A Comparison Of Thermal Model Fits in p-p, p-Pb, Pb-Pb

Jean Cleymans[1], **Boris Hippolyte**[2], **Masimba Paradza**[1], **Natasha Sharma**[3]

[1]UCT-CERN Research Centre and Department of Physics, University of Cape Town, Rondebosch 7701, South Africa

[2]Institut Pluridisciplinaire Hubert Curien and Université de Strasbourg Institute for Advanced Study, CNRS-IN2P3, Strasbourg, France

[3]Department of Physics, Panjab University, Chandigarh 160014, India

Abstract. Recent work on the particle composition (hadrochemistry) of the final state in proton-proton (p-p), proton-lead (p-Pb) and lead-lead (Pb-Pb) collisions as a function of the charged particle multiplicity ($\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}\eta$) is reviewed. It is argued that for high multiplicities (at least about 20 charged hadrons in the mid-rapidity interva) consistent results are obtained in the thermal model.

1. Use of Thermal Concepts in Heavy-Ion Collisions

The final state produced in heavy ion collisions at the Large Hadron Collider (LHC) is characterized by a large number of hadrons. In central collisions at $\sqrt{s_{NN}}$ = 5.02 TeV [1] a total of about 30 000 particles are produced as can be seen from figure 1. To analyze the properties of such a large number of particles it is quite natural to use concepts from statistical mechanics e.g. use energy density, particle density, pressure, temperature, chemical composition, etc ... As it turns out these concepts are useful at all beam energies.

In high energy collisions applications of the thermal-statistical model in the form of the hadron resonance gas model have been successful (see e.g. [2, 3] for two recent publications) in describing the composition of the final state e.g. the yields of pions, kaons, protons and other hadrons.

In this presentation the focus will be on the results of an analysis performed recently [4, 5] about the validity of statistical concepts. In particular it has been concluded there that if the multiplicity exceeds 20 hadrons at mid-rapidity then the use of statistical concepts is justified, irrespective of whether it is a p-p, a p-Pb or a Pb-Pb collision. In high energy collisions applications of the thermal-statistical model in the form of the hadron resonance gas model have been successful (see e.g. [2, 3] for two recent publications) in describing the composition of the final state e.g. the yields of pions, kaons, protons and other hadrons.

2. The Theoretical Basis for the Thermal Model

Using the Cooper-Frye formula [6], the momentum distribution of particles of type i is determined by:

$$E\frac{dN_i}{d^3p} = \frac{g_i}{(2\pi)^3}\int d\sigma_\lambda p^\lambda \exp\left(-\frac{p^\mu u_\mu}{T} + \frac{\mu_i}{T}\right), \tag{1}$$

where the integration is done over the freeze-out surface σ_λ, u_μ is the four-velocity, T and μ_i are the freeze-out temperature and chemical potential respectively Integrating this over the momenta one obtains

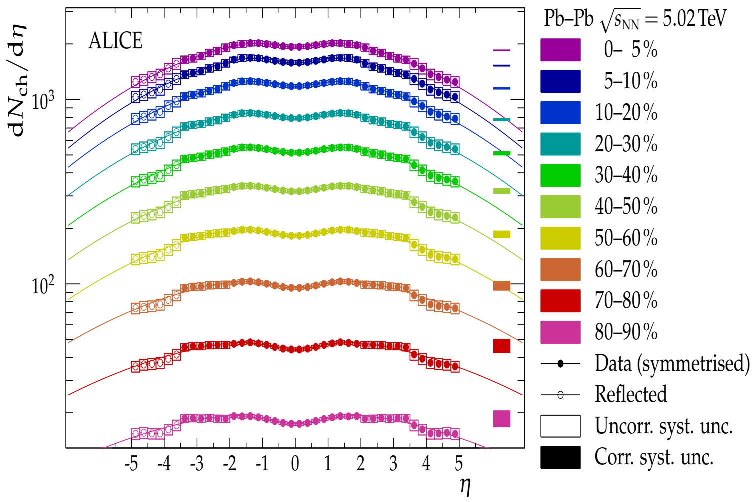

Figure 1. The number of charged particles in Pb-Pb collisions at $\sqrt{s_{NN}}$ = 5.02 TeV as a function of pseudo-rapidity [1].

$$N_i = \frac{g_i}{(2\pi)^3}\int d\sigma_\lambda \int \frac{d^3p}{E}p^\lambda \exp\left(-\frac{p^\mu u_\mu}{T}+\frac{\mu_i}{T}\right)$$

The second integral in the above expression, namely the one over the momenta must be a four-vector, since only u^λ is available as a four-vector, it leads to

$$N_i = \int d\sigma_\lambda u^\lambda n_i^0(T,\mu)$$

where n_i^0 is the density in a fireball at rest. If the temperature and chemical potential are the same along the freeze-out curve, then one obtains

$$N_i = n_i^0(T,\mu)\int d\sigma_\lambda u^\lambda$$

i.e. integrated (4π) multiplicities are the same as for a single fireball at rest (apart from the volume).

