Theoretical Description of Half-lives and Electron Spectra for Higher Order Forbidden Non-unique β Decays

Anil Kumar

and

Praveen C. Srivastava

Department of Physics, IIT Roorkee, India

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Outline

Introduction

Different types of beta decay:

- Allowed Beta Decay ($\Delta J = 0, \pm 1$, parity change = NO)
- Double beta decay
- Neutrinoless Double Beta Decay
- **Forbidden Beta Decay**
  - Unique [$\Delta J = K + 1$](K is the order of forbiddeness of the transition)
  - **Non unique [$\Delta J = K$, (in case of $K = 1$, $\Delta J = 0$)]**
    (In both cases unique and non-unique forbidden β decay, the parity change for odd-forbidden and remain same in even-forbidden)

◇ The progress of experimental techniques in the beta decay experiments made it possible to test the predictive power of the nuclear models.

◇ The decay rate depends on g_A^2 for β and g_A^4 for $\beta\beta$ decays.
$g_A \rightarrow$ **axial-vector coupling constant**

- O. S. Kirseborm $et.al.$, "Measurement of $2^+ \rightarrow 0^+$ ground-state transition in the β decay of ^{20}F", Phys. Rev. C **100**, 065805 (2019).
- Jouni T. Suhonen, "Value of the Axial-Vector Coupling Strength in β and $\beta\beta$ Decays: A Review ", Frontiers in Physics **5**, 55 (2017).

Formalism of FNU Beta Decay

Half-life:

$$t_{1/2} = \frac{\kappa}{\tilde{C}},$$

where κ is a constant with value 6147 s and $\tilde{C}$ is the dimensionless integrated shape function, which can be expressed as

$$\tilde{C} = \int_1^{w_0} C(w_e) p w_e (w_0 - w_e)^2 F_0(Z_f, w_e) dw_e,$$

here w_e is the total energy of emitted electron and w_0 is the endpoint energy of beta spectrum.

The shape factor $C(w_e)$ contains the nuclear structure information. The general form of this factor is defined as

$$C(w_e) = \sum_{k_e, k_\nu, K} \lambda_{k_e} \left[M_K(k_e, k_\nu)^2 + m_K(k_e, k_\nu)^2 - \frac{2\gamma_{k_e}}{k_e w_e} M_K(k_e, k_\nu) m_K(k_e, k_\nu) \right].$$

- H. Behrens, W. Bühring, *Electron Radial Wave Functions and Nuclear Beta-Decay* (Clarendon Press, Oxford, 1982).
- M. Haaranen, J. Kotila, and J. Suhonen, Phys. Rev. C **95** , 024327 (2017).
- M. T. Mustonen, M. Aunola, and J. Suhonen, Phys. Rev. C **73** , 054301 (2006).

Formalism of FNU Beta Decay

The quantities $M_K(k_e, k_\nu)$ and $m_K(k_e, k_\nu)$ are complicated expression of nuclear form factors and other kinematical factors. The auxiliary quantities are defined as

$$\lambda_{k_e} = \frac{F_{k_e-1}(Z_f, w_e)}{F_0(Z_f, w_e)} \quad \text{and} \quad \gamma_{k_e} = [k_e^2 - (\alpha Z_f)^2]^{1/2}.$$

Where $F_{k_e-1}(Z_f, w_e)$ is the generalized Fermi function. In the impulse approximation, the form factor coefficients are related to nuclear matrix elements (NMEs)

$$R^{LV} F^{(N)}_{KLS}(k_e, m, n, \rho) = (-1)^{K-L} g_V{}^V \mathcal{M}^{(N)}_{KLS}(k_e, m, n, \rho)$$
$$R^{LA} F^{(N)}_{KLS}(k_e, m, n, \rho) = (-1)^{K-L+1} g_A{}^A \mathcal{M}^{(N)}_{KLS}(k_e, m, n, \rho)$$

The NMEs for a transition between an initial (i) and final (f) state is given by

$${}^{V/A}\mathcal{M}^{(N)}_{KLS}(pn)(k_e, m, n, \rho) = \frac{1}{\sqrt{2J_i + 1}} \sum_{pn} {}^{V/A}m^{(N)}_{KLS}(pn)(k_e, m, n, \rho)(\psi_f \| [c_p^\dagger \tilde{c}_n]_K \| \psi_i).$$

- H. Behrens, W. Bühring, *Electron Radial Wave Functions and Nuclear Beta-Decay* (Clarendon Press, Oxford, 1982).
- M. Haaranen, J. Kotila, and J. Suhonen, Phys. Rev. C **95** , 024327 (2017).
- M. T. Mustonen, M. Aunola, and J. Suhonen, Phys. Rev. C **73** , 054301 (2006).

