Study of Kinematic Effects in Double Beta Decay

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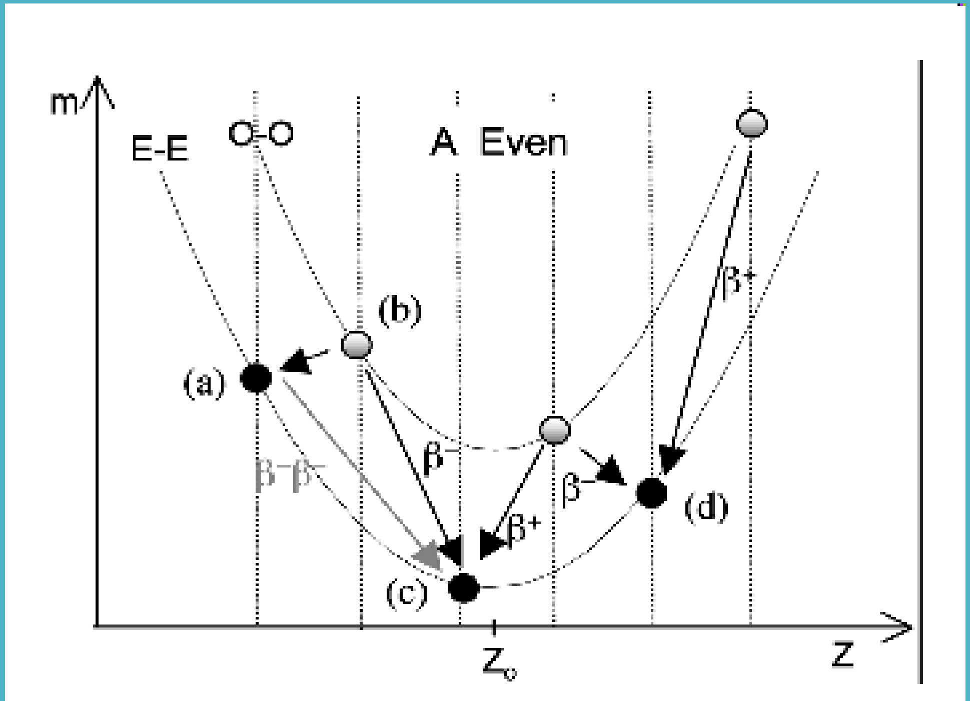
Outline

- Short introduction to double-beta decay : DBD rates
- Phase space factors formulas
- Search of Lorentz invariance violation in $2\nu\beta\beta$ decay
- Uncertainties in calculation of PSF and electron energy spectra
- Results
- Conclusions

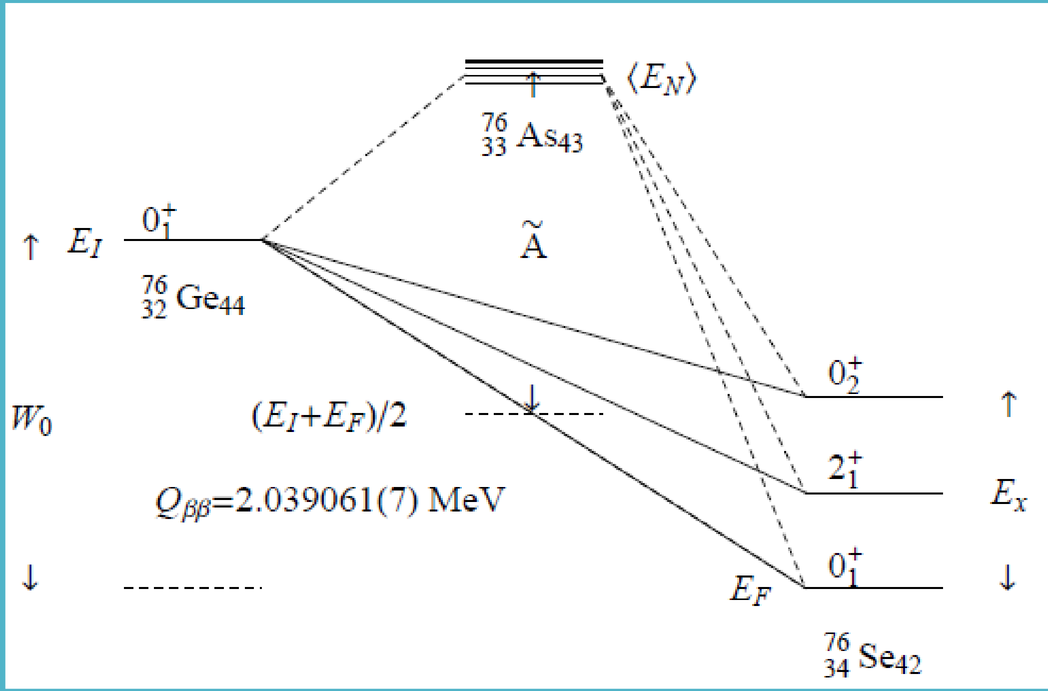
Double Beta Decay

DBD is the rarest known radioactive decay measured until now, by which an e-e nucleus transforms into another e-e nucleus with the same mass but with its nuclear charge changed by two units.

