

6th International Conference on Collective Motion in Nuclei under Extreme Conditions



29 October – 2 November 2018, Cape Town



Marcella Grasso

A beyond-mean-field description for nuclear excitation spectra: applications of the subtracted SRPA

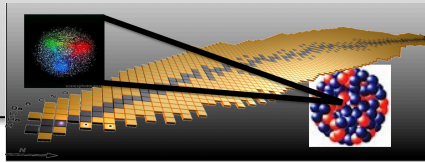


Outline

- Beyond RPA with the second RPA (SRPA) model employing EDFs
- Implementation of the SRPA model. Application of a subtraction method (SSRPA) to handle double counting, instabilities and ultraviolet divergences
- Applications. - Systematic study of the isoscalar GQR
- Beyond-mean-field effective masses
- Conclusions

EDF models currently employ, in most cases, phenomenological effective interactions adjusted at the mean-field level

Within the EDF theory: designing interactions adapted for beyond mean-field models (cancellation of double counting, regularization of divergences, ..., possibly bridging with EFT/ab initio (reducing the empirical character)



Or, specific solutions exist, for example a subtraction procedure, that we have applied within the second random-phase approximation

Tselyaev, PRC 75, 024306 (2007)

Tselyaev, PRC 88, 054301 (2013)

Gambacurta, Grasso, Engel, PRC 92, 034303 (2015)

Gambacurta, Grasso, EPJA 52, 198 (2016)

Collaborators (work on beyond mean-field)

- F. Catara (Catania Univ.), G. Co' (Lecce univ.), V. De Donno (Lecce Univ.), D. Gambacurta (ELI, Bucharest), J. Engel (North Carolina), O. Vasseur (IPN Orsay)

SRPA with zero-range and finite-range effective interactions and Implementation with a subtraction procedure

SRPA-based models

- J. Bonnard, A. Boulet (IPN Orsay), G. Colo' (Milano Univ.), U. van Kolck (IPN Orsay), D. Lacroix, (IPN Orsay), X. Roca-Maza (Milano Univ.), J. Yang (Chalmers)

Nuclear interaction designed for beyond mean field

Work on the density functional



JRA TheoS (Theoretical Support for Nuclear Facilities in Europe) Task: Development of suitable effective interactions in mean-field and BMF theories



International Laboratory LIA COLL-AGAIN
(France-Italy collaborations)

SRPA model : formally established since several decades

$$Q_v^\dagger = \sum_{ph} (X_{ph}^v a_p^\dagger a_h - Y_{ph}^v a_h^\dagger a_p) + \sum_{p < p', h < h'} (X_{php'h'}^v a_p^\dagger a_h a_{p'}^\dagger a_{h'} - Y_{php'h'}^v a_h^\dagger a_p a_{h'}^\dagger a_{p'})$$

**Excitation operators:
2p2h configurations
are included, together
with the RPA 1p1h
configurations**

- Hoshino and Arima, Phys. Rev. Lett. 37, 266 (1976)
- Knupfer and Huber, Z. Phys. A 276, 99 (1976)
- Adachi and Yoshida, Nucl. Phys. A 306, 53 (1978)
- Tohyama, Gong, Z. Phys.A 332, 269 (1989)
- Lacroix, Ayik, Chomaz, Prog. Part. Nucl. Phys. 52, 497 (2004)
- Schwesinger, Wambach, Phys. Lett. B 134, 29 (1984)
- Schwesinger, Wambach, Nucl. Phys. A 426, 253 (1984)
- Wambach, Rep. Prog. Phys. 51, 989 (1988)
- Drozd, Nishizaki, Speth, Wambach, Phys. Rep. 197, 1 (1990)
- Nishizaki and Wambach, Phys. Lett. B 349, 7 (1995)
- Nishizaki and Wambach, Phys. Rev. C 57, 1515 (1998)

**Examples of first
applications for the
calculation of
fragmentation and
spreading widths
(strong cuts in the
2p2h space, Second
Tamm-Dancoff,
truncations and
approximations in
the 2p2h sector of
the matrix)**

**In the last decade.
No approximations in 2p2h matrix elements
and large 2p2h cutoff values**

- **Papakonstantinou and Roth, Phys. Lett. B 671, 356 (2009)**
- **Papakonstantinou and Roth, Phys. Rev. C 81, 024317 (2010)**
- **Gambacurta, Grasso, and Catara, Phys. Rev. C 81, 054312 (2010)**
- **Gambacurta, Grasso, and Catara, J. Phys. G 38, 035103 (2011)**
- **Gambacurta, Grasso, and Catara, Phys. Rev. C 84, 034301 (2011)**
- **Gambacurta, Grasso, De Donno, Co, and Catara, Phys. Rev. C 86 021304(R) (2012)**

**Microscopic
interaction
(derived from
Argonne V18)**

**Phenomen.
Skyrme and
Gogny
interactions**

SRPA model

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B}^* & -\mathcal{A}^* \end{pmatrix} \begin{pmatrix} \mathcal{X}^\nu \\ \mathcal{Y}^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} \mathcal{X}^\nu \\ \mathcal{Y}^\nu \end{pmatrix}$$

Schematically: same form as RPA equations

$$\mathcal{A} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \quad \mathcal{B} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$$
$$\mathcal{X}^\nu = \begin{pmatrix} X_1^\nu \\ X_2^\nu \end{pmatrix}, \quad \mathcal{Y}^\nu = \begin{pmatrix} Y_1^\nu \\ Y_2^\nu \end{pmatrix}.$$

1 and 2:

short-hand notation for 1p1h and 2p2h

A_{11} and B_{11} : standard RPA matrices

A_{12} , A_{21} , B_{12} , and B_{21} : coupling between 1p1h and 2p2h

A_{22} and B_{22} : 2p2h sector

For cases where the interaction is density independent and A_{22} diagonal the expressions are simplified

$$\Omega^{\text{SRPA}} = \begin{bmatrix} A_{11}'(\omega) & B_{11}' \\ -B_{11}'^* & -A_{11}'^*(\omega) \end{bmatrix} \quad \Omega^{\text{RPA}} = \begin{bmatrix} A_{11}' & B_{11}' \\ -B_{11}'^* & -A_{11}'^* \end{bmatrix}$$

where the energy-dependent matrix elements are

$$A_{11}'(\omega) = A_{11}' + \sum_2 A_{12}(\omega + i\eta - A_{22})^{-1} A_{21}'$$

