From the nuclear equation of state straight to nuclear collective motion

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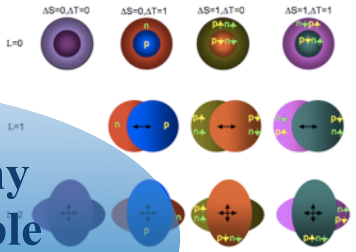
A rendering of the future RAON complex, under construction in Daejeon

Interests and motivation

RI facilities

Giant and Pygmy Resonances; stable and exotic nuclei

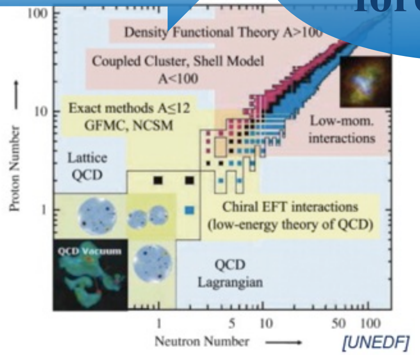


astronomy, HICs

EoS / NM properties

Underlying forces, EDF

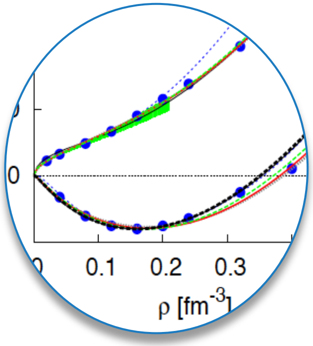
pseudodata



From EoS parameters to observables?

Some parameters are quite well constrained (within approximately 10%)

- ρ_0, E_0
- Also rather well: K_0, J



❖ Others not so well ...

- L, K_{sym}, Q_{sym}, ...

What I want to do:

Baseline EoS → Vary parameters ± at will → Check GRs properties etc

❖ **Why is this not done?**

A crisis of sorts

RAON

- Equation of state *vs* finite nuclei
  - Energy density functionals, EDFs (Skyrme, Gogny,...)
- However...
  - Only few of the hundreds of EDF models can simultaneously describe nuclear matter and finite nuclei

[M.Dutra et al.,PRC85(2012)035201; P.D.Stevenson et al., AIP Conf.Proc.1529,262]

  - Spurious correlations among parameters (e.g., K_0,m*)
  - ... while binding energies and radii "prefer" different values for the effective mass [M.Bender et al., Rev. Mod. Phys. 75,121

leads to the existence of an exact energy density functional. In practice, however, this functional is extremely complicated and establishing a useful form is more of an art than a science. One

[A.Bulgac et al., Phys. Rev. C 97,044313]

Not satisfactory!

- EDF scope either seriously misjudged **or not fully exploited**

Overview

- New strategy to unify the modeling of nuclei and homogeneous matter
  - KIDS functional
  - Controlled and converging expansion of nuclear EDF
- Proof of principle
  - Fix equation of state; map it onto an equivalent Skyrme functional; apply it straight to nuclei without changing it at all; m* doesn't matter
  - **It works!**

World first

- First explorations of nuclear response
  - Polarizability of ^{68}Ni
  - "Fluffiness" of Sn isotopes

People

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- Tae-Sun Park, **S**ungkyunkwan University
- Yeunhwan Lim, IBS (now in Texas)

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  - **D**aegu
  - **S**ungkyunkwan
- Hana Gil, Kyungpook National University
- Yongseok Oh, Kyungpook National University
- Gilho Ahn, University of Athens, Greece
- ++

NUCLEAR ENERGY DENSITY FUNCTIONAL FOR KIDS

- Natural Ansatz for energy density – inspired by QMBT / EFT
- Convenient Skyrme formalism for nuclei

KIDS Ansatz: Expansion in $k_F \sim \rho^{1/3}$

$$\mathcal{E}(\rho,\delta) = \frac{E(\rho,\delta)}{A} = \mathcal{T}(\rho,\delta) + \sum_{i=0}^{3} c_i(\delta)\rho^{1+i/3}$$

- If I have SNM and PNM, namely $c_i(0)$ and $c_i(1)$ (plus the quadratic approximation) I obtain analytically: $\{\rho_0, E_0, K_0, Q_0\}, \{J, L, K_{sym}, Q_{sym}\}$
- **And vice versa**; or I can fit to SNM/PNM pseudodata
- First, a few words on:
  - Motivation for Ansatz
  - Why 4 terms? Why low order?

for details: PP,Park,Lim,Hyun,Phys. Rev. C 97,014312 (2018)

Ansatz motivation - part I: Brueckner theory

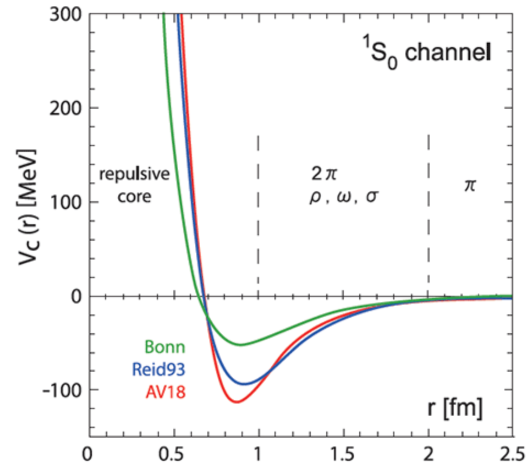
Fetter and Walecka, "Quantum theory of many-particle systems"

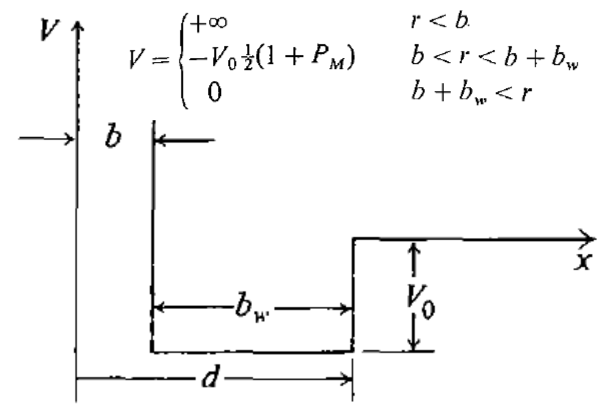
- ❖ Realistic potential: strong repulsive core plus attraction at longer range
- ❖ Apply Brueckner methodology in the calculation of nuclear matter energy

➔ Result: $k_F^2, k_F^3, k_F^4, k_F^5, k_F^6, \ldots,$ converging

- ◆ Even powers: from repulsive part
- ◆ Odd powers: from both

➔ The Fermi momentum is the relevant variable : **powers of $\rho^{1/3}$**





Ansatz motivation - part II: effective field theory

PP,Park,Lim,Hyun,Phys. Rev. C 97,014312

- Saturation density is low...
  - with respect to (effective) boson exchange range (?)
    - one-pion exchange: vanishing expectation value
    - next boson: rho with m_ρ~775MeV~$4fm^{-1}$
  - Effective Lagrangian in powers of $\mathbf{k_F/m_\rho}$
- Expansion of E/A in powers of k_F
  - ... which means, again, powers of $\rho^{1/3}$
  - The Fermi momentum as the relevant variable
  - k_F^3 and k_F^4 (i.e., coupling~$\rho^{1/3}$) known to be important for obtaining saturation [Kaiser et al.,NPA697(2002)]
- *Dilute Fermi gas: plus logarithmic terms*

H.-W. Hammer, R.J. Furnstahl / Nuclear Physics A 678 (2000) 277–294

Nuclear energy density functional for KIDS

PP,Park,Lim,Hyun,Phys. Rev. C 97,014312

Natural Ansatz for potential energy: powers of $k_F \sim \rho^{1/3}$

But how many powers? Which are relevant?