Model Space and Interactions (NSM)

Transitions	K	BR%(Exp)	Model Space and Interactions
^{46}Sc$(4^+)\rightarrow^{46}$Ti(2^+)	2	0.0036	Full $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$ KB3G[1] and GXFP1A[2] interactions.
^{59}Fe$(3/2^-)\rightarrow^{59}$Co$(7/2^-)$	2	0.18	
^{87}Rb$(3/2^-)\rightarrow^{87}$Sr$(9/2^+)$	3	100	$2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, $(1g_{9/2})^{2p,10n}$, $(2d_{5/2})^{0p,2n}$, $3s_{1/2}$, and $2d_{3/2}$ GLEPN[3] interaction.
^{24}Na$(4^+)\rightarrow^{24}$Mg(2^+)	2	0.064	Full $0d_{5/2}$, $1s_{1/2}$, and $0d_{3/2}$ USDB[4] and **DJ16A**[5] interactions.
^{36}Cl$(2^+)\rightarrow^{36}$Ar(0^+)	2	98.1	

[1] A. Poves *et. al.*, Nucl. Phys. A 694 157 (2001).
[2] M. Honma *et. al.*, Phys. Rev. C **69**, 034335 (2004).
[3] H. Mach *et al.*, Phys. Rev. C **41**, 226 (1990).
[4] B. A. Brown *et. al.*, Phys. Rev. C **74**, 034315 (2006).
[5] N. A. Smirnova *et. al.*, Phys. Rev. C **100**, 054329 (2019).

Results and Discussion

Results for

^{46}Sc, ^{59}Fe, and ^{87}Rb

Including the next-to-leading-order (NLO) terms in the shape factor.

Logft values and Half-lives of ^{46}Sc and ^{59}Fe

For the calculations of the logft and partial half-life, we have used the value of coupling constant $g_A = 1.00$ (quenched) and 1.27 (bare).
(vector coupling constant $g_V = 1.00$ from conserved vector current (CVC) hypothesis).

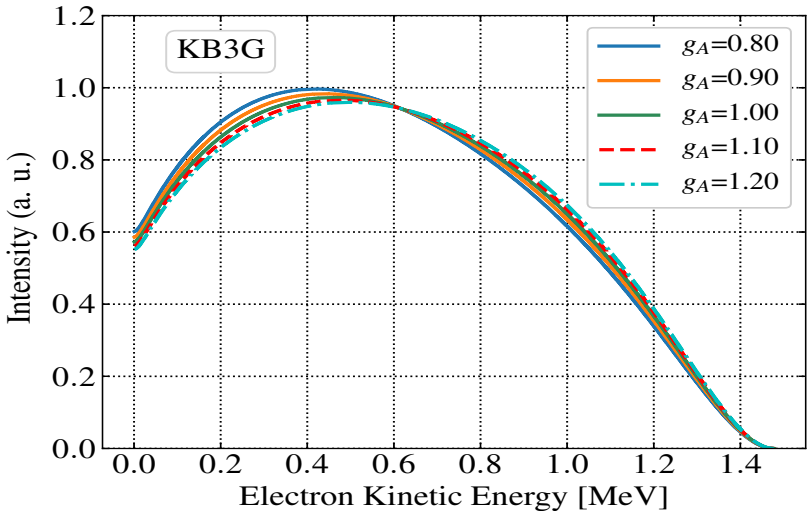
Logft values:

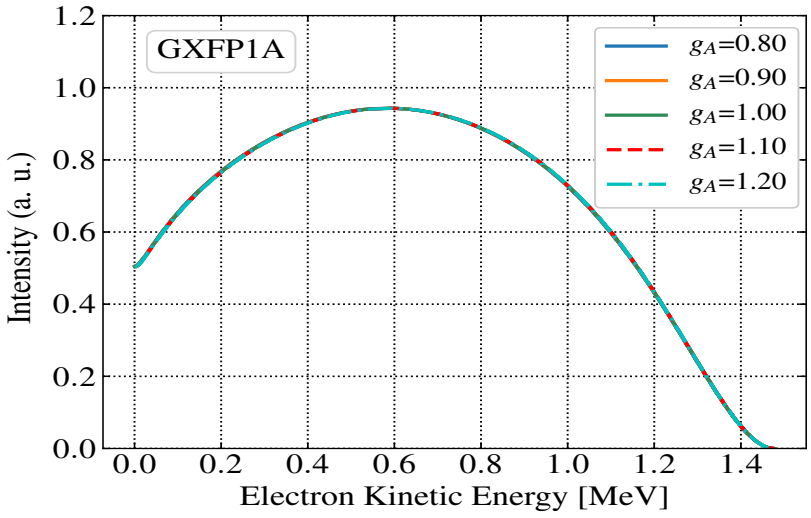
Transitions	Expt	KB3G		GXFP1A	
		g_A=1.00	g_A=1.27	g_A=1.00	g_A=1.27
^{46}Sc(4^+) $\rightarrow$ ^{46}Ti(2^+)	12.94(9)	12.568	12.425	12.652	12.445
^{59}Fe($3/2^-$) $\rightarrow$ ^{59}Co($7/2^-$)	11.14(10)	11.391	11.771	11.665	12.161

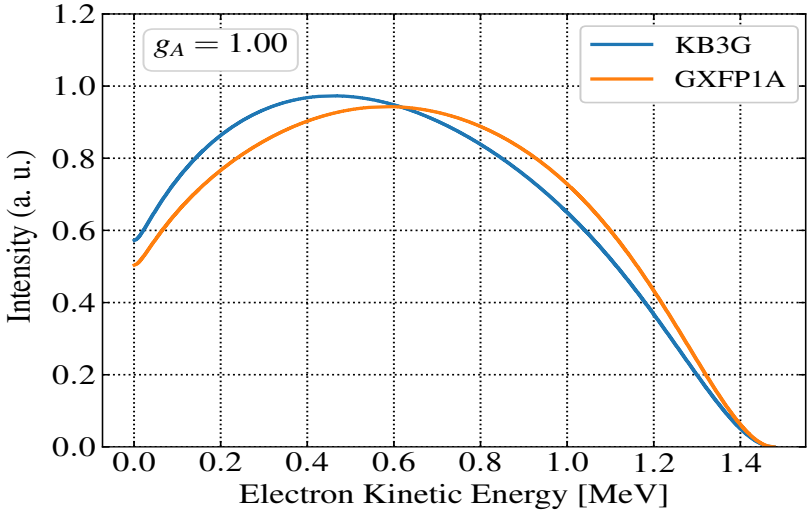
Partial half-life:

Transitions	Expt	KB3G		GXFP1A	
		g_A=1.00	g_A=1.27	g_A=1.00	g_A=1.27
^{46}Sc(4^+) $\rightarrow$ ^{46}Ti(2^+)	6.38×10^3 y	2.68×10^3 y	1.93×10^3 y	3.26×10^3 y	2.02×10^3 y
^{59}Fe($3/2^-$) $\rightarrow$ ^{59}Co($7/2^-$)	0.67×10^2 y	1.18×10^2 y	2.82×10^2 y	2.21×10^2 y	6.93×10^2 y

