A microscopic treatment of correlated nucleons: Collective properties in stable and exotic nuclei

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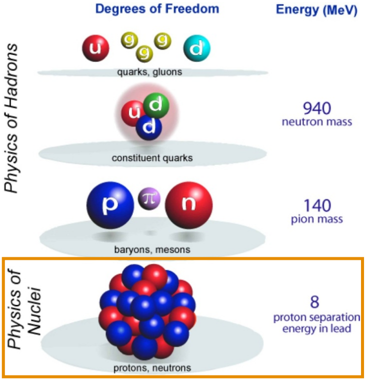
General assumptions

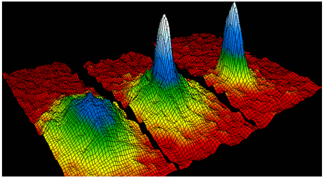


Framework

- Low-energy scales
  - $\longrightarrow$ nucleons are point-like, structureless particles
    relevant degrees of freedom = nucleons
- Solve the nuclear many-body problem
  - $\longrightarrow$ Use of effective interactions $\longrightarrow$ Energy-Density Functionals (EDF): functionals derived in most cases from effective interactions

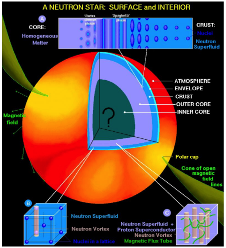
Interdisciplinarity of many-body techniques

Atomic physics



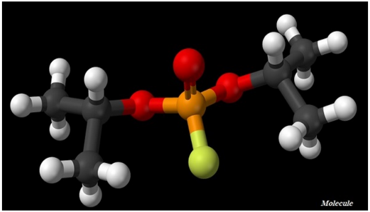
Bose-Einstein condensate of ultra-cold trapped atoms

Astrophysics



Nuclei and other nuclear systems in the crust of neutron stars

Chemistry & Condensed matter physics

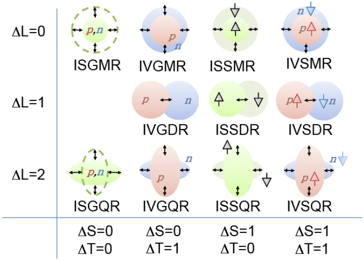


Strong analogy between Energy-Density Functionals (EDF) and Density Functional Theory (DFT)

Objectives

Challenge

Provide an accurate description of fragmentation and spreading width of excitations
⟶ Going beyond the mean-field approximation: coupling single-particle degrees of freedom with multiparticle-multihole configurations



Schematic view of giant resonances

What we want to describe

- Low-lying states
- Giant resonances

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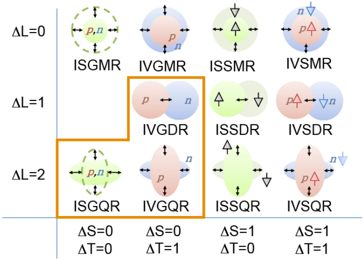

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Outline

1. **Starting point: Second RPA (SRPA) and its drawbacks**

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1. **Starting point: Second RPA (SRPA) and its drawbacks**
   - Formalism of Second RPA
   - Problems of standard SRPA
2. **Correction of SRPA: Subtraction method**
   - Principle of the subtraction procedure
   - How subtraction deals with problems of SRPA
3. **Extension of the SSRPA model**
   - Nuclei with partially-occupied orbitals
   - Use of the equal-filling approximation
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Formalism of Second RPA

Choose the ground state $|0\rangle$ as the Hartree-Fock ground state

Define the excitation operator ${Q_\nu}^\dagger$: $\begin{cases} Q_\nu^\dagger |0\rangle = |\nu\rangle \\ Q_\nu |0\rangle = 0 \end{cases}$

$$Q_\nu^\dagger := \sum_{m,i} \left(X_{mi}(\nu)\, a_m^\dagger a_i - Y_{mi}(\nu)\, a_i^\dagger a_m \right) \longrightarrow \textbf{1p1h (RPA)}$$

$$+ \sum_{\substack{m,\, n>m \\ i,\, j>i}} \left(X_{mnij}(\nu)\, a_m^\dagger a_n^\dagger a_j a_i - Y_{mnij}(\nu)\, a_i^\dagger a_j^\dagger a_n a_m \right) \longrightarrow \textbf{2p2h (SRPA)}$$

