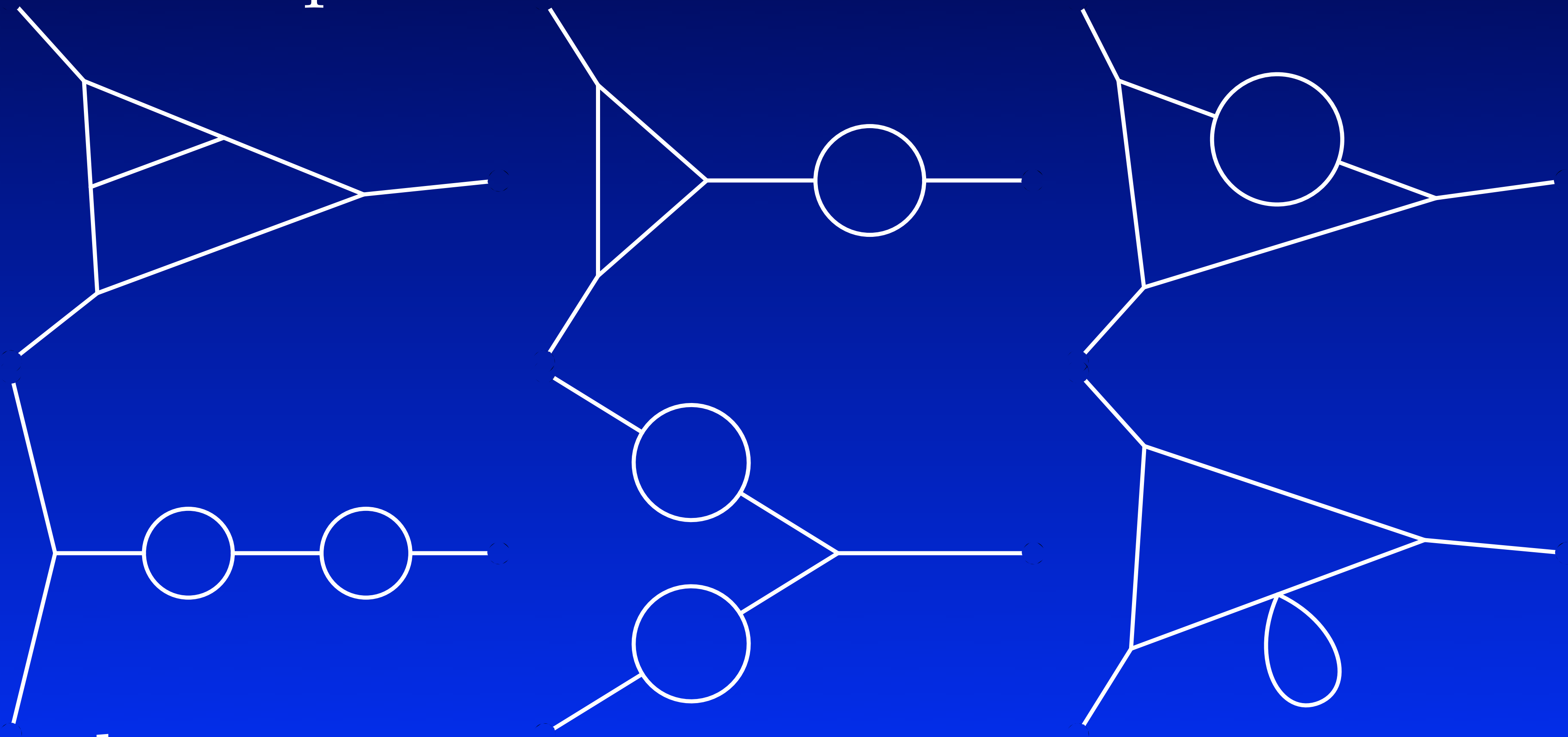
We can see that the dependence drastically changes the normal one-loop running, and lowers the unification scale considerably.



The XDs naturally lead to GUTs at scales substantially below the usual GUT scale.