Therefore after integration over p_T and y one has:

$$\frac{N_i}{N_j} = \frac{n_i^0}{n_j^0} \tag{2}$$

where, as above, n_i^0 and n_j^0 are the particle yields for particle types i and j as calculated in a fireball at rest. For this result to hold, the freeze-out temperature has to be the same for all particles (which may not always be the case).

This does not mean that the freeze-out has to be instantaneous. The only requirement is that the freeze-out temperature has to be the same along the freeze-out curve.

A well known model for the expansion of matter in a heavy ion collision combimes Bjorken scaling [7] with a transverse expansion, in this case it can be shown that one obtains after integration over p_T [8, 9, 10].

$$\frac{dN_i/dy}{dN_j/dy} = \frac{n_i^0}{n_j^0} \tag{3}$$

where N_i^0 is the particle yield as calculated in a fireball at rest. Effects of hydrodynamic flow cancel out in ratios. The volume is given by $\pi R^2 \tau$ corresponding to a cylindrical expansion lasting for a time τ in the longitudinal direction.

3. Uncertainties in the Thermal Model

Uncertainties are related to the incomplete knowledge of hadron species and resonance properties as reflected in the Particle Data Booklet [11]. Particle yields are determined from:

$$N_i = \sum_j N_j Br(j \to i). \tag{4}$$

Hence one must know how hadronic resonances decay.

As an example, the final yield of π^+'s is given by

$$N_{\pi^+} = N_{\pi^+}(\text{thermal}) + N_{\pi^+}(\text{resonance decays}) \tag{5}$$

depending on the temperature, over 80% of observed pions are due to resonance decays. In these descriptions use is made of the grand canonical ensemble and the canonical ensemble with exact strangeness conservation. In addition the use of the canonical ensemble with exact baryon, strangeness and charge conservation were also concidered.

The yields produced in heavy-ion collisions have been the subject of intense discussions over the past few years and several proposals have been made in view of the fact that the number of pions is underestimated while the number of protons is overestimated. Several proposals to improve on this have been made recently:

- Incomplete hadron spectrum [12],
- chemical non-equilibrium at freeze-out [13, 14, 15],
- modification of hadron abundances in the hadronic phase [16, 17, 18],
- separate freeze-out for strange and non-strange hadrons [19, 20, 21, 22],
- excluded volume interactions [23],
- energy dependent Breit-Wigner widths [24],
- use the phase shift analysis to take into account repulsive and attractive interactions [25, 26],
- use the K-matrix formalism to take interactions into account [27].

These proposals improve the agreement with the observed yields and furthermore, some of them change the chemical freeze-out temperature, T_{ch} in only a minimal way like those presented recently in [24, 26]. In the present analysis Therefore the basic structure of the thermal model was kept with a single freeze-out temperature and focus on the resulting thermal parameters T_{ch}, γ_s and the radius. All calculations were done using the latest version of THERMUS [28] [1].

The results [4, 5] show some interesting new features:

- the grand canonical ensemble, the ensemble with strict strangeness conservation and the one with strict baryon number, strangeness and charge conservation agree very well for the particle composition in Pb-Pb collisions, they also agree well for p-Pb collisions but marked differences for p-p collisions are present. These differences disappear as the multiplicity of charged particles increases in the final state. Thus, p-p collisions with high multiplicities agree with what is seen in large systems like p-Pb and Pb-Pb collisions. Quantitatively this agreement starts when there are at least 20 charged hadrons in the mid-rapidity interval being considered. It also throws doubt on the applicability of the thermal model as applied to p-p collisions with low multiplicity.
- The convergence of the results in the three ensembles lends support to the idea that one reaches a thermodynamic limit where the results are independent of the ensemble being used.

[1] B. Hippolyte and Y. Schutz, https://github.com/thermus-project/THERMUS

4. Ensembles considered in the thermal model

Three different ensembles based on the thermal model were compared. In the following equations use is made of the Boltzmann approximation, but the actual numerical calculations use quantum statistics as far as possible.

