

# TOROIDAL MODE: FROM GIANT RESONANCE TO INDIVIDUAL STATES

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# Motivation and content

The **toroidal dipole resonance** (TDR) is interesting in many aspects:

- a remarkable example of **electric intrinsic vortical** motion in nuclei, which **does not** contribute to the continuity equation

$$\dot{\rho} + \vec{\nabla} \cdot \vec{j}_{nuc} = 0$$

Repko, P.-G. Reinhard, VON, J. Kvasil,  
PRC, 87, 024305 (2013).

- source of **pygmy dipole resonance** (PDR)
- constitutes low-energy part of ISGDR
- Problems with TDR experiment: there are only indirect  $(\alpha, \alpha')$  data.
- New alternative way:  
recent calculations show that in **light** nuclei should exist **individual dipole toroidal states (TS)** with  $I^\pi K = 1^-1$ .

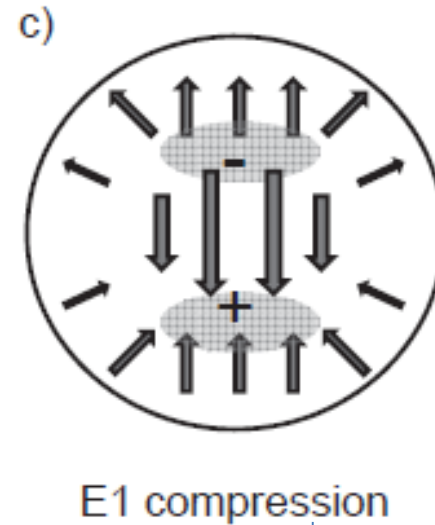
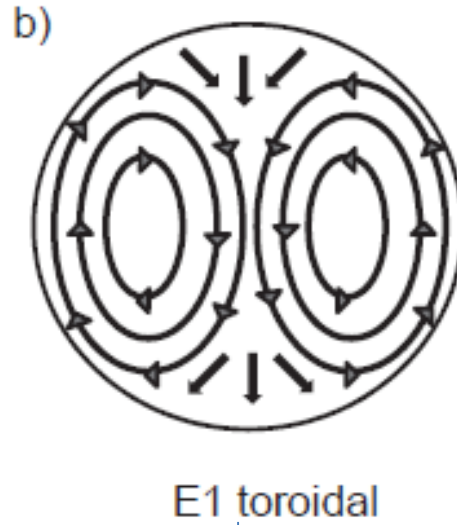
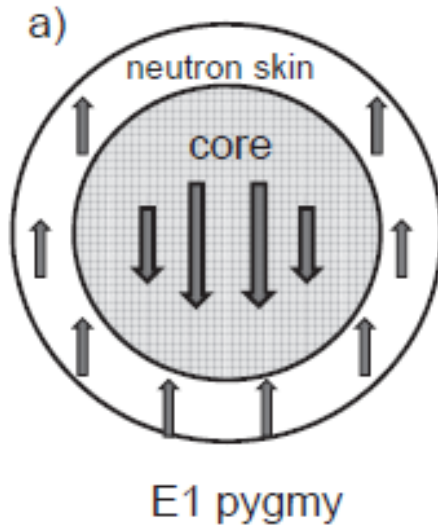
VON, A. Repko, J. Kvasil and P.-G. Reinhard,  
PRL 120, 182501 (2018)

# Exotic dipole resonances

R. Mohan et al (1971),

V.M. Dubovik (1975)  
S.F. Semenko (1981)

M.N. Harakeh (1977)  
S. Stringari (1982)



Alternative source of information on nuclear incompressibility

Dominate in E1(T=0) excitation channel  
(due to suppression of dominant E1(T=1) motion)

irrotational

vortical

irrotational

$$E = 50 \div 60 A^{-1/3} \text{ MeV}$$

$$E = 50 \div 70 A^{-1/3} \text{ MeV}$$

$$E = 132 A^{-1/3} \text{ MeV}$$

Reviews:

N. Paar et al, Rep. Prog. Phys. 70 691 (2007);

D. Savran et al, Prog. Part. Nucl. Phys. 70, 210 (2013)

VON, J. Kvasil, A. Repko, W. Kleinig, and P.-G. Reinhard, Phys. Atom. Nucl. 79, 842 (2016).


- Different kinds of dipole oscillations with fixed c.m.

### Toroidal E1 operator:

$$\hat{M}_{tor}(E1\mu) = \frac{1}{10\sqrt{2}c} \int d\vec{r} \left[ r^3 + \frac{5}{3} r \langle r^2 \rangle_0 \right] \vec{Y}_{11\mu}(\hat{\vec{r}}) \cdot \underbrace{[\vec{\nabla} \times \hat{\vec{j}}_{nuc}(\vec{r})]}_{\text{vortical flow}}$$

### Compression E1 operator:

$$\hat{M}_{com}(E1\mu) = -\frac{i}{10c} \int d\vec{r} \left[ r^3 - \frac{5}{3} r \langle r^2 \rangle_0 \right] Y_{1\mu} \underbrace{[\vec{\nabla} \cdot \hat{\vec{j}}_{nuc}(\vec{r})]}_{\text{irrotational flow}} \quad \dot{\rho} + \vec{\nabla} \cdot \vec{j}_{nuc} = 0$$



$$\hat{M}'_{com}(E1\mu) = \int d\vec{r} \hat{\rho}(\vec{r}) \left[ r^3 - \frac{5}{3} r \langle r^2 \rangle_0 \right] Y_{1\mu}$$