Two-loop

Calculating the 2-loop diagrams we have $\mathcal{O}(100)$ topologies, as opposed to $\mathcal{O}(10)$ at 1-loop:



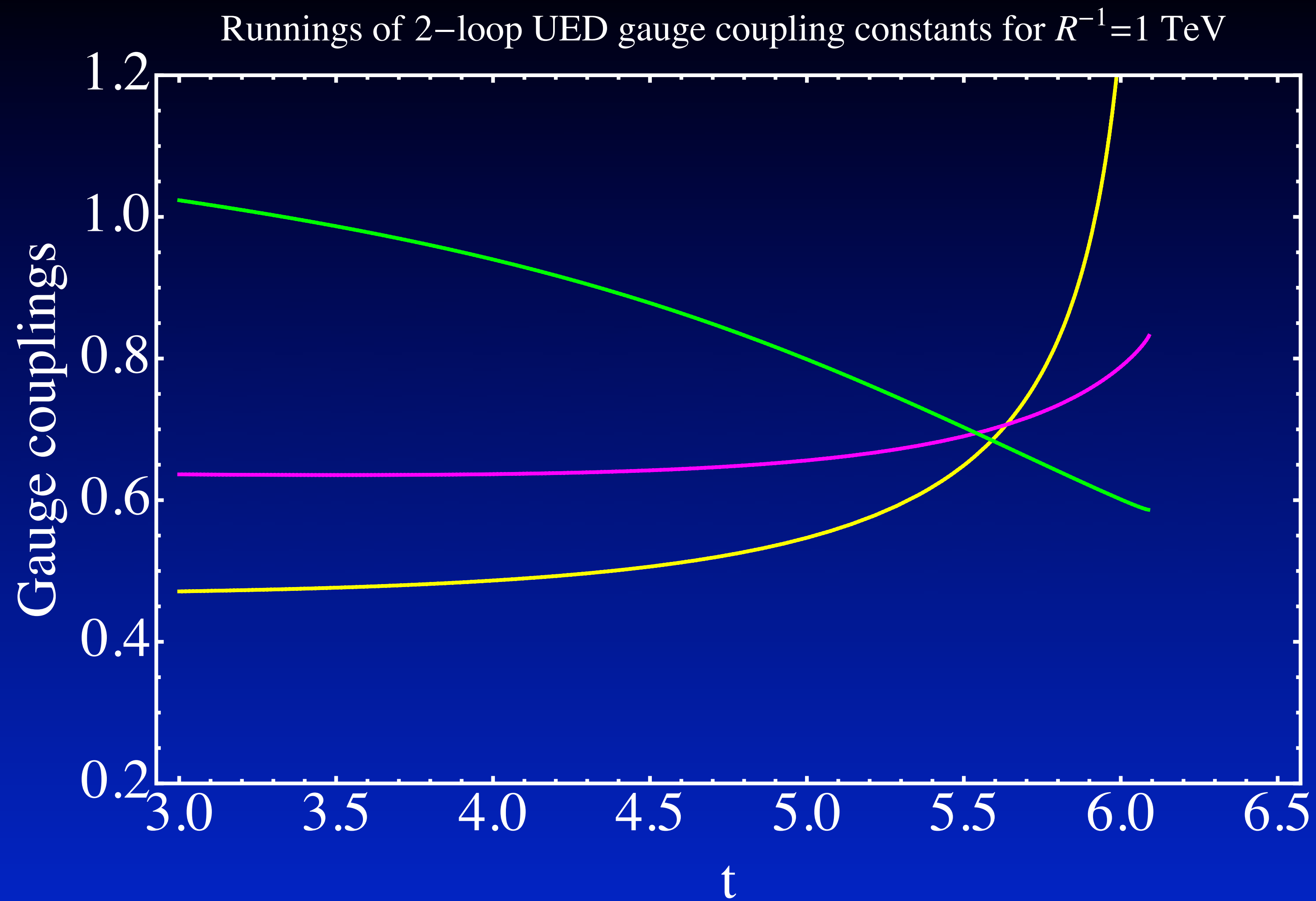
plus many, many more...