β Spectrum of ^{46}Sc$(4^+) \rightarrow {}^{46}Ti(2^+)$

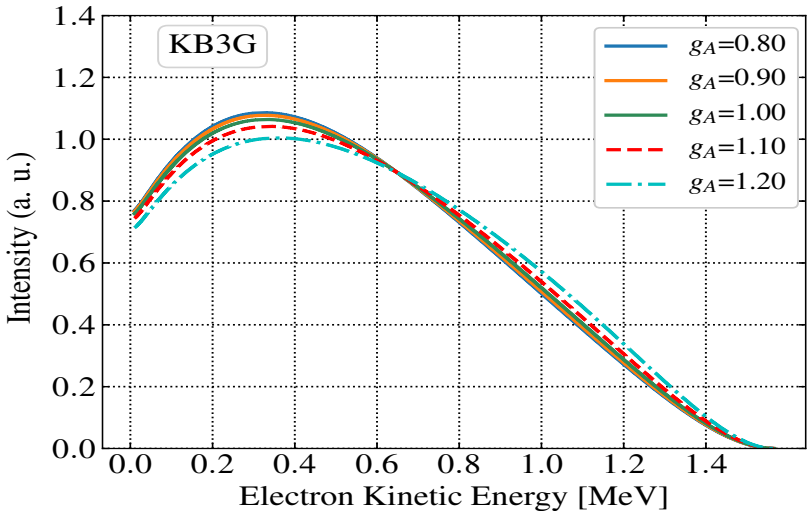


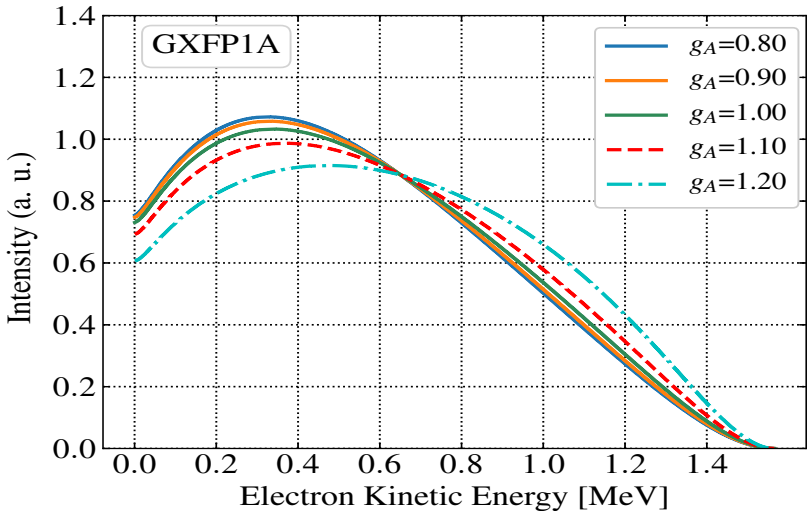


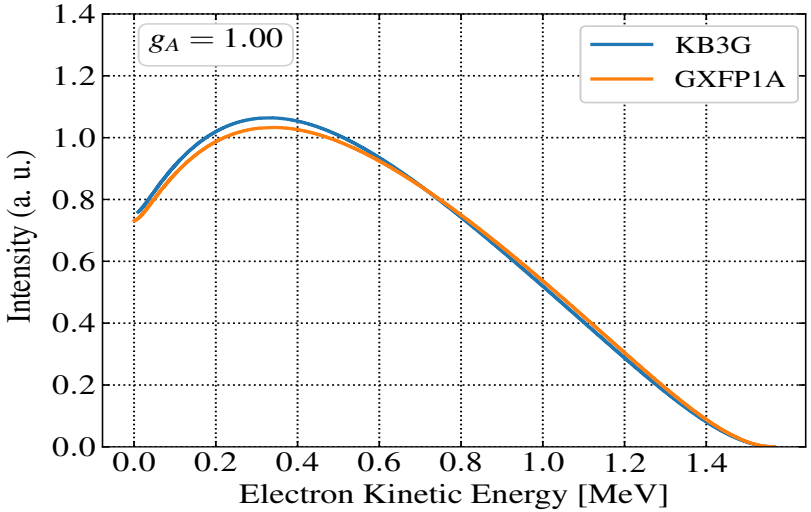


Normalized computed electron spectra from nuclear shell model for 2nd-FNU β^- decay of ^{46}Sc corresponding to different values of g_A (g_V=1.00).

β Spectrum of ^{59}Fe$(3/2^-) \rightarrow {}^{59}Co(7/2^-)$





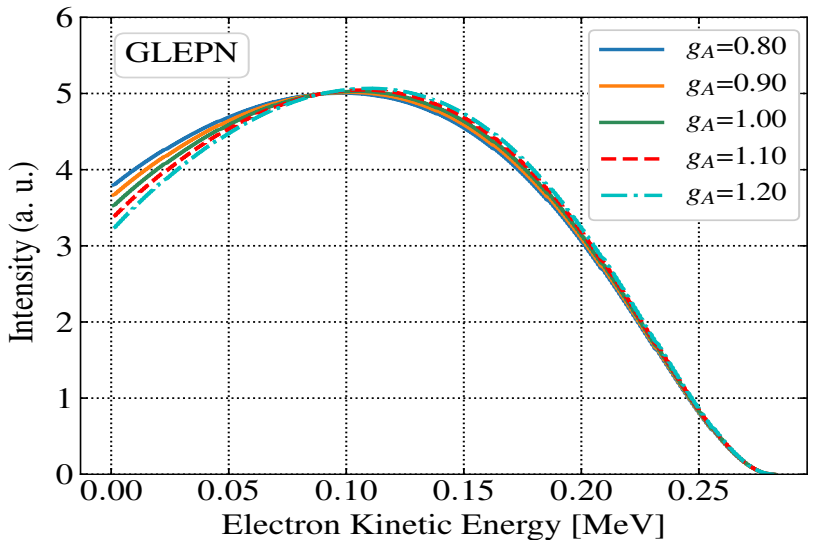


Normalized computed electron spectra from nuclear shell model for 2nd-FNU β^- decay of ^{59}Fe corresponding to different values of g_A (g_V=1.00).