It occurs whatever single β decay can not occur due to energetical reasons or it is highly forbidden by angular momentum selection rules



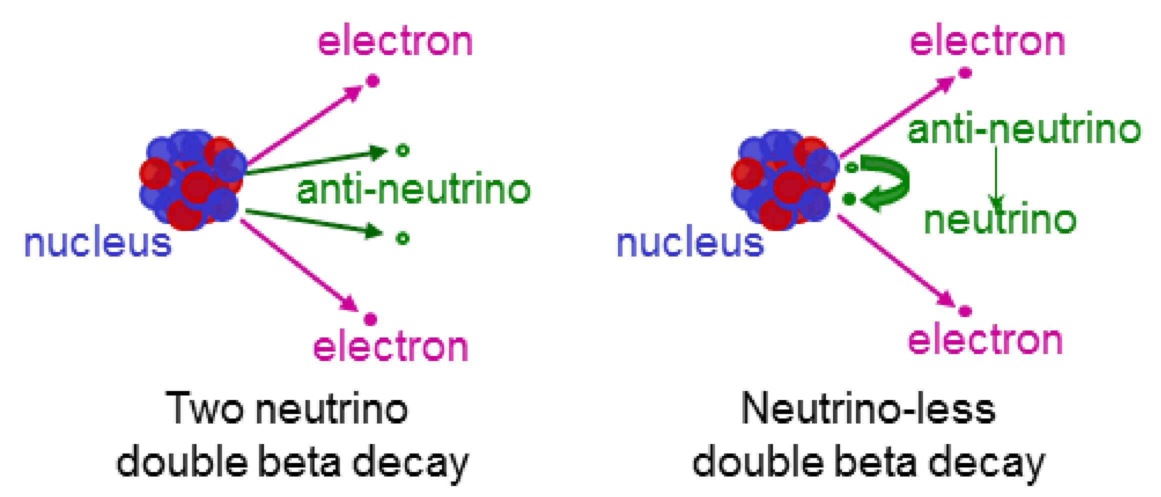
(a) and (d) are stable against β decay,
but unstable against: $\beta^- \beta^-$ for (a) and $\beta^+\beta^+$ for (d)

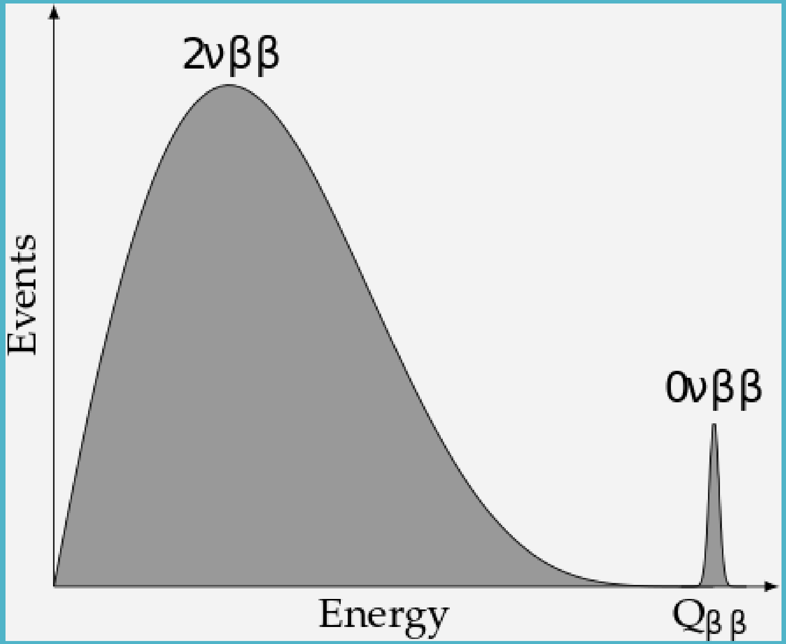


35 isotopes decaying with double electron emission
Several isotopes decaying with double positron emission

Double Beta Decay processes

Decay modes	*Channels*		
$2\nu\beta\beta$	*two electrons*	$2\nu\beta^-\beta^-$	$0\nu\beta^-\beta^-$
$0\nu\beta\beta$	*two positrons*	$2\nu\beta^+\beta^+$	$0\nu\beta^+\beta^+$
	EC/electron	$2\nu EC\beta^+$	$0\nu EC\beta^+$
	EC/EC	$2\nu ECEC$	$0\nu ECEC$





Importance of the DBD study

Neutrino properties:

- character Dirac or Majorana?
- mass scale (absolute mass)
- mass hierarchy
- how many flavors? Sterile neutrinos?

Checking symmetries

Lepton number, CP, Lorentz

Constraints of BSM parameters

associated with different mechanisms/scenarios that may contribute to the neutrinoless DBD occurrence

DBD lifetimes

$[T^{2\nu}_{1/2}]^{-1} = G^{2\nu}(Q_{\beta\beta}, Z) \quad \times \quad g_A^4 \quad \times \quad |m_e c^2 M^{2\nu}|^2$ *2νββ*

$[T^{0\nu}_{1/2}]^{-1} = \sum_k [G^{0\nu}(Q_{\beta\beta}, Z) \quad \times \quad g_A^4 \quad \times \quad |M_k^{0\nu}|^2 \quad \times \quad \langle\eta_k\rangle]$ *0νββ*

$\downarrow$ atomic physics PSF — $\downarrow$ nuclear physics NME — $\downarrow$ particle physics BSM

$$M_{2\nu} = \sum_N \frac{\langle 0_F^+ \| \tau^+\sigma \| 1_N^+\rangle\langle 1_N^+ \| \tau^+\sigma \| 0_I^+\rangle}{\frac{1}{2}W_0 + E_N - E_I}$$

$$M^{0\nu} = M^{0\nu}_{GT} - \left(\frac{g_V}{g_A}\right)^2 M^{0\nu}_F + M^{0\nu}_T$$

$G^{(2,0)\nu}(E_0, Z)$ phase space factors (PSF)

$M^{(2,0)\nu}$ = nuclear matrix elements (NME)

$\sum_k M^{0\nu}{}_k = |M^{0\nu}{}_\nu|^2 \langle m_\nu\rangle^2 + |M^{0\nu}{}_N|^2 \langle m_N\rangle^2 + |M^{0\nu}{}_\lambda|^2 \langle\eta_\lambda\rangle^2 + |M^{0\nu}{}_q|^2 \langle\eta_q\rangle^2 + \ldots$

$\langle\eta_l\rangle$ = BSM parameter specific the 0νββ mechanism; g_A = axial-vector constant

Precise calculations of PSF and NME are needed to predict lifetimes, derive neutrino parameters, extract information on neutrino properties

