The gallium anomaly reassessed – Formulating neutrino detection problems in a Bayesian framework

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Joel Kostensalo

University of Jyväskylä

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Why should we revisit the gallium anomaly?

- It has been long unexplained.
- It has been suggested that new physics such as the existence of one or more eV-scale *sterile neutrino* could be behind these discrepancies.
- Disagreement between experiment and theory has been reported at the 2–3σ level
- The previous estimates treat systematic uncertainties in the two GALLEX and SAGE experiments as independent
- The most quoted estimate for the significance is based on a theoretical calculation with a factor 10 too large value for the first excited state contribution (*note: no erratum*)
- Taking into account uncertainties related to the solar neutrino background etc. is hard in a frequentistic framework based on normal distributions

Bayesian neutrino physics

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Statistical analysis of Giunti and Laveder (2011) found a statistically significant difference between the experiments and the shell model prediction of Haxton at the 3.0σ level.

The statistical significance has been reported at 2.3σ for the recent shell model results by J.K. *et al.* and at 2.6σ for the (p,n)-reaction based estimates of Bahcall, and at 3.0σ for the charge-exchange results of Frekers *et al*.

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Problems

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- Formulating the comparison with the experimental results in a statistically rigorous way

Solutions

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- It is possible to build a hierarchical model and compare the experimental and theoretical results using a Bayesian approach. This allows us to take into account all the uncertainties in a practically implementable way.

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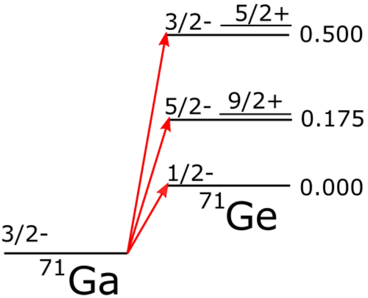
- Gs-to-gs cross section can be deduced from beta decay of ^{71}Ge
- For the excited states other methods must be used (calculations, CERs)
- Bahcall used (p,n)-BGTs (more specifically half of the old upper limit <0.056 for $\mathrm{BGT}_{5/2-}/\mathrm{BGT}_{\mathrm{g.s.}}$)



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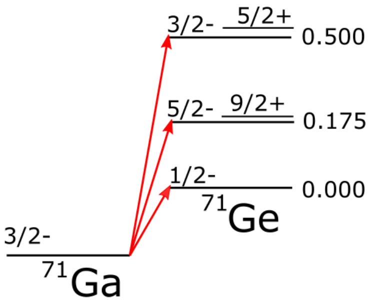

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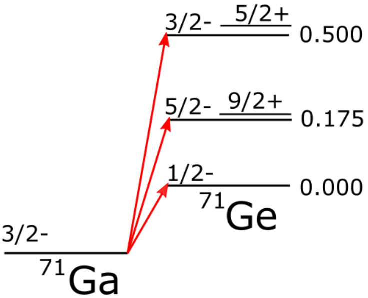

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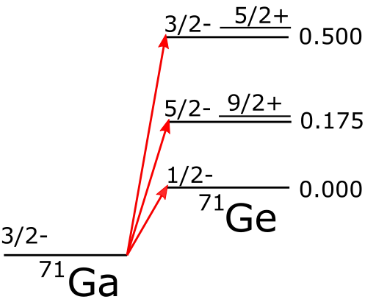

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- Reproduces the excitation spectrum relatively well
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Shell model cross sections:

$$5.67 \pm 0.10 \times 10^{-45}\ \text{cm}^2 \quad (^{51}\text{Cr})$$

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Bahcall cross sections:

$$5.81^{+0.21}_{-0.16} \times 10^{-45}\ \text{cm}^2 \quad (^{51}\text{Cr})$$

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Cross section can be expressed as

$$\sigma = \sigma_{gs}\left(1 + \xi_{5/2^-}\frac{\mathrm{BGT}_{5/2^-}}{\mathrm{BGT}_{gs}} + \xi_{3/2^-}\frac{\mathrm{BGT}_{3/2^-}}{\mathrm{BGT}_{gs}}\right)$$