- Grand canonical ensemble (GCE), the conservation of quantum numbers is implemented using chemical potentials. The quantum numbers are conserved on the average. The partition function depends on thermodynamic quantities and the Hamiltonian describing the system of N hadrons:

$$Z_{GCE} = \mathrm{Tr}\left[e^{-(H-\mu N)/T}\right] \tag{6}$$

which, in the framework of the thermal model considered here, leads to

$$\ln Z_{GCE}(T,\mu,V) = \sum_i g_i V \int \frac{d^3p}{(2\pi)^3} \exp\left(-\frac{E_i-\mu_i}{T}\right) \tag{7}$$

in the Boltzmann approximation, g_i is the degeneracy factor of hadron i, V is the volume of the system, μ_i is the chemical potential associated with the hadron. The yield is given by:

$$N_i^{GCE} = g_i V \int \frac{d^3p}{(2\pi)^3} \exp\left(-\frac{E_i}{T}\right), \tag{8}$$

where we have put the chemical potentials equal to zero, as relevant for the beam energies at the Large Hadron Collider considered here. The decays of resonances have to be added to the final yield

$$N_i^{GCE}(\mathrm{total}) = N_i^{GCE} + \sum_j Br(j\to i) N_j^{GCE}. \tag{9}$$

- Canonical ensemble with exact implementation of strangeness conservation, we will refer to this as the strangeness canonical ensemble (SCE). There are chemical potentials for baryon number B and charge Q but not for strangeness:

$$Z_{SCE} = \mathrm{Tr}\left[e^{-(H-\mu N)/T}\delta_{(S,\sum_i S_i)}\right] \tag{10}$$

The delta function imposes exact strangeness conservation, requiring overall strangeness to be fixed to the value S, in this paper we will only consider the case where overall strangeness is zero, $S=0$. This change leads to [29]:

$$Z_{SCE} = \frac{1}{(2\pi)}\int_0^{2\pi} d\phi e^{-iS\phi} Z_{GCE}(T,\mu_B,\lambda_S) \tag{11}$$

where the fugacity factor is replaced by

$$\lambda_S = e^{i\phi} \tag{12}$$

$$N_i^{SCE} = V\frac{Z_i^1}{Z_{S=0}^C}\sum_{k,p=-\infty}^{\infty} a_3^p a_2^k a_1^{-2k-3p-s} I_k(x_2) I_p(x_3) I_{-2k-3p-s}(x_1), \tag{13}$$

where $Z_{S=0}^C$ is the canonical partition function

$$Z_{S=0}^C = e^{S_0}\sum_{k,p=-\infty}^{\infty} a_3^p a_2^k a_1^{-2k-3p} I_k(x_2) I_p(x_3) I_{-2k-3p}(x_1),$$

where Z_i^1 is the one-particle partition function calculated for $\mu_S = 0$ in the Boltzmann approximation. The arguments of the Bessel functions $I_s(x)$ and the parameters a_i are introduced as,

$$a_s = \sqrt{S_s/S_{-s}} \ , \ \ x_s = 2V\sqrt{S_s S_{-s}}, \tag{14}$$

where S_s is the sum of all $Z_k^1(\mu_S = 0)$ for particle species k carrying strangeness s. As previously, the decays of resonances have to be added to the final yield

$$N_i^{SCE}(\text{total}) = N_i^{SCE} + \sum_j Br(j \to i) N_j^{SCE}. \tag{15}$$

- Canonical ensemble with exact implementation of B, S and Q conservation, we will refer to this as the full canonical ensemble (FCE). In this ensemble there are no chemical potentials. The partition function is given by:

$$Z_{FCE} = \mathrm{Tr}\left[e^{-H/T}\delta_{(B,\sum_i B_i)}\delta_{(Q,\sum_i Q_i)}\delta_{(S,\sum_i S_i)}\right] \tag{16}$$

$$Z_{FCE} = \frac{1}{(2\pi)^3}\int_0^{2\pi} d\alpha e^{-iB\alpha}\int_0^{2\pi} d\psi e^{-iQ\psi}\int_0^{2\pi} d\phi e^{-iS\phi} Z_{GCE}(T,\lambda_B,\lambda_Q,\lambda_S) \tag{17}$$

where the fugacity factors have been replaced by

$$\lambda_B = e^{i\alpha}, \quad \lambda_Q = e^{i\psi}, \quad \lambda_S = e^{i\phi}. \tag{18}$$

As before, the decays of resonances have to be added to the final yield

$$N_i^{FCE}(\text{total}) = N_i^{FCE} + \sum_j Br(j \to i) N_j^{FCE}. \tag{19}$$

In this case the analytic expression becomes very lengthy and we refrain from writing it down here, it is implemented in the THERMUS program [28].