- Fit to homogeneous matter pseudodata
  - Variational Monte Carlo (APR, FP)
- Statistical analysis of fit quality; naturalness
- Keep only the important terms! No overtraining

$$\mathcal{E}(\rho,\delta) = \frac{E(\rho,\delta)}{A} = \mathcal{T}(\rho,\delta) + \sum_{i=0}^{3} c_i(\delta)\rho^{1+i/3}$$

- SNM: 3 terms suffice in converging hierarchy ($c_3(0)=0$)
- PNM: 4 terms necessary (*different preferences*)

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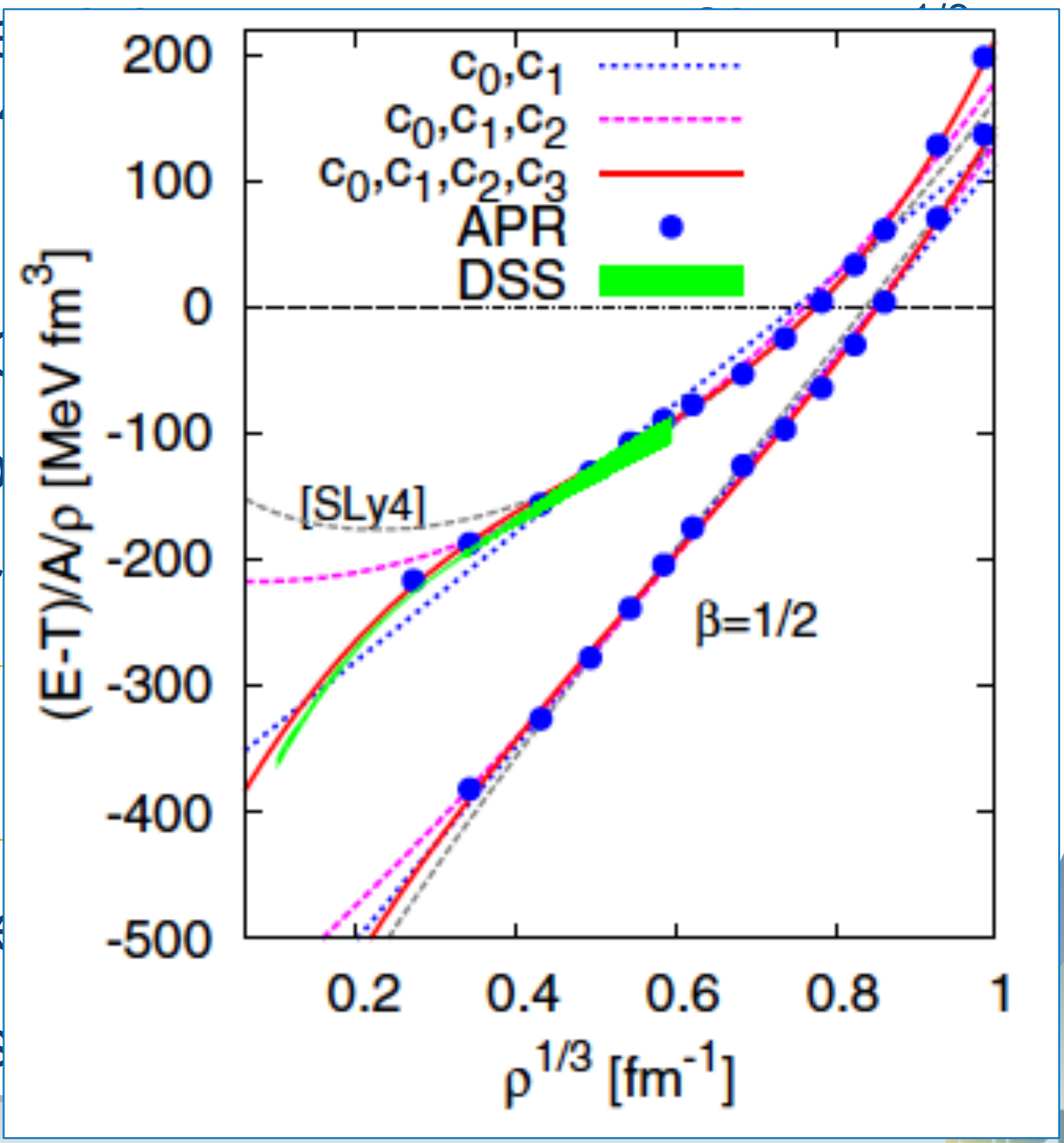

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*** for APR, more terms could lead to overfitting*

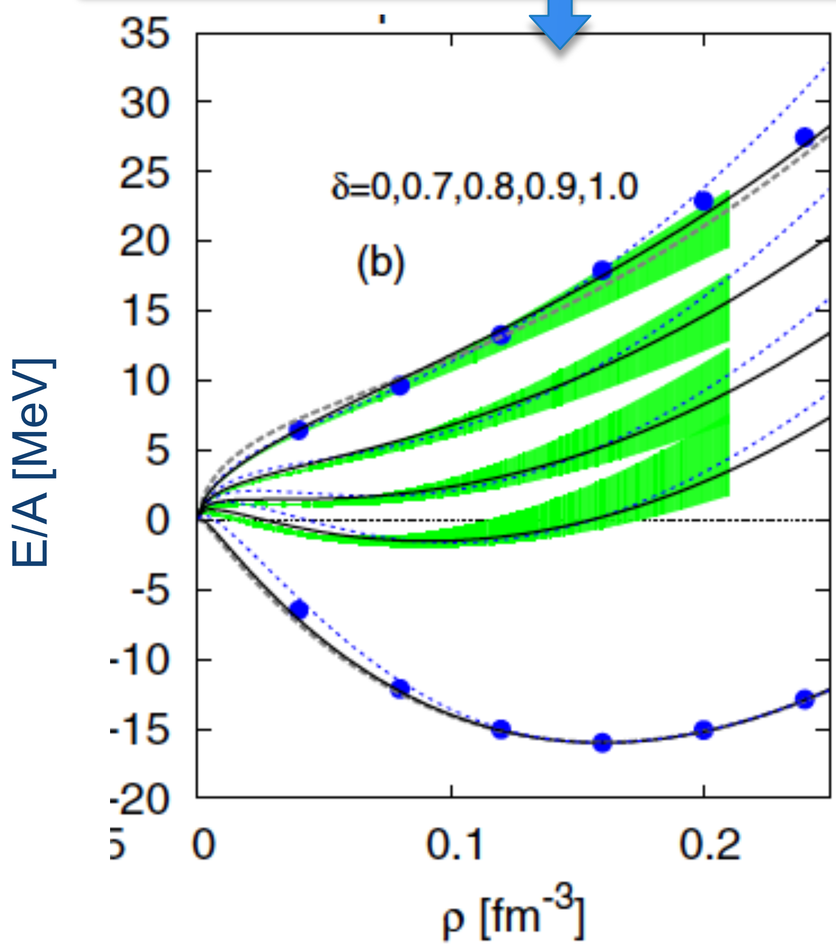
Baseline: Parameter set KIDS-ad2

- Symmetric nuclear matter:
  - Set ρ_0=0.16 fm -3, E_0=-16MeV, K_0 = 240 MeV
  - Determine $c_{0,1,2}(0)$ (analytical expressions)
  - Leads to Q_0=-373 MeV
- Pure neutron matter:
  - Fit $c_{0,1,2,3}(1)$ to the APR pseudodata for PNM
  - Resulting symmetry-energy parameters:

