

Small System Corrections to Thermal Field Theory and pQCD Energy Loss

I Kolbé¹, W A Horowitz¹, and S Mogliacci¹

¹Department of Physics, R. W. James Building, University Road, University of Cape Town, Rondebosch, 7700, South Africa

E-mail: isobel.kolbe@gmail.com

Abstract. The ominous absence of partonic energy loss in small colliding systems is forcing the heavy-ion community to sharpen its theoretical tools. In particular, all the main forms of pQCD energy loss depend on the assumption that the brick of QGP is large, but it is clear that any droplet of QGP that may be created in high multiplicity $p/d + A$ or peripheral AA collisions cannot be much larger than the mean free path, $\lambda_{mfp}^{\text{QCD}} \sim 1$ fm. We present an illuminating and exhaustive numerical analysis of the recent extension of DGLV energy loss to all separation distances, and show a need for stronger theoretical control over the nature of the medium at early times and short separation distances. Prompted by this demand for an understanding of the small system corrections to medium properties that are traditionally computed in an infinite medium, we then present a thermal field theoretical calculation of the thermodynamic properties of a toy model. We consider a single, massless, scalar field and impose Dirichlet boundary conditions in order to investigate the qualitative effect of a boundary on the properties of the medium, finding significant deviations from the standard Stefan-Boltzmann results, even at relatively large L .

1. Introduction

A long program of high energy heavy-ion experiments at the Large Hadron Collider (LHC), complimented by similar experiments at the Relativistic Heavy Ion Collider (RHIC), have produced a wealth of data concerning the nature of what is believed to be a droplet of deconfined quark matter - the Quark Gluon Plasma (QGP). It is widely accepted that the QGP is produced in experiments involving collision systems that are considered to be large, such as nearly head-on lead-lead (PbPb) or gold-gold (AuAu) collisions (collectively, AA) [1–5]. The heavy-ion community's faith in the presence of the QGP in such large systems is due to the presence of a number of signatures of the QGP, most of which are well described within the paradigm of the production of a strongly coupled, almost perfect fluid. However, the heavy-ion community has now spent half a decade unable to resolve a major inconsistency in its understanding of the QGP; The inconsistency arises from the fact that many of the traditional signatures of the QGP are present also in smaller colliding systems such as proton-proton (pp), proton- or deuteron-gold and proton-lead (p/dA), and peripheral AA collisions, with the notable exception of one key observable - the partonic energy loss of color-charged objects traversing the QGP[6].

Resolving the question of the absence of energy loss in the apparent presence of the QGP requires a range of cross-checks and refinements. It is clear that one must attempt to understand the problem from both an experimental and a theoretical perspective. While major advances have been made on the experimental side, both in measuring the observed signatures more carefully (see, for instance, [7]) and understanding the biases related to measuring energy loss in the traditional manner [8], theoretical advances have been slow out of the blocks. Although some models have been put forward that might

explain the presence of a QGP signature in the absence of a QGP, see for instance [9], they are unable to satisfactorily compliment the self-consistent picture of the presence of the QGP in large colliding systems.

A logical first step to understanding the absence of energy loss in small systems from a theoretical standpoint would be to simply compute the expected energy loss, since energy loss studies in large systems have lead to an understanding of the path-length dependence of energy loss [10, 11]. Such studies have been performed [12], but over-predict the energy loss in small systems. Their inability to reproduce the data might be explained by the simple fact that all major pQCD energy loss calculations assume that the system (L) is large compared to the mean free path (λ_{mfp}) of the parton.

In these proceedings we present two very different attempts at understanding the nature of a small droplet of QGP. First, we briefly review the recent computation of a correction to DGLV (Djordjevic, Gyulassy, Levai, Vitev) pQCD energy loss which relaxes an assumption whereby the first scattering of a hard parton traversing a QGP is assumed to be far from its production within the QGP. Armed with such a correction, we present a numerical investigation of the energy loss formula, finding a crucial dependence of the energy loss on the physics of short separation distances between the production of the hard parton and its first scattering.

In the second part of the proceedings we compute the thermodynamic properties of a toy model, consisting of a single, free, massless, scalar field, which has been subjected to a “geometrical confinement” in which Dirichlet boundary conditions are place on the field. We find that the fundamental properties of a quantum field are altered when it is confined to a small space.