In all three case we have also taken into account the strangeness saturation factor γ_s [30] which enters as a multiplicative factor, raised to the power of the strangeness content, in the particle yields. Keeping this factor fixed at one does not change the fixed message, only the resulting value of χ^2 is increased indicating a worsening of the fits.

These three ensembles are applied to p-p collisions at 7 TeV in the central region of rapidity [31], to p-Pb collisions at 5.02 TeV [32, 33] and to Pb-Pb collisions at 2.76 TeV [34, 35, 36] with particular focus on the dependence on the charged particle multiplicity. It is well known that in this kinematic region, one has particle - antiparticle symmetry and therefore there is no net baryon density and also no net strangeness. The different ensembles nevertheless give different results because of the way they are implemented. A clear size dependence is present in the results of the ensembles. In the thermodynamic limit they should become equivalent. Clearly there are other ensembles that could be investigated and also other sources of finite volume corrections.

A similar analysis was done in [37, 38, 39] for p-p collisions at 200 GeV but without the dependence on charged multiplicity.

For p-Pb and Pb-Pb collisions the Ω measurements were included in the analysis, so that six particle species were considered for p-Pb and Pb-Pb. It was also checked checked explicitly that for the five bins in p-p collisions where the Ω has also been measured, there is no difference in the outcome for the values of T_{ch}, γ_s and the radius.

As shown in [40, 41] the ϕ meson is not described very well and has not been included.

5. Comparison of different statistical ensembles.

In figure 2 we show the chemical freeze-out temperature as a function of the multiplicity of hadrons in the final state [31]. As explained in the previous section the freeze-out temperature has been calculated using three different ensembles. The highest values are obtained using the canonical ensemble with exact conservation of three quantum numbers, baryon number B, strangeness S and charge Q, all of them being set to zero as is appropriate for the central rapidity region in p-p collisions at 7 TeV. In this ensemble the temperature drops strongly from the lowest to the highest multiplicity.

The lowest values for $T_{\rm ch}$ are obtained when using the grand canonical ensemble, in this case the conserved quantum numbers are again zero. The results are clearly different from those obtained in the previous ensemble, especially in the low multiplicity intervals. They gradually approach each other and they become equal at the highest multiplicities.

For comparison with the previous two cases we also calculated $T_{\rm ch}$ using the canonical ensemble with only strangeness S being exactly conserved using the method presented in [29]. In this case the results are close to those obtained in the grand canonical ensemble, with the values of $T_{\rm ch}$ always slightly higher than in the grand canonical ensemble. Again for the highest multiplicity interval the results become equivalent.

The canonical BSQ ensemble leads to results which are incompatible with those obtained from Lattice Gauge Theory [42] which indicate that hadrons cannot exist above the critical temperature which has been estimated to be about 156$\pm$1.5 MeV. As can be seen in figure 2, even though all the ensembles produce different results, for high multiplicities the results converge to a common value close to 160 MeV. In figure 3 we show results for the strangeness saturation factor γ_s [30]. In this case we obtain again quite substantial differences in each one of the three ensembles considered. The highest values being found in the canonical ensemble with exact strangeness conservation. Note that the values of γ_s become compatible with unity, i.e. with chemical equilibrium for all light flavors. It is to be noticed the for multiplicities the value of γ_s is slightly above unity (by almost 10%) but is compatible with full chemical equilibrium within one standard deviation. In figure 4 the radius at chemical freeze-out obtained in the three ensembles is presented. As in the previous figures, the results become independent of the ensemble chosen for the highest multiplicities while showing clear differences for low multiplicities.

The results show that there is a strong correlation between some of the parameters. The very high temperature obtained in the canonical BSQ ensemble (FCE) correlates with the small radius in the same ensemble. Particle yields increase with temperature but a small volume decreases them, hence the correlation between the parameters.

The fits to the hadronic yields obtained in p-p collisions at 7 TeV in five different centrality bins are shown in figure4 and 5. The upper panels show the yields while the lower panels show the ratios of the measured data divided by the fit values for the three different ensembles considered here. The three lines corresponding to the fits are often very close to each other and overlap, hence they are not always visible on the figures.