### Toroidal and compression operators are coupled:

$$\hat{M}_{tor}(E1\mu) = -\frac{i}{2\sqrt{3}c} \int d\vec{r} \hat{\vec{j}}_{nuc}(\vec{r}) \cdot \vec{\nabla} \times (\vec{r} \times \vec{\nabla}) \left[ r^3 - \frac{5}{3} r \langle r^2 \rangle_0 \right] Y_{1\mu}(\hat{\vec{r}})$$

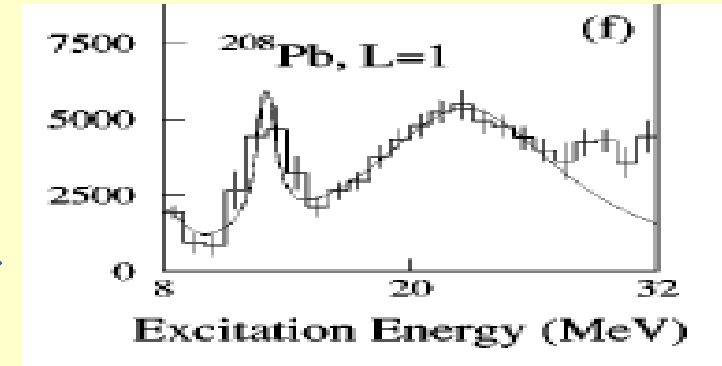
# TDR and CDR constitute low- and high-energy ISGDR branches (?)

## Experiment: $(\alpha, \alpha')$

- $^{208}\text{Pb}$
- D.Y. Youngblood et al, 1977
  - H.P. Morsch et al, 1980
  - G.S. Adams et al, 1986
  - B.A. Devis et al, 1997
  - H.L. Clark et al, 2001
  - D.Y. Youngblood et al, 2004
  - M.Uchida et al, PRC 69, 051301(R) (2004)

Familiar treatment  $\longrightarrow$

LE (toroidal)      HE (compression)



TDR      CDR

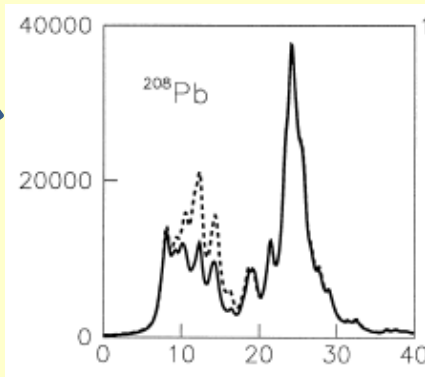
A.Repko, P.-G. Reinhard, V.O.N. and J. Kvasil, PRC 87, 024305 (2013).

There are also exp ISGDR data in

$^{56}\text{Fe}$ ,  $^{58,60}\text{Ni}$ ,  $^{90}\text{Zr}$ ,  $^{116}\text{Sn}$ ,  $^{144}\text{Sm}$ , ...

## Theory:

- G. Colo et al, PLB 485, 362 (2000)
- D. Vretenar et al, PRC, 65, 021301(R) (2002)
- N. Paar et al, Rep. Prog. Phys. 70 691 (2007);



Perhaps Uchida observed at 10-17 MeV not TDR but CDR fraction coupled to TDR. Main TDR peak should lie lower at ~ 7-9 MeV.

The direct observation of TDR in  $(\alpha, \alpha')$  can be disputed in general since  $(\alpha, \alpha')$  is mainly determined by transition density while toroid depends on the vortical transition current.