Phase space for 2νββ

$$\frac{dW_{2\nu}}{d\cos\theta_{12}} = |M_{2\nu}|^2 F_{2\nu}^{(0)} + |M_{2\nu}|^2 F_{2\nu}^{(1)} \cos\theta_{12} \quad G_{2\nu}^{(i)} = \frac{F_{2\nu}^{(i)}}{g_A^4 (m_e c^2)^2}$$

$$G^{2\nu} = C_1 \int_0^Q d\epsilon_1 \int_0^{Q-\epsilon_1} d\epsilon_2 \int_0^{Q-\epsilon_1-\epsilon_2} d\omega_1$$
$$\times F(Z_f,\epsilon_1)F(Z_f,\epsilon_2)\sqrt{\epsilon_1(\epsilon_1+2)}(\epsilon_1+1)\sqrt{\epsilon_2(\epsilon_2+2)}(\epsilon_2+1)$$
$$\times \omega_1^2 (Q-\epsilon_1-\epsilon_2-\omega_1)^2 \left(\langle K_N\rangle^2 + \langle L_N\rangle^2 + \langle K_N\rangle\langle L_N\rangle\right)$$

$$\langle K_N\rangle = \frac{1}{\epsilon_1+\omega_1+\langle E_N\rangle - E_I + 1} + \frac{1}{\epsilon_2+\omega_2+\langle E_N\rangle - E_I + 1}$$

$$\langle L_N\rangle = \frac{1}{\epsilon_1+\omega_2+\langle E_N\rangle - E_I + 1} + \frac{1}{\epsilon_2+\omega_1+\langle E_N\rangle - E_I + 1}$$

$$C_1 = (\tilde{A}^2 G_F^4 |V_{ud}|^4 m_e^9)/(96\pi^7 \ln 2) \quad \tilde{A}^2 = [Q/2 + \langle E_N\rangle - E_I + 1]^2$$

$F(Z,\varepsilon)$ = Fermi function

$(\varepsilon_1,\varepsilon_2)/(\omega_1,\omega_2)$ =electron/neutrino energies

E_I = parent g.s. energy;

$<E_N>$ = average energy of intermediate states

$$\frac{dG_{2\nu}^{(0)}}{d\epsilon_1}$$ *single electron spectra*

$$\frac{dG_{2\nu}^{(0)}}{d(\epsilon_1+\epsilon_2-2m_ec^2)}$$ *summed energy electron spectra*

$$\alpha(\epsilon_1) = \frac{dG_{2\nu}^{(1)}/d\epsilon_1}{dG_{2\nu}^{(0)}/d\epsilon_1}$$ *electron angular correlation*

Additional information from kinematic quantities:

Single state dominance (summation over N reduces to one state and $<E_N> = 1^+$)

Transitions to excited 2^+ states (single and summed energy electron spectra from transitions to g.s and to 2+ states differ each other)

Mechanisms (for 0νββ – single electron spectra and angular correlation are different each other for light ν exchange mechanisms and RH contributions)

Lorentz invariance violation in weak decays

- The general framework characterizing LIV is the Standard Model Extension (SME).
- In minimal SME (operators dimension ≤ 4) there are operators that couples to ν_s and affect ν flavor oscillations, ν velocity or ν phase spaces (β, ββ decays).
- Until recently tests for LIV involving ν_s were perform in ν oscillation experiments.
- LV can also be investigated in β and ββ decays by a precise analysis of electron spectra.
- Since 2016 deviations to Lorentz symmetry began to be also investigated in DBD experiments: EXO NEMO3, CUPID0, GERDA.
- There is a q-independent operator (counter-shaded operator), that doesn't affect ν oscillations, and hence can not be detected in LBL neutrino experiments, but can affect the summed energy spectra of electrons or the energy electron spectra and electron angular correlations (which can be investigated in experiments with tracking systems that can reconstruct the direction of the two emitted electrons).
- In the absence of observation of LIV deviations one can constrain the $\mathring{a}^{(3)}_{of}$ coefficient that governs the isotropic (like-time) component of the counter-shaded operator.

The coupling of the neutrino to the counter-shaded operator modifies the neutrino momentum from the standard expression.

This, further, modifies the *2νββ transition amplitude, so the decay rate can be written as a sum of the standard term and a perturbation due to LVββ*

$\Gamma^{(2\nu)} = \Gamma_0^{(2\nu)} + d\Gamma^{(2\nu)}$

$\Gamma_0^{(2\nu)} = G_0^{2\nu}(E_0, Z) \times g_A^4 \times |m_e c^2 \, M^{2\nu}|^2$

$d\Gamma^{(2\nu)} = dG^{2\nu}(E_0, Z) \times g_A^4 \times |m_e c^2 \, M^{2\nu}|^2$

$G_0^{2\nu} = C \int_0^Q d\varepsilon_1 F(Z, \varepsilon_1) [\varepsilon_1(\varepsilon_1 +2)]^{1/2} (\varepsilon_1 +1) \int_0^{Q-\varepsilon 1} d\varepsilon_2 F(Z, \varepsilon_2) [\varepsilon_2(\varepsilon_2 +2)]^{1/2} (\varepsilon_2 +1)(Q- \varepsilon_1- \varepsilon_2)^5$

$dG^{2\nu} = 10 å^{(3)}{}_{of} \, C \int_0^Q d\varepsilon_1 F(Z, \varepsilon_1) [\varepsilon_1(\varepsilon_1 +2)]^{1/2} (\varepsilon_1 +1) \int_0^{Q-\varepsilon 1} d\varepsilon_2 F(Z, \varepsilon_2) [\varepsilon_2(\varepsilon_2 +2)]^{1/2} (\varepsilon_2 +1)(Q- \varepsilon_1- \varepsilon_2)^4$

$C = G_F^4 |V_{ud}|^4 m_e / 240\pi^7 \quad t_{1,2} = \varepsilon_{1,2} - 1;$

$G_0^{2\nu}$, $dG^{2\nu}$ can be calculated in different approximations:

J.S. Diaz, PRD**89**(2014)
EXO, PRD **93** (2016)
CUPID0, PRD**100**(2019)
GERDA, Thesis2017