β spectrum of ^{87}Rb$(3/2^-) \rightarrow ^{87}Sr(9/2^+)$

	Expt	GLEPN	
		g_A=1.00	g_A=1.27
$\log ft$	17.499(10)	16.134	16.224
Half-life	4.81×10^{10} y	0.21×10^{10} y	0.25×10^{10} y

Normalized computed electron spectra from nuclear shell model for 2nd-FNU β^- decay of ^{87}Rb corresponding to different values of g_A (g_V=1.00).



Results for

^{24}Na and ^{36}Cl

Including only leading-order (LO) terms in the shape factor.

Anil Kumar, P. C. Srivastava, J. Kostensalo, and J. Suhonen (In preparation).

Nuclear Matrix Elements of ^{36}Cl

	^{36}Cl$(2^+)\rightarrow^{36}$Ar(0^+)	
	USDB	DJ16A
${}^{V}\mathcal{M}^{(0)}_{211}$	0	0
${}^{V}\mathcal{M}^{(0)}_{211}$(CVC)	**-0.029375±0.0005**	**-0.015943±0.0010**
${}^{V}\mathcal{M}^{(0)}_{220}$	-5.892542	-3.483430
${}^{V}\mathcal{M}^{(0)}_{220}(1,1,1,1)$	-7.250832	-4.357072
${}^{V}\mathcal{M}^{(0)}_{220}(2,1,1,1)$	-6.955989	-4.195245
${}^{A}\mathcal{M}^{(0)}_{221}$	-1.249043	-2.025348
${}^{A}\mathcal{M}^{(0)}_{221}(1,1,1,1)$	-1.496326	-2.412741
${}^{A}\mathcal{M}^{(0)}_{221}(2,1,1,1)$	-1.426626	-2.297321

Nuclear Matrix Elements of ^{24}Na

	^{24}Na$(4^+)\rightarrow^{24}$Mg(2^+)	
	USDB	DJ16A
${}^{V}\mathcal{M}^{(0)}_{211}$	0	0
${}^{V}\mathcal{M}^{(0)}_{211}$(CVC)	**0.023790±0.0001**	**-0.018446±0.0002**
${}^{V}\mathcal{M}^{(0)}_{220}$	-0.431273	-0.131891
${}^{V}\mathcal{M}^{(0)}_{220}(1,1,1,1)$	-0.530979	-0.123441
${}^{V}\mathcal{M}^{(0)}_{220}(2,1,1,1)$	-0.509588	-0.110404
${}^{A}\mathcal{M}^{(0)}_{221}$	-0.430287	-0.482638
${}^{A}\mathcal{M}^{(0)}_{221}(1,1,1,1)$	-0.524687	-0.577264
${}^{A}\mathcal{M}^{(0)}_{221}(2,1,1,1)$	-0.502493	-0.550486
${}^{A}\mathcal{M}^{(0)}_{321}$	-1.459626	-0.758772