2. pQCD Energy Loss in a Small System

In these proceedings the aim is simply to present the most recent findings of a calculation which involved computing the short separation distance correction to standard DGLV energy loss, and to offer them as motivation for the subsequent calculation in section 3. The reader is referred to the original DGLV calculation [13], the Masters thesis [14] in which details of the calculation of the correction are given and the upcoming publication already available online [15], for further details. For clarity, we treat the high- p_T eikonal parton produced at an initial point (t_0, z_0, \mathbf{x}_0) inside a finite QGP brick, where we have used \mathbf{p} to mean transverse 2D vectors, $\vec{\mathbf{p}} = (p_z, \mathbf{p})$ for 3D vectors and $p = (p^0, \vec{\mathbf{p}}) = [p^0 + p^z, p^0 - p^z, \mathbf{p}]$ for four vectors in Minkowski and light cone coordinates, respectively. As in the DGLV calculation, we consider the n th target to be a Gyulassy-Wang Debye screened potential [16] with Fourier and color structure given by

$$V_n = 2\pi\delta(q^0)\frac{4\pi\alpha_s}{\vec{q}_n^2 + \mu^2}e^{-i\vec{q}_n\cdot\vec{x}_n}T_{a_n}(R) \otimes T_{a_n}(n), \quad (1)$$

where the color exchanges are handled using the applicable $SU(N_c)$ generator, $T_a(n)$ in the d_n dimensional representation of the target, or $T_a(R)$ in the d_R dimensional representation of the high- p_T parent parton.

In light cone coordinates the four-momenta of the emitted gluon, the final high- p_T parton, and that exchanged with the medium Debye quasiparticle are, respectively,

$$k = \left[xP^+, \frac{m_g^2 + \mathbf{k}^2}{xP^+}, \mathbf{k} \right], \quad p = \left[(1-x)P^+, \frac{M^2 + \mathbf{k}^2}{(1-x)P^+}, \mathbf{q} - \mathbf{k} \right], \quad q = [q^+, q^-, \mathbf{q}], \quad (2)$$

where the initially produced high- p_T particle of mass M has large momentum $E^+ = P^+ = 2E$ and negligible other momentum components. Notice that we include the QCD analogue of the Ter-Mikayelian plasmon effect [17], a color dielectric modification of the gluon dispersion relation, with an effective emitted gluon mass m_g [13, 18]. See figure 1 for a visualization of the relevant momenta.

Following [13] we define $\omega \approx xE^+/2 = xP^+/2$, from which a shorthand for energy ratios will prove useful notationally: $\omega_0 \equiv \mathbf{k}^2/2\omega$, $\omega_i \equiv (\mathbf{k} - \mathbf{q}_i)^2/2\omega$, $\omega_{(ij)} \equiv (\mathbf{k} - \mathbf{q}_i - \mathbf{q}_j)^2/2\omega$, and $\tilde{\omega}_m \equiv (m_g^2 + M^2x^2)/2\omega$.

