Precision Determination of the Light Quark Masses from QCD Finite Energy Sum Rules

Alexes K. Mes

Department of Physics, University of Cape Town, Private Bag X3, Rondebosch 7701, South Africa

E-mail: `MSXALE002@myuct.ac.za`

Abstract. The light quark masses are determined from a QCD finite energy sum rule, using the pseudoscalar correlator to six-loop order in perturbative QCD, with the leading vacuum condensates and higher order quark mass corrections included. Both the fixed order perturbation theory (FOPT) method and contour improved perturbation theory (CIPT) method are explored. The results in the latter framework exhibit good convergence and stability in the window $s_0 =$ 3.0 5.0 GeV^2 for the strange quark and $\mathrm{s}_0 = 1.5$ 4.0 GeV^2 for the up and down quarks; where s_0 is the radius of the integration contour in the complex s-plane. The results are: $\overline{\mathrm{m}}_s(2\ \mathrm{GeV}) = 91.8 \pm 9.9$ MeV, $\overline{\mathrm{m}}_u(2\ \mathrm{GeV}) = 2.6 \pm 0.4$ MeV, $\overline{\mathrm{m}}_d(2\ \mathrm{GeV}) = 5.3 \pm 0.4$ MeV, and the sum $\overline{\mathrm{m}}_{ud} \equiv (\overline{\mathrm{m}}_u + \overline{\mathrm{m}}_d)/2$, is $\overline{\mathrm{m}}_{ud}(2\ \mathrm{GeV}) = 3.9 \pm 0.3$ MeV. These proceedings critically explore how the current results - computed precisely in a modern computer language, Mathematica - compare to past determinations in literature.

1. Introduction

Quantum Chromodynamics is a confining theory, meaning that quarks do not exist in nature as single, free particles; but, are rather found in hadronic bound states. Due to this quark and gluon confinement it is impossible to determine the quark masses with the same techniques as one would use to determine the mass of non-confined particles.

There are two major approaches in Quantum Chromodynamics that can be used to calculate the quark masses at some given energy scale. Broadly this divides into a numerical approach (lattice QCD [1], [2]) and an analytical approach (QCD sum rules [3,4]). Lattice QCD discretizes space and time to reduce the infinite number of field variables in QCD to a finite countable number within the path-integral formulation of Quantum Field Theory [5]. QCD sum rules separate the perturbative and nonperturbative contributions in QCD, where the latter are described by quark and gluon condensates which are present. The sum rule method relates low energy hadronic quantities (which are measurable) to high energy expressions in QCD, such that unmeasurable QCD parameters become a function of known hadronic information.

Currently, there are numerous sum rule frameworks. The finite energy sum rule method (FESR), in which QCD sum rules are placed within the complex squared energy plane, is the approach taken to determine these results.

There are two frameworks in which the contour integral in FESR can be performed: fixed order perturbation theory (FOPT) and contour improved perturbation theory (CIPT). Fixed order keeps – as its name suggests – the strong coupling, $\alpha_s(s)$, frozen on the integration contour, and performs the renormalization group improvement after integration. On the other hand, in contour improved perturbation theory the strong coupling is running and the renormalization group improvement is implemented before integration.

2. Method and Calculations

In this determination, the preferred calculation was done by considering the pseudoscalar correlator within the framework of contour improved perturbation theory. Considering the criteria of stability and convergence, this framework was favoured over fixed order perturbation theory for both the case of the correlator determining the up and down quark mass and the correlator determining the strange quark mass. Respectively, these masses were found to be $\overline{m}_u(2\,\mathrm{GeV}) = (2.6 \pm 0.4)\,\mathrm{MeV}$, $\overline{m}_d(2\,\mathrm{GeV}) = (5.3 \pm 0.4)\,\mathrm{MeV}$ and $\overline{m}_s(2\,\mathrm{GeV}) = (91.8 \pm 9.9)\,\mathrm{MeV}$ in the CIPT framework. The reader is referred to *Light Quark Masses from QCD Finite Energy Sum Rules* by A.K. Mes [6] where details of the method behind these calculations are provided.