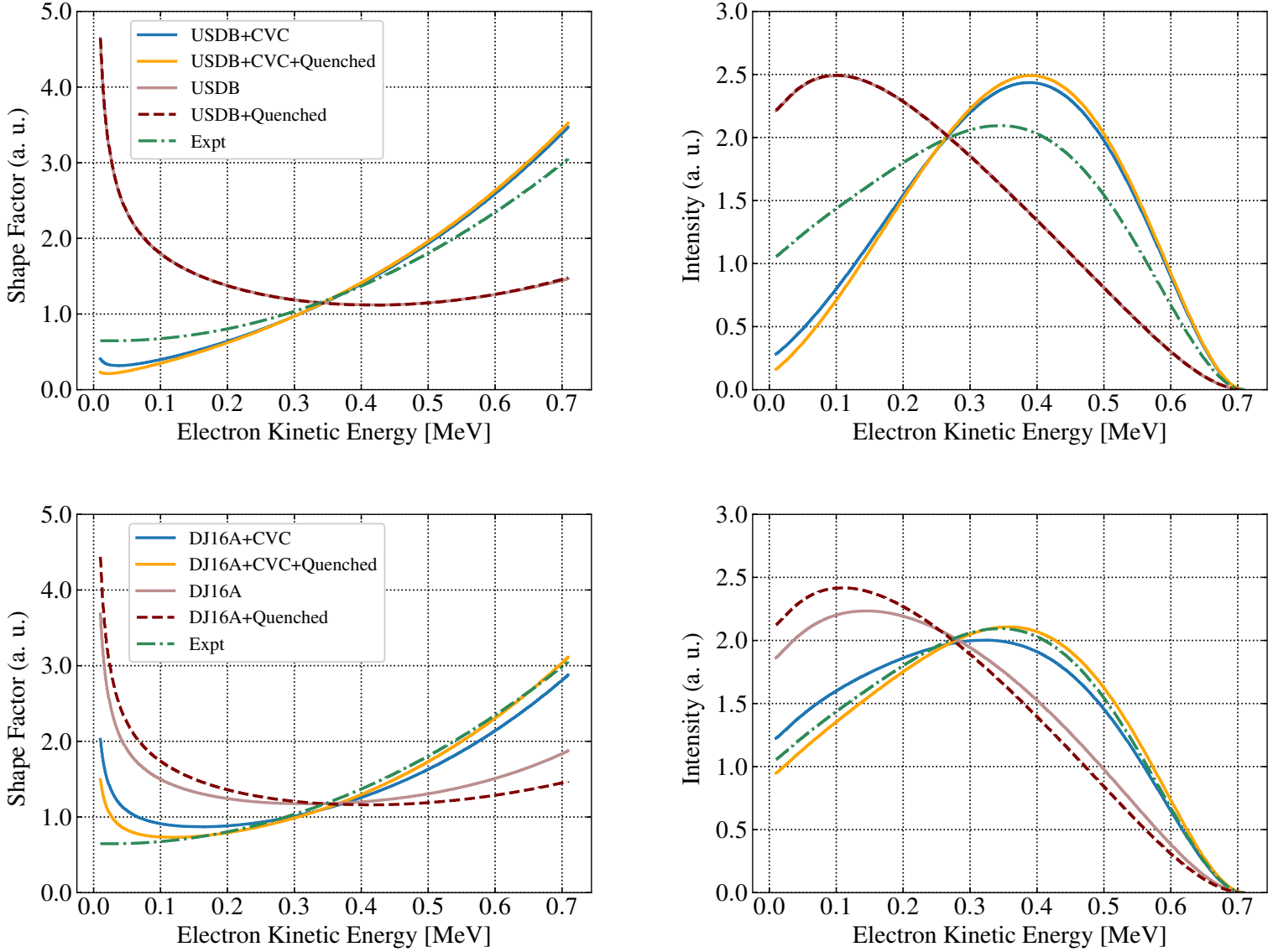
Logft Values of ^{24}Na and ^{36}Cl

For the calculations of the logft values, we have used the bare value of weak coupling constants $g_A = 1.27$ and $g_V = 1.00$.

	logft(SM)		
	USDB	DJ16A	Expt
^{36}Cl(2^+)$\rightarrow$^{36}Ar(0^+)	12.635	13.978	13.321(3)
^{24}Na(4^+)$\rightarrow$^{24}Mg(2^+)	12.237	12.881	11.340(4)