J=33MeV, L=49MeV, K_{sym}=-157MeV, Q_{sym}=586MeV

Interpolations and extrapolations

Calculations with chiral interactions reproduced, although they were not used for fitting



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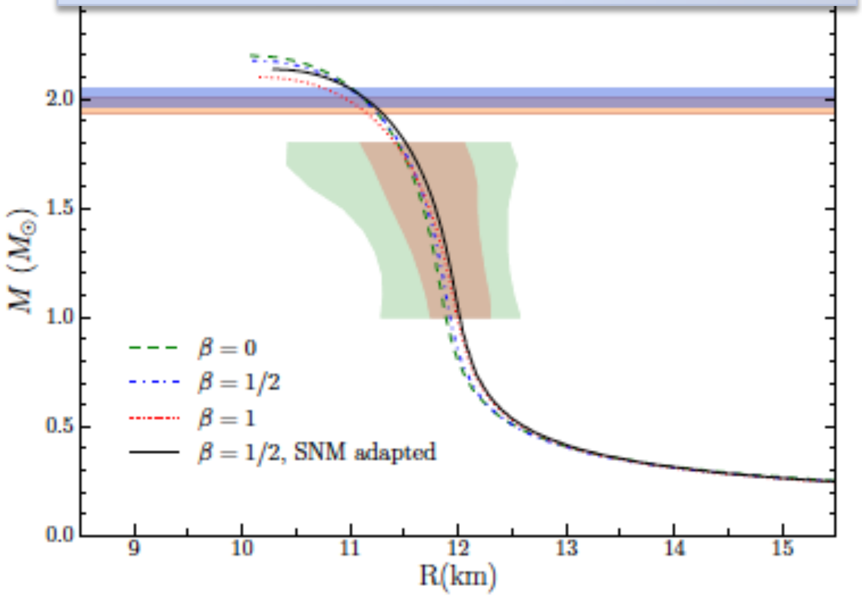
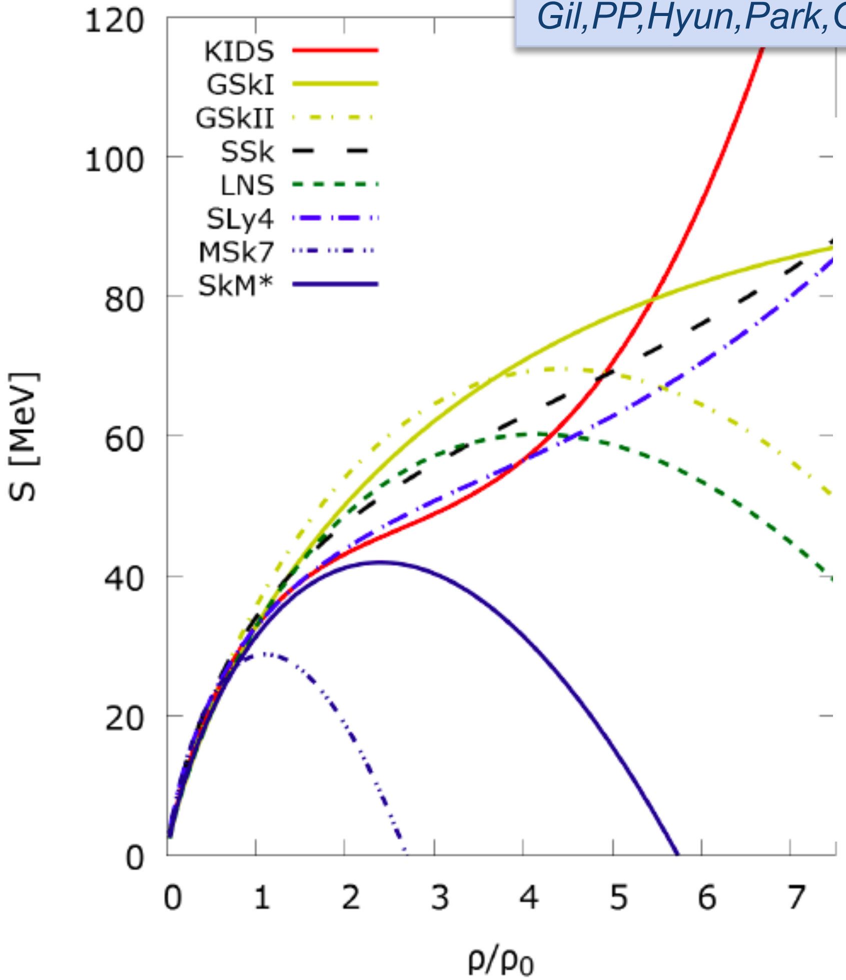
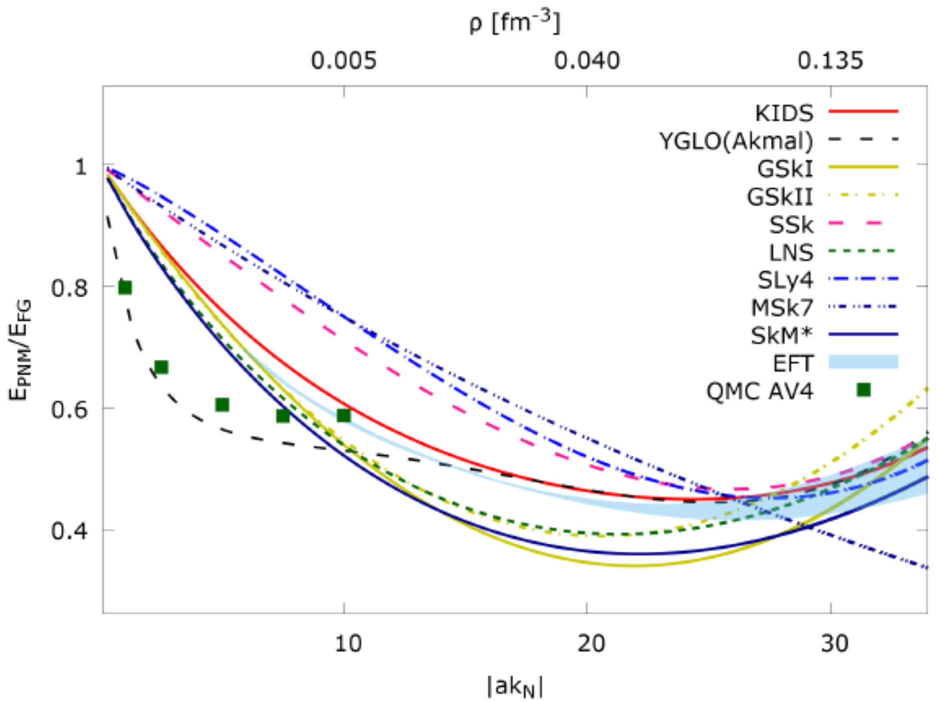