Second-order energy-dependent self-energy insertion $\Sigma(\omega)$ -> leads to a beyond mean-field model and provides the description of spreading widths and fragmentation (in addition to the single-particle Landau damping) through de coupling with 2p2h

Drawbacks of the SRPA model (two are general and two are generated by the choice of specific interactions)

- **(Too) strong shift to lower energies with respect to the RPA spectrum**
- **Instabilities (Thouless theorem)**

General

EDFs

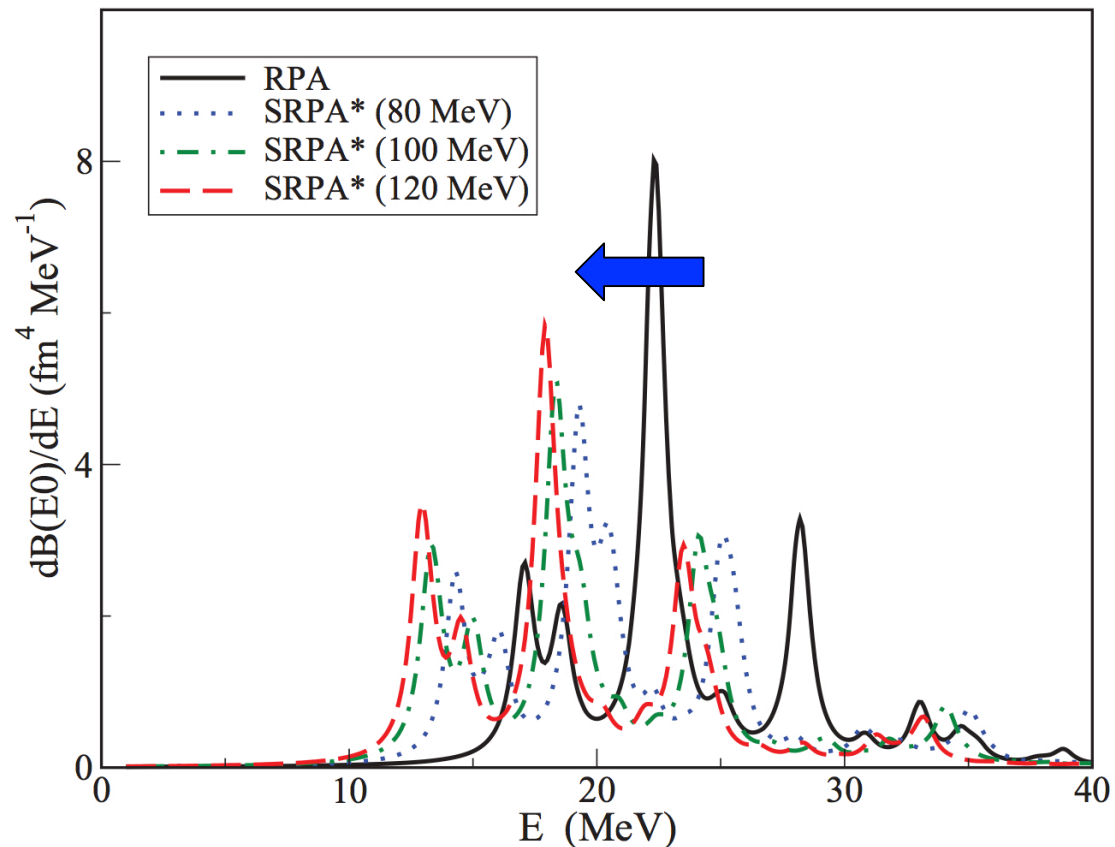
Recent studies about instabilities and double counting:

- Tselyaev, Phys. Rev. C 88, 054301 (2013)
- Papakonstantinou, Phys. Rev. C 90, 024305 (2014)

With the Gogny force (density-dependent contact term in the construction of the residual interaction) - ^{16}O

Isoscalar monopole response. The cutoff is in 2p2h configurations (in parentheses)

Shift, double counting, and cutoff dependence



Gambacurta, Grasso, et al., Phys. Rev. C 86, 021304 (R) (2012)

EDF and double counting for extensions of RPA (Tselyaev)

- Correlations implicitly included in the functional (because of the adjustment of the parameters at mean-field level to reproduce some observables). In the spirit of DFT: **‘exact’ functional to be used for mean-field-type calculations**
- Thus, this functional must produce a **static RPA response function which is the ‘exact’ zero-energy response function.** The RPA static polarizability should be regarded as the **‘exact’ one.**
- Any modification of the response function (to go beyond the mean field) should be zero in the static limit to avoid double counting of correlations



$$\Pi^{SRPA}(0) = \Pi^{RPA}(0) = -2m_{-1}^{RPA}$$
$$\alpha^{RPA} = -\Pi(0) = 2 \sum_{\nu} \frac{|\langle \nu | F | 0 \rangle|^2}{E_{\nu} - E_0} = 2m_{-1}^{RPA}$$

Adachi,
Lipparini,
NPA 489, 445
(1988)

This is achieved by subtracting the self-energy calculated at zero energy to the energy-dependent self-energy
 $\Sigma(E) - \Sigma(0)$ (Tselyaev 2013)

Stability condition in RPA (Thouless theorem, Nucl. Phys. 21, 225 (1960), Nucl. Phys. 22, 78 (1961))

If the HF state minimizes the expectation value of the Hamiltonian
-> the RPA stability matrix is positive semi-definite (real eigenvalues and eigenvectors with positive eigenvalues have positive norm)

- **Double counting** 
- **Instabilities (Thouless theorem)** 
- **Strong shift downwards of energies**
(with respect to RPA) and divergences
(with zero-range forces) ?

$$\Sigma(E) - \Sigma(0)$$



The second-order self-energy is responsible for the divergence. The subtraction removes it.

