Axial Vector form factors for neutrino-nucleus scattering

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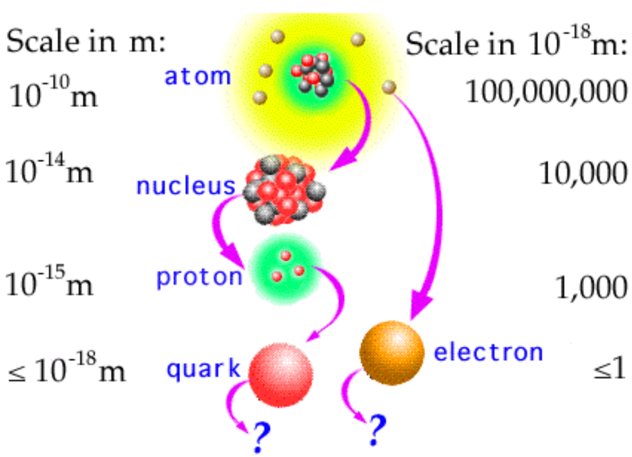

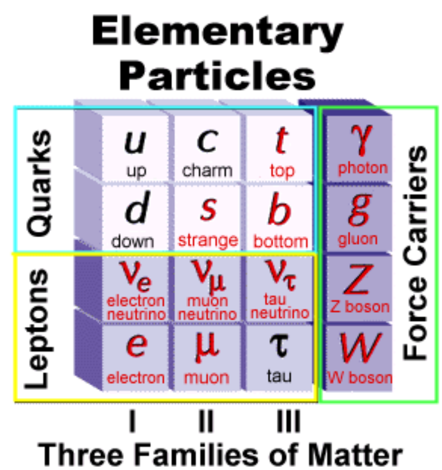

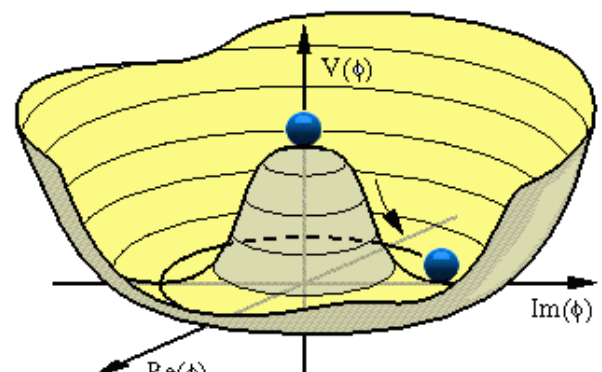

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GOAL: High precision results for axial, electric and magnetic form factors versus Q^2 needed for determining x-section of (ν, e, μ) scattering off nuclei

METHOD: Use large scale simulations of lattice QCD to calculate the matrix elements of the axial and vector currents within the nucleon state.

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References

Charges: Gupta et al, PRD.98 (2018) 034503
AFF: Gupta et al, PRD 96 (2017) 114503
AFF: Jang et al, PRL 124 (2020) 072002
VFF: Jang et al, PRD 100 (2020) 014507

The ν-n differential cross-section:

$$\frac{d\sigma}{dQ^2}\begin{pmatrix}\nu_l + n \to l^- + p \\ \bar{\nu}_l + p \to l^+ + n\end{pmatrix} = \frac{M^2 {G_F}^2 cos^2\theta_c}{8\pi {E_\nu}^2}\left\{A(Q^2) \pm B(Q^2)\frac{(s-u)}{M^2} + C(Q^2)\frac{(s-u)^2}{M^4}\right\},$$

$$\begin{aligned}
A(Q^2) &= \frac{(m^2+Q^2)}{M^2}\Big[(1+\tau)F_A^2 - (1-\tau)F_1^2 + \tau(1-\tau)F_2^2 + 4\tau F_1 F_2 \\
&\quad - \frac{m^2}{4M^2}\left((F_1+F_2)^2 + (F_A+2F_P)^2 - 4\left(1+\frac{Q^2}{4M^2}\right)F_P^2\right)\Big], \\
B(Q^2) &= \frac{Q^2}{M^2}F_A(F_1+F_2), \\
C(Q^2) &= \frac{1}{4}(F_A^2 + F_1^2 + \tau F_2^2).
\end{aligned}$$