Study	Method	$\frac{\mathrm{BGT}_{5/2^-}}{\mathrm{BGT}_{gs}}$	$\frac{\mathrm{BGT}_{3/2^-}}{\mathrm{BGT}_{gs}}$
Krofcheck et al.	(p,n)	<0.057	0.126 ± 0.023
Bahcall		0.028	0.146
Frekers et al.	$(^3\mathrm{He},\mathrm{t})$	0.039 ± 0.030	0.202 ± 0.016
Kostensalo et al.	ISM	0.033 ± 0.017	0.016 ± 0.008
Haxton	ISM (tr.)	0.33 ± 0.28	0.016

In (p,n)-reactions the interference between the Gamow-Teller (GT) and tensor (T) NMEs is described by the effective linear combination

$$\langle f\|O_{(p,n)}\|i\rangle = \langle f\|O_{\mathrm{GT}}\|i\rangle + \delta\langle f\|O_{L=2}\|i\rangle\,, \quad (1)$$

where i (f) is the initial (final) nuclear state and $\delta \approx 0.1$ is the mixing parameter.

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Table: Results for ^{71}Ga with $\delta = 0.097$.

State	$\langle f \| O_{\mathrm{GT}} \| i \rangle$	$\langle f \| O_{L=2} \| i \rangle$	$\mathrm{BGT}_{\beta}^{\mathrm{SM}}$	$\mathrm{BGT}_{(\mathrm{p,n})}^{\mathrm{SM}}$
$1/2^-_{\mathrm{g.s.}}$	-0.795	0.465	0.158	0.141
$5/2^-_1$	0.144	-1.902	0.0052	0.0004
$3/2^-_1$	0.100	0.0482	0.0025	0.0027

- There is a known large destructive interference for the $5/2^-$ state (Haxton 1998)
- New calculations show that there is a smaller destructive interference for the ground state
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The basic idea of the model is to view a neutrino detection experiment as a repeated *Bernoulli trial*. One trial consists of a single ^{71}Ga atom in a 1 cm^2 area, and a single neutrino hitting a uniformly distributed random spot. If an interaction happens, this constitutes a "success".

We can use the fact Beta distribution is a conjugate prior and select a highly uninformative prior, such as Beta$(1/2, 1)$ for the cross section.

Since the cross section is $<< 1$, it is easy to see that the relative uncertainty of the cross section $\sqrt{\mathrm{Var}(\sigma)}/\mathbb{E}[\sigma] \propto 1/\sqrt{\mathrm{events}}$. It is also easy to see that the total number of events is a sufficient statistic for the cross section $\Rightarrow$ this contains all the information relevant for the determination of σ.