By following Tselayev 2013 ->

It is possible to rewrite the equations (after subtraction)
in a non energy dependent SRPA form:

$$\mathcal{A}_F^S = \begin{pmatrix} A_{11'} + \sum_2 A_{12}(A_{22'})^{-1}A_{21'} + \sum_2 B_{12}(A_{22'})^{-1}B_{21'} & A_{12} \\ A_{21} & A_{22'} \end{pmatrix}$$
$$\mathcal{B}_F^S = \begin{pmatrix} B_{11'} + \sum_2 A_{12}(A_{22'})^{-1}B_{21'} + \sum_2 B_{12}(A_{22'})^{-1}A_{21'} & B_{12} \\ B_{21} & B_{22'} \end{pmatrix}$$

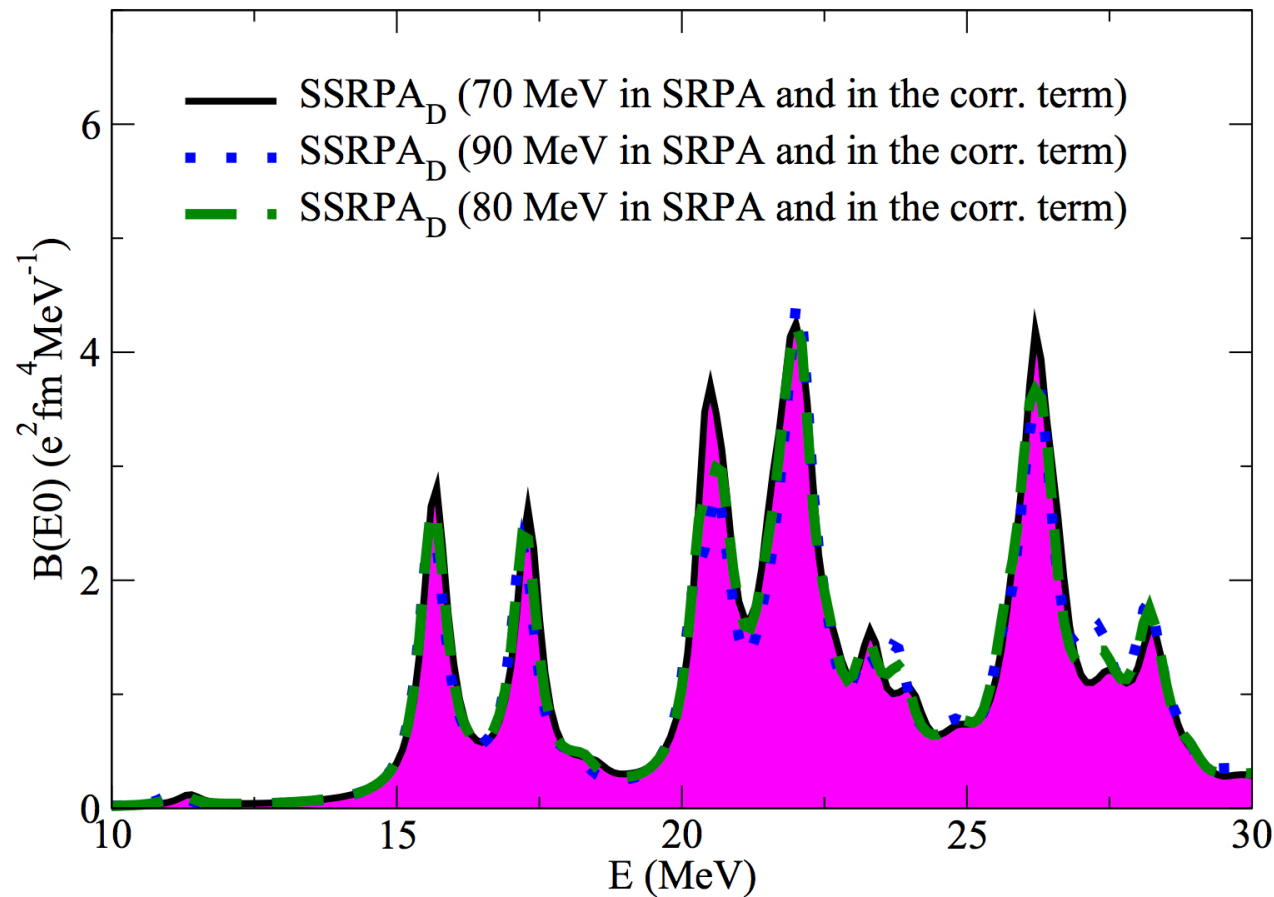
S -> subtracted

F -> full scheme (inversion of the matrix $A_{22'}$)

Gambacurta, Grasso, Engel, PRC 92, 034303 (2015)

Robust prediction. No cutoff dependence ISGMR for ^{16}O . SGI parametrization

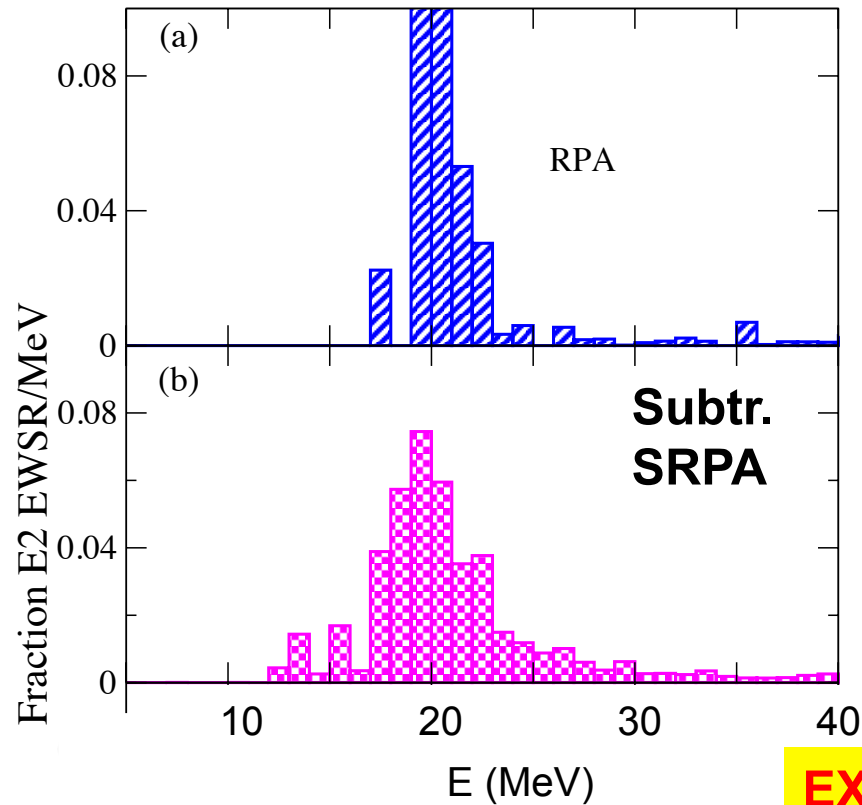
^{16}O



Gambacurta, Grasso, Engel, PRC 92, 034303 (2015)