F_A = axial form factor
$G_E = F_1 - \tau F_2$ Electric
$G_M = F_1 + F_2$ Magnetic
$\tau = Q^2/4M^2$
$M = M_p$ = 939 MeV

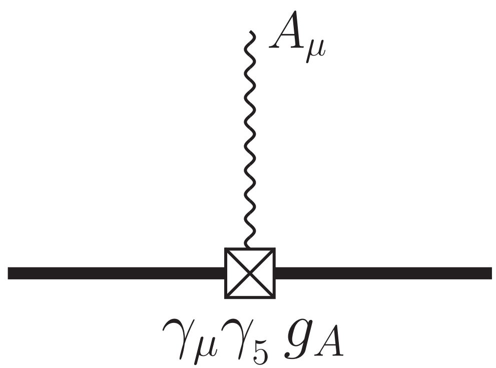
$\langle N A_\mu N \rangle \to$ linear combination of F_A , $\tilde{F}_P$
$\langle NPN \rangle \to G_P$
$\langle N V_\mu N \rangle \to G_E$, G_M

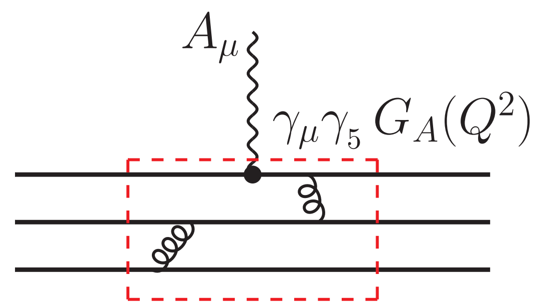
Cohesive strategy for (e, μ, ν)-Z scattering
☛5 Form Factors, g_A, μ , g_p^*

- $G_E(Q^2)$ Electric
- $G_M(Q^2)$ Magnetic
- $G_A(Q^2)$ Axial
- $\tilde{G}_P(Q^2)$ Induced pseudoscalar
- $G_P(Q^2)$ Pseudoscalar
- The lattice methodology is the same
- Precise experimental data exist for $G_E(Q^2)$ and $G_M(Q^2)$
- Axial ward identity relates $G_A(Q^2)$, $\tilde{G}_P(Q^2)$, $G_P(Q^2)$

- $G_E(Q^2 = 0)$ $= 1$ Conserved vector charge
- $G_M(Q^2 = 0)$ $= \boldsymbol{\mu} = \boldsymbol{4.7058}$ Magnetic moment
- $G_A(Q^2 = 0)$ $= g_A = 1.277(2)$ Axial charge
- $\tilde{G}_P\left(Q^2 = 0.88m_\mu^2\right) = g_p^* = 8.06(55)$ Induced pseudoscalar charge

Axial-vector form factors





$\propto 1/Q^4$

Calculate the 3 form factors on the lattice

- Axial: G_A
- Induced pseudoscalar: $\tilde{G}_P$
- Pseudoscalar: G_P

$$\langle N(p_f) | A^\mu(q) | N(p_i) \rangle = \bar{u}(p_f) \left[\gamma^\mu G_A(q^2) + q_\mu \frac{\tilde{G}_P(q^2)}{2M} \right] \gamma_5 u(p_i)$$

$$\langle N(p_f) | P(q) | N(p_i) \rangle = \bar{u}(p_f) G_P(q^2) \gamma_5 \, u(p_i)$$

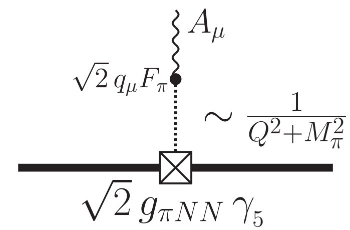
G_A, $\tilde{G}_P$, G_P must satisfy the PCAC relation: $\partial_\mu A_\mu = 2mP$