Comparisons with other models

Gil,PP,Hyun,Park,Oh,arXiv:1805.11321





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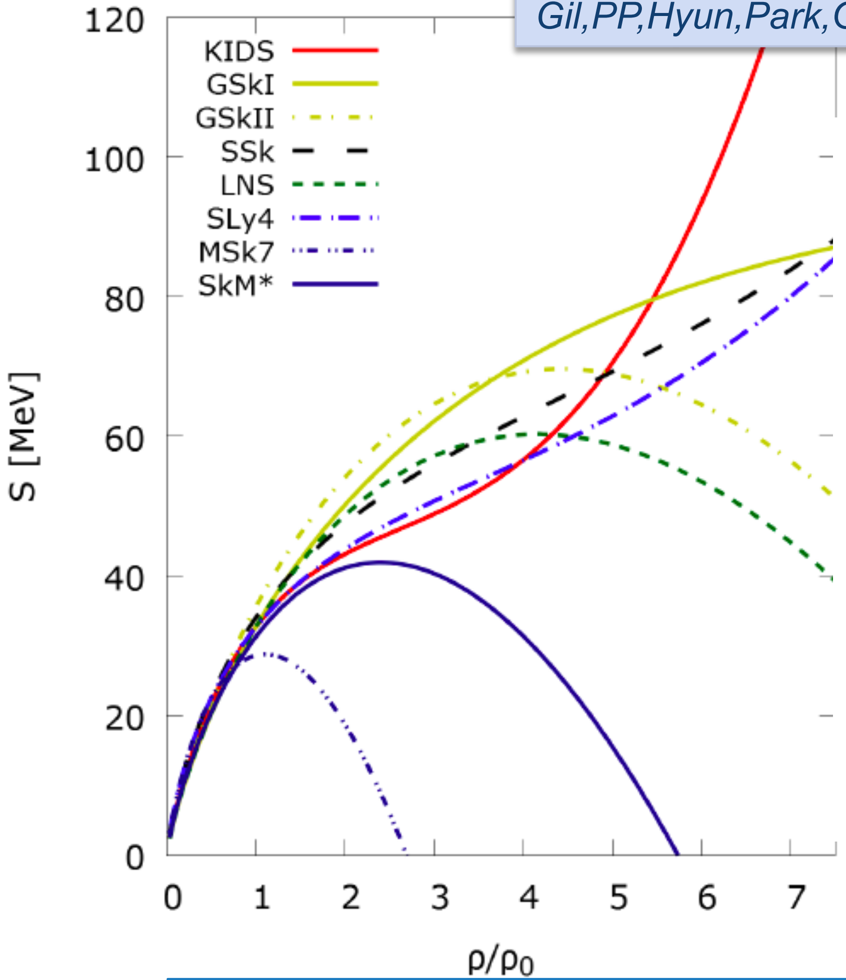

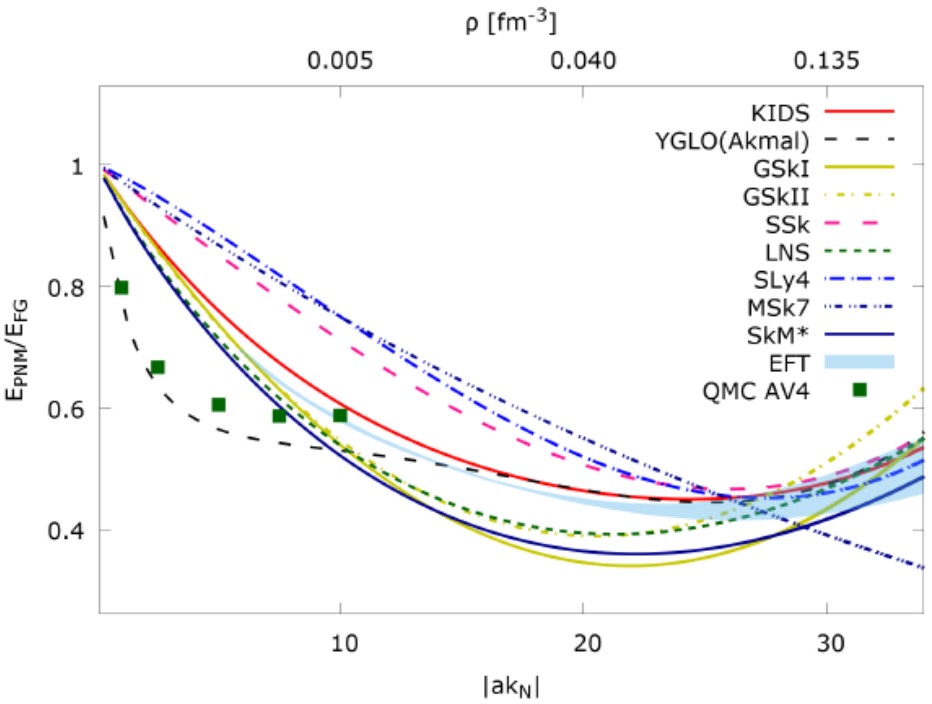

Excellent! Can we now take this straight to nuclei?

PROOF OF PRINCIPLE: APR TAKEN TO NUCLEI

- Mapping to equivalent Skyrme functionals
- Nuclear structure
- Nuclear response (first results / preliminary)

Skyrme parameters by reverse engineering

$$v_{i,j} = (t_0 + y_0 P_\sigma)\delta(\mathbf{r}_{ij}) + \frac{1}{2}(t_1 + y_1 P_\sigma)[\delta(\mathbf{r}_{ij})\mathbf{k}^2 + \text{h.c.}]$$
$$+(t_2 + y_2 P_\sigma)\mathbf{k}' \cdot \delta(\mathbf{r}_{ij})\mathbf{k} + iW_0\, \mathbf{k}' \times \delta(\mathbf{r}_{ij})\, \mathbf{k} \cdot (\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)$$
$$+\frac{1}{6}\sum_{n=1}^{3}(t_{3n} + y_{3n}P_\sigma)\rho^{n/3}\delta(\mathbf{r}_{ij}), \qquad (3)$$

Minimal Skyrme-type “force”

$$t_0 = \frac{8}{3}c_0(0)\,, \quad y_0 = \frac{8}{3}c_0(0) - 4c_0(1),$$
$$t_{3n} = 16c_n(0)\,, \quad y_{3n} = 16c_n(0) - 24c_n(1), \quad (n \neq 2$$
$$t_{32} = 16c_2(0) - \frac{3}{5}\left(\frac{3}{2}\pi^2\right)^{2/3}\theta_s,$$
$$y_{32} = 16c_2(0) - 24c_2(1) + \frac{3}{5}(3\pi^2)^{2/3}\left(3\theta_\mu - \frac{\theta_s}{2^{2/3}}\right)$$

with

$$\theta_s \equiv 3t_1 + 5t_2 + 4y_2\,, \quad \theta_\mu \equiv t_1 + 3t_2 - y_1 + 3y_2\,.$$

unconstrained from homogenous matter ➔ vary freely

But the total $c_2(0)$, $c_2(1)$ will remain unchanged!