Quadrupole excitations. Spreading width (SGII)

^{16}O



Centroid: 20.73 MeV
Width: 2.42 MeV

Centroid: 20.21 MeV
Width: 4.05 MeV

Gambacurta, Grasso, Engel,
PRC 92, 034303 (2015)

EXP: Lui, Clark, Youngblood,
PRC 64, 064308 (2001)

Centroid: 19.76 MeV
Width: 5.11 MeV

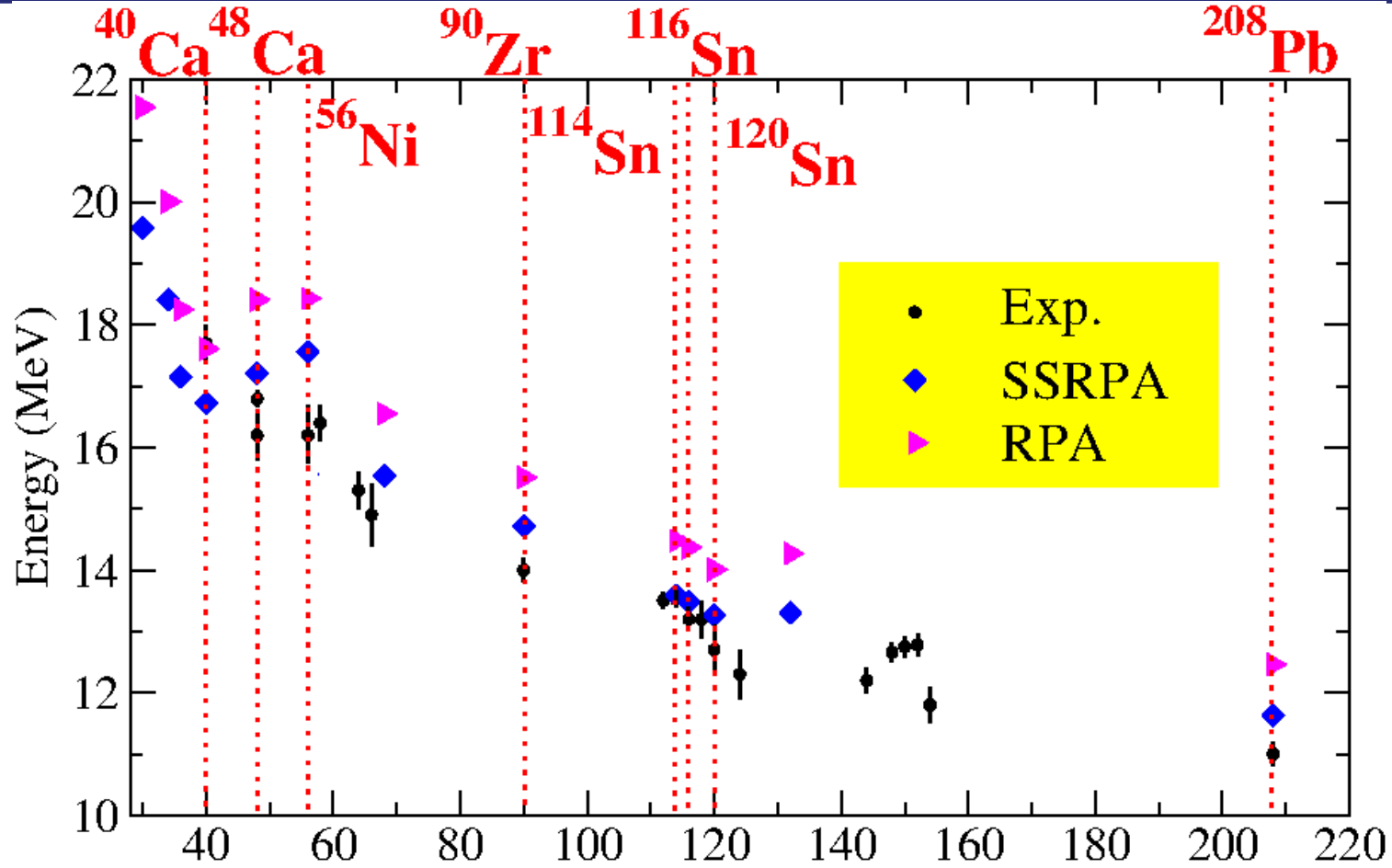
SOME RECENT APPLICATIONS

- ◆ **Dipole excitations and dipole polarizability in ^{48}Ca (Danilo Gambacurta talk)**
Gambacurta, Grasso, Vasseur, PLB 777, 163 (2018)

- ◆ **Systematic study of GQRs**
Vasseur, Gambacurta, Grasso, PRC 98, 044313 (2018)

- ◆ **Beyond-mean-field effects on effective masses**
Grasso, Gambacurta, Vasseur, arXiv:1807.04039

Isoscalar GQRs from ^{30}Si to ^{208}Pb
Centroids (folding with narrow Lorentzian distributions and adjusting on it a Lorentzian distribution)