**NEED IN NEW EXPERIMENTS!**

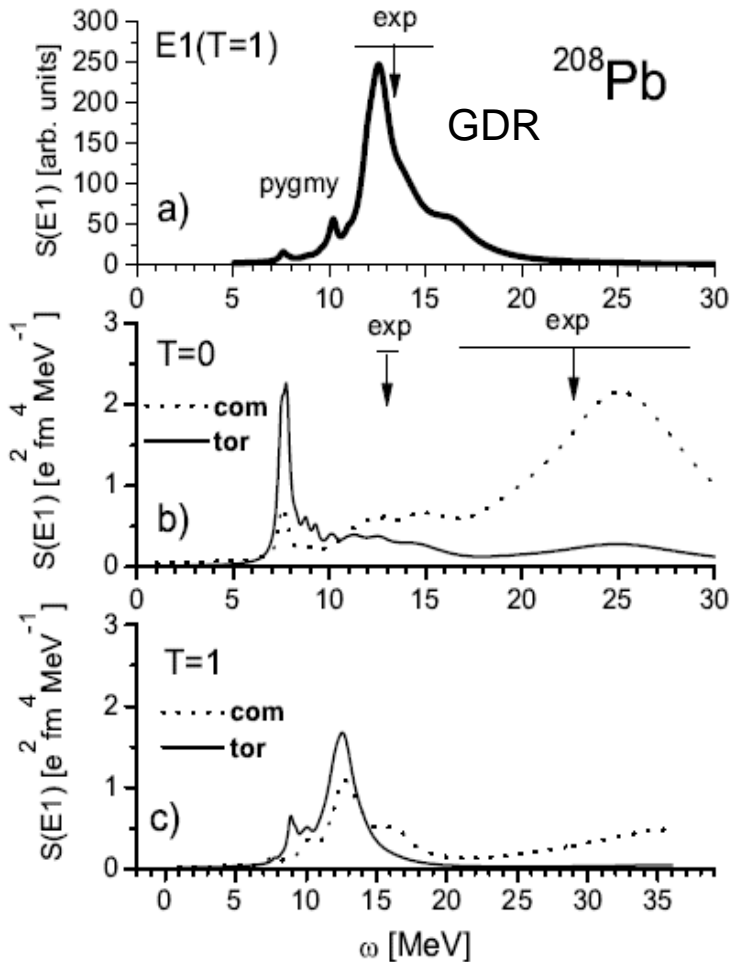
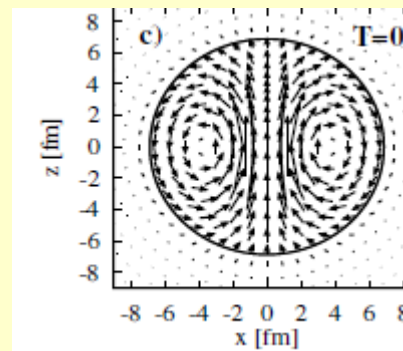
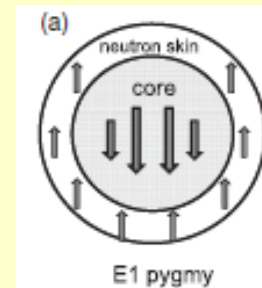
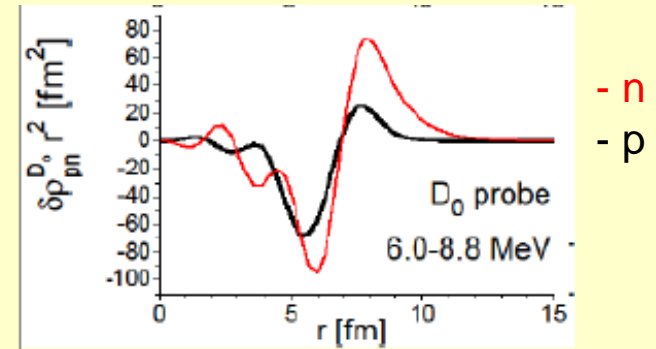
# Strength functions

SLy6

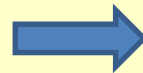
A. Repko, P.G. Reinhard, VON, J. Kvasil, PRC, 87, 024305 (2013)

## PDR region hosts TDR and CR!

Typical PDR transition density:

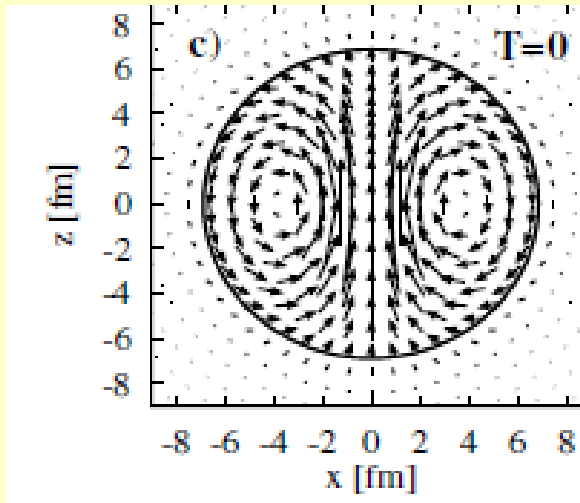


Nucleon current in the PDR region is mainly toroidal!



**Skyrme RPA:** 208Pb

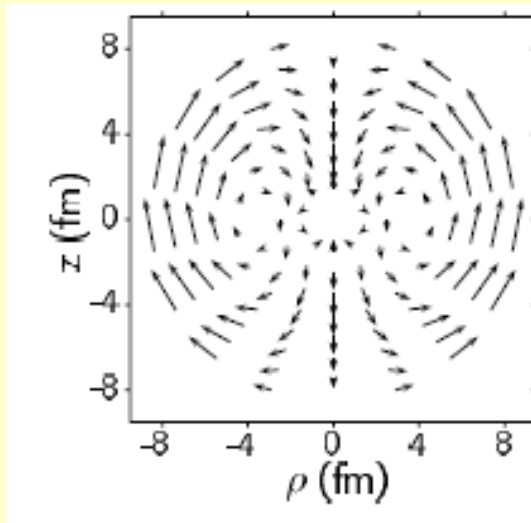
Repko, P.-G. Reinhard, VON, J. Kvasil,  
PRC, 87, 024305 (2013).