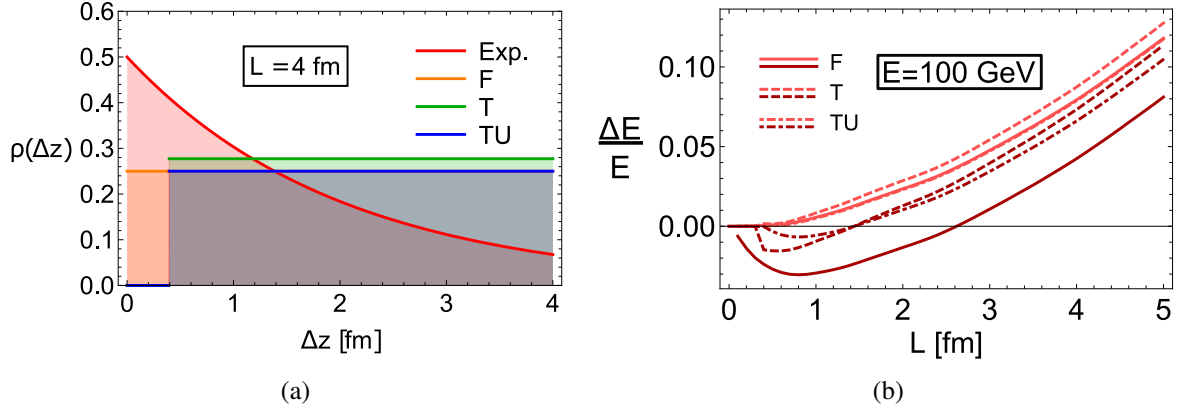


Figure 2: (a) The four scattering center distributions discussed in the text, see equation (4), for a brick of QGP of length $L = 4$ fm. (b): The relative energy loss of a bottom quark with energy $E = 100$ GeV, as a function of length of the path traversed through the brick of QGP, showing the original DGLV result (light) and the all separation distance result (dark), for three different choices of scattering center distribution.

(solid), Truncated (dashed), and Truncated Un-renormalized (dot-dashed) step functions, for both the original DGLV (light) and our all separation distance result (dark). Notice that, although the DGLV result is not sensitive to the choice of scattering center distribution, the all separation distance result is extremely sensitive to the physics of short separation distances. This sensitivity persists out to large (~ 5 fm) path lengths. The reader is referred to [15] for extensive discussion surrounding the mass and energy dependence of the all separation correction as well as an investigation into the nature of the sensitivity to short separation times. It is clear from extreme sensitivity of the energy loss to the physics of small separation distances that a better understanding of the nature of a small droplet of QGP is required.

3. Thermodynamics of a Small System

In order to investigate the nature of a quantum field that is subjected to spatial constraints, we present results from an investigation that involved considering a single, free, massless, scalar field upon which are placed Dirichlet boundary conditions. We then compute the thermodynamic properties of such a field. The reader is referred to our upcoming publication which is already online for details of the calculations [19]. For clarity we outline the calculation here.

In the standard thermal field theoretic formalism [20], the partition function may be written as a Feynman path integral

$$\mathcal{Z}(T, V) \propto \int_{\phi(0)}^{\phi(\beta)} [\mathcal{D}\phi] \exp \left\{ - \int_0^\beta d\tau \int_V d^{D-1}x \mathcal{L} \right\} \Big|_{\phi(\beta)=\phi(0)} \quad (5)$$

In the usual Matsubara approach, one would now compute a Fourier decomposition of the field by compactifying both the temporal and the spatial dimensions onto circles. We propose compactifying instead those spatial dimensions that one would want to consider as having finite extent onto lengths L_i . For pedagogical reasons, we will compactify one spatial dimension at a time, but the second and third compactifications follow simply and are presented in detail in [19]. We therefore consider the following Fourier decomposition of a single, free, massless scalar field which has finite extent in the x_1 -direction.

$$\phi(\tau, z_1, \mathbf{x}) = \sum_{n \in \mathbb{Z}} \sum_{\ell_1 \in \mathbb{N}} \sum_{\mathbf{k} \in \mathbb{Z}^{D-2}} \sqrt{\frac{2\beta}{L_1 R^{D-2}}} \exp \{ i\omega_n \tau - i\omega_{\mathbf{k}} \cdot \mathbf{x} \} \sin \{ \omega_{\ell_1} z_1 \} \tilde{\phi}_{n, \ell_1}(\omega_{\mathbf{k}}) \quad (6)$$