This leads to additions to the β functions of the gauge couplings:

$$
\begin{aligned}
(4\pi)^4 g_1^{-3}\beta_{g_1}^{(2)} &= (S(t)^2-1)\left(\frac{199}{50}g_1^2+\frac{27}{10}g_2^2+\frac{44}{5}g_3^2\right.\\
&\quad \left.-\mathrm{Tr}\left(\frac{17}{5}Y_u^\dagger Y_u+Y_d^\dagger Y_d+3Y_e^\dagger Y_e\right)\right),\\
(4\pi)^4 g_2^{-3}\beta_{g_2}^{(2)} &= (S(t)^2-1)\left(\frac{9}{10}g_1^2+\frac{35}{6}g_2^2+12g_3^2\right.\\
&\quad \left.-\mathrm{Tr}\left(3Y_u^\dagger Y_u+3Y_d^\dagger Y_d+Y_e^\dagger Y_e\right)\right),\\
(4\pi)^4 g_3^{-3}\beta_{g_3}^{(2)} &= (S(t)^2-1)\left(\frac{11}{10}g_1^2+\frac{9}{2}g_2^2-26g_3^2\right.\\
&\quad \left.-4\mathrm{Tr}\left(Y_u^\dagger Y_u+Y_d^\dagger Y_d\right)\right).
\end{aligned}
$$

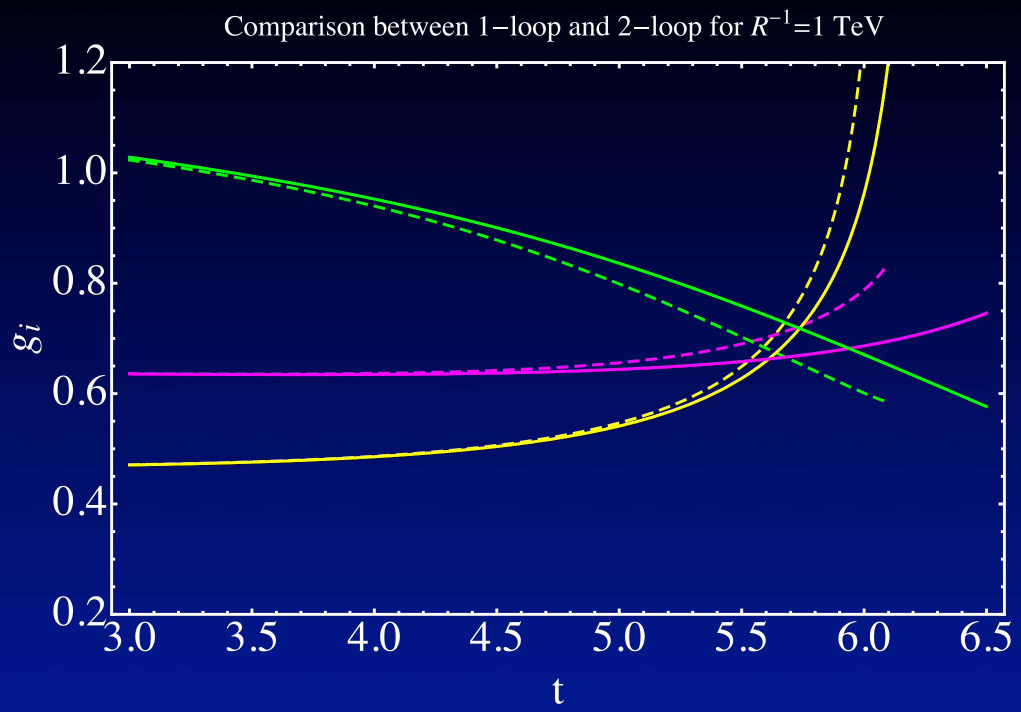
At 1-loop the RGEs are disentangled. However, for 2-loops the RGEs of the gauge and Yukawas are entangled.