SRPA equation: $$\overbrace{\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix}}^{\text{Stability matrix}} \begin{pmatrix} X(\nu) \\ Y(\nu) \end{pmatrix} = \hbar\omega_\nu \begin{pmatrix} G & 0 \\ 0 & -G^* \end{pmatrix} \begin{pmatrix} X(\nu) \\ Y(\nu) \end{pmatrix}$$

with $A = \begin{pmatrix} (A_{mi,nj}) & (A_{pk,minj}) \\ (A_{minj,pk}) & (A_{minj,pqkl}) \end{pmatrix}$ and $X(\nu) = \begin{pmatrix} X_1(\nu) \\ X_2(\nu) \end{pmatrix}$

$A_{mi,nj} = \delta_{mn}\delta_{ij}(\epsilon_m - \epsilon_i) + v_{mjin}$ in the HF limit

Problems of standard SRPA

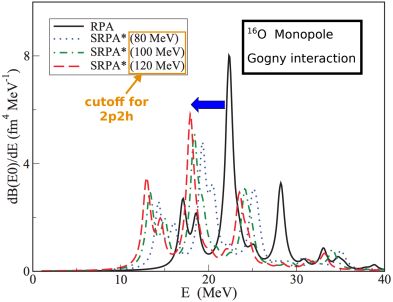
- Very strong shift to low energies
- Instabilities

} Thouless theorem does not hold in SRPA

Papakonstantinou, *Phys. Rev. C* **90**, 024305 (2014)

- Double-counting of correlations
- Cutoff dependence with zero-range interactions

} Use of an effective interaction "EDF problems"



Gambacurta, Grasso *et al.*, *Phys. Rev. C* 86, 021304 (R) (2012)

Outline

Principle of the subtraction procedure

- The SRPA equation can be written as an energy-dependent RPA-like problem

- Removal of double-counting of correlations is achieved by imposing that the stability matrix in this energy-dependent RPA-like problem be equal, at zero energy, to the RPA stability matrix

  Tselyaev, *Phys. Rev. C* **88**, 054301 (2013)

- This translates into a subtraction of only 1p1h elements in , *e.g.* A_{11} becomes

$$A'_{11} := A_{11} - \Sigma(0)$$

$\Sigma(0)$: 2^{nd}-order self-energy at zero excitation energy

$$\Sigma(0) = -A_{12}A_{22}^{-1}A_{12}^{\dagger} + B_{12}({}^{t}A_{22})^{-1}B_{12}^{\dagger}$$

- The subtraction has been successfully applied, dealing with all the drawbacks of SRPA

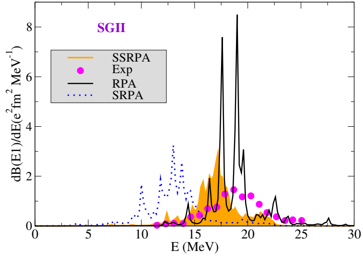
  Gambacurta, Grasso, J. Engel, *Phys. Rev. C* **92**, 034303 (2015)

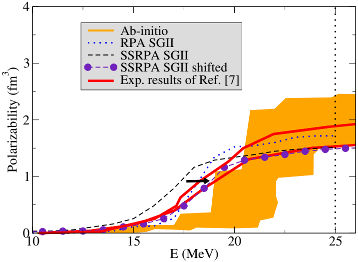
  Gambacurta, Grasso, *Eur. Phys. J. A* **52**, 198 (2016)

How subtraction deals with problems of SRPA

Electric dipole strength and polarizability in ^{48}Ca

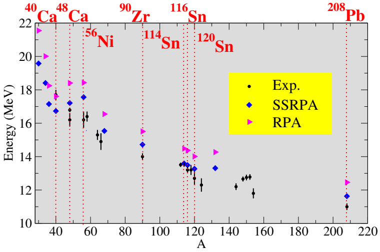
Gambacurta, Grasso, Vasseur, *Phys. Lett. B* **777**, 163-168 (2018)





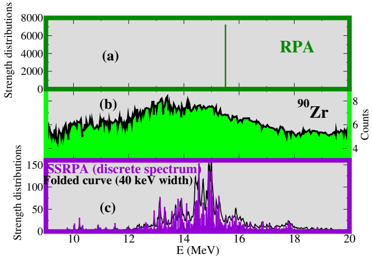
Quadrupole response: Systematics on medium- and heavy-mass nuclei