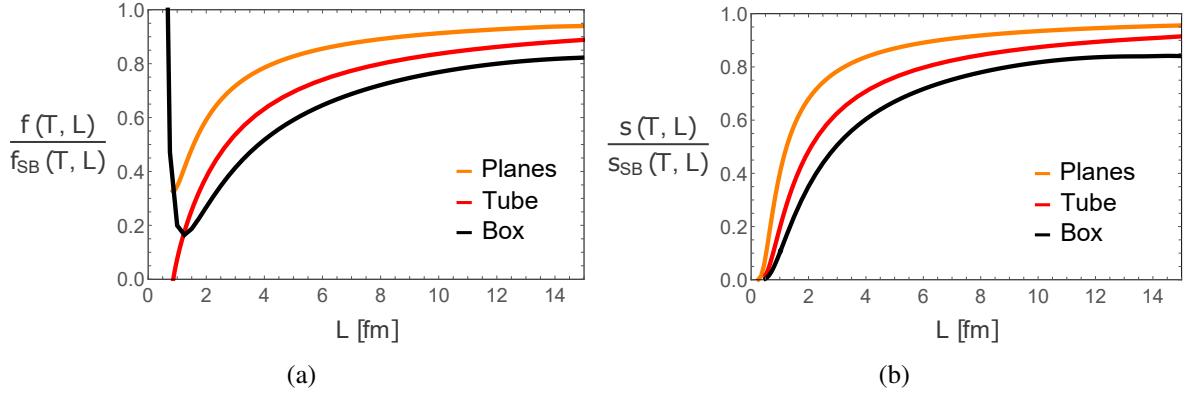


Figure 3: The free energy density (a) and entropy density (b) of a single, free, massless scalar field with one (orange), two (red) and three (black) spatial dimensions subjected to Dirichlet boundary conditions and rescaled to the Stefan-Boltzmann result.

One may now follow the usual procedure in which equation (6) is substituted into equation (5) and the partition function computed by taking all summations in infinite dimensions to integrals. We deviate from the usual procedure in that we retain those summations that are considered to occur in finite dimensions (here, in the case of only one compactified dimension, the summation over ℓ_1). These remaining summations must then be computed using a combination of Dimensional Regularisation and Epstein-Zeta Regularization. We have computed two numerically equivalent forms of the partition function that have different analytical forms and therefore different convergence properties. We present here only one of the results and only for the case of one compactified dimension, the reader is referred to [19] for more details.

The partition function may be used to compute any thermodynamic quantity. For instance, the free energy density and the entropy density

$$f = -\frac{T}{V} \text{Ln} \mathcal{Z}, \quad s = \frac{1}{V} \left. \frac{\partial F}{\partial T} \right|_{\{L_k\}}. \quad (7)$$

The free energy density of a single, free, massless scalar field which is subjected to Dirichlet boundary conditions in one dimension is then given by

$$f^{(1)} = -\frac{\pi^2}{1440L_1^4} - \frac{T^2}{2L_1^2} \sum_{\ell=1}^{\infty} \left[\ell \text{Li}_2 \left(e^{-\frac{\pi \ell}{TL_1}} \right) \right] - \frac{T^3}{2\pi L_1} \sum_{\ell=1}^{\infty} \left[\text{Li}_3 \left(e^{-\frac{\pi \ell}{TL_1}} \right) \right]. \quad (8)$$

By compactifying one dimension, we are considering a situation in which the scalar field is confined between two infinite parallel plates. However, one may easily compute the cases for an infinite tube (compactifying two dimensions) or a fully compactified box. The reader is referred to [19] for the full expressions for the tube and box cases. We present the free energy density and the entropy density for these three cases in figure 3.

Further results from this study are presented in the present proceedings by S. Mogliacci. However, it is already clear from these first results that the nature of a quantum field is changed fundamentally by the presence of a boundary, since a correction of almost 20% is found in the box case, even for asymptotically large L .

4. Conclusions

The fact that partonic energy loss has not been observed in small systems while many other signatures of the QGP appear to be present, has not yet been satisfactorily understood. One major stumbling block

to understanding the absence of energy loss in small colliding systems is the fact that there is very little theoretical control over finite systems.

We have presented the final results from a numerical study of a partonic energy loss formula that was derived in the DGLV formalism to include the effects of short separation distances between the production of a hard parton and its first scattering within the medium. We showed a strong dependence on the physics of separation distances on the order of the Debye screening length. We then presented preliminary results involving the study of a single, free, massless scalar field subjected to Dirichlet boundary conditions, revealing substantial deviations from the Stefan-Boltzmann limit, even for large systems.

These findings suggest that, not only are the properties of a droplet of QGP fundamentally different when the fields are subjected to finite boundary conditions, but also that the dynamics of energy loss are altered at short time scales. These effect need to be investigated quantitatively if the heavy-ion community is to understand the absence of partonic energy loss in small systems.

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