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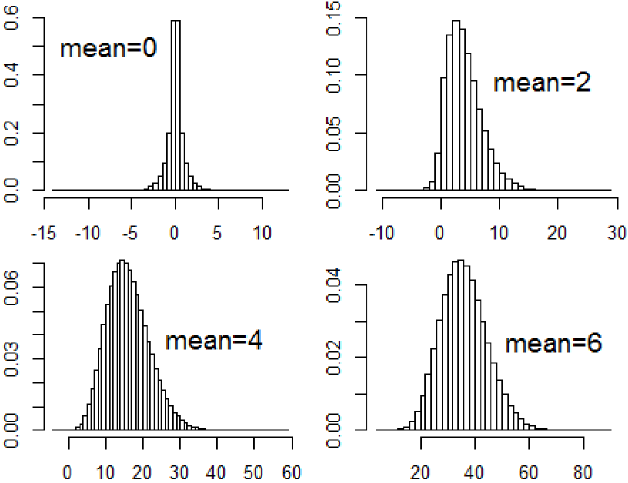
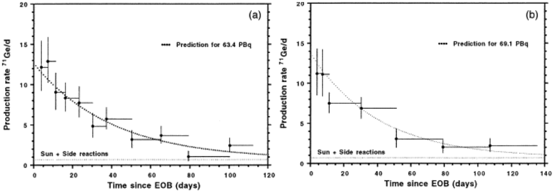
	Trials (10^{44})	Events
G1	$703.9^{+12.21}_{-17.76}$(stat.) $^{+3.520}_{-3.942}$(syst.)	389.76 ± 38.28
G2	$775.6^{+37.04}_{-17.76}$(stat.) $^{+3.878}_{-4.343}$(syst.)	365.93 ± 41.82
S1	$6.766 \times (72.6 \pm 0.2) \times (1.9114 \pm 0.022)^{+5.7\%}_{-5.6\%}$(syst.)	518.21 ± 62.93
S2	$6.603 \times (72.6 \pm 0.2) \times (1.513 \pm 0.007)^{+5.4\%}_{-5.2\%}$(syst.)	$401.58^{+36.51}_{-32.86}$



In the original fits of SAGE and GALLEX solar neutrino backgrounds (0.27/day and 0.67 ± 0.11/day) were included in the maximum likelihood fits of each individual run resulting in *negative rates* in 1σ error bars.

However, the events are interchangeable, so knowing which events belonged to which run doesn't give additional information. Instead one can just subtract the solar-background from the events as a Poisson distributed random variable.

The calculations were done with R using JAGS (Just Another Gibbs Sampler).



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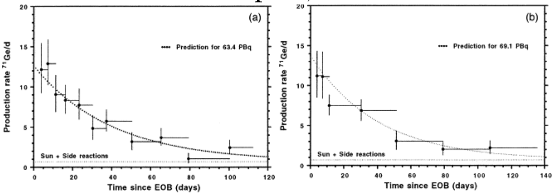

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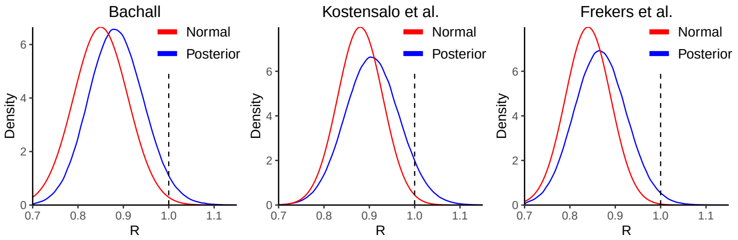
Theory	Posterior ETI	"Significance"	Normal appr.
Bachall	0.936	1.85σ	2.6σ
Bachall corr.	0.894	1.62σ	
Kostensalo et al.	0.873	1.52σ	2.3σ
Frekers et al.	0.974	2.22σ	3.0σ
Frekers et al. corr.	0.942	1.90σ	
Comb. theory	0.915	1.72σ	



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- P-values can indicate how incompatible the data are with a specified statistical model.
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American Statistical Assosiation guidelines

In the Bayesian framework one can also assess probabilities for things which do not have frequentistic interpretations, such as *What is the probability that sterile neutrinos exist?*

The statistical significance of the reactor anomaly is usually reported to be about 2–3σ. Reliable estimates are extremely difficult. The gallium anomaly is about 1.72σ.

Let's make the subjective assumption that the only reason one would think sterile neutrinos exist is the two anomalies. For new physics we usually require a discrepancy of 5σ. This can be reformulated by assuming *prior odds*

$$\frac{p(\mathrm{NP}=1)}{p(\mathrm{NP}=0)} = \frac{1-\Phi(4.4)}{\Phi(4.4)}$$

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Assuming one could come up with a consistent model for sterile neutrinos (or something else) which could explain both these anomalies, the odds ratio for new physics is

OR(NP) = 0.005 (RAA 2 sigma)   OR(NP) = 0.090 (3 sigma)

For previous gallium anomaly estimates the odds ratios would have been (RAA 2σ)

0.021 (Kostensalo et al.),   0.05 (Bahcall),   0.17 (Frekers et al.)