6. Discussion and Conclusions

In this paper we have investigated three different ensembles to analyze the variation of particle yields with the multiplicity of charged particles produced in proton-proton collisions at the center-of-mass energy of $\sqrt{s}$ = 7 TeV [31], p-Pb collisions at 5.02 TeV [32, 33] and Pb-Pb collisions at 2.76 TeV [34, 35, 36].

The basic structure of the thermal model as presented in [28] was kept and the focus was on the resulting thermal parameters T_{ch}, γ_s and the radius and their dependence on the final state multiplicity. It is to be noted in this regards that recent improvements on the treatment of the particle yields do not lead to substantial changes of the chemical freeze-out temperature, T_{ch} [24, 26]. The results show two new interesting features:

- a comparison of the grand canonical ensemble, the ensemble with strict strangeness conservation and the one with strict baryon number, strangeness and charge conservation agree very well for large systems like p-Pb and Pb-Pb, but show marked differences for p-p collisions. These differences tend

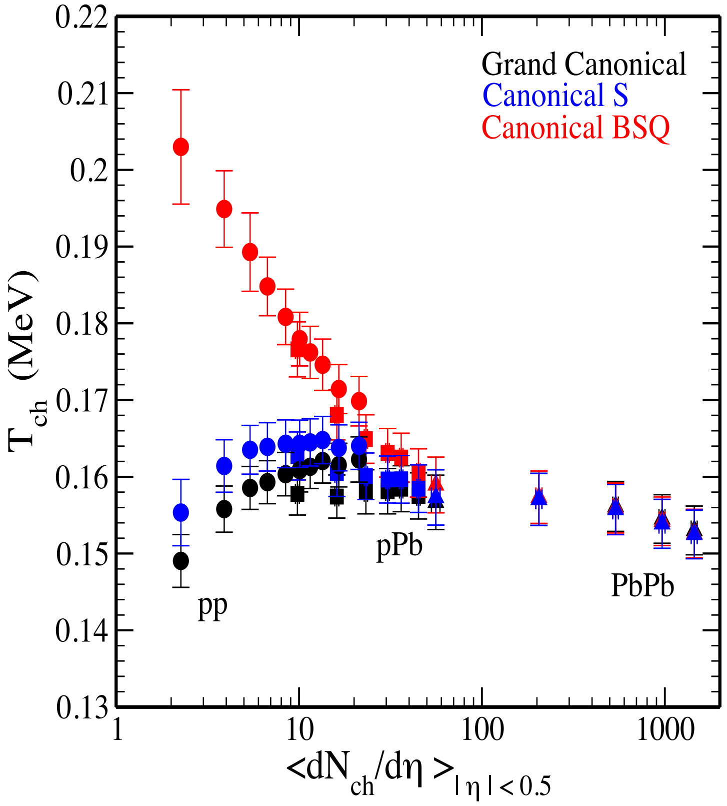

Figure 2. The chemical freeze-out temperature, $T_{\rm ch}$, obtained for three different ensembles. The black points are obtained using the grand canonical ensemble, the blue points use exact strangeness conservation while the red points have built-in exact baryon number, strangeness and charge conservation. Circles are for p-p collisions at 7 TeV [31], squares are for p-Pb collisions at 5.02 TeV [32, 33] while triangles are for Pb-Pb collisions at 2.76 TeV [34, 35, 36].

to disappear as the multiplicity of charged particles increases in the final state of p-p collisions. This supports the fact that p-p collisions with high multiplicities agree with what is seen in large systems like Pb-Pb. Quantitatively this starts happening when there are more than 20 charged hadrons in the mid-rapidity interval being considered. It also throws doubt on the applicability of the thermal model in low multiplicity p-p collisions.

- The convergence of the results in the three ensembles lends support to the notion a thermodynamic limit is reached where results are independent of the ensemble being used.

It is of interest to note that all three ensembles lead to the same results when the multiplicity of charged particles $dN_{\rm ch}/d\eta$ exceeds 20 at mid-rapidity. This could be interpreted as reaching the thermodynamic limit since the three ensembles lead to the same results. It would be of interest to extend this analysis to higher beam energies and higher multiplicity intervals.