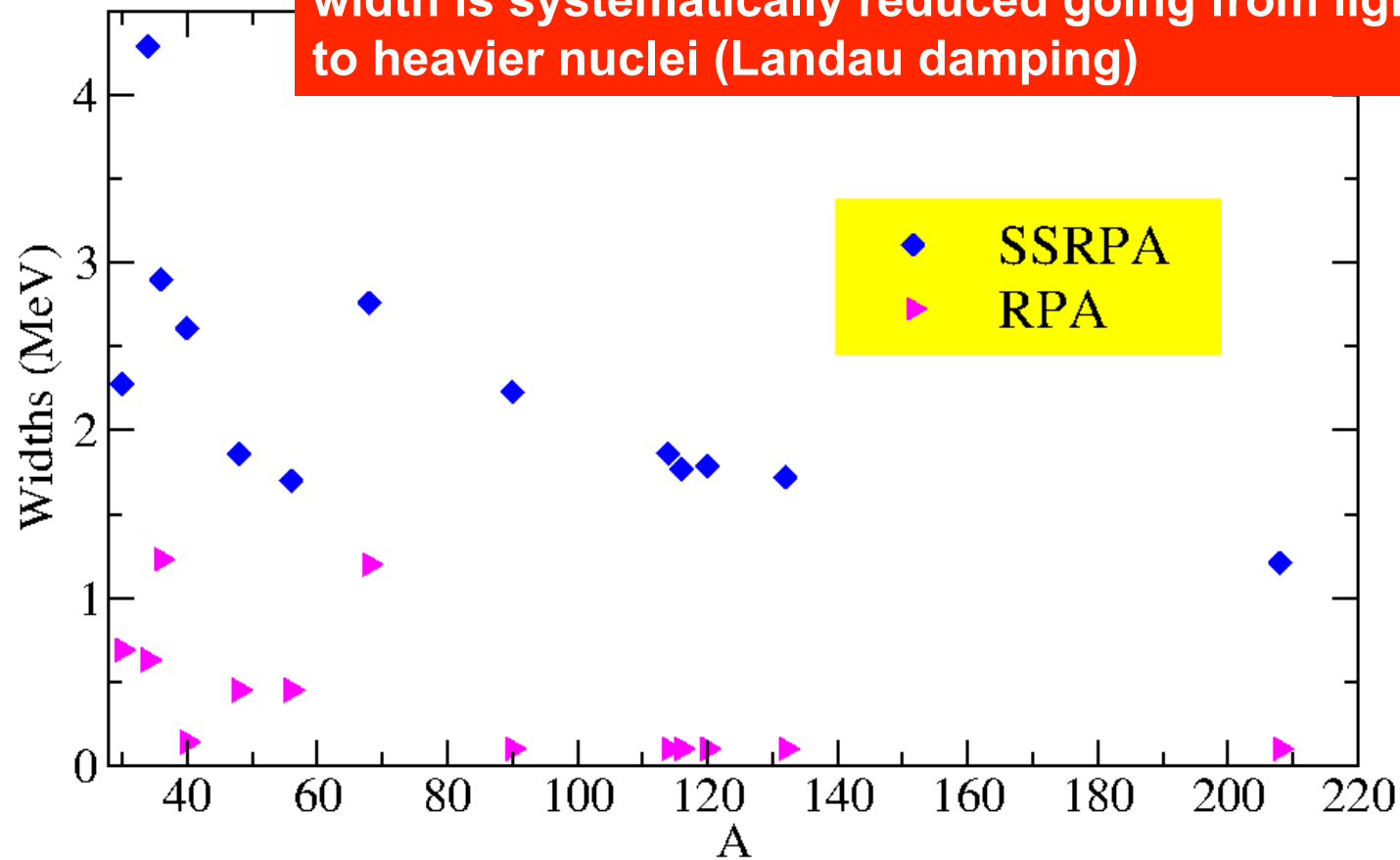


Vasseur, Gambacurta, Grasso,
 PRC 98, 044313 (2018)

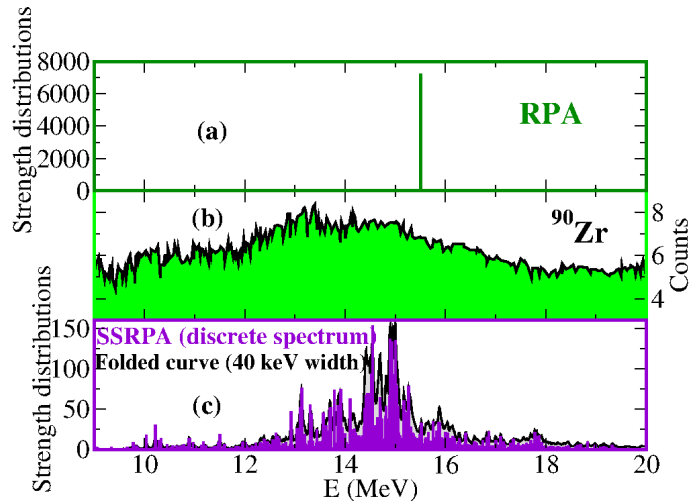
Globally: better agreement with the experimental data compared to RPA

Isoscalar GQRs. Widths (folding with narrow Lorentzian distributions and adjusting on it a Lorentzian distribution)

General trend, found both in RPA and in SSRPA: the width is systematically reduced going from lighter to heavier nuclei (Landau damping)



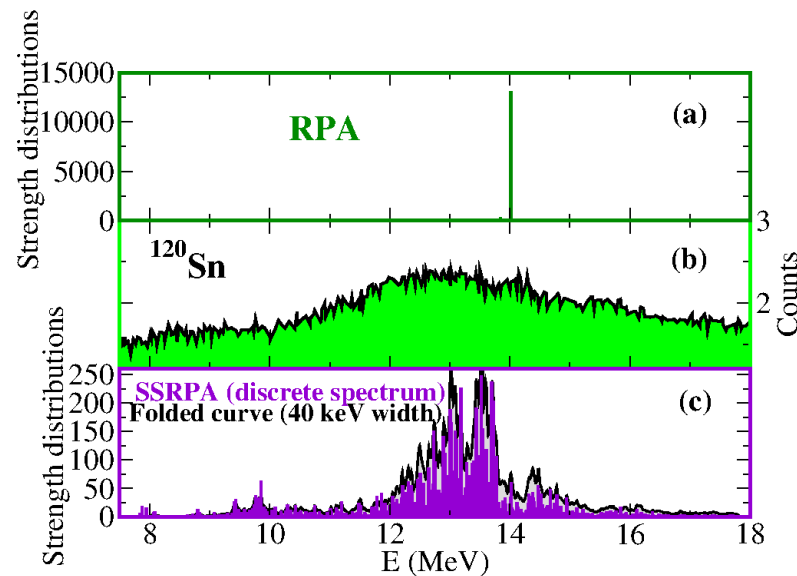
High-resolution proton inelastic scattering (p,p') spectra measured at iThemba LABS



SSRPA: discrete spectra and folded spectra with a Lorentzian of width equal to 40 keV (equal to the experimental energy resolution)

**Exp. data:
Shevchenko et al, PRL 93,
122501 (2004)**

**Vasseur, Gambacurta, Grasso,
PRC 98, 044313 (2018)**



Effective masses m^*

- Landau's theory of Fermi liquids: the system of interacting particles is described through quasiparticles having an effective mass m^*
- Study of m^* (relevant for the properties related to the propagation of particles in a medium): broad interest in many-body physics. Impact on, for example:
 - Density of states in a many-body system
 - Specific heat of a low-temperature Fermi gas
 - Maximum mass of a neutron star
 - Energies of axial compression or breathing modes in atomic gases and in nuclei (Isoscalar Giant Quadrupole Resonances)