The light quark masses presented here are in agreement with recent sum rule and lattice QCD determinations in this field. Figures (1) and (2) show how these results compare with the current literature.

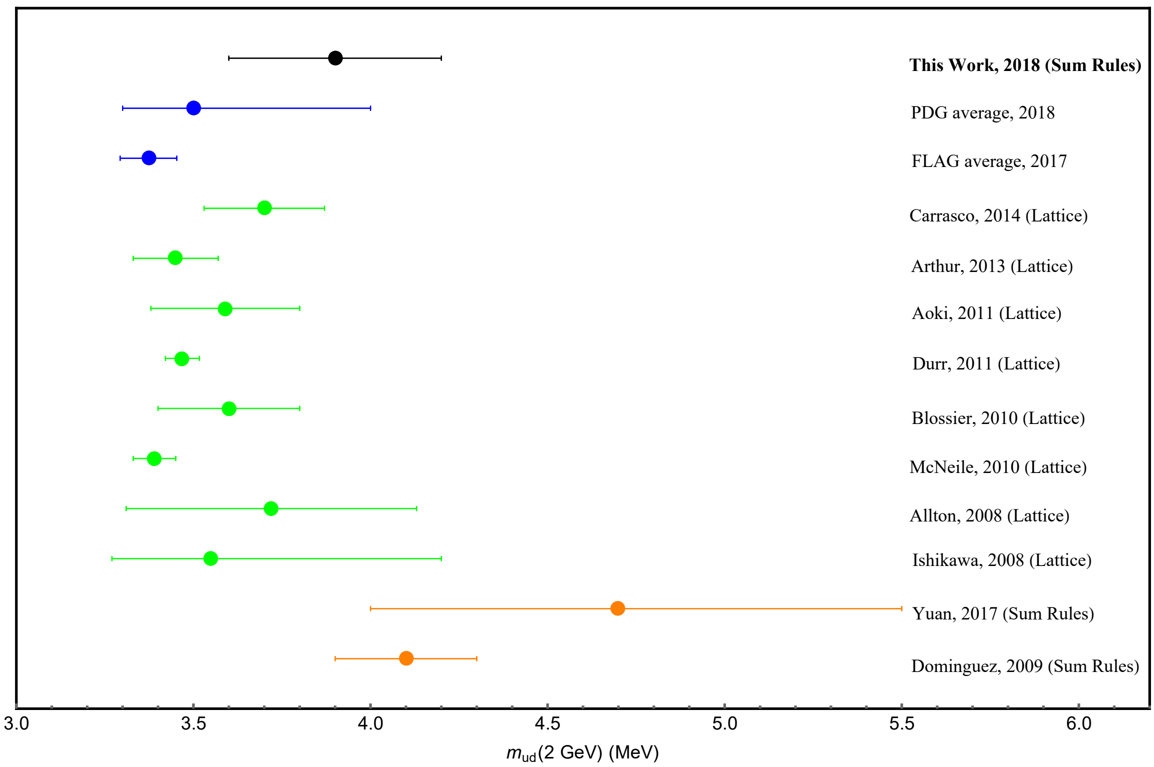

Figure 1. The up and down quark mass obtained in [6] compared to the PDG [7] and FLAG [2] averages, as well as the most recent sum rule results [8, 9] and lattice QCD results [10–17].

There is a trend for lattice QCD determinations to generally yield lower up and down quark mass values than sum rule determinations. Although this is observed, it is not apparent what the cause is - especially since the same trend is not observed in the case of the strange quark mass (figure (2)). From figure (1) we can see that this determination – through its original aspects and various improvements (examined in the next section) – is in good agreement with the recent numerical determinations; thereby reducing the existing tension between QCD sum rules and lattice QCD determinations.