- $F(Z, \varepsilon) = (2\pi\eta)[1-\exp(- 2\pi\eta)]^{-1}$, $\eta = \pm\alpha Z\varepsilon/q$
- $F(Z, \varepsilon) = 4(2qR_A)^{2(\gamma-1)} \exp(\pi\eta) \left|\frac{\Gamma(\gamma+i\eta)}{\Gamma(2\gamma+1)}\right|^2 \qquad \gamma = \mathrm{Sqrt}\,[1 - (\alpha Z)^2]$
- using exact electron functions obtained by solving Dirac equations

$$G^{2\nu} = C_1 \int_0^Q d\epsilon_1 \int_0^{Q-\epsilon_1} d\epsilon_2 \int_0^{Q-\epsilon_1-\epsilon_2} d\omega_1$$
$$\times F(Z_f,\epsilon_1)F(Z_f,\epsilon_2)\sqrt{\epsilon_1(\epsilon_1+2)}(\epsilon_1+1)\sqrt{\epsilon_2(\epsilon_2+2)}(\epsilon_2+1)$$
$$\times \omega_1^2(Q-\epsilon_1-\epsilon_2-\omega_1)^2\left(\langle K_N\rangle^2+\langle L_N\rangle^2+\langle K_N\rangle\langle L_N\rangle\right)$$

$$\delta G^{2\nu} = 10\mathring{a}_{of}^{(3)} C_2 \int_0^Q d\epsilon_1 \int_0^{Q-\epsilon_1} d\epsilon_2 \int_0^{Q-\epsilon_1-\epsilon_2} d\omega_1$$
$$\times F(Z_f,\epsilon_1)F(Z_f,\epsilon_2)\sqrt{\epsilon_1(\epsilon_1+2)}(\epsilon_1+1)\sqrt{\epsilon_2(\epsilon_2+2)}(\epsilon_2+1)$$
$$\times \omega_1(Q-\epsilon_1-\epsilon_2-\omega_1)^2\left(\langle K_N\rangle^2+\langle L_N\rangle^2+\langle K_N\rangle\langle L_N\rangle\right)$$

$$C_1 = (\tilde{A}^2 G_F^4 |V_{ud}|^4 m_e^9)/(96\pi^7 \ln 2)$$
$$C_2 = (\tilde{A}^2 G_F^4 |V_{ud}|^4 m_e^8)/(240\pi^7 \ln 2)$$

$$\langle K_N\rangle = \frac{1}{\epsilon_1+\omega_1+\langle E_N\rangle-E_I} + \frac{1}{\epsilon_2+\omega_2+\langle E_N\rangle-E_I}$$
$$\langle L_N\rangle = \frac{1}{\epsilon_1+\omega_2+\langle E_N\rangle-E_I} + \frac{1}{\epsilon_2+\omega_1+\langle E_N\rangle-E_I}$$

$$G^{2\nu} = C_3 \int_0^Q d\epsilon_1 \int_0^{Q-\epsilon_1} d\epsilon_2 F(Z_f,\epsilon_1)F(Z_f,\epsilon_2)$$
$$\times \sqrt{\epsilon_1(\epsilon_1+2)}(\epsilon_1+1)\sqrt{\epsilon_2(\epsilon_2+2)}(\epsilon_2+1)(Q-\epsilon_1-\epsilon_2)^5$$

$$\delta G^{2\nu} = 10\mathring{a}_{of}^{(3)} C_4 \int_0^Q d\epsilon_1 \int_0^{Q-\epsilon_1} d\epsilon_2 F(Z_f,\epsilon_1)F(Z_f,\epsilon_2)$$
$$\times \sqrt{\epsilon_1(\epsilon_1+2)}(\epsilon_1+1)\sqrt{\epsilon_2(\epsilon_2+2)}(\epsilon_2+1)(Q-\epsilon_1-\epsilon_2)^4$$

$$\langle K_N\rangle \simeq \langle L_N\rangle \simeq \frac{2}{E_I-\langle E_N\rangle-(Q/2+1)}$$

$$C_3 = (G_F^4 |V_{ud}|^4 m_e^9)/(240\pi^7 \ln 2)$$

$$C_4 = (G_F^4 |V_{ud}|^4 m_e^8)/(240\pi^7 \ln 2)$$

Formulas in previous analyses

Approximations of the Fermi functions

$$F(Z_f,\epsilon) = F^{NR}(Z_f,\epsilon) = \frac{2\pi\eta}{1-e^{-2\pi\eta}}$$

Non-relativistic treatment
*Primakov&Rosen RPP **22**(1959)*

$$\frac{d\Gamma}{dK} = \Lambda \cdot (K^5 + 10K^4 + 40K^3 + 60K^2 + 30K) \times [(Q_{\beta\beta}-K)^5 + 10\tilde{a}_{of}^{(3)}(Q_{\beta\beta}-K)^4]$$

$$\Lambda = \frac{G_F^4 g_A^4 |V_{ud}|^4 F_{PR}^2(Z) m_e^{11}}{7200\pi^7} |\mathcal{M}^{2\nu}|^2$$

$$F(Z_f,\epsilon) = F_0(Z_f,\epsilon) = 4(2pR_A)^{2(\gamma-1)} e^{\pi\eta} \frac{|\Gamma(\gamma+i\eta)|^2}{[\Gamma(2\gamma+1)]^2}$$

Relativistic treatment: solution of a Dirac equation in a point charge Coulomb potential
*Suhonen&Civitarese, PR **301** (1998)*

$$F(Z_f,\epsilon) = \frac{f_1^2(\epsilon,R_A) + g_{-1}^2(\epsilon,R_A)}{2p^2}$$

Fermi function built up from the radial solution of a Dirac equation in a Coulomb-type potential
*Kotila&Iachello,PRC**85**(2012);*
*Stoica&Mirea PRC**88**(2013); RRP63(2015)*