PCAC ($\partial_\mu A_\mu = 2\widehat{m}$P) requires

$$2\widehat{m}G_P(Q^2) = 2M_N G_A(Q^2) - \frac{Q^2}{2M_N}\tilde{G}_P(Q^2)$$

Pion pole-dominance (PPD) hypothesis

$$\widetilde{G_P}(Q^2) = G_A(Q^2)\left[\frac{4M_N^2}{Q^2 + M_\pi^2}\right]$$



If pion pole-dominance holds

$\Rightarrow$ there is only one independent form factor

Goldberger-Trieman relation at $Q^2 = 0$

$$F_\pi \; g_{\pi NN} = M_N \; g_A$$

Extracting the ground state matrix elements from 3-point functions

$$\langle N(p_f)|A_\mu(q)|N(p_i)\rangle$$

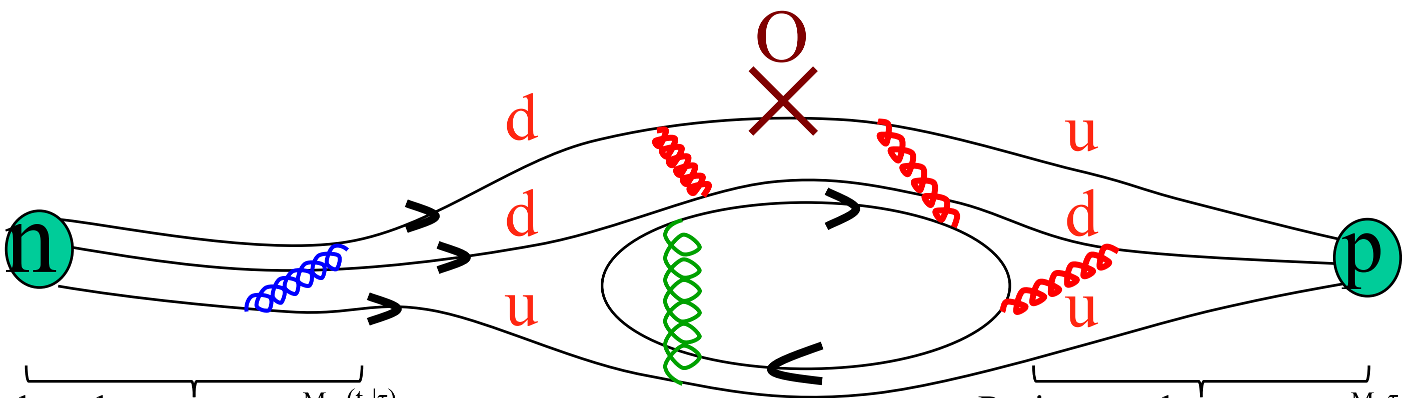
$$\langle N(p_f)|P(q)|N(p_i)\rangle$$

$$\langle N(p_f)|V_\mu(q)|N(p_i)\rangle$$

QM tells us $\langle A_\mu\rangle = \int d^3x\, \overline{\psi}(x) A_\mu\, \psi(x)$

Lattice QCD gives us $\psi(x)$
as an "integral over paths"

Calculating matrix elements using Lattice QCD



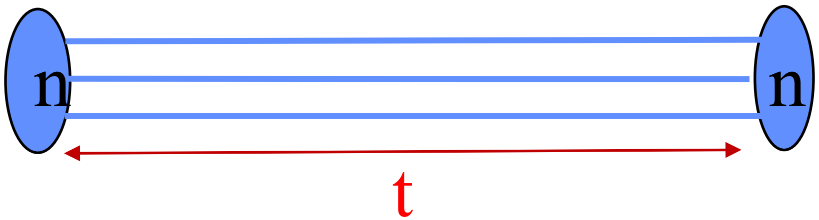
Ensemble average over background gauge configurations

$$\langle\Omega|\hat{N}(t,p')\hat{O}(\tau,p'-p)\hat{N}(0,p)|\Omega\rangle =$$

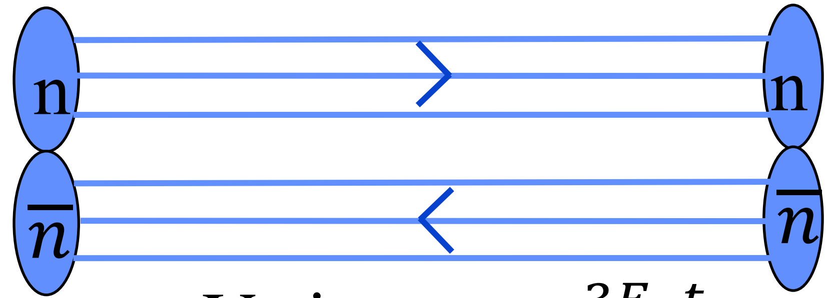
$$\sum_{i,j}\langle\Omega|\hat{N}(p')|N_j\rangle e^{-\int dt\,H}\langle N_j|\hat{O}(\tau,p'-p)|N_i\rangle e^{-\int dt\,H}\langle N_i|\hat{N}(p)|\Omega\rangle =$$