- Similar results for Ca, Ni, Zr, Sn, Sm, Yb, U

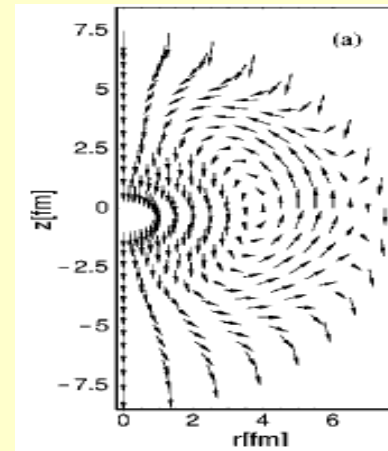
**QPM:** 208Pb

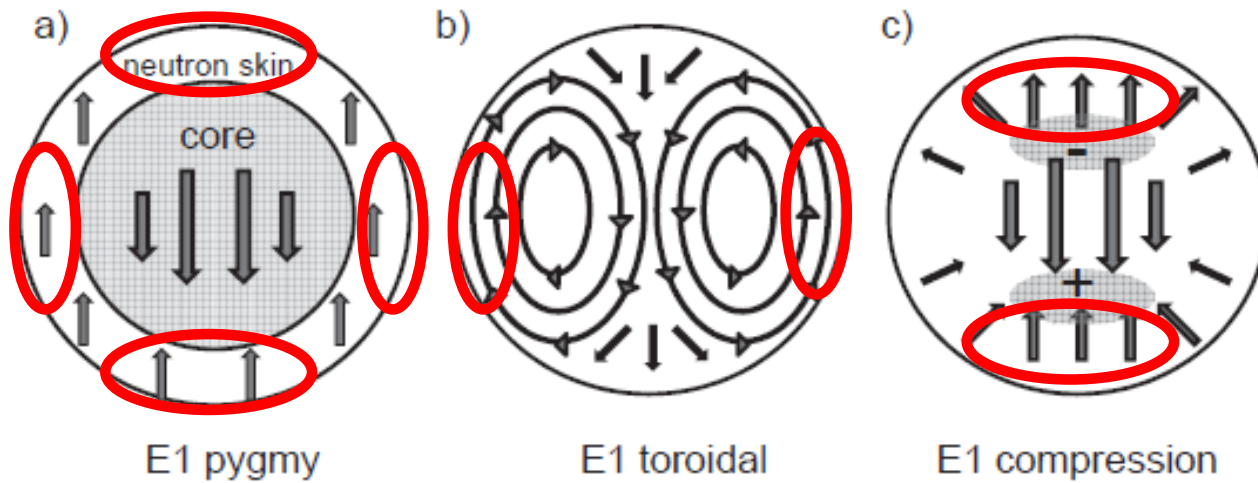
N.Ryezayeva et al, PRL 89, 272502 (2002).



**Relativistic RPA:** 116Sn

D. Vretenar et al, PRC 65, 021301R (2002).





VON, J. Kvasil, A. Repko, W. Kleinig, P.-G. Reinhard, **Phys. Atom. Nucl.**, 79, 842 (2016).

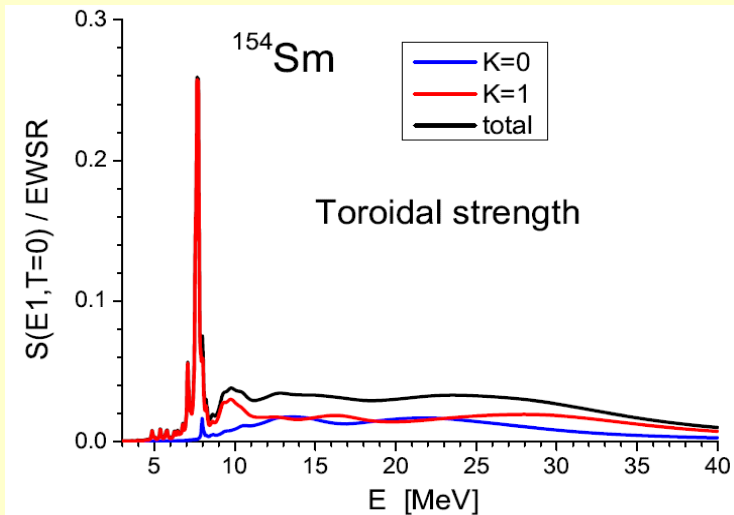
- PDR can be viewed as a local peripheral part of TDR and CDR
- Such a treatment of PDR does not affect a bulk of previous theoretical and experimental results for PDR. They remain to be valid.
- However we obtain a better knowledge what is really happens with dipole states in the PDR region.



# Deformation features of TDR

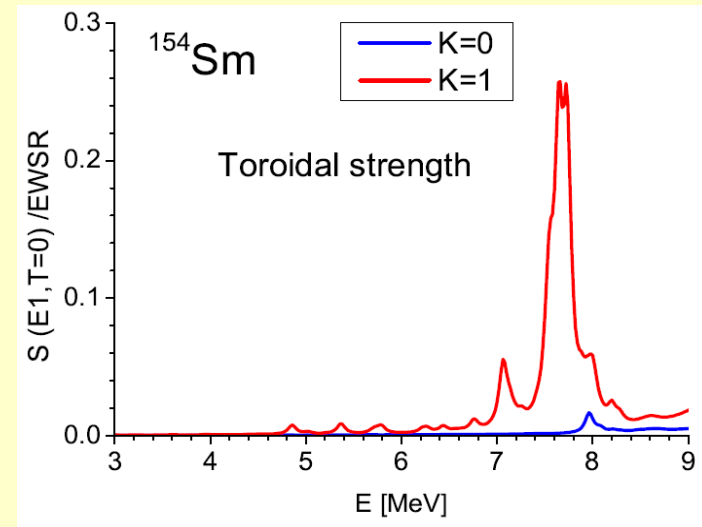
$^{154}\text{Sm}$ , SLy6  $\beta_2^{\text{exp}} = 0.339$