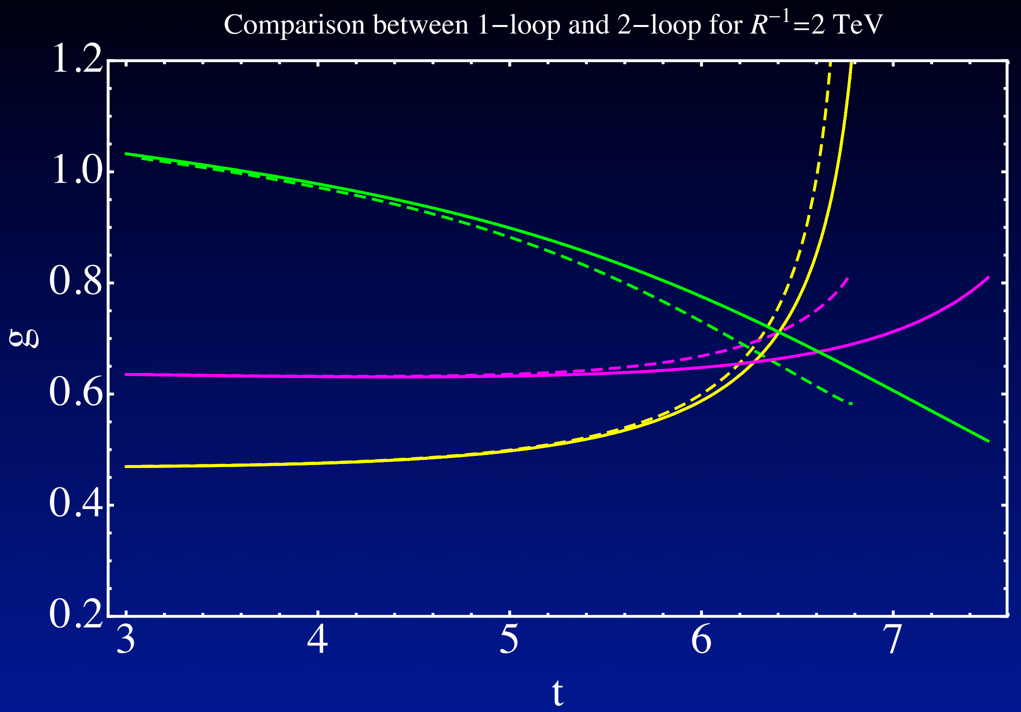
So we expect a few percent change on the evolution of gauge couplings due to the appearance of Yukawa couplings.



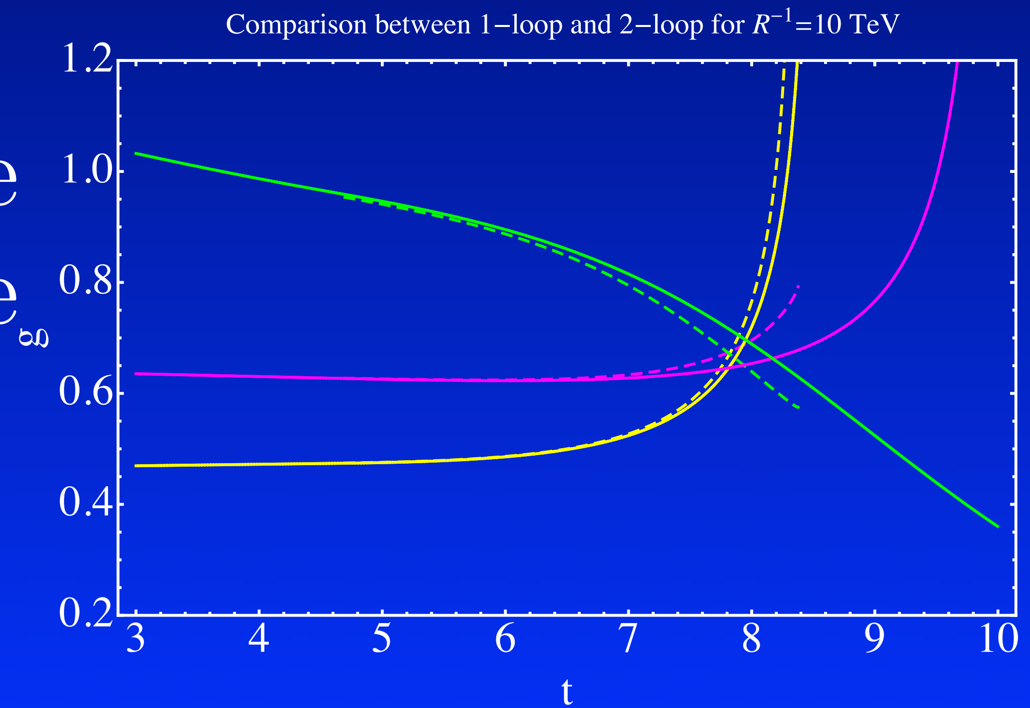
As expected the XD brings down the unification scale to a lower scale.