$$\sum_{i,j}\langle\Omega|\hat{N}(p')|N_j\rangle e^{-E_j(t-\tau)}\underbrace{\langle N_j|\hat{O}(\tau,p'-p)|N_i\rangle}\, e^{-E_i\tau}\langle N_i|\hat{N}(p)|\Omega\rangle$$

Spectrum from the 2-point function Γ^2



Signal in Γ^2: $e^{-E_N t}$



Variance: $e^{-3E_\pi t}$

$$\Gamma^2(t) = |A_0|^2 e^{-M_0 t} + |A_1|^2 e^{-M_1 t} + |A_2|^2 e^{-M_2 t} + |A_3|^2 e^{-M_3 t} + \ldots.$$

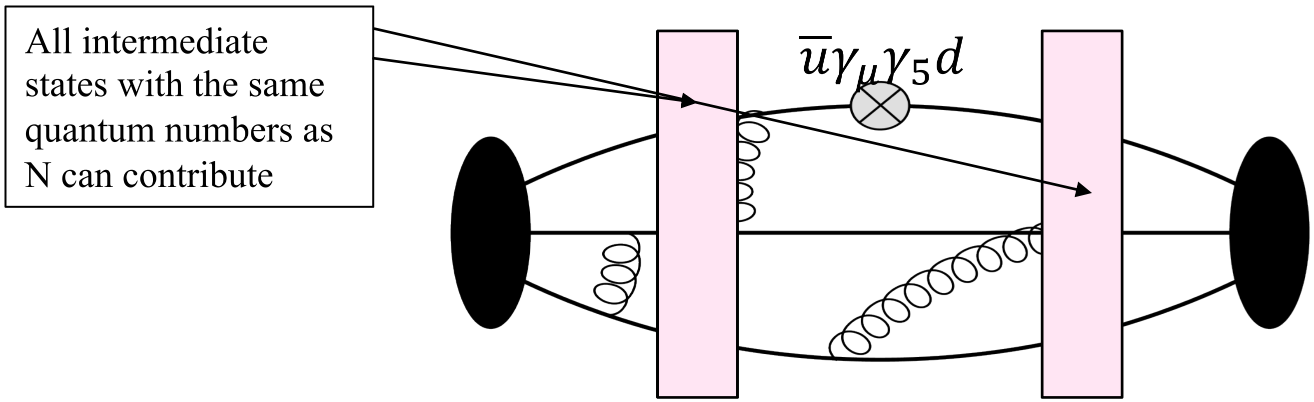
Fit the data for $\Gamma^2(t)$ versus t to extract

M_0, M_1, … masses of the ground & excited states
A_0, A_1, … corresponding amplitudes

- The signal degrades exponentially $e^{-(M_N - 1.5 M_\pi)t}$
- To resolve a <u>*small*</u> mass gap $(M_1 - M_0)$ needs large t

Excited states & violation of PCAC

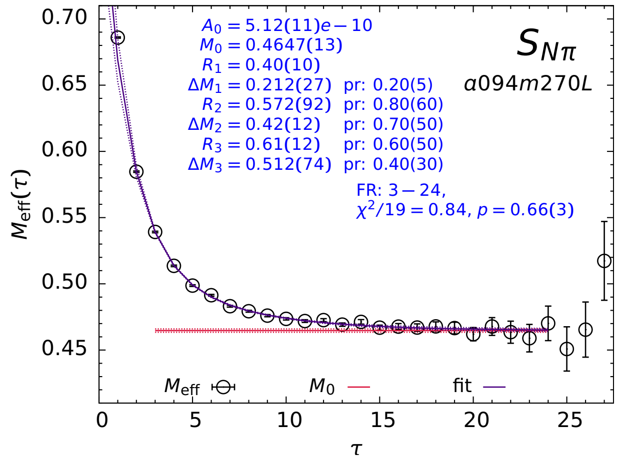
Challenge: To get the matrix elements in the ground state of the nucleon, the contributions of all excited states have to be removed.