Energy-weighted strength functions

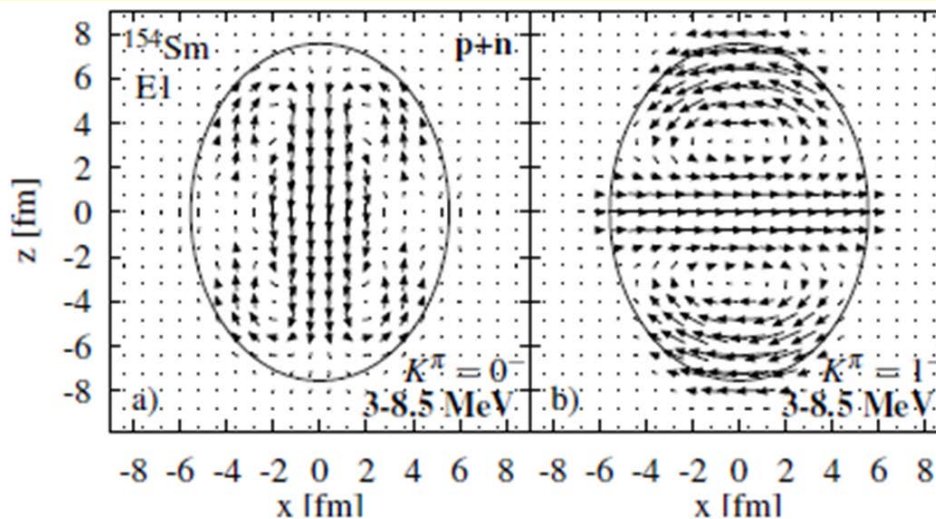


# Skyrme QRPA, SLy6

A. Repko, J. Kvaail, VON,, P.-G. Reinhard,  
EPJA, 53, 221 (2017)



K=1 dominates !!!



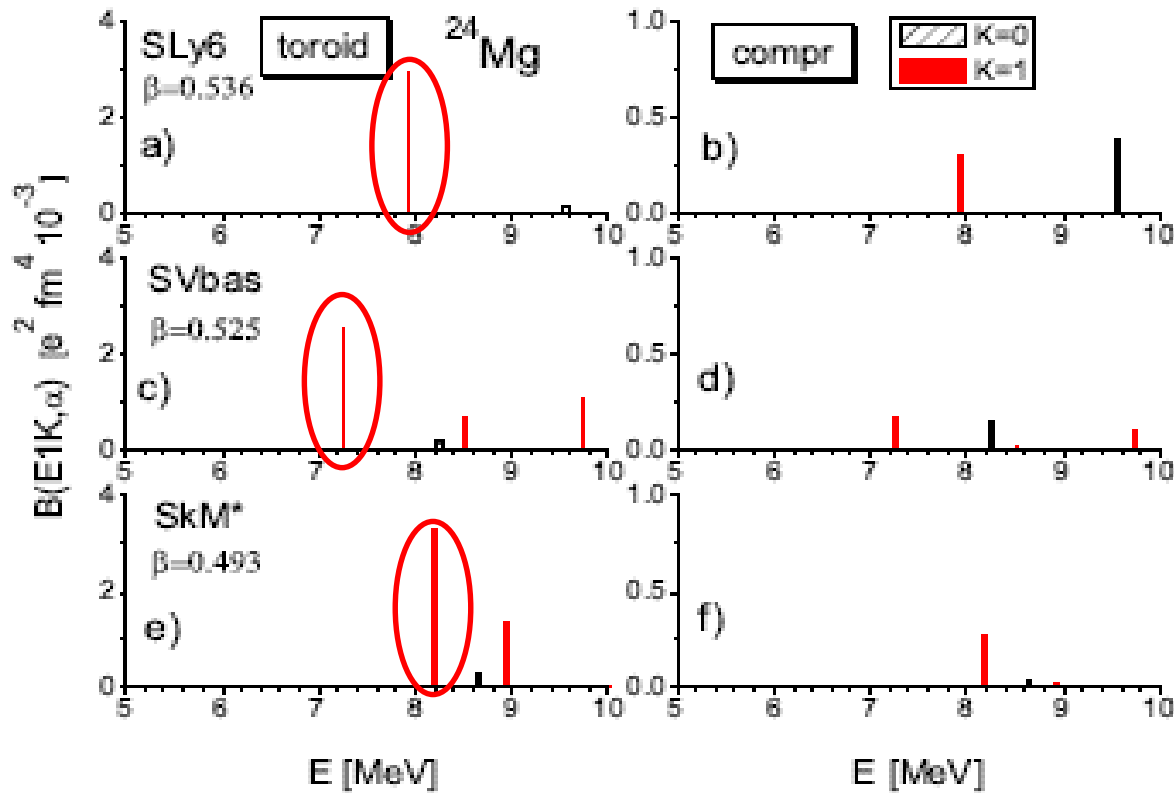
Similar results for other prolate nuclei.

K=1 dominance can in principle be used as **TDR fingerprint** in future experiments.

If prolate deformation is so important, then what we will get in nuclei with a huge axial deformation, like  $^{24}\text{Mg}$ ?