Effective masses in Fermi liquids

First dynamic measurement of the polaron effective mass

PRL **103**, 170402 (2009)

PHYSICAL REVIEW LETTERS

week ending
23 OCTOBER 2009

Collective Oscillations of an Imbalanced Fermi Gas: Axial Compression Modes and Polaron Effective Mass

S. Nascimbène, N. Navon, K. J. Jiang, L. Tarruell,* M. Teichmann,† J. McKeever,‡ F. Chevy, and C. Salomon
Laboratoire Kastler Brossel, CNRS, UPMC, École Normale Supérieure, 24 rue Lhomond, 75231 Paris, France

The Fermi polaron is an impurity immersed in a Fermi sea (strongly imbalanced Fermi gases) *.

Based on the **Landau theory of Fermi liquids**, the energy spectrum of the polaron is similar to that of a free particle. Using the **local-density approximation**, the frequency ω^* of the polaron is

$$\frac{\omega^*}{\omega} = \sqrt{\frac{1-A}{m^*/m}}$$

ω is the frequency of the trap (harmonic oscillator), A is a dimensionless quantity that characterizes the attraction of the impurity by the other atoms

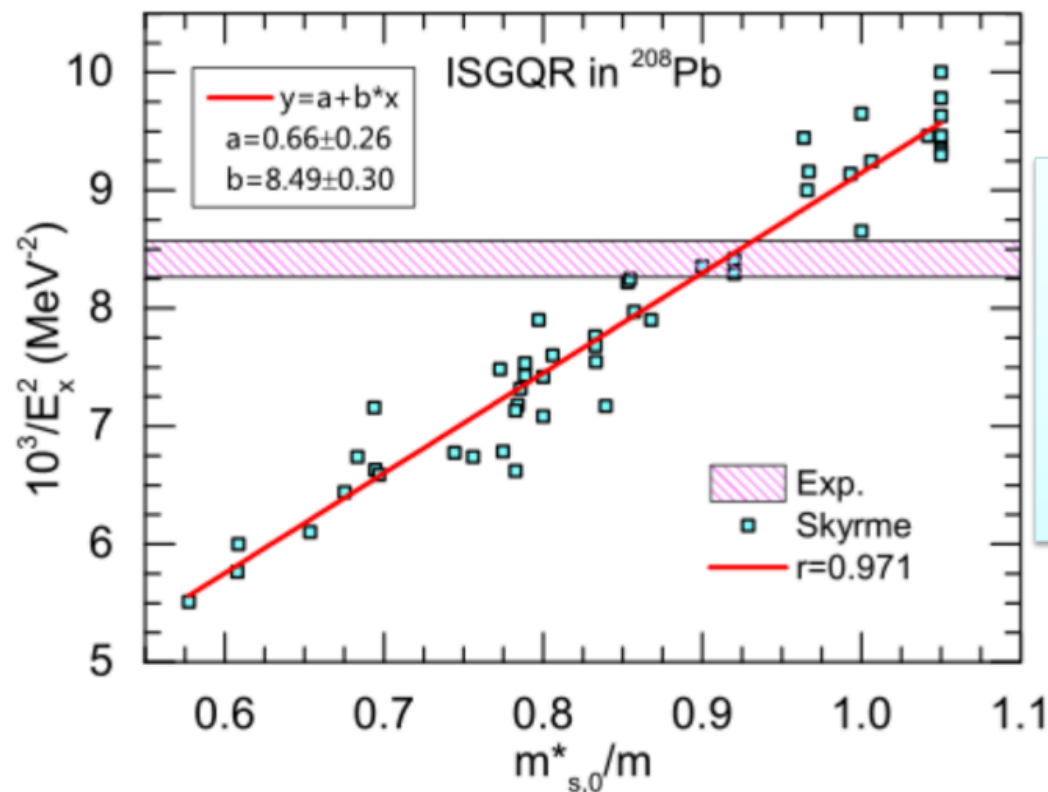
Lobo, Recati, Giorgini, Stringari, PRL **97**, 200404 (2006)

* Analogous calculations for nuclear systems:
Forbes et al., PRC **89**, 041301 (R) (2014)
Roggero et al., PRC **92**, 054303 (2015)

Effective masses in Fermi liquids

The axial breathing mode in nuclear physics corresponds to the isoscalar GQR.