Issue: How to determine the spectrum of all states that contribute significantly to a given 3-point function?

Challenges

- Cannot go to large τ because the signal/noise degrades as $e^{-(M_N - 1.5 M_\pi)\tau}$
  - 2-pt: τ~ 2fm ; 3-pt: τ~1.5fm



- $\widehat{N}$ couples to the nucleon, all its excitations and multi-hadron states with the same quantum numbers

- As $\vec{q} \to 0$, the tower of physical $N\pi$, $N\pi\pi$ states becomes arbitrarily dense above ~1210 MeV

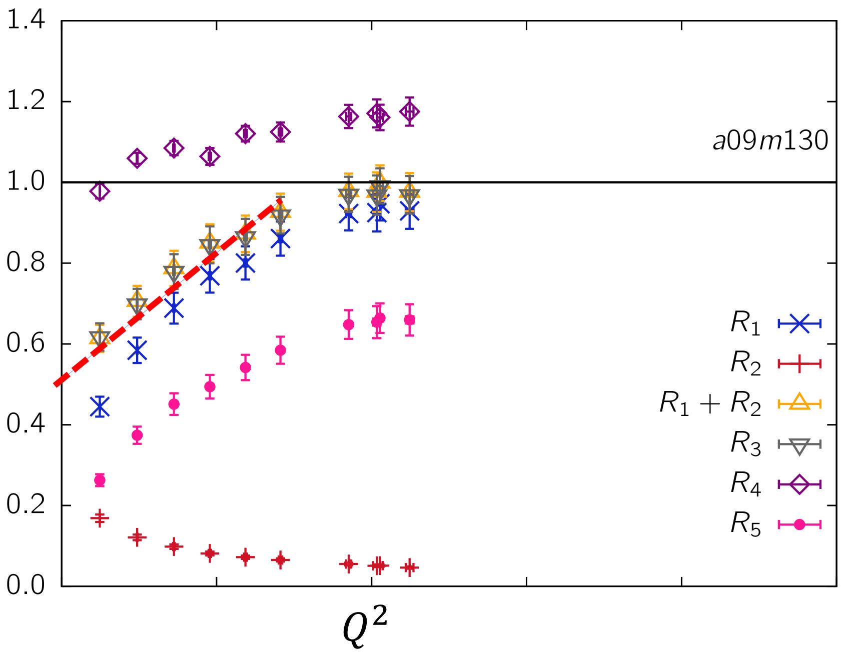
- The excited states that gives significant contribution to a give ME are not known *a priori*

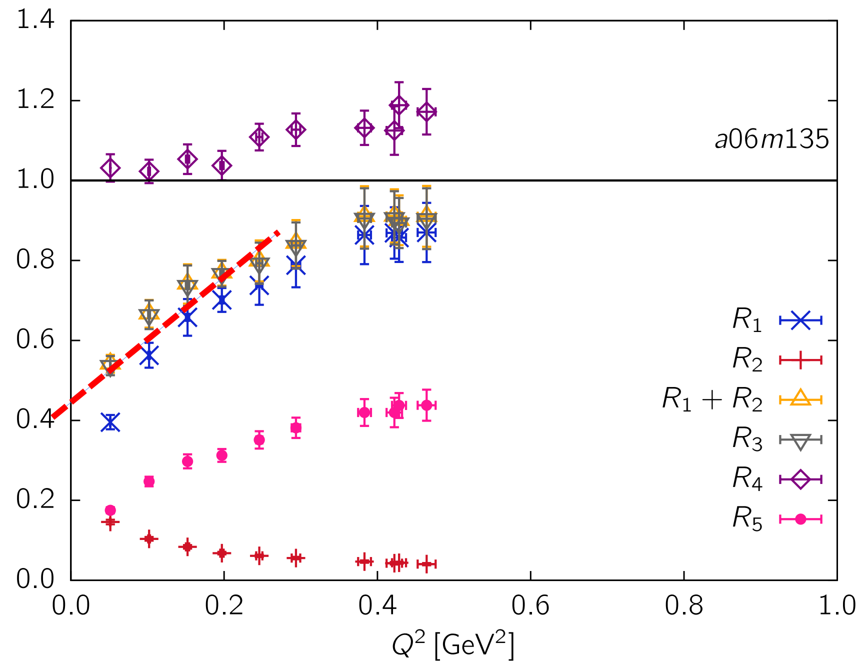
2017 Showed Violation of PCAC

Gupta et al, PRD 96 (2017) 114503

$$\text{PCAC:}\ \ R_1 + R_2 = \frac{\hat{m}\, G_P}{M_N G_A} + \frac{Q^2 \tilde{G}_P}{4M_N^2} = 1$$