Procedure

For given KIDS functional $c_i(0)$, $c_i(1)$ (i.e., fixed SNM, PNM)

- Chose effective masses (agnostic - vary at will)
- All t_i, y_i are now known except t_1,t_2,x_1,x_2
- The two combinations θ_s,θ_μ also known (eff. masses)
- **Two independent free parameters plus spin-orbit W_0**
  - Fit only to ^{40}Ca, ^{48}Ca, ^{208}Pb
  - Only bulk properties: E/A, charge radius: 6 data

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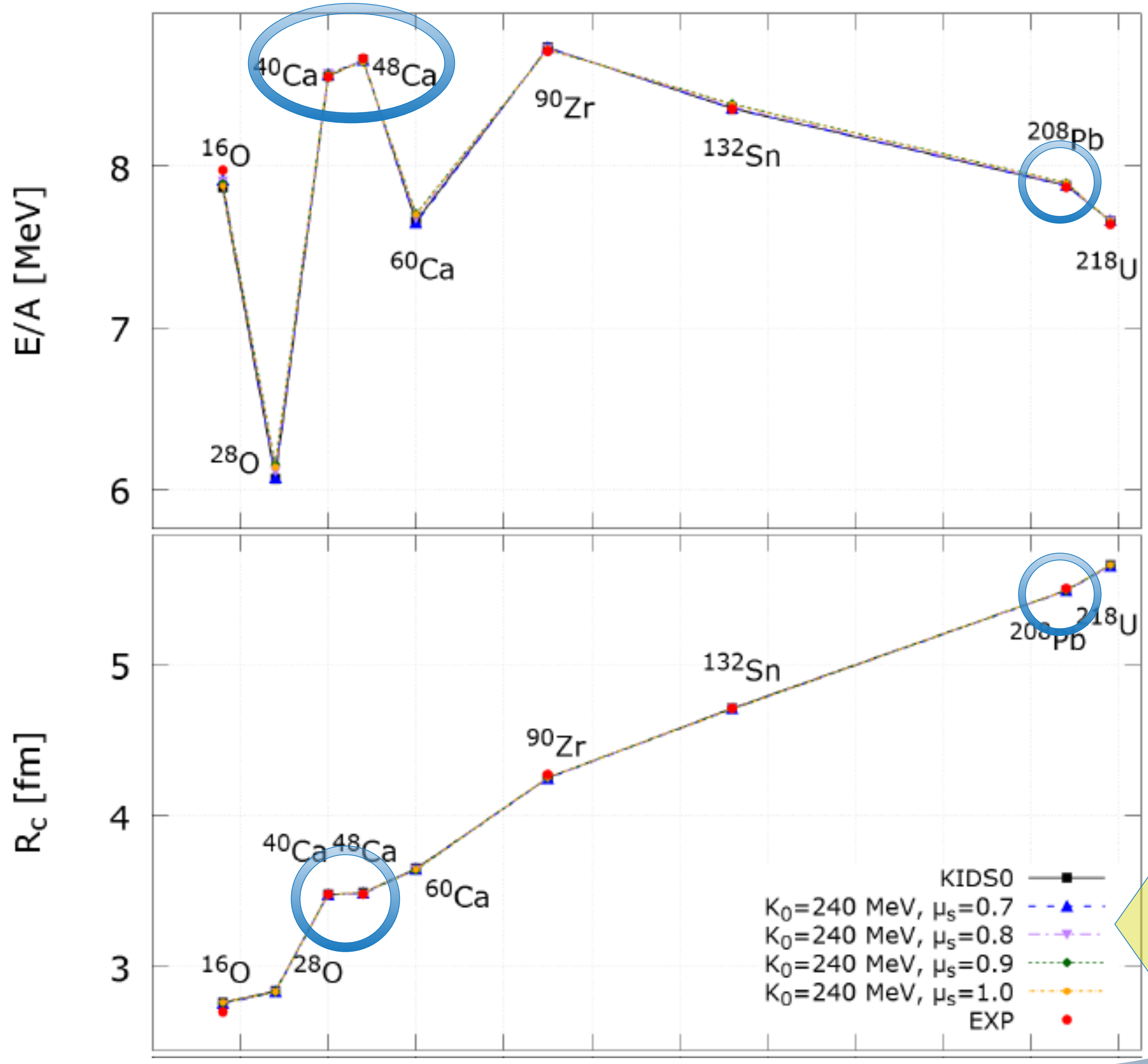
Let's begin with the baseline KIDS-ad2 set:

Empirical saturation point + APR for neutron matter

Binding energy, charge radii



predictions independent of the effective mass assumed

Gil,PP,Hyun,Park,Oh,arXiv:1805.11321

Some numbers

Model	t_0 / y_0	t_1 / y_1	t_2 / y_2	t_{31} / y_{31}	t_{32} / y_{32}	t_{33} / y_{33}	(a_1, a_2, a_3)	W_0	$D_E[\%]$ / $D_R[\%]$	^{60}Ca: E/A [MeV] / R_c [fm]
KIDS0	−1772.04	275.72	−161.50	12216.73	571.08	0.00	(1/3, 2/3, 1)	108.35	0.32	7.6561
	−127.52	0.000	0.000	−11969.99	29485.52	−22955.28			0.56	3.6465
K240,1.0,0.82	−1772.04	270.52	−355.95	12216.73	642.12	0.00	(1/3, 2/3, 1)	97.61	0.41	7.6993
	−127.52	235.79	374.20	−11969.99	29224.07	−22955.28			0.56	3.6416
K240,0.7,0.82	−1772.04	448.99	−279.45	12216.73	−2572.65	0.00	(1/3, 2/3, 1)	135.24	0.26	7.6464
	−127.52	−345.72	234.74	−11969.99	41318.69	−22955.28			0.44	3.6494
K240,0.9,1.00	−1772.04	315.97	−527.58	12216.73	−191.34	0.00	(1/3, 2/3, 1)	107.58	0.38	7.6933
	−127.52	−56.87	480.10	−11969.99	36289.12	−22955.28			0.57	3.6370
K220,1.0,0.82	−1938.71	281.04	−479.05	15900.76	−2750.91	0.00	(1/3, 2/3, 1)	88.96	0.52	7.7701
	−294.19	236.07	388.03	−8285.96	25831.04	−22955.28			0.94	3.6524
K220,0.7,0.82	−1938.71	466.23	−439.68	15900.76	−5965.68	0.00	(1/3, 2/3, 1)	133.36	0.44	7.6807
	−294.19	−247.10	422.10	−8285.96	37925.67	−22955.28			0.82	3.6663
GSkI [32]	−1855.45	397.23	264.63	13858.00	−2694.06	−319.87	(1/3, 2/3, 1)	169.57	0.20	7.6294
	−219.02	−698.59	−478.13	1747.29	3200.69	146.94			0.68	3.6640
SLy4 [37]	−2488.91	486.82	−546.39	13777.00			(1/6, −, −)	122.69	0.38	7.7030
	−2075.75	−167.37	546.39	18652.68					0.91	3.6734