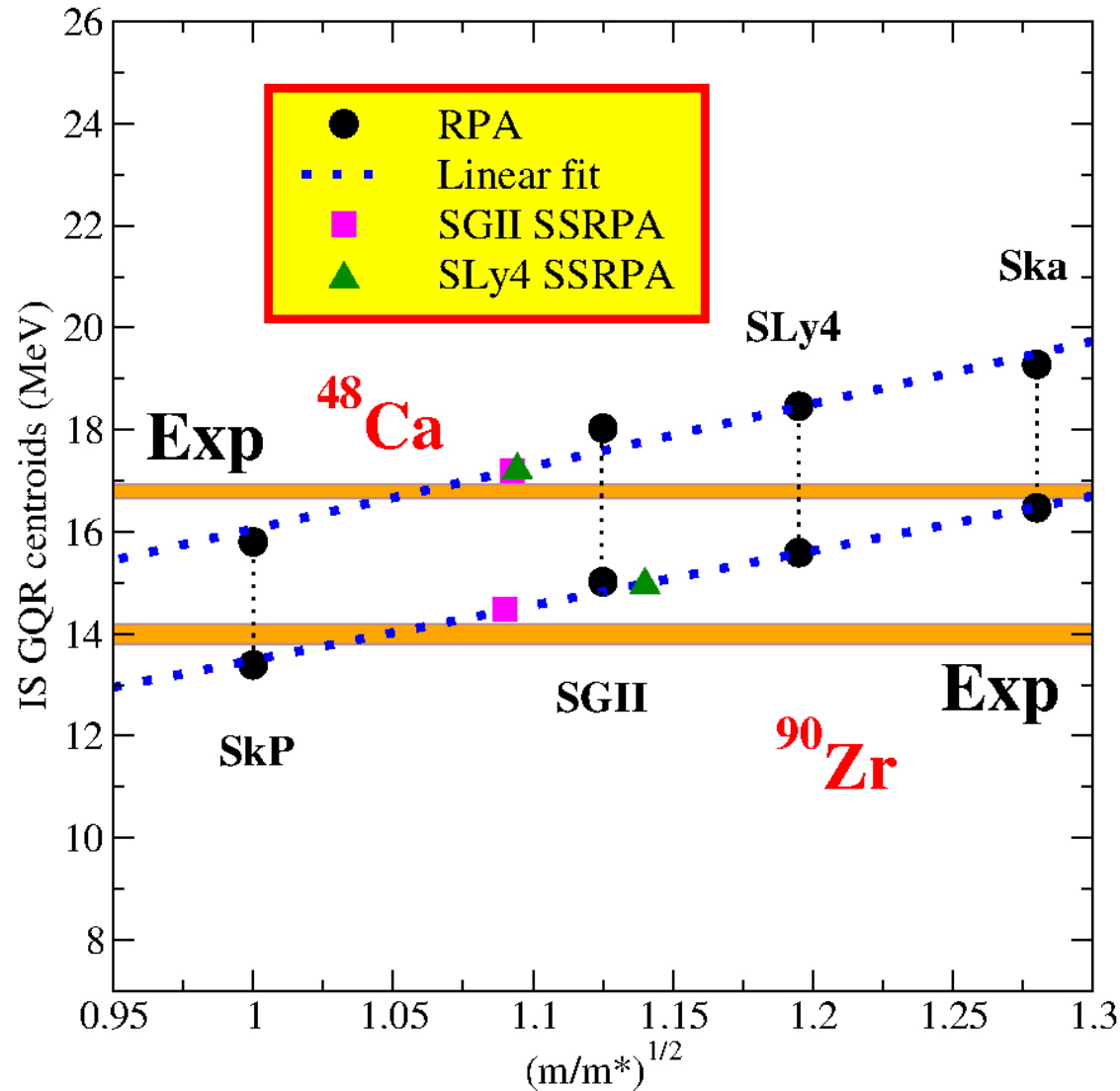
Based on the Landau theory of Fermi liquids, relation between the centroid energy of the IS GQR and $(m/m^*)^{1/2}$ known and used



Bao-An Li et al., Prog. Part. Nucl. Phys. 99, 29 (2018)

Blaizot, Phys. Rep. 64, 171 (1980)

Beyond-mean-field (SSRPA) effective masses in the nuclear Fermi liquid from axial breathing modes



Grasso, Gambacurta,
Vasseur,
arXiv:1807.04039

SSRPA extraction of the effective mass

Definition of effective mass:

$$\frac{1}{m^*} = \frac{dE}{dk} \frac{1}{\hbar^2 k}$$

for a particle of energy E and momentum k , with

$$E = \frac{\hbar^2 k^2}{2m} + \Sigma_k + \Sigma_{k,E}.$$

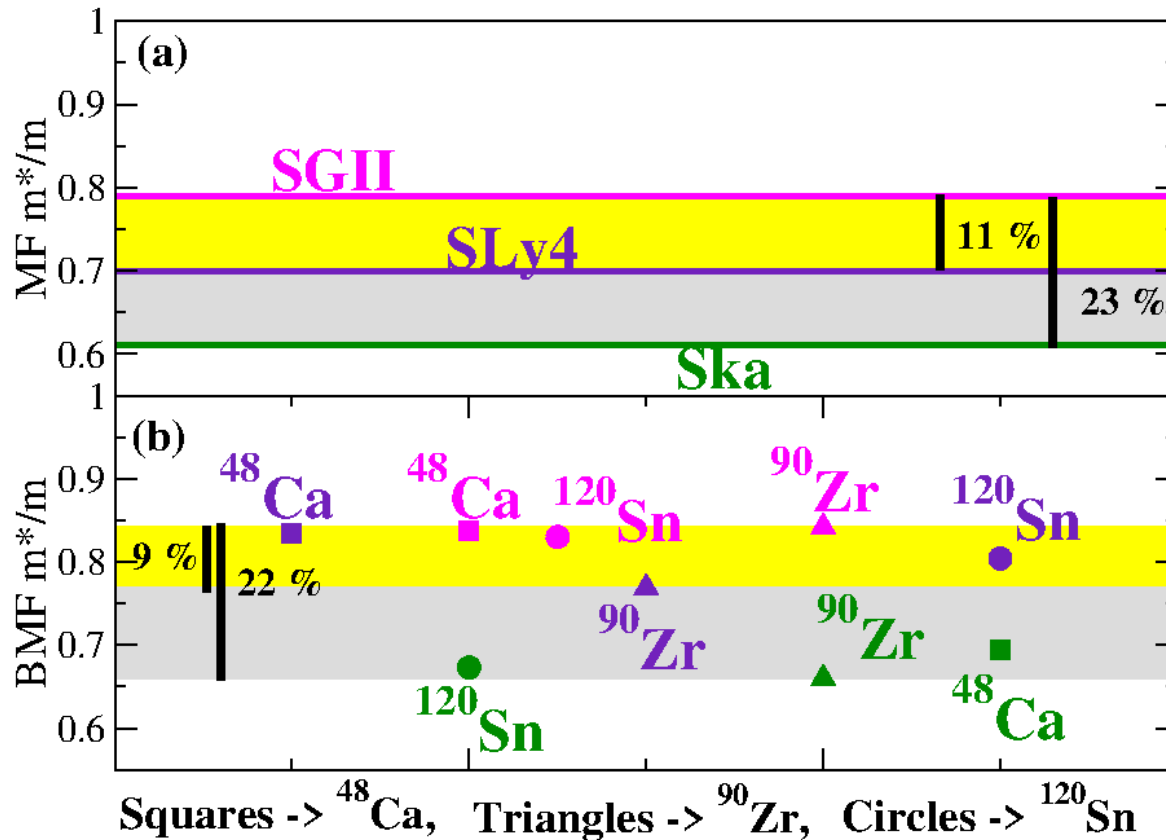
k-mass (leading order of the Dyson equation and E-mass -> beyond mean field, energy dependence of the self-energy)

$$\begin{aligned} \frac{m^*}{m} &= \left(1 - \frac{\partial \Sigma_{k,E}}{\partial E}\right) \cdot \left(1 + \frac{m}{\hbar^2 k} \frac{\partial \Sigma_k}{\partial k}\right)^{-1} \\ &\equiv \frac{m_E^*}{m} \cdot \frac{m_k^*}{m}, \end{aligned}$$

One may extract, for each nucleus and for each interaction, an estimation of the E-mass (equal to 1 at the mean-field level).