PCAC violated if one uses the spectrum from 2-point function





$a = 0.087\ fm$

$M_\pi = 138\ MeV$

$a = 0.057\ fm$

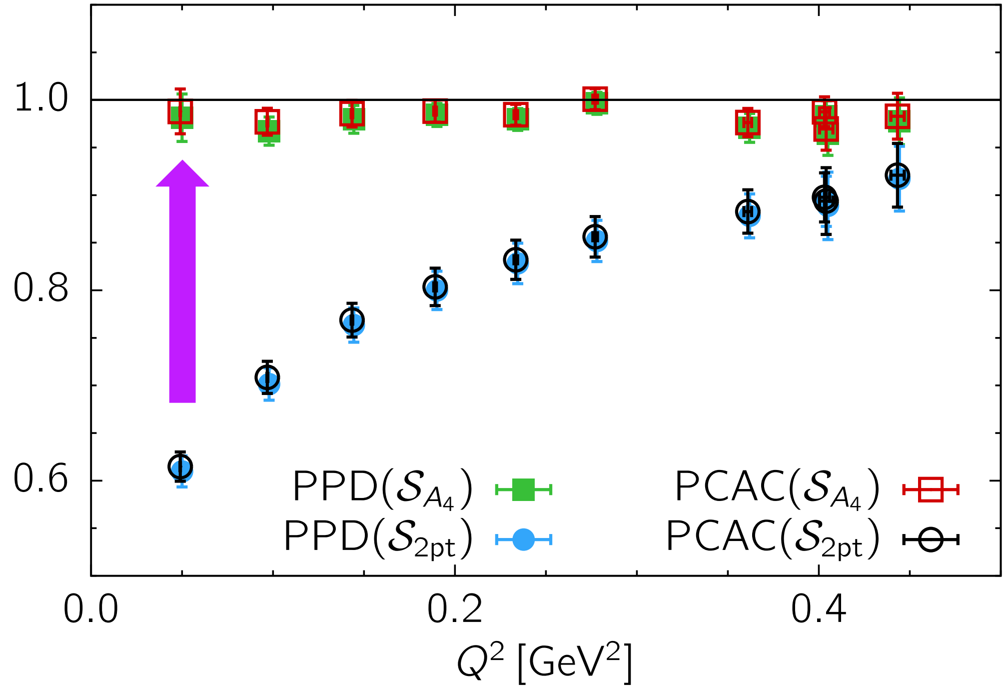
$M_\pi = 136\ MeV$

2019: Resolution of PCAC and PPD

Jang et al, PRL 124 (2020) 072002

On including low mass $N_{p=0}\pi_p$ and $N_p\pi_{-p}$ excited states neglected in previous works, showed PCAC and PPD are satisfied