$^{24}\text{Mg}$

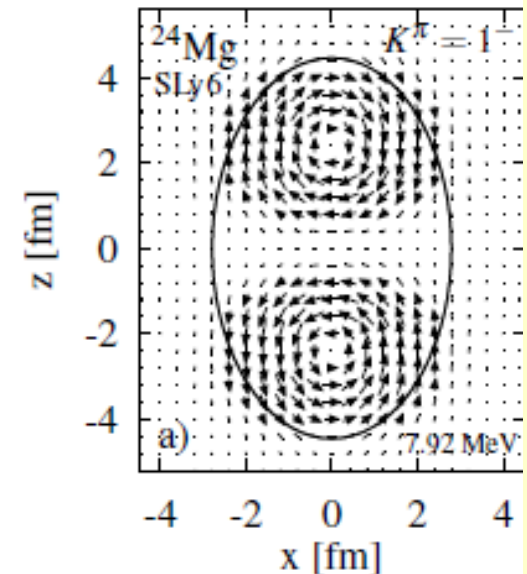
$$\beta_2^{\text{exp}} = 0.605$$



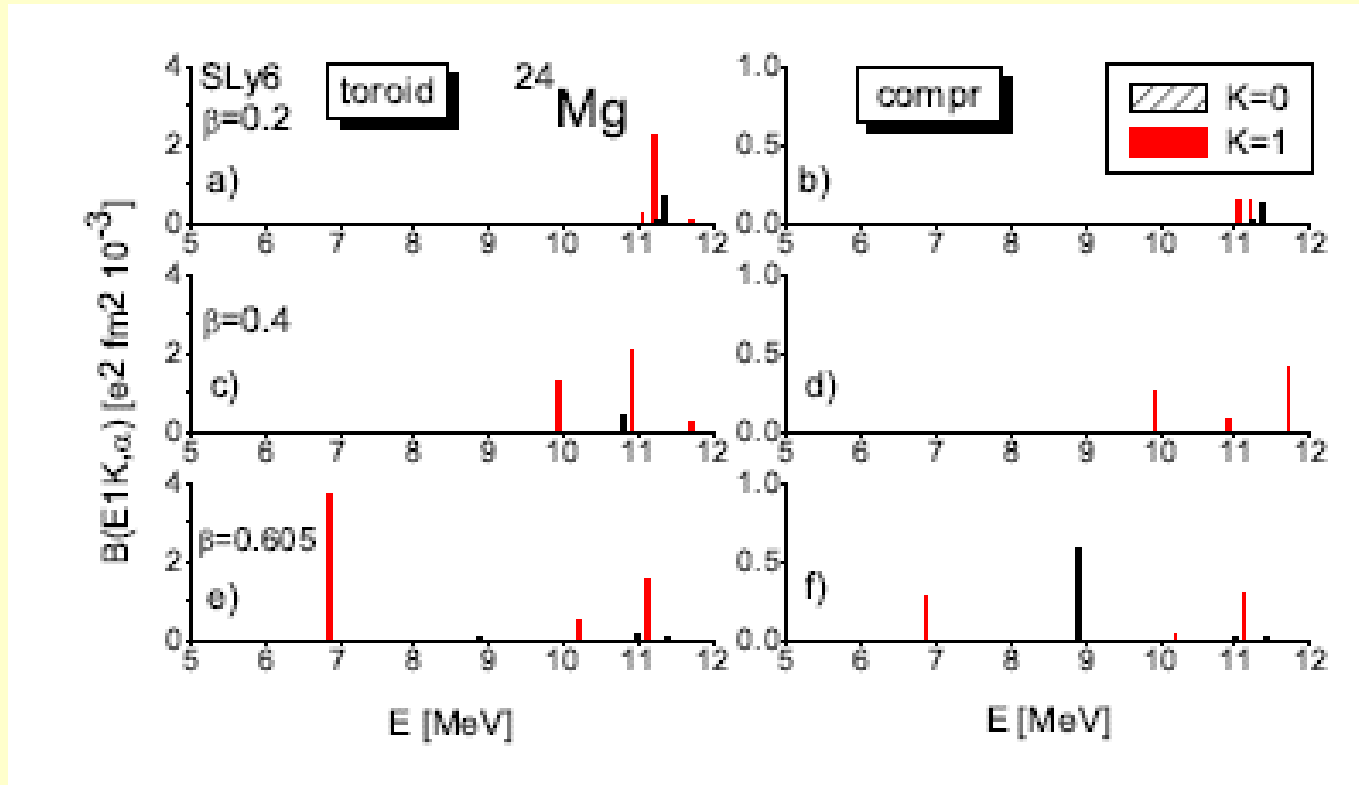
QRPA results for  
SLy6,  
SVbas,  
SkM\*

Persistence of the main result:  
the **lowest** toroidal  $K=1$  peak

The remarkable example of  
individual toroidal state!



## Dependence on deformation



TS becomes lowest **due to of the large axial prolate deformation.**

$K=1$  peak is:

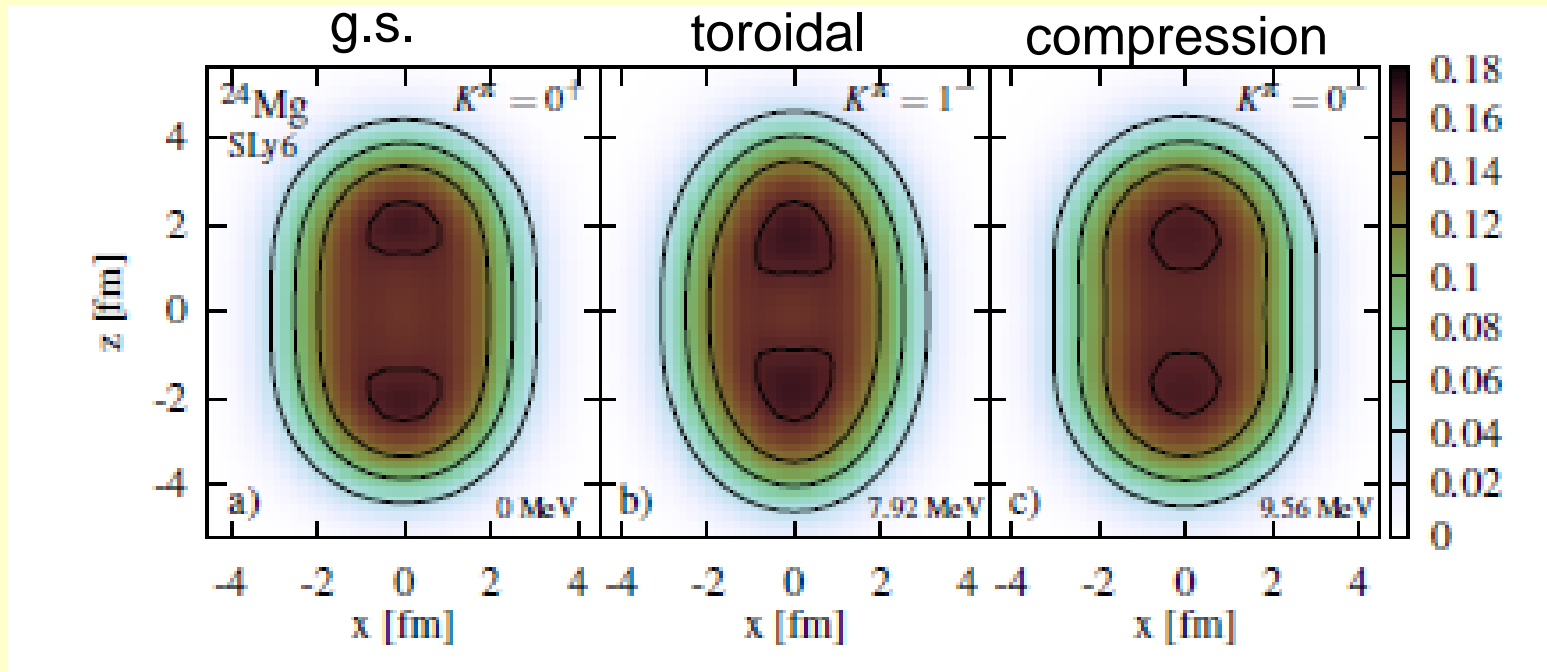
- the **lowest** dipole state
- well separated from other states

To get individual lowest TS, **two rigorous requirements** should be held:

- huge prolate deformations
- sparse low-energy spectrum

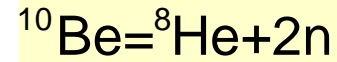
This can be realized just in light deformed nuclei

## Relation to cluster structure of $^{24}\text{Mg}$



Densities for the ground, toroidal and compression states

- Excitation energies near  $\alpha$  particle threshold  $S_\alpha = 9.3$  MeV
- Cluster structure in all three states.
- Conversion of the vortex ring into vortex-antivortex pair.
- Toroidal flow looks as rotation of two clusters in the opposite direction.  
Appearance of specific vortical rotational bands?

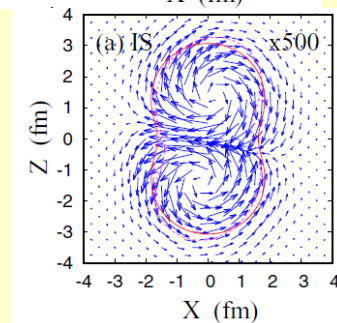
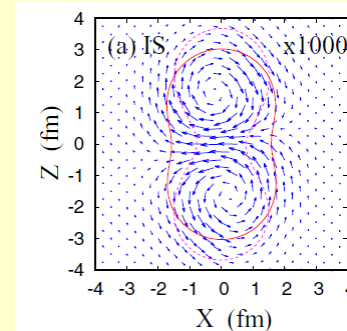
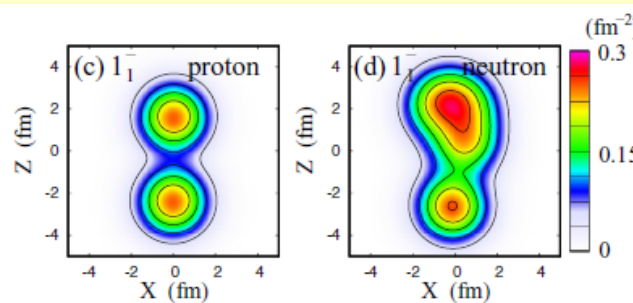
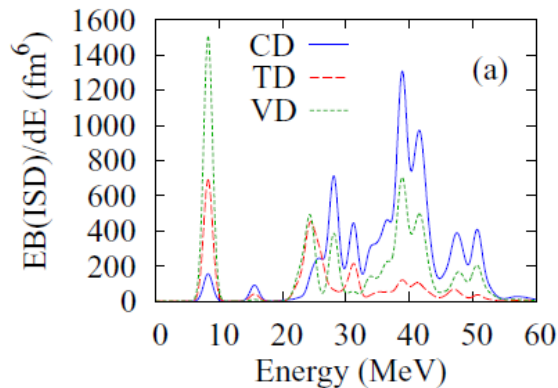


**Toroidal, compressive, and  $E1$  properties of low-energy dipole modes in  $^{10}\text{Be}$**

Yoshiko Kanada-En'yo and Yuki Shikata

Department of Physics, Kyoto University, Kyoto 606-8502, Japan

We studied dipole excitations in  $^{10}\text{Be}$  based on an extended version of the antisymmetrized molecular dynamics, which can describe 1p-1h excitations and large amplitude cluster modes. Toroidal and compressive dipole operators are found to be good probes to separate the low-energy and high-energy parts of the isoscalar dipole excitations, respectively. Two low-energy  $1^-$  states, the toroidal dominant  $1_1^-$  state at  $E \sim 8$  MeV and the  $E1$  dominant  $1_2^-$  state at  $E \sim 16$  MeV, were obtained. By analysis of transition current densities, the  $1_1^-$  state is understood as a toroidal dipole mode with exotic toroidal neutron flow caused by rotation of a deformed  $^6\text{He}$  cluster, whereas the  $1_2^-$  state is regarded as a neutron-skin oscillation mode, which are characterized by surface neutron flow with inner isoscalar flow caused by the surface neutron oscillation against the  $2\alpha$  core.