On par with functionals fitted to many nuclear data

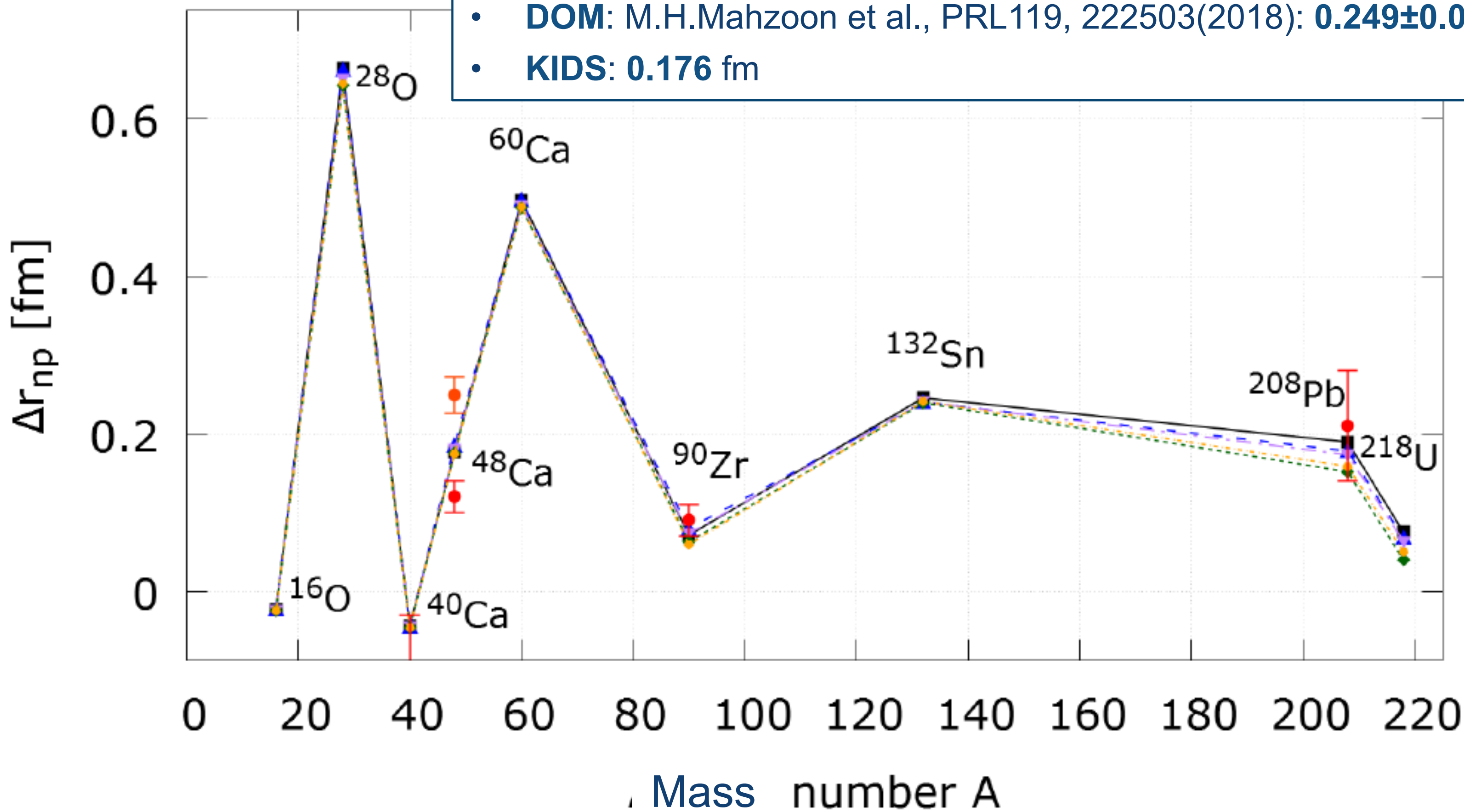
$$D_O = \frac{1}{N_{\rm nucl}} \sum_{i=1}^{N_{\rm nucl}} \left| \frac{O_i^{\rm expt} - O_i^{\rm cal}}{O_i^{\rm expt}} \right|$$

Gil,PP,Hyun,Park,Oh,arXiv:1805.11321

Neutron skin thickness

<u>neutron skin of 48Ca:</u>

- **CCM**: G.Hagen et al., Nature Phys. 12,186(2016): **0.12-0.15** fm
- **DOM**: M.H.Mahzoon et al., PRL119, 222503(2018): **0.249±0.023** fm
- **KIDS**: **0.176** fm



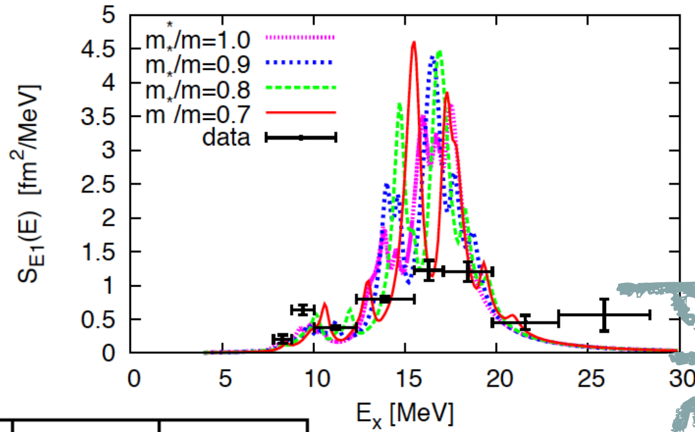
Data: antiprotonic atoms, PREX (^{208}Pb), DOM (^{48}Ca, upper)

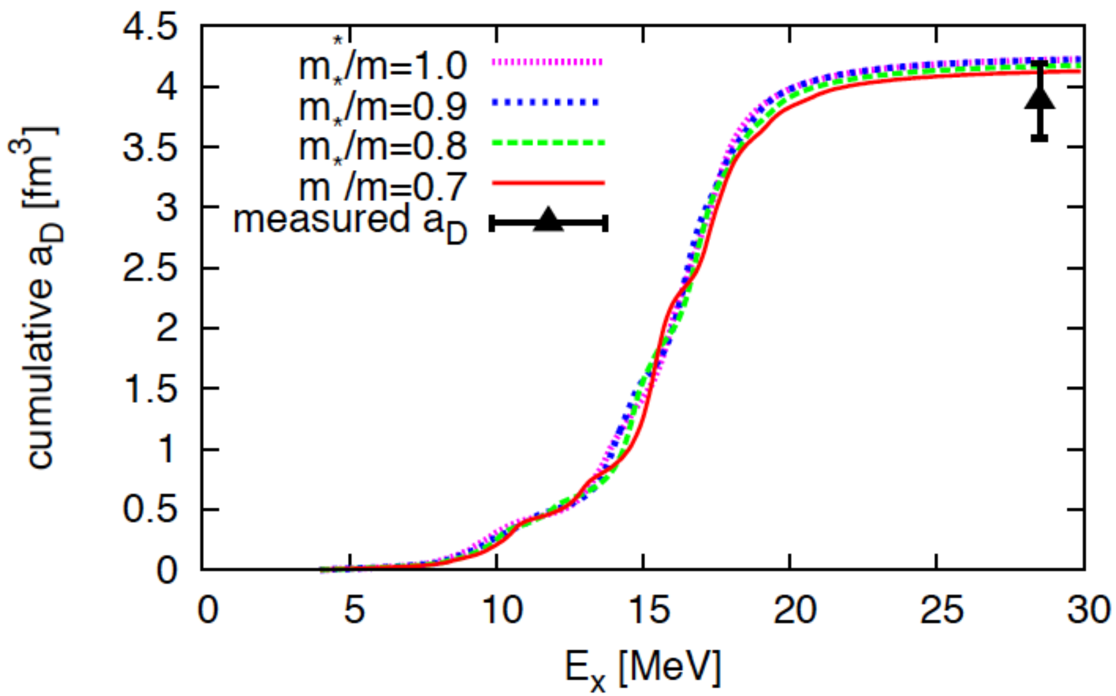
Predictions of APR EoS for the neutron skin thickness!