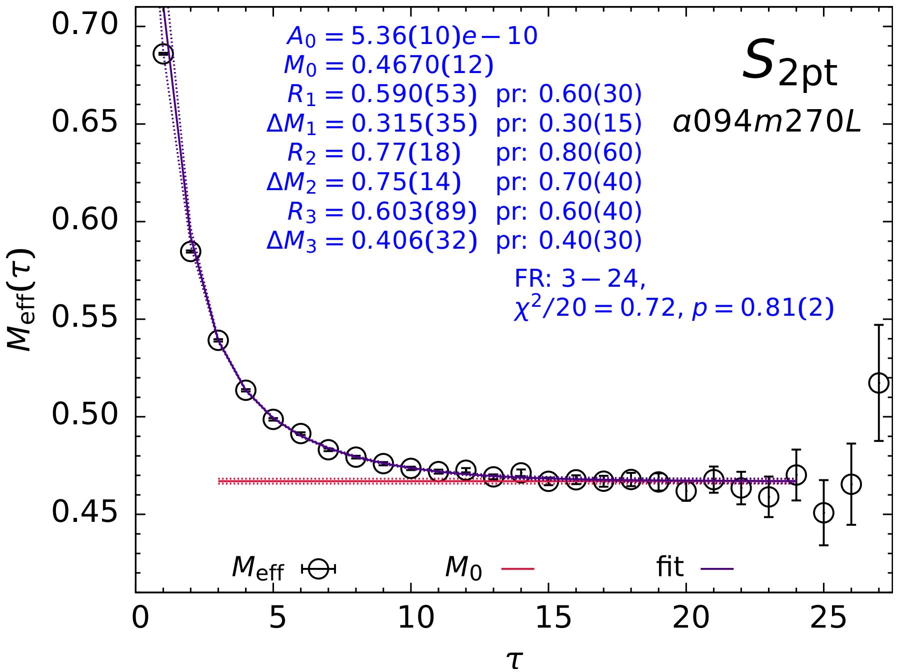
$$\frac{\hat{m}G_P}{M_N G_A} + \frac{Q^2\tilde{G}_P}{4M_N^2 G_A} = 1$$

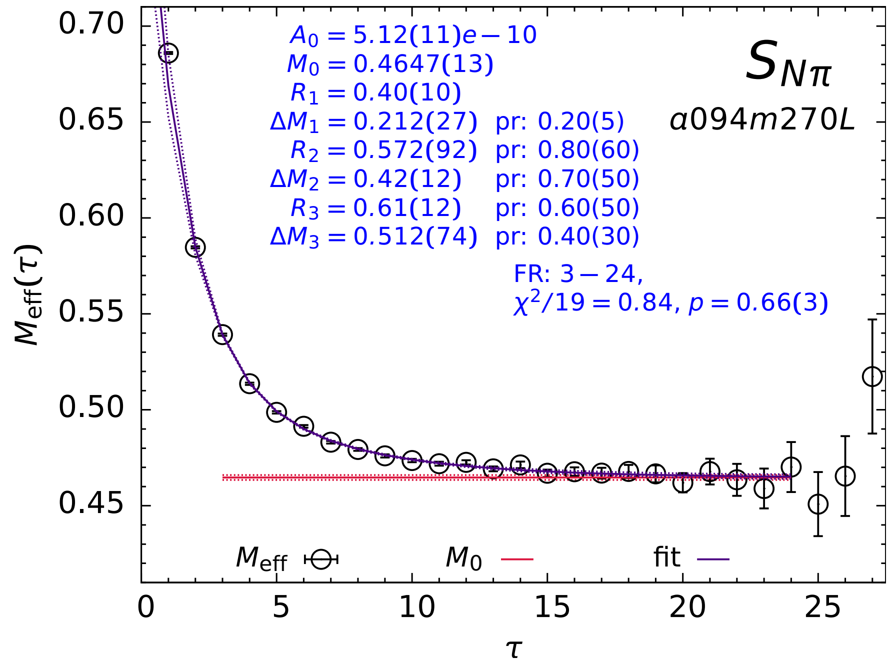


The E_1 of $N_{p=0}\pi_p$ and $N_p\pi_{-p}$ are determined from fits to the $\langle NA_4N\rangle$ 3-point axial correlation function!!

Spectrum from the 2-point function

Comparing two 4-state fits

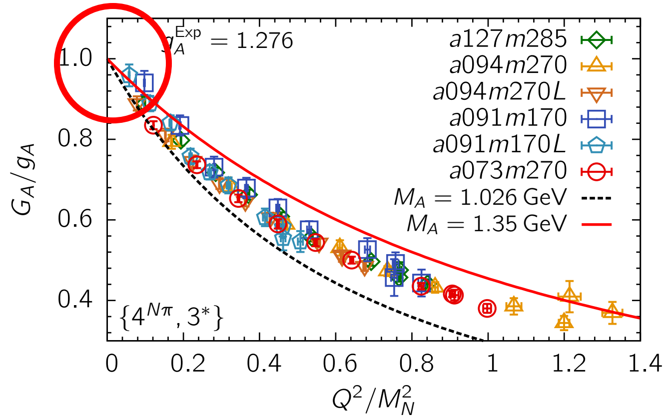




Large region of parameter values give similar χ^2/dof

Resulting Change in:
$G_A \sim 1$—$4\ \%$
$\tilde{G}_P \sim 15$—$25\ \%$
$G_P \sim 20$—$30\ \%$