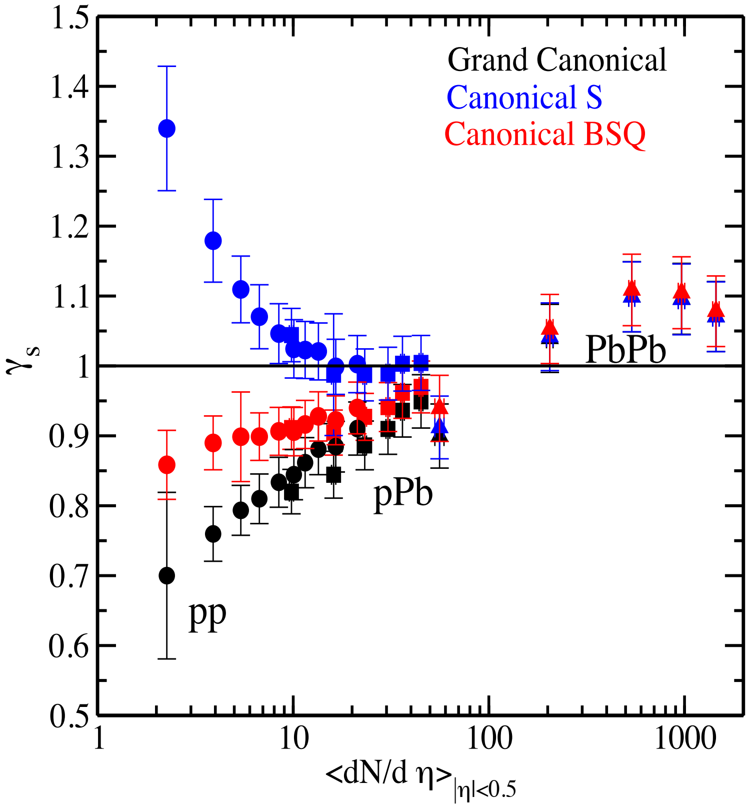

Figure 3. The strangeness saturation factor γ_s obtained for three different ensembles. The black points were obtained using the grand canonical ensemble, the blue points uses exact strangeness conservation while the red points have built-in exact baryon number, strangeness and charge conservation. Circles are for p-p collisions at 7 TeV [31], squares are for p-Pb collisions at 5.02 TeV [32, 33] while triangles are for Pb-Pb collisions at 2.76 TeV [34, 35, 36].

Acknowledgments

One of us (J.C.) gratefully thanks the National Research Foundation of South Africa for financial support. N.S. acknowledges the support of SERB Ramanujan Fellowship (D.O. No. SB/S2/RJN- 084/2015) of the Department of Science and Technology of India. B.H. acknowledges the support of the Université de Strasbourg Institute for Advanced Study.

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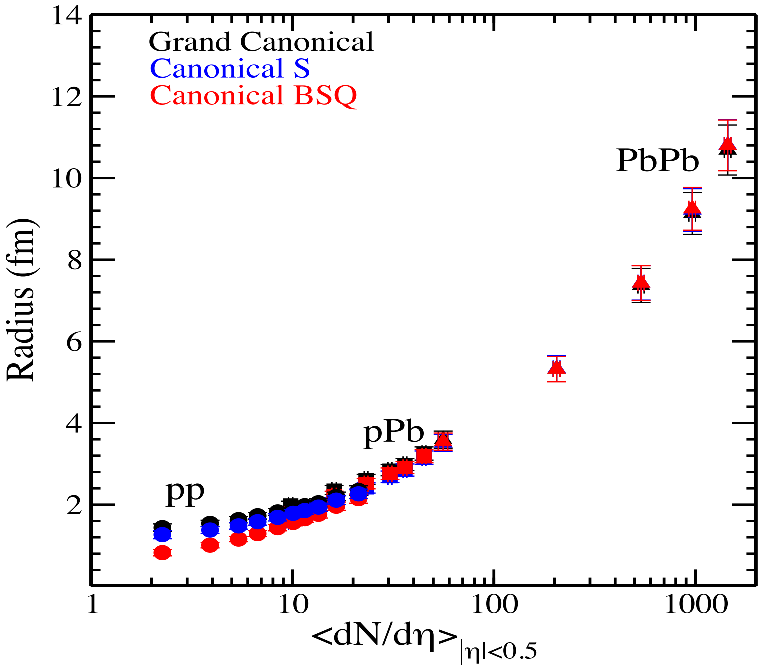

Figure 4. The chemical freeze-out radius obtained for three different ensembles. The black points were obtained using the grand canonical ensemble, the blue points uses exact strangeness conservation while the red points have built-in exact baryon number, strangeness and charge conservation. Circles are for p-p collisions at 7 TeV [31], squares are for p-Pb collisions at 5.02 TeV [32, 33] while triangles are for Pb-Pb collisions at 2.76 TeV [34, 35, 36].

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