Gil,PP,Hyun,Park,Oh,arXiv:1805.11321

KIDS-ad2: Predictions for ^{68}Ni (not fitted)

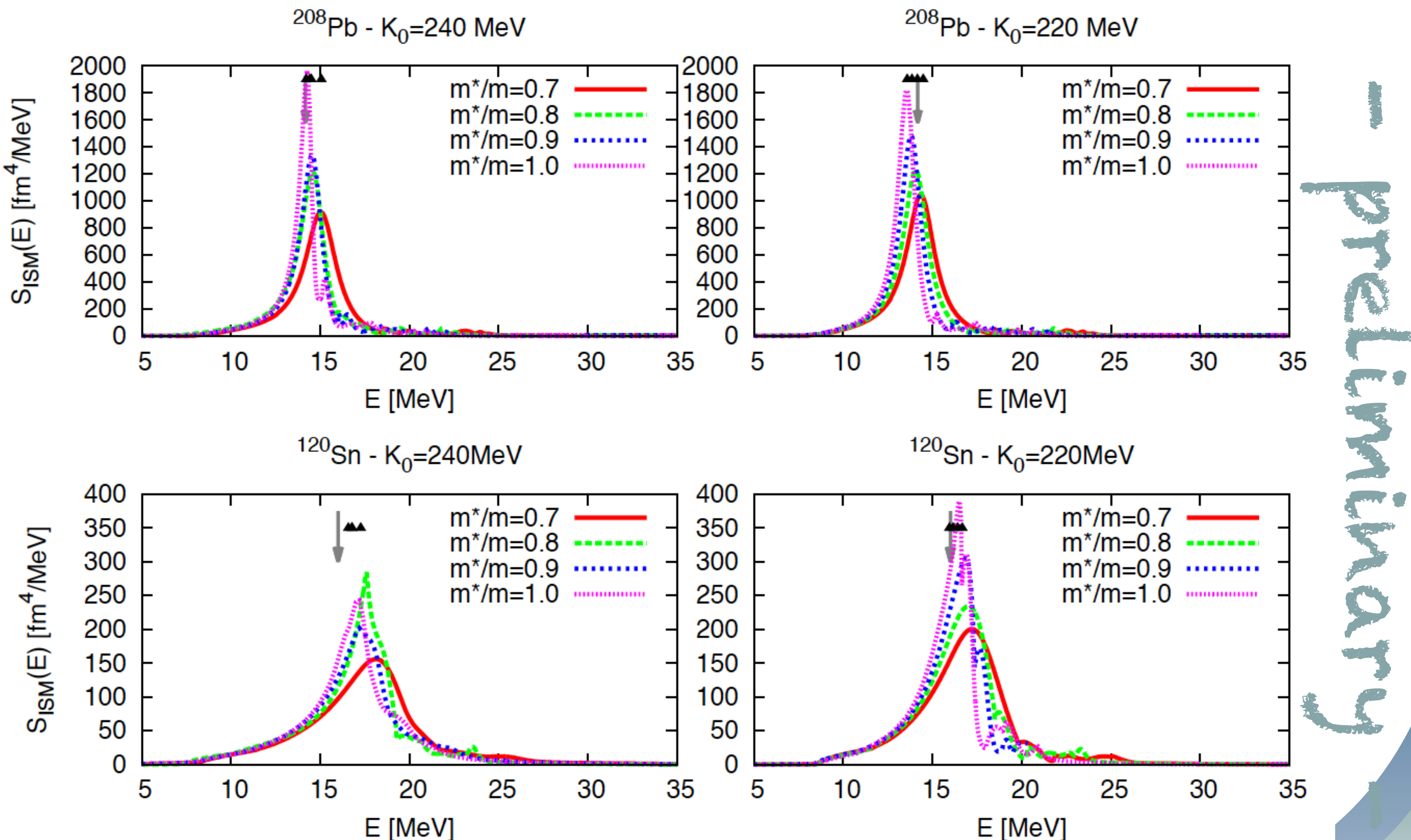
- Binding energy per particle:
  - KIDS-ad2: **8.68~8.69** MeV [*]
  - AME2016: **8.68247(4)** MeV
- Dipole polarizability:





[*] *for m*/m=1.0~0.7:8.68794; 8.68176; 8.68838; 8.68912 MeV*
[**] *a_D measurementT.Aumann and D.Rossi, private communication*

Compression mode and the effective mass



KIDS: Analytical relations

❖ Symmetric nuclear matter:

- $\{\rho_0, E_0, K_0\}$ ➔3x3 system ➔$\{c_i(0);\ i=0,1,2;\ c_3(0)=0\}$
  - *Feasible but unnecessary:*
  - *$\{\rho_0, E_0, K_0, Q_0\}$ ➔4x4 system ➔ $\{c_i(0);\ i=0,1,2,3\}$*