But now only slightly lower than before.

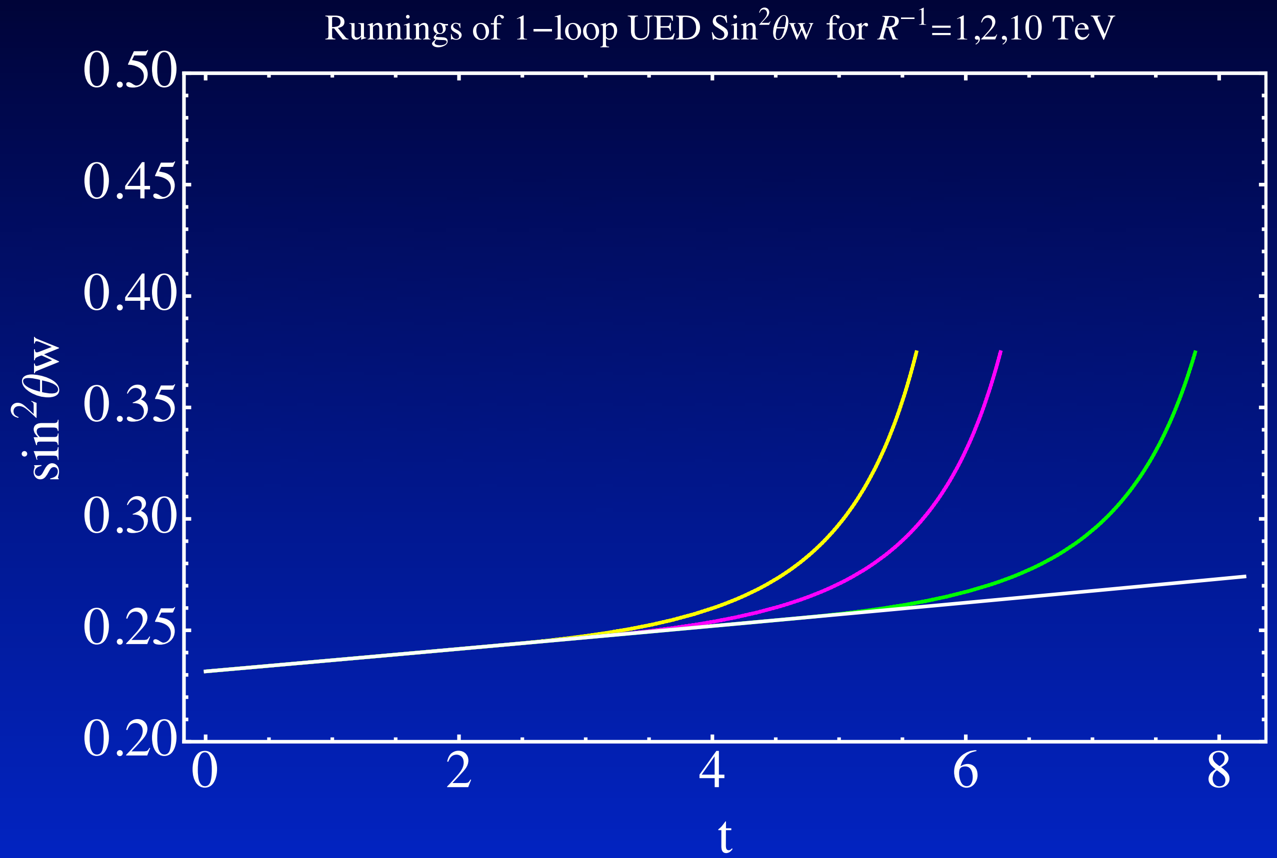




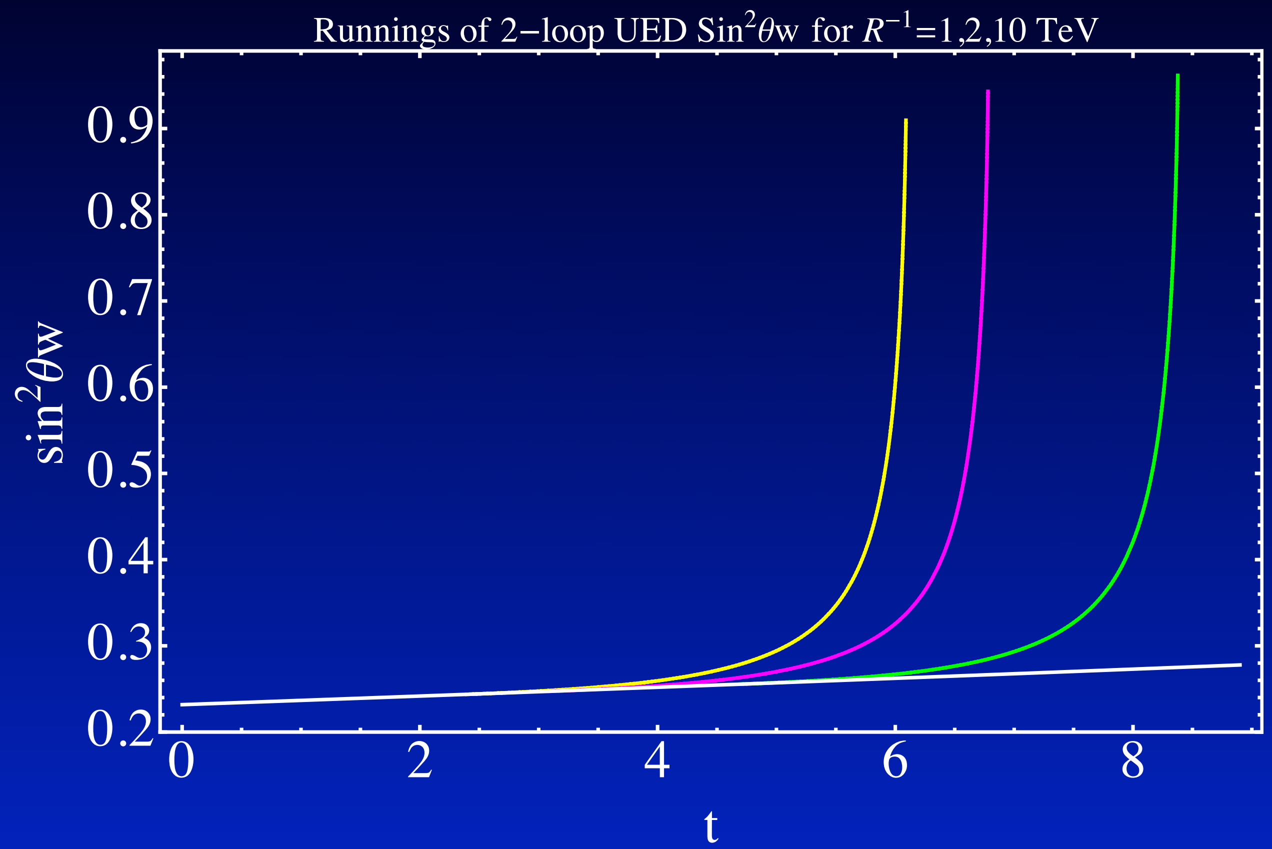
This also might change g_3 because of the large size of Y_t
(see cont. work).



Weinberg angle



In a previous work we studied how the Weinberg angle ran, observing a sizable increase in its value



The addition of 2-loop effects seems to enhance this substantially

Conclusions

In XD models the 1-loop running of couplings is insufficient, as higher order corrections can be just as important at scales a few times above R^{-1}.

The addition of 2-loops seems to give better, but slightly lower gauge unification.