Calculation of the phase space factors for DBD

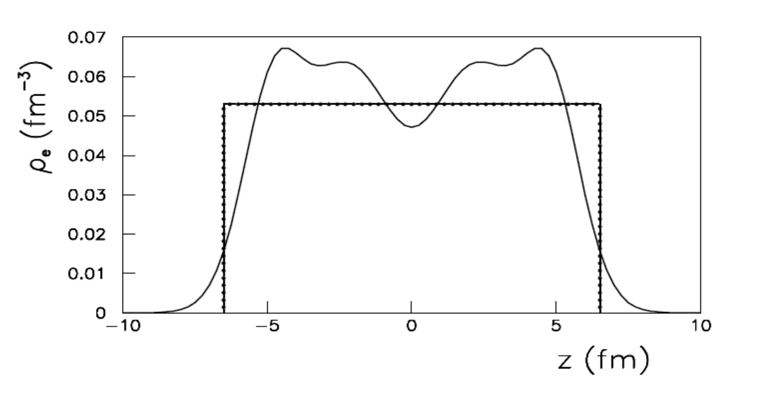
$$G^{\beta\beta}_{2\nu}(0^+ \to 0^+) = \frac{2\tilde{A}^2}{3\ln 2 g_A^4 (m_e c^2)^2} \int_{m_e c^2}^{Q^{\beta\beta}+m_e c^2} d\epsilon_1 \int_{m_e c^2}^{Q^{\beta\beta}+2m_e c^2-\epsilon_1} d\epsilon_2 \int_0^{Q^{\beta\beta}+2mc_e^2-\epsilon_1-\epsilon_2} d\omega_1 f_{11}^{(0)} w_{2\nu}(\langle K_N\rangle^2+\langle L_N\rangle^2+\langle K_N\rangle\langle L_N\rangle)$$

$$f_{11}^{(0)} = | f^{-1-1} |^2 + | f_{11} |^2 + | f^{-1}{}_1 |^2 + | f_1{}^{-1} |^2$$

$$f^{-1-1} = g_{-1}(\epsilon_1) g_{-1}(\epsilon_2) \ ; \ f_{11} = f_1(\epsilon_1) f_1(\epsilon_2),$$
$$f^{-1}{}_1 = g_{-1}(\epsilon_1) f_1(\epsilon_2) \ ; \ f_1{}^{-1} = f_1(\epsilon_1) g_1(\epsilon_2)$$

$$\frac{dg_\kappa(\epsilon,r)}{dr} = -\frac{\kappa}{r} g_\kappa(\epsilon,r) + \frac{\epsilon - V + m_e c^2}{c\hbar} f_\kappa(\epsilon,r)$$
$$\frac{df_\kappa(\epsilon,r)}{dr} = -\frac{\epsilon - V - m_e c^2}{c\hbar} g_\kappa(\epsilon,r) + \frac{\kappa}{r} f_\kappa(\epsilon,r)$$

$$V(Z,r) = \begin{cases} -\frac{Z\alpha\hbar c}{r}, & r \geq R_A \\ -Z(\alpha\hbar c)\left(\frac{3-(r/R_A)^2}{2R_A}\right), & r < R_A \end{cases}$$



$$\rho_e(\vec{r}) = \sum (2j_i+1) v_i^2 \, |\psi_i(\vec{r})|^2$$

$$V(r) = \alpha\hbar c \int \frac{\rho_e(\vec{r'})}{|\vec{r} - \vec{r'}|} d\vec{r'}$$

$$\frac{d^2\phi}{dx^2} = \frac{\phi^{3/2}}{\sqrt{x}}$$

Ingredients: Fermi functions from exact electron w.f. obtained as solutions of a Dirac equation with Coulomb-type potential built with a realistic proton density in nucleus, with inclusion of FNS and screening effects

Results

Table 1. Phase space factors computed with Fermi functions in approximation schemes A, B, C. Values are obtained with Equations (9,10). Q-values are also displayed.

		$G^{2\nu}$ in units of $10^{-21} yr^{-1}$		
Nucleus	Q^{a}(keV)	A	B	C
^{48}Ca	4268.070 ± 0.076	12051.21 ± 2.02	15941.38 ± 2.66	15212.54 ± 2.54
^{76}Ge	2039.059 ± 0.014	25.755 ± 0.002	52.834 ± 0.003	48.384 ± 0.003
^{82}Se	2997.9 ± 0.3	828.96 ± 0.75	1779.02 ± 1.58	1597.40 ± 1.42
^{100}Mo	3034.40 ± 0.17	1241.83 ± 0.62	3822.34 ± 1.89	3312.62 ± 1.63
^{110}Pd	2017.85 ± 0.64	40.32 ± 0.11	163.07 ± 0.43	138.57 ± 0.37
^{116}Cd	2813.50 ± 0.13	776.81 ± 0.32	3318.62 ± 1.33	2767.61 ± 1.11
^{130}Te	2527.515 ± 0.260	344.24 ± 0.31	1891.38 ± 1.66	1538.53 ± 1.35
^{136}Xe	2458.13 ± 0.41	287.56 ± 0.42	1797.71 ± 2.55	1441.47 ± 2.04
		$\delta G^{2\nu}/(10\mathring{a}^{(3)}_{of})$ in units of $10^{-21} yr^{-1} \mathrm{MeV}^{-1}$		
Nucleus	Q^{a}(keV)	A	B	C
^{48}Ca	4268.070 ± 0.076	5318.08 ± 0.81	7002.59 ± 1.06	6674.90 ± 1.01
^{76}Ge	2039.059 ± 0.014	21.754 ± 0.001	44.195 ± 0.002	40.446 ± 0.002
^{82}Se	2997.9 ± 0.3	497.02 ± 0.40	1054.26 ± 0.84	945.27 ± 0.75
^{100}Mo	3034.40 ± 0.17	733.70 ± 0.33	2219.94 ± 0.99	1920.26 ± 0.85
^{110}Pd	2017.85 ± 0.64	34.09 ± 0.08	135.41 ± 0.32	114.94 ± 0.27
^{116}Cd	2813.50 ± 0.13	489.48 ± 0.18	2046.83 ± 0.74	1703.76 ± 0.61
^{130}Te	2527.515 ± 0.260	238.08 ± 0.19	1277.20 ± 1.00	1036.90 ± 0.81
^{136}Xe	2458.13 ± 0.41	203.69 ± 0.27	1241.58 ± 1.57	993.53 ± 1.26