We have found an enhancement of the E-mass between 6 and 16% with SSRPA (nucleus and interaction dependence)

Beyond-mean-field effective masses. **Theoretical error**



Grasso, Gambacurta, Vasseur, arXiv:1807.04039

Mean field \rightarrow dispersion related to the used interaction
 Beyond-mean-field \rightarrow in addition, nucleus dependence. However,
theoretical error not larger than for the mean-field case

Effect on the single-particle excitation spectrum

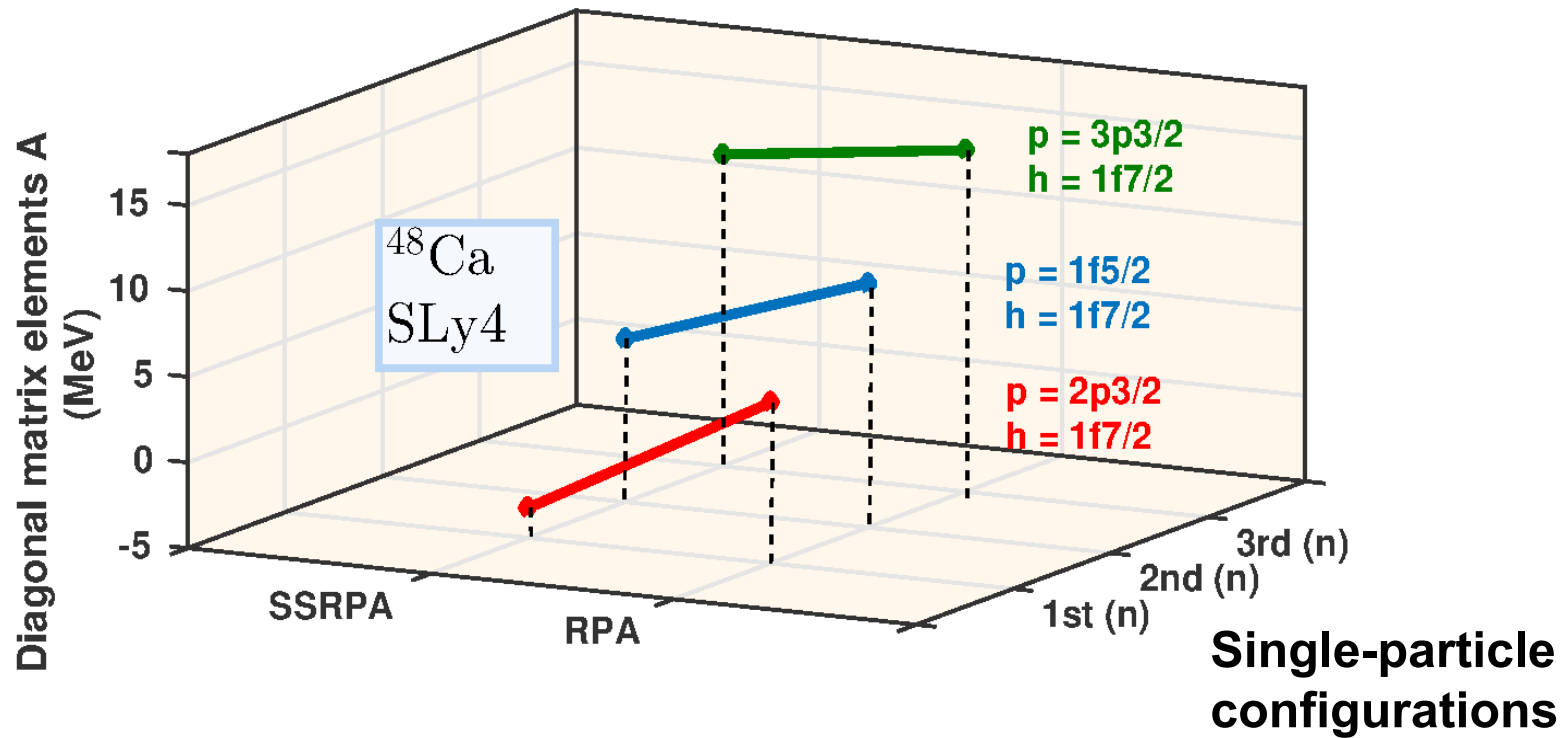
$$\mathcal{A}_F^S = \begin{pmatrix} A_{11'} - \sum_2 A_{12}(A_{22'})^{-1}A_{21'} + \sum_2 B_{12}(A_{22'})^{-1}B_{21'} & A_{12} \\ A_{21} & A_{22'} \end{pmatrix}$$
$$\mathcal{B}_F^S = \begin{pmatrix} B_{11'} + \sum_2 A_{12}(A_{22'})^{-1}B_{21'} + \sum_2 B_{12}(A_{22'})^{-1}A_{21'} & B_{12} \\ B_{21} & B_{22'} \end{pmatrix}$$

S -> subtracted

F -> full scheme (inversion of the matrix $A_{22'}$)

Gambacurta, Grasso, Engel, PRC 92, 034303 (2015)

Beyond-mean-field effective masses. Effective compression of the single-particle spectrum



$$A_{1,1}^{RPA} \rightarrow A_{1,1}^{SSRPA}(E) = [\epsilon_p - \epsilon_h]_{MF} + \bar{V}_{phhp} + \sum_{2,2'} \frac{A_{ph,2} A_{2',ph}}{E + i\eta - A_{2,2'}} + \sum_{2,2'} \frac{A_{ph,2} A_{2',ph}}{A_{2,2'}}.$$

Summary

- Implementation of the SRPA model by a subtraction procedure: double counting, stability condition (correction of the shift with respect to the RPA), convergence with respect to the cutoff
- Some recent applications:
 - ◆ **Systematic study of GQRs** (compared to RPA: centroids globally in better agreement with the experimental data; enhancement of the widths owing to the description of the spreading width)
 - ◆ **Beyond-mean-field effect on the effective mass** (extraction of an enhanced effective mass produced by beyond-mean-field effects)