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- The previous estimates for the significance of the gallium anomaly are off due to an error in the calculations and due to unrealistic approximations.
- The new shell model calculations explain partially the difference between the charge-exchange BGTs and the shell model GALLEX/SAGE results.
- After taking into account the number events, correlations in the systematic errors etc. there is no evidence for a difference between the theoretical predictions and the GALLEX/SAGE results.
- A final step one can take is to get all the original data and construct a hierarchical model which takes all the available information into account. The uncertainty estimates for the number of events still relies on the numbers obtained by GALLEX/SAGE using the traditional approximations.
- The probability of sterile neutrinos existing is about 1%.

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John N. Bahcall Phys. Rev. C **56**, 3391 (1997).

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Frekers et al. PLB 706 134–138 (2011).

University of Jyväskylä

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- Carlo Giunti

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Thank you!

Transition	$\langle f \| O_{GT} \| i \rangle$	$\langle f \| O_{L=2} \| i \rangle$	BGT_{β}^{SM}	$BGT_{(p,n)}^{SM}$
$3/2^- \to 1/2^-$ (0 keV)	−0.451	0.348	0.051	0.044
$3/2^- \to 5/2^-$ (175 keV)	0.082	−2.23	0.017	0.0045
$3/2^- \to 3/2^-$ (500 keV)	0.056	0.104	0.0008	0.0011

$$\frac{0.082^2}{2 \times 3/2 + 1} = 0.0017$$

Results in a 15% contribution from the 5/2- state when it should be 1.5%

Possible problems in extracting the BGT value:

- Extraction of the $[J_{pro}\ J_{tar}\ J_{rel}] = [110]$ component at $0°$. Is the nuclear structure input valid and what are the uncertainties related to this?
- Relating the $[J_{pro}\ J_{tar}\ J_{rel}] = [110]$ component at $0°$ to the GT strength: possible significant contributions from $L = 2$ matrix element.

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Angular distribution analysis

"One-body transition densities (OBTDs) were calculated in the shell-model code NuShellX using the GXPF1a interaction in the full fp-model space"

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$BGT_{1/2-}$ 0.390
$BGT_{5/2-}$ 0.001
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Requires $g_A \approx 0.6$ to reproduce the experimental half life of ^{71}Ge.

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With these the ground-state transition is 92%, transition to $5/2^-$ state is 40%, and the transition to $3/2^-$ state is 87 % [110]. How accurate are these?

Bayesian neutrino physics

Joel Kostensalo

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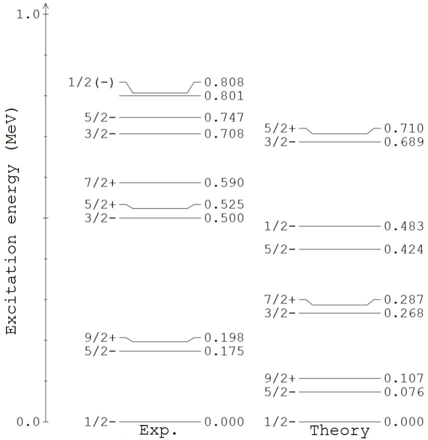
1.0
0.0
Excitation energy (MeV)
1/2(-) 0.808
0.801
5/2- 0.747
3/2- 0.708
7/2+ 0.590
5/2+ 0.525
3/2- 0.500
9/2+ 0.198
5/2- 0.175
1/2- 0.000
Exp.
5/2+ 0.710
3/2- 0.689
1/2- 0.483
5/2- 0.424
7/2+ 0.287
3/2- 0.268
9/2+ 0.107
5/2- 0.076
1/2- 0.000
Theory