Table A2. Phase space factors (upper part) and their deviations (lower part) computed with numerical Fermi functions and with inclusion of kinematic factors. Relative uncertainties associated with Q-values are displayed in column 3, in terms of σ_Q/Q. Differences introduced by the omission of kinematic factors: $\xi = (G - G^{\mathrm{No\ VS}})/G$ and $\delta\xi = (\delta G - \delta G^{\mathrm{No\ VS}})/\delta G$ are also displayed in the last column ($G^{\mathrm{No\ VS}}$ and $\delta G^{\mathrm{No\ VS}}$ are the ones from Table 1).

Nucleus	$G(10^{-21}\mathrm{y}^{-1})$	$\sigma_G/G\ \left(\frac{\sigma_Q}{Q}\right)$	$\xi(\%)$
^{48}Ca	15443.23	8.83	1.494
^{76}Ge	48.50	8.32	0.231
^{82}Se	1604.65	8.58	0.452
^{100}Mo	3325.32	8.53	0.382
^{110}Pd	138.79	8.20	0.158
^{116}Cd	2775.55	8.43	0.286
^{130}Te	1541.74	8.32	0.208
^{136}Xe	1444.19	8.29	0.188
Nucleus	$\delta G/10\mathring{a}_{0f}(10^{-21}y^{-1}\mathrm{MeV}^{-1})$	$\sigma_{\delta G}/\delta G\ \left(\frac{\sigma_Q}{Q}\right)$	$\delta\xi(\%)$
^{48}Ca	6797.21	7.94	1.800
^{76}Ge	40.56	7.43	0.284
^{82}Se	950.49	7.69	0.549
^{100}Mo	1929.23	7.64	0.465
^{110}Pd	115.16	7.31	0.195
^{116}Cd	1709.73	7.54	0.349
^{130}Te	1039.56	7.43	0.255
^{136}Xe	995.83	7.39	0.231

Ratio $G^{2\nu}/\delta G^{2\nu}$ and the energies where LIV are expected to be maximal

Nucleus	K_m(keV) [9]	K_m(keV)	$\mathring{a}_{of}^{(3)} G^{2\nu}/\delta G^{2\nu}$ (10^{-6}GeV)
^{48}Ca	1980	2002	227.199 ± 0.048
^{76}Ge	810	818	119.562 ± 0.009
^{82}Se	1300	1297	168.823 ± 0.195
^{100}Mo	1320	1294	172.365 ± 0.111
^{110}Pd	–	777	120.518 ± 0.420
^{116}Cd	1200	1165	162.339 ± 0.085
^{130}Te	1050	1013	148.307 ± 0.170
^{136}Xe	1020	980	145.023 ± 0.269

Summed energy spectra of electrons in the approximations A, B and C

A= NR approx. is inadequate in precise electron spectra analyses

B = approx. (analytical Fermi function); non-inclusion of FNS and screening effects: differences up to 30% as compared with “exact” Fermi function