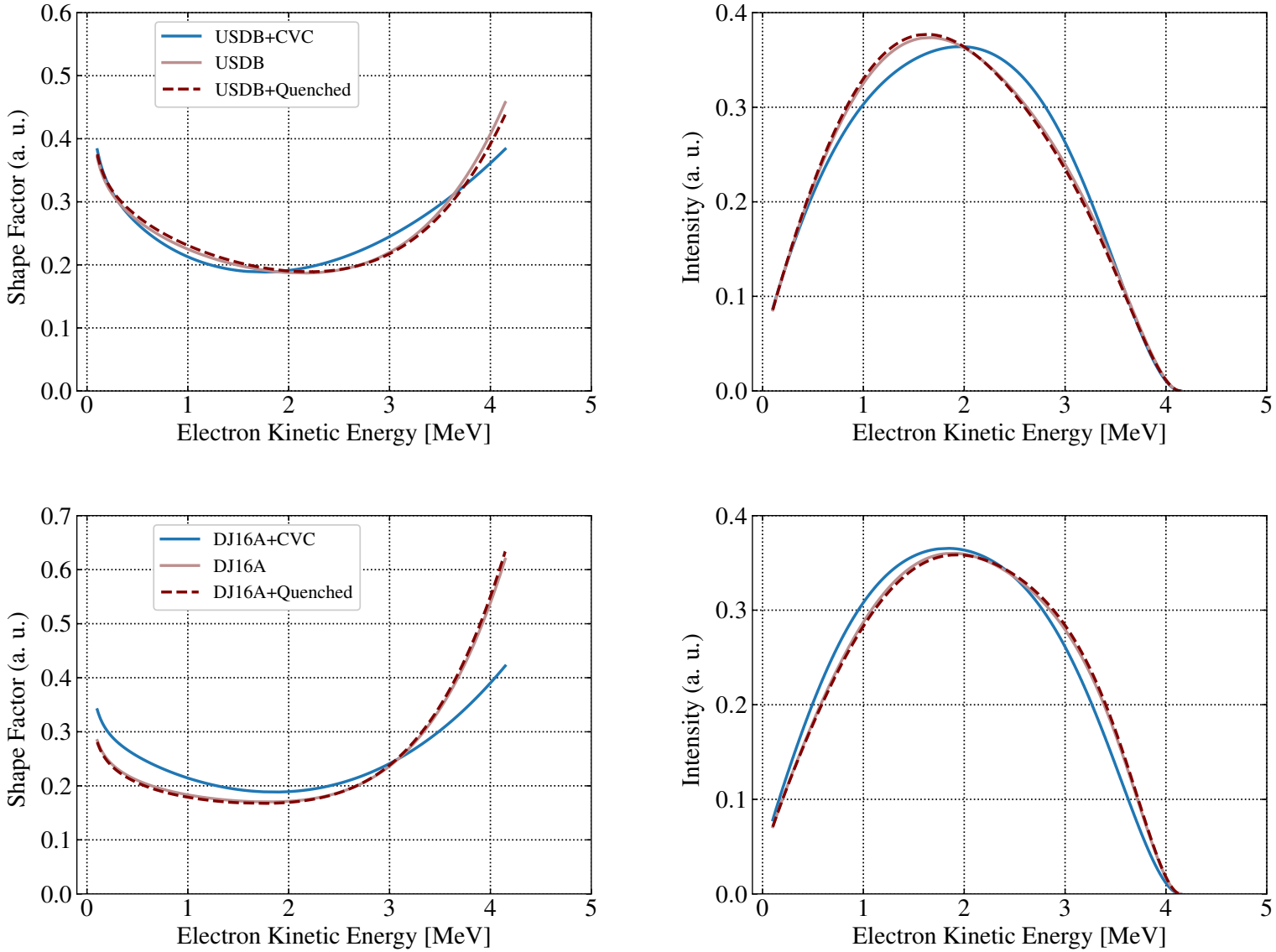
	logft(SM+CVC)		
	USDB	DJ16A	Expt
^{36}Cl(2^+)$\rightarrow$^{36}Ar(0^+)	13.221	13.108	13.321(3)
^{24}Na(4^+)$\rightarrow$^{24}Mg(2^+)	11.367	11.331	11.340(4)

β Spectrum of ^{36}Cl$(2^+) \rightarrow ^{36}$Ar(0^+)



Experimental Data→ H. Rotzinger *et. al.*, J Low Temp Phys **151**, 1087 (2008).

β Spectrum of ^{24}Na$(4^+) \rightarrow {}^{24}Mg(2^+)$



Summary

- Use the established and well-tested Hamiltonians to calculate the nuclear matrix elements for all transitions.
- Study of high-forbidden non-unique β^- decays (^{46}Sc, ^{59}Fe, and ^{87}Rb) with including next-to-leading-order terms in the shape factors.
- The shape of β spectrum changes with the axial-vector coupling constant g_A.
- The shape factor and electron spectra of the β^- decay for ^{36}Cl and ^{24}Na strongly depend on the matrix element ${}^{V}\mathcal{M}^{(0)}_{211}$.
- For ^{24}Na, there is no experimental data available for the shape factor and the electron spectra. Thus, our theoretical results might be very useful for comparison in the future experiments.

Thank You