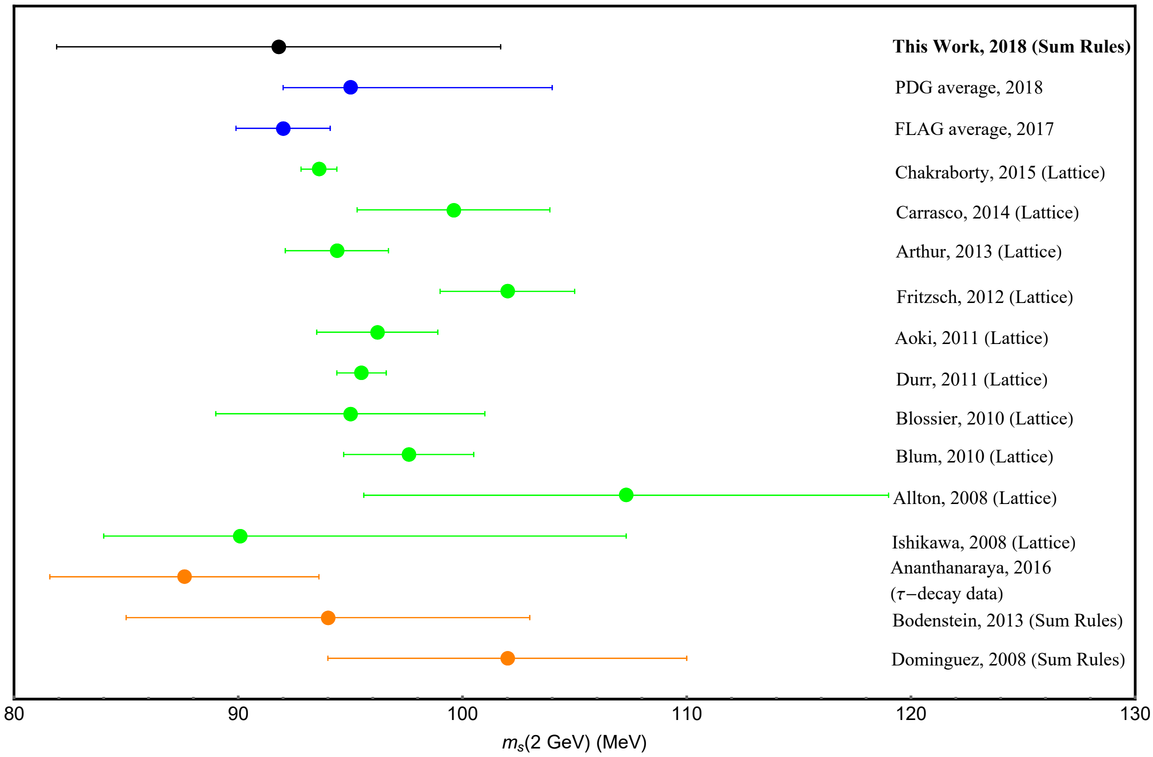

Figure 2. The strange quark mass obtained in [6] compared to the PDG [7] and FLAG [2] averages, as well as the most recent analytical results [18–20] (the Ananthanarayan et al. determination extracts the strange quark from τ-decay spectral moments using renormalization techniques, while the latter two are sum rule determinations) and lattice QCD results [10–14, 16, 17, 21–23].

The strange quark determination presented here is in good agreement with most recent results in the literature.

Examining figures (1) and (2), the question naturally emerges: *How do the precision determinations presented here differ computationally from previous sum rule calculations of the light quark masses?*, or phrased differently *What are the original aspects and improvements of the precision determinations presented here over other sum rule determinations?*. This question is addressed in the following section.

3. Comparisons

In 2018, The Particle Data Group [7] considered two phenomenological determinations by Yuan et al. [8] and Dominguez et al. [9] when determining the world average of the up and down quark masses. The latest calculation of $\overline{m}_{ud}$ by Yuan et al. [8], published in 2017, determines the up and down quark masses in the I = 0 scalar channel within the Borel transform framework. It further differs from the current determination since it includes an instanton contribution as a nonperturbative effect in the theoretical representation of the correlation function.