Current Status: Axial



Issues:

- The pattern of excited states cannot be determined from $\langle NA_4N\rangle$ at $(Q^2 = 0)$. Thus analysis of g_A is not yet fully resolved
- In principle there is a tower of $N_p\pi_{-p}$ states. We have only included the dominant one (consistent with chiral perturbation theory)
- Statistical precision falls at large Q^2

Future:

- Determine a robust way to extract $G_A(Q^2 = 0) \equiv g_A$
- Further improve control over excited-states
- Increase the range of values of lattice spacing and pion masses to reduce the associated systematics
- Incorporate/Develop methods to extend the range of Q^2

Electric & Magnetic form factors

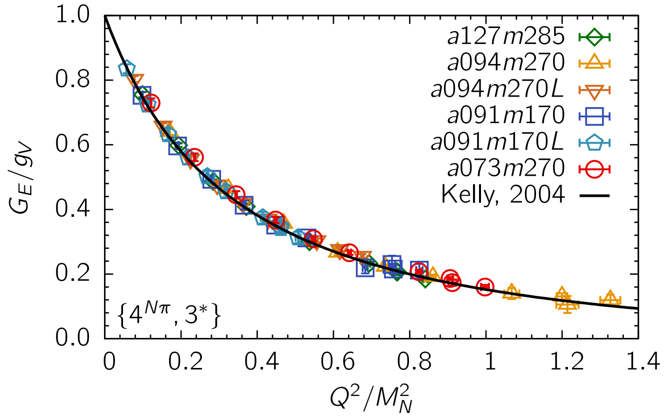
Matrix Elements of $V_\mu \rightarrow$ Dirac (F_1) and Pauli (F_2) form factors

$$\langle N(p_f) | V^\mu(q) | N(p_i) \rangle = \bar{u}(p_f) \left[\gamma^\mu F_1(q^2) + \sigma^{\mu\nu} q_\nu \frac{F_2(q^2)}{2M} \right] u(p_i)$$

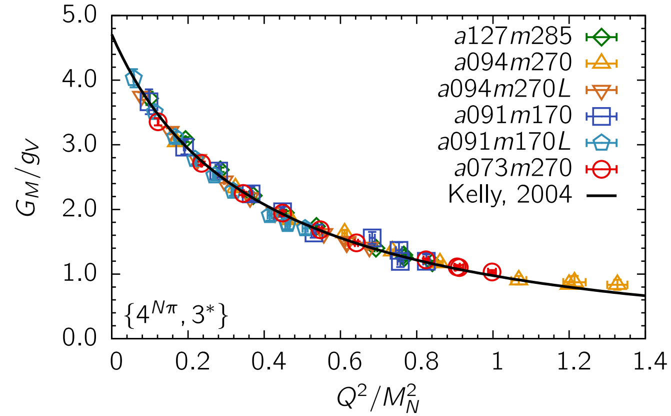
Define Sachs Electric (G_E) and Magnetic (G_M) form factors

$$G_E(q^2) = F_1(q^2) - \frac{q^2}{4M^2} F_2(q^2), \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

Current Status: Electric & Magnetic



Electric



Magnetic

- The extraction of electric and magnetic form factors is insensitive to the details of the excited states
- The form factors do not show significant dependence on the lattice spacing or the quark mass
- Good agreement with the Kelly curve. Confirms lattice methodology
- Improve precision and get data over larger range of parameter values

Future

- Continue to develop a robust analysis strategy for removing excited states in all nucleon matrix elements
  - Charges, Form factors
  - nEDM
- Flavor diagonal operators for (“neutral current” interactions, dark matter, proton spin, momentum fraction, …)
- Generate and analyze ensembles to improve chiral and continuum extrapolation for
  - Clover-on-clover formulation
  - Increase statistics on Clover-on-HISQ calculations
- Perform a comprehensive analysis of scattering data using lattice results for $G_E(Q^2)$, $G_M(Q^2)$, $G_A(Q^2)$, $\tilde{G}_P(Q^2)$