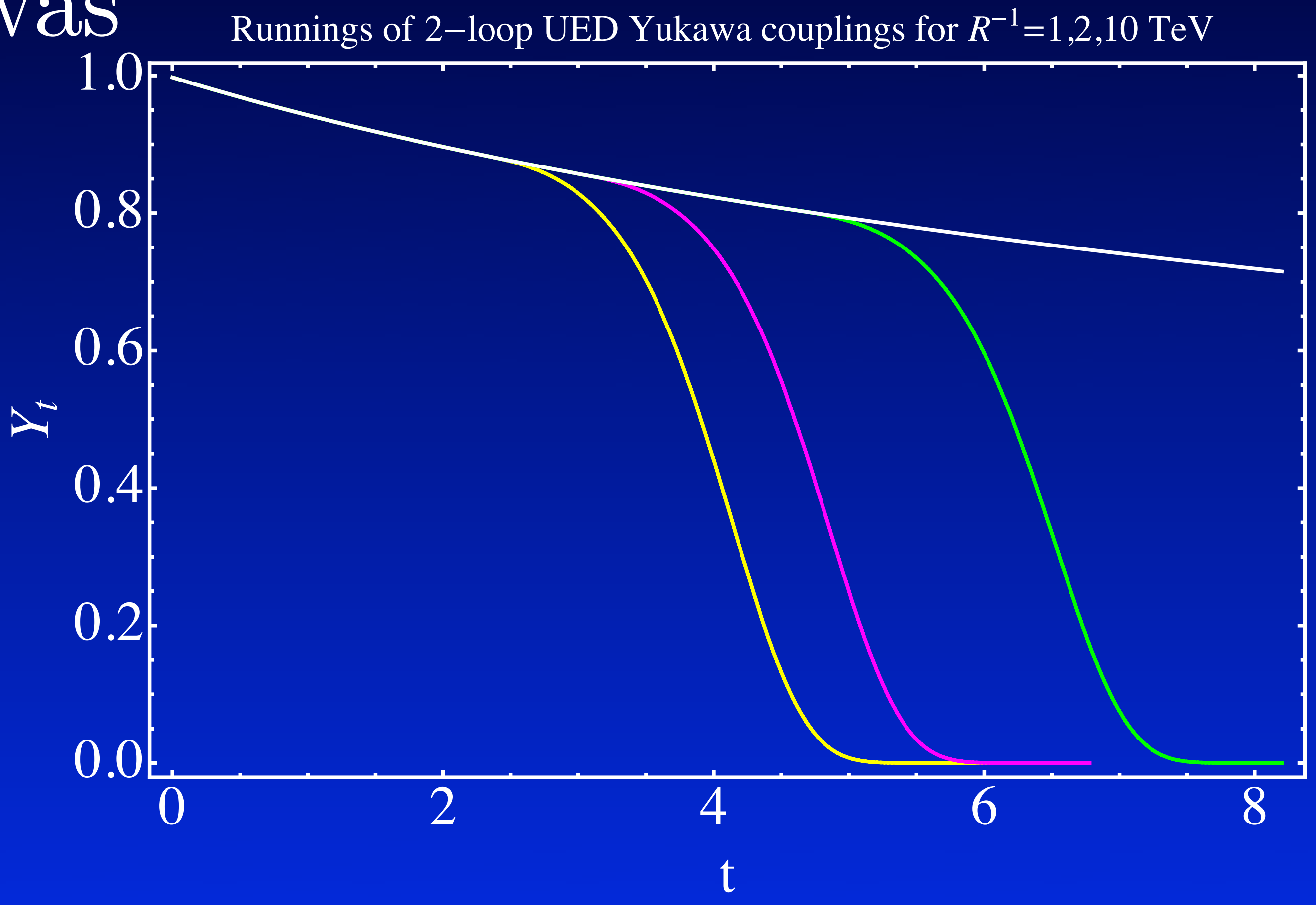
The RGEs appear to stable to higher order corrections.

The running of the Weinberg angle seems to be much greater.

Further study is required to see how much the phenomenology will change. As these types of corrections now include Yukawas, and we need to ensure Y_t remains perturbative

Continuing work

We wish to extend this to studies of the Yukawas



where preliminary work indicates that 2-loop corrections seem to plateau

How will this change the CKM parameters runnings?

What of other UED models, such as SUSY versions?

Etc.

Thank-you