The previous determination by Dominguez et al. [9] – published in 2009 and also included in the Particle Data Group world average of $\overline{m}_{ud}$ – is a FESR determination of the up and down quark masses. Since this determination was also done by considering a pseudoscalar correlator, it might (on face value) seem as if [9] is very similar to the determination presented here. However, they are fundamentally different in a number of ways.

One important, noteworthy difference is how the determinations are computationally implemented. In the current precision determination the calculations have been computed in Mathematica (see [6] for the annotated notebooks), a modern 64-bit computer language with natural support for precision real and complex numbers. At a minimum, using the machine precision corresponds to 16 digits of mantissa, although higher multiples of the machine precision can be used. In comparison, the determination by Dominguez et al. [9] was performed using Fortran 90 with code edited and updated from Fortran 77; respectively these are 1991 and 1977 computer languages. Typically the early standard Fortran versions only contain single precision (numbers accurate to 7 digits of mantissa), while double precision (typically 14 digits of mantissa) can be specified for real numbers. Complex numbers present more of an issue, as the double precision must be split between the real and imaginary components. Notwithstanding this, it is the confluence of these different working numerical precisions in earlier Fortran languages that can typically result in an accumulated error. The issue of numerical precision can make a notable impact on the results. For example, in CIPT where the renormalization group equations must be solved at 10 000 points around the circular contour, with each point relying on the previous value of the strong coupling as an initial condition, the opportunity for numerical carried error is pronounced. In Mathematica the numerical underflow in such a situation can be managed with greater care.

Considering that the determination by Dominguez et al. [9] hitherto represented the most recent FESR determination of the up and down quark masses, and formed part of the world average of these quark masses as determined by the Particle Data Group [7]; it is worrying that such old computer languages were used, especially since modern alternatives are readily available. The current determination [6], where all calculations were performed in Mathematica, represents a significant advancement in achieved numerical precision of FESR determinations of the up and down quark masses.

Furthermore, the analysis of different kernels, examining the issue of the convergence of the perturbative QCD expansion, a different implementation of the running QCD coupling and a more careful error analysis are some of the considerable improvements accomplished by this present determination - the details on each of these issues can be found in [6].

Turning towards the strange quark mass, in 2018 the Particle Data Group [7] considered three phenomenological determinations Ananthanarayan et al. [18], Bodenstein et al. [19], Dominguez et al. [20] when determining the world average of the strange quark mass. The latest publication by Ananthanarayan et al. [18], published in 2016, uses a renormalization group summed perturbation theory and relates this to τ-decay spectral function data in order to extract the strange quark mass. This is not a FESR sum rule determination of $\overline{m}_s$, and as such will not be examined further here. The previous determination by Bodenstein et al. [19], published in 2013, is a FESR determination of the strange quark mass. The determination is performed in Mathematica and the convergence of the perturbative QCD expansion is examined. However, the calculations in [19] are only performed in the framework of FOPT. Here, in this current determination [6], both FOPT and CIPT are considered, leading to a different conclusion about the preferred framework in which to calculate the strange quark mass (in terms of stability and convergence) being made. The 2008 determination by Dominguez et al. [20], was calculated by the same collaboration and around the same time as the determination by Dominguez et al. [9]. Consequently it suffers from all the issues pertaining to the determination in [9] discussed previously.

4. Conclusion

The work outlined here and presented in detail in [6] gives the most accurate up, down and strange quark mass determination from a QCD finite energy sum rule to date. It is further worth noting that a publication of the up and down quark mass determinations resulting from the author's work in [6], can be found in the Journal of High Energy Physics [24].

The corresponding Mathematica notebooks to the current precision determinations in [6] can be found in the GitHub repository `https://github.com/AlexesMes/light-quark-masses`. It is still uncommon in the field of theoretical physics to make the code available with a published paper. Historically, this was not possible as it made publications unreadable and too expensive to print. However, these reasons are no longer applicable in the present day, and the author has made her code available, since she believes in the importance of modern research and open collaboration.

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