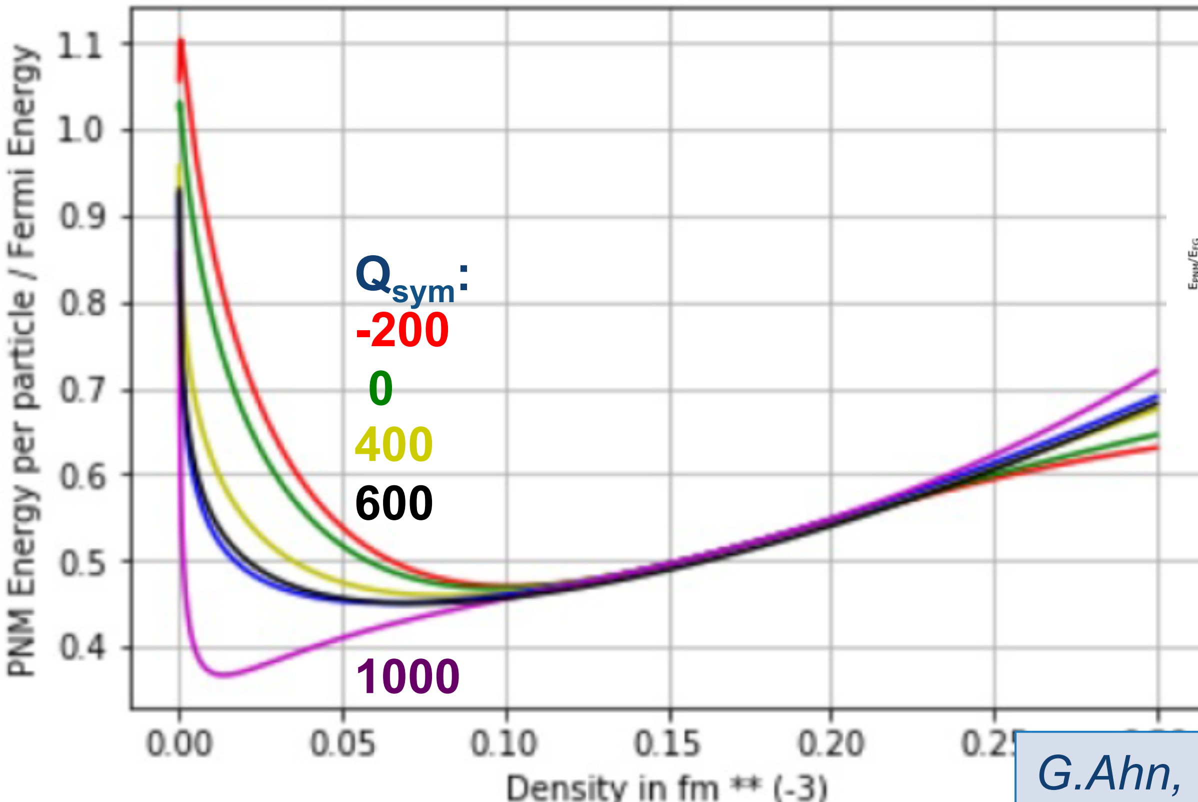
❖ Symmetry energy:

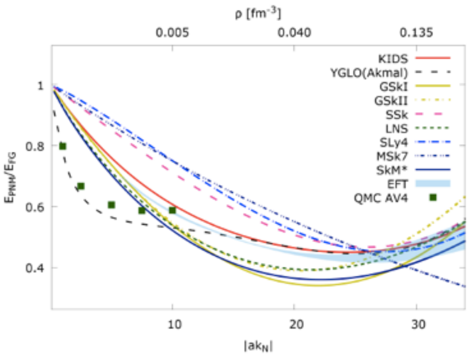
- $\{J, L, K_{sym}, Q_{sym}\}$ ➔4x4 system ➔ $\{[c_i(1)-c_i(0)];\ i=0,1,2,3$

Let us keep SNM, J, L, K_{sym} steady and equal to the KIDS-ad2 values; vary Q_{sym}; and solve for $c_i(1)$

Exploring symmetry energy parameters

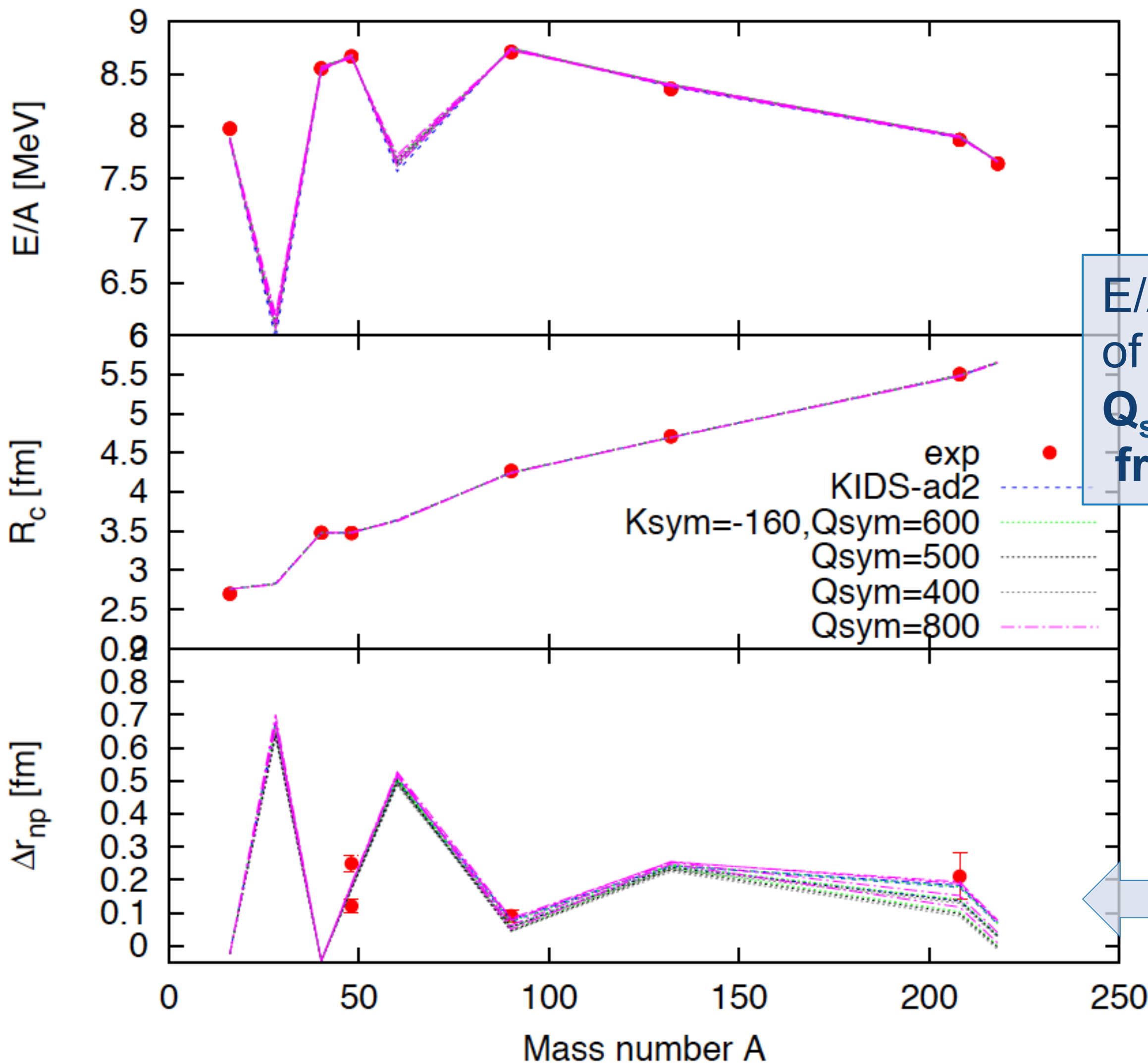
- Dilute neutron matter





G.Ahn, MSc Thesis (NKUA, 2018)

Exploring symmetry energy parameters



E/A, R_c independent of Q_{sym} ✔

Q_{sym} not constrainable from such data

Neutron skin thickness vs Q_{sym}?

To conclude...

- The versatile KIDS functional
  - A converged expansion of the nuclear EoS/EDF in terms of the Fermi momentum
- Reverse-engineer a Skyrme functional for applications in nuclei
  - Bulk and static properties are found independent of the effective mass!
  - **We can vary the bulk EoS and m* independently**
- Effective mass and ...
  - Polarizability of ^{68}Ni (static)
  - “Fluffiness” of Sn isotopes

preliminary / in progress

- ... many other explorations to come

Thank you!

Single-particle levels