C = exact Fermi functions, screening effect, “realistic” Coulomb-type potential

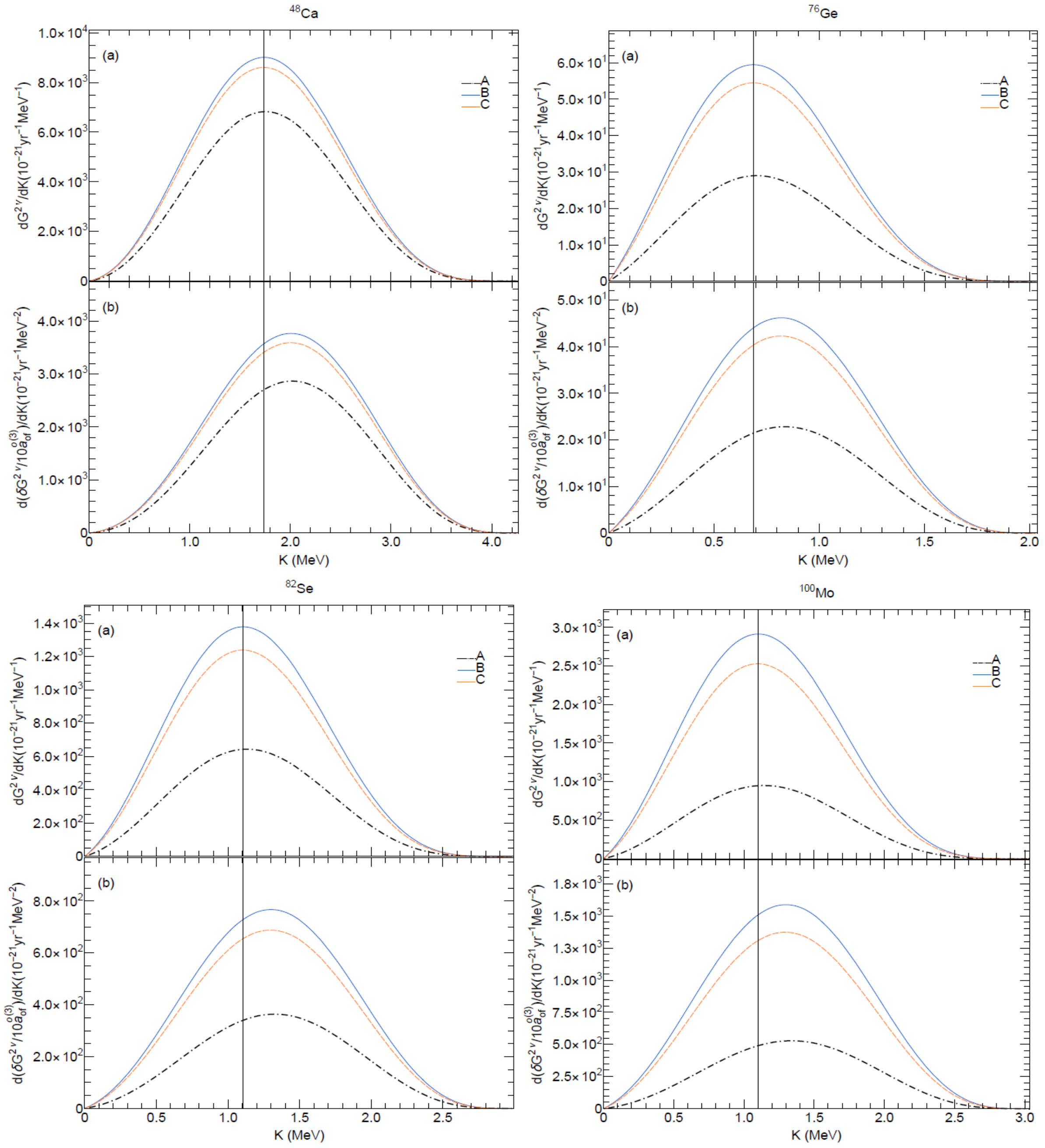


Figure 1. Summed energy spectra of electrons in the standard $2\nu\beta\beta$ decay (a) and

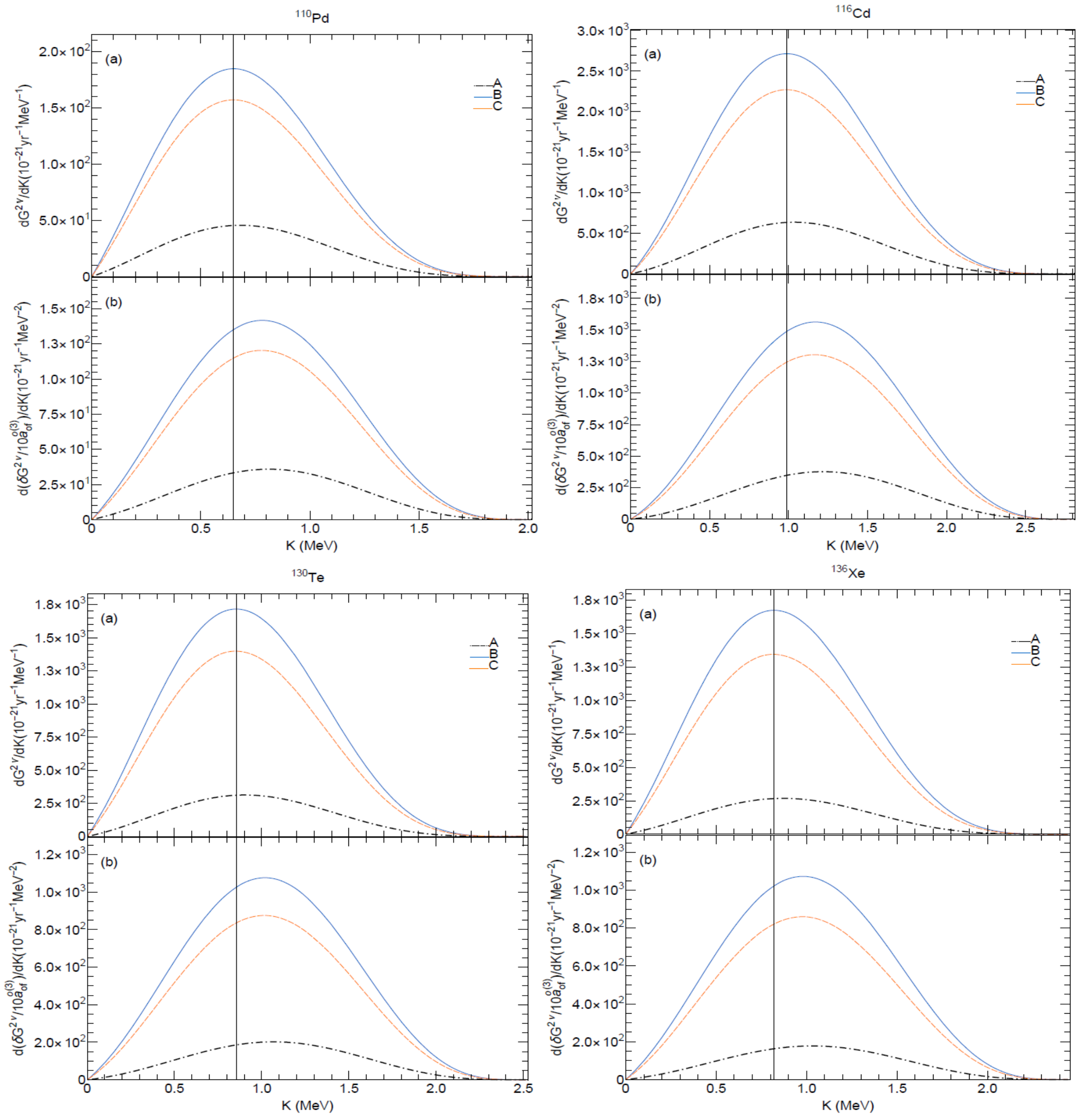


Figure 2. Summed energy spectra of electrons in the standard $2\nu\beta\beta$ decay (a) and their deviations due to LIV (b) for the nuclei ^{110}Pd, ^{116}Cd, ^{130}Te and ^{136}Xe.