AMD

cluster  
GCM

the lowest dipole state  
 $1^\pi K = 1^- 1_1$  is toroidal!

- [21] J. Kvasil, V. O. Nesterenko, W. Kleinig, P.-G. Reinhard, and P. Vesely, *Phys. Rev. C* **84**, 034303 (2011).
- [22] A. Repko, P.-G. Reinhard, V. O. Nesterenko, and J. Kvasil, *Phys. Rev. C* **87**, 024305 (2013).
- [23] V. O. Nesterenko, J. Kvasil, A. Repko, W. Kleinig, and P.-G. Reinhard, *Phys. At. Nucl.* **79**, 842 (2016).

How to check experimentally these theoretical predictions?

What is the suitable reaction to observe TM in experiment?

Which signature can be used for unambiguous identification of TM?

What about (e,e')?

Here we meet the problem: impact of the magnetization current

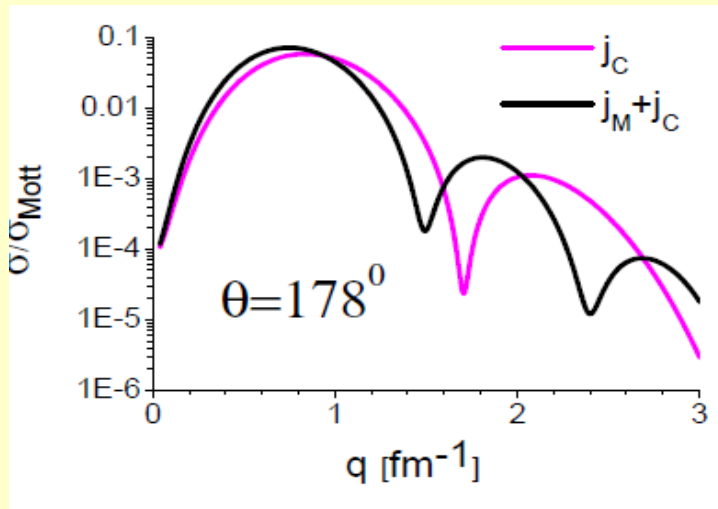
$$\hat{M}_{tor}(E1\mu) = \frac{1}{10\sqrt{2}c} \int d\vec{r} [r^3 + \frac{5}{3}r \langle r^2 \rangle_0] \vec{Y}_{11\mu}(\hat{r}) \cdot [\vec{\nabla} \times \hat{j}_{nuc}(\vec{r})]$$

Nuclear current

$$\hat{j}_{nuc}(\vec{r}) = \frac{e\hbar}{m} \sum_{q=n,p} (\hat{j}_{con}^q(\vec{r}) + \hat{j}_{mag}^q(\vec{r}))$$

$$\hat{j}_{con}^q(\vec{r}) = -ie_{eff}^q \sum (\delta(\vec{r} - \vec{r}_k) \vec{\nabla}_k - \vec{\nabla}_k \delta(\vec{r} - \vec{r}_k)) \rightarrow \text{toroidal flow}$$

$$\hat{j}_{mag}^q(\vec{r}) = \frac{g_s^q}{2} \gamma \sum_{k \ni q} \vec{\nabla}_k \times \hat{s}_{qk} \delta(\vec{r} - \vec{r}_k), \quad \gamma = 0.7$$



PWBA for 24Mg

Impact of  $\vec{j}_M$  current is significant at  $q > 1 \text{ fm}^{-1}$ , and makes toroidal effect unresolved.

## Possible routs to observe the toroidal mode:

- 1) Indirect (CM  $\rightarrow$  TM) excitation of TM in  $(\alpha, \alpha')$  using the CM/TM coupling.
- 2)  $(e, e' \gamma) \rightarrow$  scattering angle of the photon can depend on the nuclear flow.
- 3) Using polarizability: toroidal flow can polarize the outcoming electron.
- 4) To use not current distributions but other TDR features (like in the case of the scissors M1 mode).

The search of TM is a part of the **fundamental** problem.

On the TM example, we see that modern experiment is not yet able to measure and identify electric **intrinsic vortical** excitations.

Nuclear vortical dynamics is still terra incognita.



# Conclusions

- ★ Toroidal dipole resonance (TDR) is interesting in many aspects:
  - example of electric intrinsic vortical motion,
  - TDR is a possible origin of PDR.

However TDR has problems with direct experimental observation.

- ★ We propose a new route: investigation of **individual** toroidal states (TS) in light nuclei. Light nuclei seem to be very promising for this aim. Interesting relations with a cluster structure.
- ★ The quest of TM is a part of the general **fundamental** problem: ability of modern theory and experiment to explore **vortical** nuclear flow.

Thank you for attention!