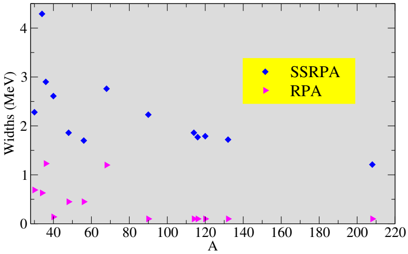
Vasseur, Gambacurta, Grasso, *Phys. Rev. C* **98**, 044313 (2018)



- Better reproduction of experimental values of centroids in SSRPA (for most nuclei)
- Coupling between 1p1h and 2p2h configurations (absent in RPA) $\longrightarrow$ spreading

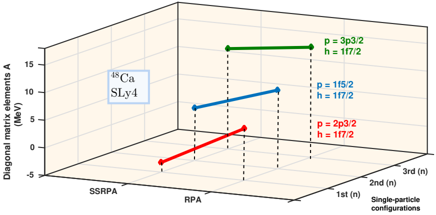
How subtraction deals with problems of SRPA

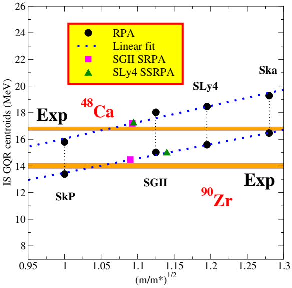




Grasso, Gambacurta, Vasseur, *Phys. Rev. C* **98**, 051303(R) (2018)

Effective masses in nuclear matter from axial breathing modes in ^{48}Ca and ^{90}Zr





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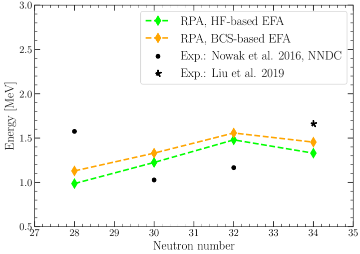
- Second step: calculate the occupation numbers in BCS
  $\longrightarrow$ Estimation of pairing correlations

Low-energy quadrupole strength in Ar isotopes

Exp. from: Liu et al., *Phys. Rev. Lett.* **122**, 072502 (2019);
Nowak et al., *Phys. Rev. C* **93**, 044335 (2016); NNDC (BNL)



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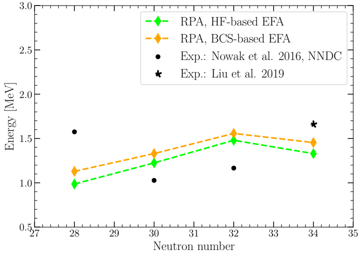

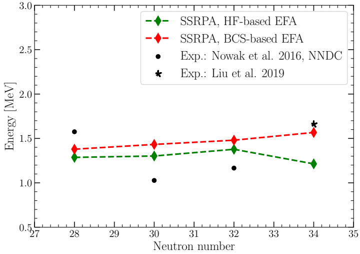

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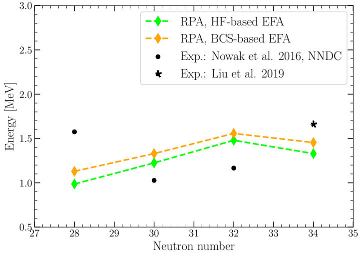

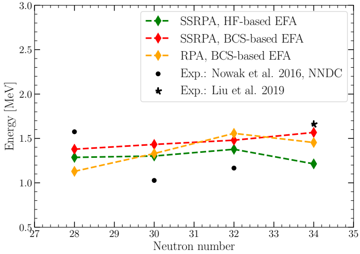

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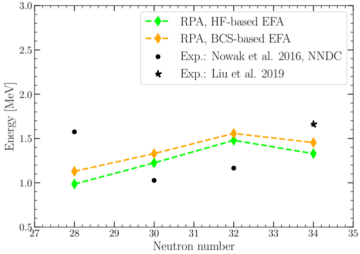

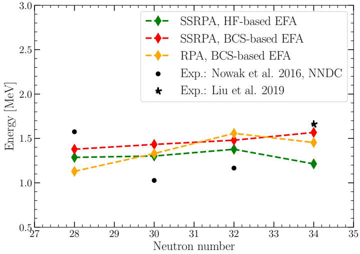

- A first attempt to include pairing correlations in SRPA
- The two rises corresponding to shell closures at $N = 28$ and $N = 34$ are not reproduced...
- ... but both the inclusion of 2p2h configurations and of pairing correlations improve the agreement with experiements

Summary

- Improve the description of fragmentation and width of resonances
  $\longrightarrow$ Need of a beyond-mean field description
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  1. Use the EFA to reach nuclei with non fully-occupied orbitals
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Thank you for your attention!