Electron spectra

$$\frac{dG^{(0)}_{2\nu}}{d\epsilon_1}$$

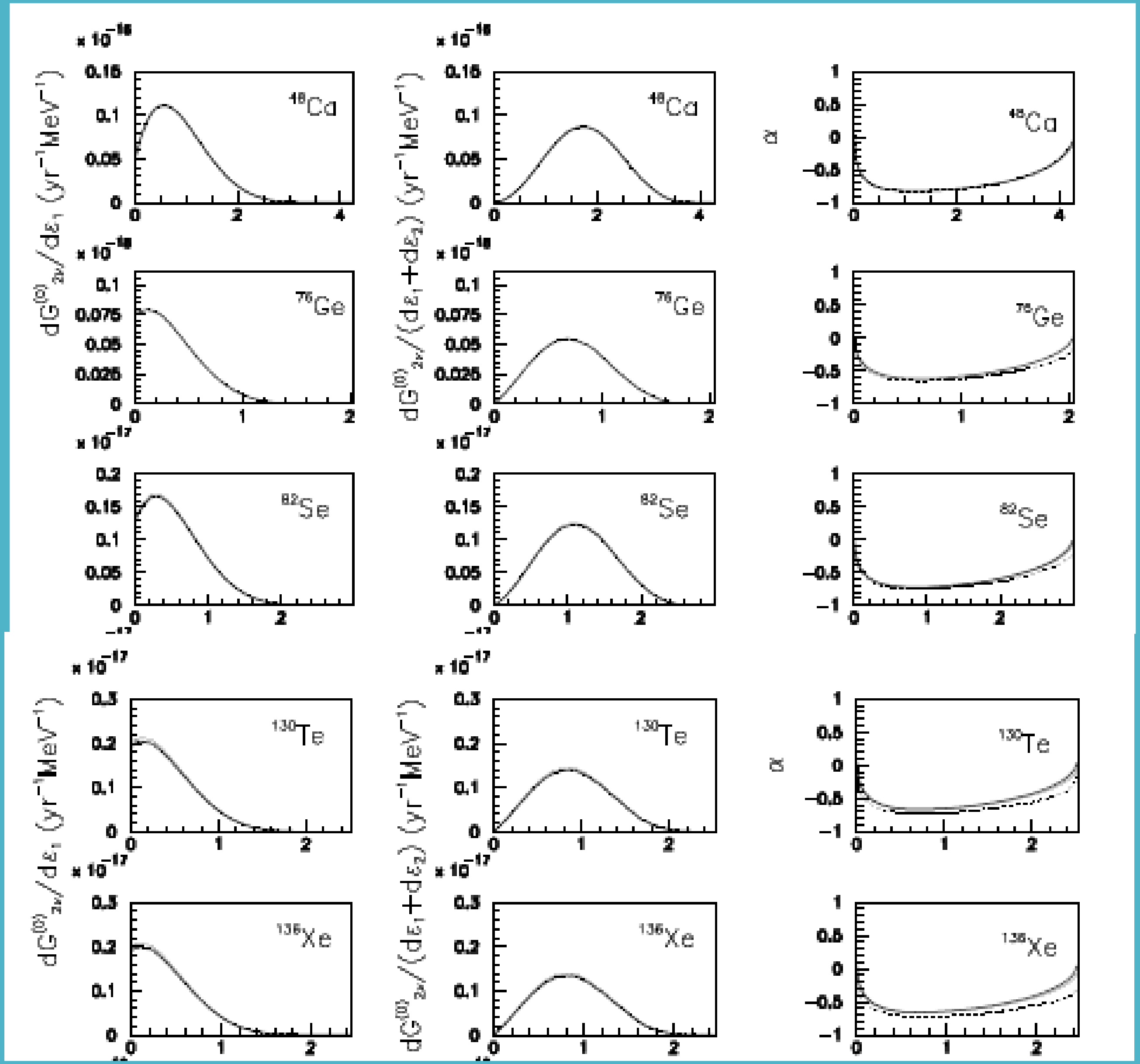
summed energy electron spectra

$$\frac{dG^{(0)}_{2\nu}}{d(\epsilon_1+\epsilon_2-2m_ec^2)}$$

electron angular correlations

$$\alpha(\epsilon_1)=\frac{dG^{(1)}_{2\nu}/d\epsilon_1}{dG^{(0)}_{2\nu}/d\epsilon_1}$$

calculated with method C



Further improvement of the accuracy of DBD kinematic calculations

$$g_{-1}(\varepsilon) = \int_0^\infty w(r)\, g_{-1}(\varepsilon, r)\, r^2\, dr$$

$$f_1(\varepsilon) = \int_0^\infty w(r)\, f_1(\varepsilon, r)\, r^2\, dr$$

$$F(Z, \varepsilon) = \int_0^\infty w(r)\, [g^2_{-1}(\varepsilon,r) + f^2_1(\varepsilon, r)]\, r^2\, dr$$

$$w(r) = \delta\,(r-R)$$

$$w(r) = |R_{nl}(r)|^2$$

$w(r) = w(r,a) \sim 1 + \exp[(r-R)/a]$,

a=diffuseness of the nuclear surface

Conclusions

- Kinematic factors involved in DBD study are important since they can provide us with useful information about transitions to excited 2^+ states, SSD, mechanisms of $0\nu\beta\beta$ an , more recent, LIV effects

- LIV can also be investigated by searching deviations from the standard theoretical predictions of the summed energy spectra of electrons

- This implies improved precision in measuring and analysis of the electron spectra and requires accurate theoretical predictions of these spectra.

- We perform calculations of the PSF and the other kinematic quantities derived from them: electron spectra, summed energy spectra and angular distributions of the emitted electrons in $2\nu\beta\beta$. Ingredients: exact electron w.f. obtained as solutions of a Dirac equation with Coulomb-type potential built up from a realistic proton distribution in the daughter nucleus, with inclusion of FNS and screening effects.

- We estimated and discuss the sources of uncertainties in such calculations as: obtaining of Fermi functions (most important), $Q_{\beta\beta}$ (we propose Q-values, obtained with a recommended average procedure, from measurements reported in literature) and the K_N and L_N terms (used more accurate formulas of them)

- Further improvement of accuracy is possible by considering improved weighting function for calculation of the radial electron w.f. obtained from solving the Dirac equation.

- Similar calculations, for beta decays, EC processes, etc. are possible.
  We can provide experimentalists with numerical data for their spectra simulations.

Thank you for your attention